

**HIGLEY UNIFIED SCHOOL DISTRICT
INSTRUCTIONAL ALIGNMENT**

6th Grade Math 3rd Quarter

Unit 4: Ratios and Unit Rates (6 weeks)

Topic A: Representing and Reasoning About Ratios

In this unit, students are introduced to the concepts of ratio and rate. In Topic A, the focus is on the concept of ratios. Student’s previous experience solving problems involving multiplicative comparisons, such as “*Max has three times as many toy cars as Jack,*” (4.OA.2) serves as the conceptual foundation for understanding ratios as a multiplicative comparison of two or more numbers used in quantities or measurements (6.RP.1). Students develop fluidity in using multiple forms of ratio language and ratio notation. They construct viable arguments and communicate reasoning about ratio equivalence as they solve ratio problems in real world contexts (6.RP.3). As the first topic comes to a close, students develop a precise definition of the value of a ratio $a:b$, where $b \neq 0$ as the value a/b , applying previous understanding of fraction as division (5.NF.3). They can then formalize their understanding of equivalent ratios as ratios having the same value.

Big Idea:

- Reasoning with ratios involves attending to and coordinating two quantities.
- A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit.
- Forming a ratio as a measure of a real-world attribute involves isolating that attribute from other attributes and understanding the effect of changing each quantity on the attribute of interest.
- A number of mathematical connections link ratios and fractions.
- Ratios can be meaningfully reinterpreted as quotients.

Essential Questions:

- How can you represent a ratio between two quantities?
- How does ratio reasoning differ from other types of reasoning?
- What is a ratio?
- What is a ratio as a measure of an attribute in a real-world situation?
- How are ratios related to fractions?
- How are ratios related to division?

Vocabulary

Ratio, equivalent ratios, value of a ratio, associated ratio, tape diagram, part-to-part ratio, part-to-whole ratio

Grade	Domain	Standard	AZ College and Career Readiness Standards	Explanations & Examples	Resources
6	RP	1	<p>A. Understand ratio concepts and use ratio reasoning to solve problems.</p> <p>Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. <i>For example, “The ratio of wings to beaks in</i></p>	<p>Explanation:</p> <p>A ratio is a comparison of two quantities which can be written as</p> <p style="text-align: center;">a to b, $\frac{a}{b}$, or $a:b$.</p>	<p>Eureka Math: M1 Lessons 1-8</p> <p>Big Ideas: Section 5.1</p>

the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”

6.MP.2. Reason abstractly and quantitatively.

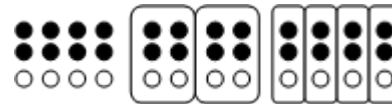
6.MP.6. Attend to precision.

A ratio is the comparison of two quantities or measures. The comparison can be part-to-whole or part-to-part.

Example:

A comparison of 8 black circles to 4 white circles can be written as the ratio of 8:4 and can be regrouped into 4 black circles to 2 white circles (4:2) and

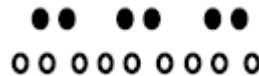
2 black circles to 1 white circle (2:1).



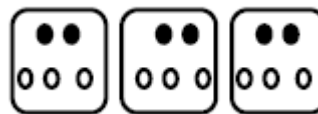
Students should be able to identify all these ratios and describe them using “For every..., there are ...”

Example:

A comparison of 6 guppies and 9 goldfish could be expressed in any of the following forms: 6/9, 6 to 9 or 6:9. If the number of guppies is represented by black circles and the number of goldfish is represented by white circles, this ratio could be modeled as



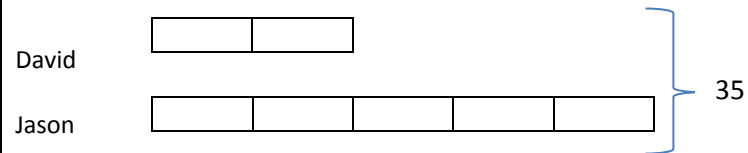
These values can be regrouped into 2 black circles (goldfish) to 3 white circles (guppies), which would reduce the ratio to, 2/3, 2 to 3 or 2:3.



Students should be able to identify and describe any ratio using “For every ___, there are ___” In the example above, the ratio could be expressed saying, “For every 2 goldfish, there are 3 guppies”.

6	RP	3a	<p>A. Understand ratio concepts and use ratio reasoning to solve problems.</p> <p>Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.</p> <p>a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.</p> <p><i>6.MP.1.</i> Make sense of problems and persevere in solving them.</p> <p><i>6.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>6.MP.4.</i> Model with mathematics</p> <p><i>6.MP.5.</i> Use appropriate tools strategically.</p> <p><i>6.MP.7.</i> Look for and make use of structure.</p>	<p>Explanations:</p> <p>What is ratio and ratio reasoning? <i>Ratios are not numbers in the typical sense. They cannot be counted or placed on a number line. They are a way of thinking and talking about relationships between quantities.</i></p> <ul style="list-style-type: none"> Students are frequently exposed to equivalent ratios in multiplication tables. For example $1/3$ is often stated as equivalent to $3/9$, which is a true statement. This relationship of equivalence can be very challenging for students to understand because it appears that they are not numerically the same. However, from a ratio perspective 1 to 3 has the same relationship as 3 to 9. In this way, students are thinking about a ratio relationship between two quantities. <table border="1" data-bbox="984 787 1673 922"> <tbody> <tr> <td></td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> <td>8</td> </tr> <tr> <td>1</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> <td>8</td> </tr> <tr> <td>2</td> <td>2</td> <td>4</td> <td>6</td> <td>8</td> <td>10</td> <td>12</td> <td>14</td> <td>16</td> </tr> <tr> <td>3</td> <td>3</td> <td>6</td> <td>9</td> <td>12</td> <td>15</td> <td>18</td> <td>21</td> <td>24</td> </tr> </tbody> </table> <ul style="list-style-type: none"> Ratio reasoning involves attending to covariation. This means that <u>students must hang onto the idea that one quantity changes or varies in relation to another quantity</u>. For example, 1 cup of sugar is used for every 3 cups of flour in a recipe. IF 2 cups of sugar are used, THEN the flour must change or vary in the same way. (IF--THEN statements might help children process the idea of a relationship between quantities.) In this case, the amount of sugar doubled, so the amount of flour should also double. Students must hold onto the idea that a change in one quantity creates a need for change in the other quantity. While this reasoning is fairly intuitive for adults, it is not always easy for children to grasp. Many opportunities to reason about ratio helps them develop the ability to attend to covariation. 		1	2	3	4	5	6	7	8	1	1	2	3	4	5	6	7	8	2	2	4	6	8	10	12	14	16	3	3	6	9	12	15	18	21	24	<p>Eureka Math: M1 Lessons 1-8</p> <p>Big Ideas: Section 5.1</p>
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3	3	6	9	12	15	18	21	24																																	

Ex: David and Jason have marbles in a ratio of 2:3. Together, they have a total of 35 marbles. How many marbles does each boy have?



Tape diagrams are visual models that use rectangles to represent the parts of a ratio. Since they are a visual model, drawing them requires attention to detail in the setup. In this problem David and Jason have numbers of marbles in a ratio of 2:3. This ratio is modeled here by drawing 2 rectangles to represent David's portion, and 3 rectangles to represent Jason's portion. The rectangles are uniform in size and lined up, e.g., on the left hand side, for easy visual reference. The large bracket on the right denotes the total number of marbles David and Jason have (35). It is clear visually that the boys have 5 rectangles worth of marbles and that the total number of marbles is 35. This information will be used to solve the problem.

- 5 rectangles = 35 marbles
 - Dividing the number of marbles by 5
- 1 rectangle = 7 marbles
 - This information will be used to solve the problem.
- David has 2 rectangles and $2 \times 7 = 14$ marbles. Therefore David has 14 marbles.
- Jason has 3 rectangles and $3 \times 7 = 21$ marbles. Therefore Jason has 21 marbles.

Example 2:

Ratios can also be used in problem solving by thinking about the total amount for each ratio unit.

			<p>The ratio of cups of orange juice concentrate to cups of water in punch is 1: 3. If James made 32 cups of punch, how many cups of orange did he need?</p> <p><i>Solution:</i> Students recognize that the total ratio would produce 4 cups of punch. To get 32 cups, the ratio would need to be duplicated 8 times, resulting in 8 cups of orange juice concentrate.</p>	
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6th Grade Math 3rd Quarter

Unit 4: Ratios and Unit Rates

Topic B: Collections of Equivalent Ratios

With the concept of ratio equivalence formally defined, students explore collections of equivalent ratios in real world contexts in Topic B. They build ratio tables and study their additive and multiplicative structure (**6.RP.3a**). Students continue to apply reasoning to solve ratio problems while they explore representations of collections of equivalent ratios and relate those representations to the ratio table (**6.RP.3**). Building on their experience with number lines, students represent collections of equivalent ratios with a double number line model. They relate ratio tables to equations using the value of a ratio defined in Topic A. Finally, students expand their experience with the coordinate plane (**5.G.1, 5.G.2**) as they represent collections of equivalent ratios by plotting the pairs of values on the coordinate plane.

Big Idea:

- Reasoning with ratios involves attending to and coordinating two quantities.
- A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit.
- Forming a ratio as a measure of a real-world attribute involves isolating that attribute from other attributes and understanding the effect of changing each quantity on the attribute of interest.
- Ratios are comparisons using division.
- Graphs and equations represent relationships between variables.

Essential Questions:

- How can you use ratios in real-life problems?
- How is a ratio used to compare two quantities or values?
- Where can examples of ratios be found?
- How can I model equivalent ratios?
- How do you determine which variable is independent/dependent in a two variable equation that represents a real-life context?
- What affect does changing the independent variable have on the dependent variable?
- How can quantitative relationships be represented?
- How is the coefficient of the dependent variable related to the graph and/or table of values?

Vocabulary			Ratio, equivalent ratios, value of a ratio, associated ratio, double number line, ratio table, coordinate plane, equation in two variables, independent variable, dependent variable, analyze, discrete, continuous																																								
Grade	Domain	Standard	AZ College and Career Readiness Standards	Explanations & Examples			Resources																																				
6	RP	3a	<p>A. Understand ratio concepts and use ratio reasoning to solve problems.</p> <p>Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.</p> <p>a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.</p> <p>6.MP.1. Make sense of problems and persevere in solving them.</p> <p>6.MP.2. Reason abstractly and quantitatively.</p> <p>6.MP.4. Model with mathematics</p> <p>6.MP.5. Use appropriate tools strategically.</p> <p>6.MP.7. Look for and make use of structure.</p>	<p>Explanations:</p> <p>Ratios and rates can be used in ratio tables and graphs to solve problems. Previously, students have used additive reasoning in tables to solve problems. To begin the shift to proportional reasoning, students need to begin using multiplicative reasoning. To aid in the development of proportional reasoning the cross-product algorithm is not expected at this level. When working with ratio tables and graphs, <i>whole number</i> measurements are the expectation for this standard.</p> <p>What is ratio and ratio reasoning?</p> <p><i>Ratios are not numbers in the typical sense. They cannot be counted or placed on a number line. They are a way of thinking and talking about relationships between quantities.</i></p> <ul style="list-style-type: none"> Students are frequently exposed to equivalent ratios in multiplication tables. For example $\frac{1}{3}$ is often stated as equivalent to $\frac{3}{9}$, which is a true statement. This relationship of equivalence can be very challenging for students to understand because it appears that they are not numerically the same. However, from a ratio perspective 1 to 3 has the same relationship as 3 to 9. In this way, students are thinking about a ratio relationship between two quantities. <table border="1" data-bbox="942 1117 1631 1252"> <tbody> <tr> <td></td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> <td>8</td> </tr> <tr> <td>1</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> <td>8</td> </tr> <tr> <td>2</td> <td>2</td> <td>4</td> <td>6</td> <td>8</td> <td>10</td> <td>12</td> <td>14</td> <td>16</td> </tr> <tr> <td>3</td> <td>3</td> <td>6</td> <td>9</td> <td>12</td> <td>15</td> <td>18</td> <td>21</td> <td>24</td> </tr> </tbody> </table> <ul style="list-style-type: none"> Ratio reasoning involves attending to covariation. This means that <u>students must hang onto the idea that one quantity changes or varies in relation to another quantity.</u> For example, 1 cup of sugar is used for every 3 cups of flour in a recipe. IF 2 				1	2	3	4	5	6	7	8	1	1	2	3	4	5	6	7	8	2	2	4	6	8	10	12	14	16	3	3	6	9	12	15	18	21	24	<p>Eureka Math: M1 Lessons 9-15</p> <p>Big Ideas: Sections 5.2 and 5.4 (non-rate problems only)</p>
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cups of sugar are used, **THEN** the flour must change or vary in the same way. (**IF--THEN statements might help children process the idea of a relationship between quantities.**) In this case, the amount of sugar doubled, so the amount of flour should also double. Students must hold onto the idea that a change in one quantity creates a need for change in the other quantity. While this reasoning is fairly intuitive for adults, it is not always easy for children to grasp. Many opportunities to reason about ratio helps them develop the ability to attend to covariation.

Example 1:

At Books Unlimited, 3 paperback books cost \$18. What would 7 books cost? How many books could be purchased with \$54.

Solution: To find the price of 1 book, divide \$18 by 3. One book costs \$6. To find the price of 7 books, multiply \$6 (the cost of one book times 7 to get \$42. To find the number of books that can be purchased with \$54, multiply \$6 times 9 to get \$54 and then multiply 1 book times 9 to get 9 books. Students use ratios, unit rates and multiplicative reasoning to solve problems in various contexts, including measurement, prices, and geometry. Notice in the table below, a multiplicative relationship exists between the numbers both horizontally (times 6) and vertically (ie. $1 \cdot 7 = 7$; $6 \cdot 7 = 42$). Red numbers indicate solutions.

Number of Books (n)	Cost (C)
1	6
3	18
7	42
9	54

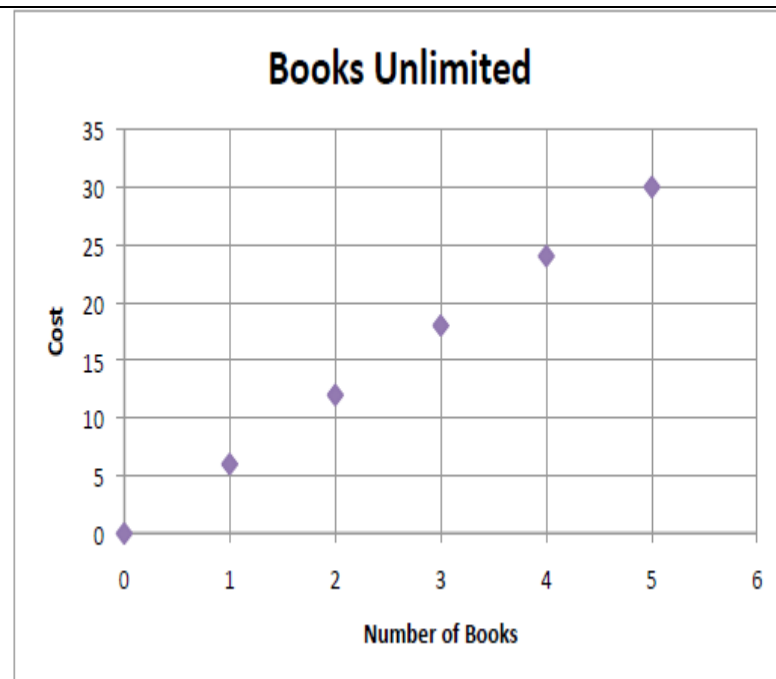
Students use tables to compare ratios. Another bookstore offers paperback books at the prices below. Which bookstore has the best

buy? Explain your answer.

Number of Books (n)	Cost (C)
4	20
8	40

To help understand the multiplicative relationship between the number of books and cost, students write equations to express the cost of any number of books. Writing equations is foundational for work in 7th grade. For example, the equation for the first table would be $C = 6n$, while the equation for the second bookstore is $C = 5n$. The numbers in the table can be expressed as ordered pairs (number of books, cost) and plotted on a coordinate plane.

Students are able to plot ratios as ordered pairs. For example, a graph of Books Unlimited would be:



Example 2:

Ratios can also be used in problem solving by thinking about the total amount for each ratio unit.

The ratio of cups of orange juice concentrate to cups of water in punch is 1: 3. If James made 32 cups of punch, how many cups of orange did he need?

Solution: Students recognize that the total ratio would produce 4 cups of punch. To get 32 cups, the ratio would need to be duplicated 8 times, resulting in 8 cups of orange juice concentrate.

Example 3:

Using the information in the table, find the number of yards in 24 feet.

Feet	3	6	9	15	24
Yards	1	2	3	5	?

Solution:

There are several strategies that students could use to determine the solution to this problem:

- Add quantities from the table to total 24 feet (9 feet and 15 feet); therefore the number of yards in 24 feet must be 8 yards (3 yards and 5 yards).
- Use multiplication to find 24 feet: 1) 3 feet x 8 = 24 feet; therefore 1 yard x 8 = 8 yards, or 2) 6 feet x 4 = 24 feet; therefore 2 yards x 4 = 8 yards.

Example 4:

Compare the number of black circles to white circles. If the ratio remains the same, how many black circles will there be if there are 60 white circles?

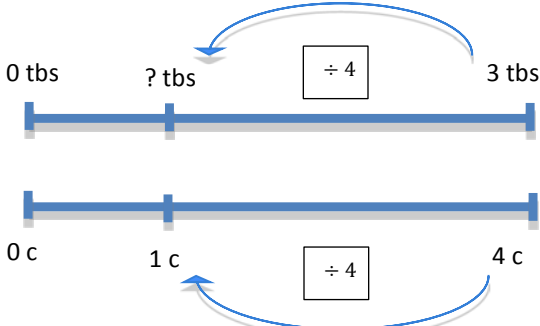


Black	4	40	20	60	?
White	3	30	15	45	60

Solution:

There are several strategies that students could use to determine the solution to this problem:

- Add quantities from the table to total 60 white circles (15 + 45). Use the corresponding numbers to determine the number of black circles (20 + 60) to get 80 black circles.
- Use multiplication to find 60 white circles (one possibility 30 x

				<p>2). Use the corresponding numbers and operations to determine the number of black circles (40×2) to get 80 black circles.</p> <p>Example 5:</p> <p>A recipe calls for 3 tablespoons of butter for every 4 cups of sugar. How many tablespoons of butter would you use for 1 cup of sugar?</p> <p><i>Solution:</i></p>  <p>$\frac{3}{4}$ tablespoons of butter would be used for every cup of sugar.</p>	
6	EE	9	<p>C. Represent and analyze quantitative relationships between dependent and independent variables.</p> <p>Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. <i>For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.</i></p>	<p>Explanation:</p> <p>The purpose of this standard is for students to understand the relationship between two variables, which begins with the distinction between dependent and independent variables. The independent variable is the variable that can be changed; the dependent variable is the variable that is affected by the change in the independent variable. Students recognize that the independent variable is graphed on the x-axis; the dependent variable is graphed on the y-axis.</p> <p>Students recognize that not all data should be graphed with a line. Data that is discrete would be graphed with coordinates only. Discrete data is data that would not be represented with fractional parts such as people, tents, records, etc. For example, a graph illustrating the cost per person would be graphed with points since part of a person would</p>	<p>Eureka Math: M4 Lesson 31-32</p> <p>Big Ideas: Section 7.4</p>

		<p>Connection to 6.RP.3</p> <p>6.MP.1. Make sense of problems and persevere in solving them.</p> <p>6.MP.2. Reason abstractly and quantitatively.</p> <p>6.MP.3. Construct viable arguments and critique the reasoning of others.</p> <p>6.MP.4. Model with mathematics.</p> <p>6.MP.7. Look for and make use of structure.</p> <p>6.MP.8. Look for and express regularity in repeated reasoning</p>	<p>not be considered. A line is drawn when both variables could be represented with fractional parts.</p> <p>Students are expected to recognize and explain the impact on the dependent variable when the independent variable changes (As the x variable increases, how does the y variable change?) Relationships should be proportional with the line passing through the origin. Additionally, students should be able to write an equation from a word problem and understand how the coefficient of the dependent variable is related to the graph and /or table of values.</p> <p>Students can use many forms to represent relationships between quantities. Multiple representations include describing the relationship using language, a table, an equation, or a graph. Translating between multiple representations helps students understand that each form represents the same relationship and provides a different perspective on the function.</p> <p>Students look for and express regularity in repeated reasoning (MP.8) as they generate algebraic models (MP.4) to represent relationships.</p> <p>Examples:</p> <ul style="list-style-type: none"> A school is having a walk-a-thon for a fund raiser. Each student in the walk-a-thon must find sponsors to pledge \$2.00 for each mile the student walks. Sponsors want to know how much money they would owe given the total distance the students would walk. <p><i>Solution:</i></p> <p>Table of Values:</p> <p>Horizontal</p> <table border="1" data-bbox="953 1252 1707 1295"> <tr> <td>Number of Miles Walked (Independent Variable known as x)</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> </tr> <tr> <td>Total Dollars Sponsor Would Owe (Dependent Variable known as y)</td> <td>0</td> <td>2</td> <td>4</td> <td>6</td> <td>8</td> <td>10</td> <td>12</td> </tr> </table> <p>Vertical</p>	Number of Miles Walked (Independent Variable known as x)	0	1	2	3	4	5	6	Total Dollars Sponsor Would Owe (Dependent Variable known as y)	0	2	4	6	8	10	12	
Number of Miles Walked (Independent Variable known as x)	0	1	2	3	4	5	6													
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Miles	Dollars
0	0
1	2
2	4
3	6
4	8
5	10
6	12

The table helps students recognize the pattern in the function.

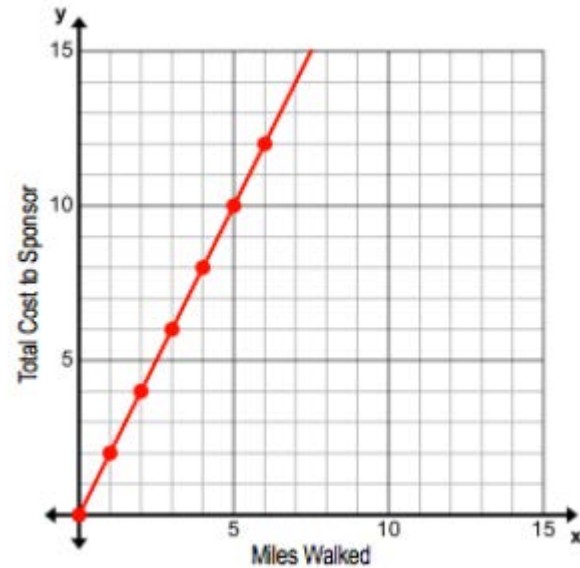
Let m = the number of miles walked

Let D = the total cost to the sponsor

$$D = 2m$$

Graph:

Students can graph the quantitative relationship on a coordinate plane. When graphing the data, the horizontal x -axis represents the independent variable of miles walked, and the vertical y -axis represents the dependent variable of total dollars the sponsor owes. The graph gives the students a visual image that helps them describe the relationship between miles walked and total money owed. When connected, the points form a straight line, which means it is a linear function. The rate of change is constant meaning that for every mile walked, there is a two-dollar cost for the sponsor.



Let m = the number of miles walked

Let D = the total cost to the sponsor

$$D = 2m$$

When representing quantitative relationships on a graph it is important to discuss whether the plotted points should or should not be connected. When graphing things that cannot be broken into smaller parts, like number of cars and riders per car, the points should not be connected. When graphing things that can be broken into smaller parts, like miles walked and dollars owed, the points should be connected. In other words, if it is reasonable within the context to have a value at any point on the line, the points should be connected. If it is not reasonable within the context to have a value at any point on the line, the points should not be connected.

Equation:

The students can translate the verbal statement to develop an equation that represents the quantitative relationship of the context.

The total sponsor cost equals miles walked times \$2.00.

Or

$$\text{Dollars} = \$2.00 \times \text{miles}$$

Or

$$D = 2m$$

Additional Examples:

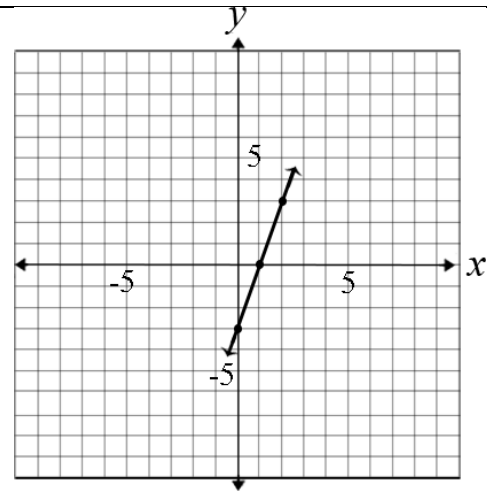
- What is the relationship between the two variables? Write an expression that illustrates the relationship.

<i>X</i>	1	2	3	4
<i>Y</i>	2.5	5	7.5	10

Solution:

$$y = 2.5x$$

- Use the graph below to describe the change in *y* as *x* increases by 1.



Solution:

As x increases by 1 y increases by 3.

6th Grade Math 3rd Quarter

Unit 4: Ratios and Unit Rates

Topic C: Unit Rates

In Topic C, students build further on their understanding of ratios and the value of a ratio as they come to understand that a ratio of 5 miles to 2 hours corresponds to a rate of 2.5 miles per hour, where the *unit rate* is the numerical part of the rate, 2.5, and *miles per hour* is the newly formed unit of measurement of the rate (6.RP.2). Students solve unit rate problems involving unit pricing, constant speed, and constant rates of work (6.RP.3b). They apply their understanding of rates to situations in the real world. Students determine unit prices and use measurement conversions to comparison shop, and decontextualize constant speed and work situations to determine outcomes. Students combine their new understanding of rate to connect and revisit concepts of converting among different-sized standard measurement units (5.MD.1). They then expand upon this background as they learn to manipulate and transform units when multiplying and dividing quantities (6.RP.3d). Topic C culminates as students construct tables of independent and dependent values in order to analyze equations with two variables from real-life contexts. They represent equations by plotting the values from the table on a coordinate grid (5.G.A.1, 5.G.A.2, 6.RP.A.3a, 6.RP.A.3b, 6.EE.C.9). They interpret and model real-world scenarios through the use of unit rates and conversions.

Big Idea:

- Rates and Ratios are comparisons using division.
- A rate is a comparison of two different things or quantities; the measuring unit is different for each value.
- Graphs and equations represent relationships between variables.

Essential Questions:

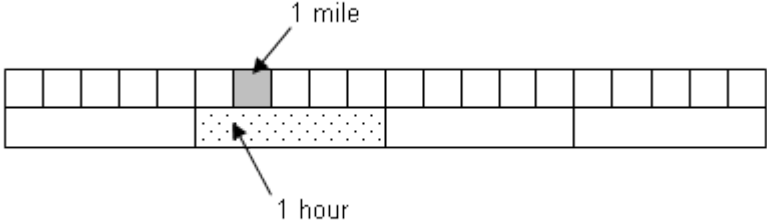

- How can you use rates to describe changes in real-life problems?
- How is a ratio or rate used to compare two quantities or values? Where can examples of ratios and rates be found?
- How can I model and represent rates and ratios?
- How are unit rates helpful in determining whether two ratios are equivalent?
- How do you determine which variable is independent/dependent in a two variable equation that represents a real-life context?
- What affect does changing the independent variable have on the dependent variable?
- How can quantitative relationships be represented?
- How is the coefficient of the dependent variable related to the graph and/or table of values?

Vocabulary

Rate, unit rate, unit price, value of a ratio, equivalent ratios, associated rates, rate unit, conversion table, constant rate, equation in two variables, independent variable, dependent variable, analyze, discrete, continuous

Grade	Domain	Standard	AZ College and Career Readiness Standards	Explanations & Examples	Resources
6	RP	2	A. Understand ratio concepts and use ratio reasoning to solve problems.	Explanation: Students build further on their understanding of ratios and the value of a ratio as they come to understand that a ratio of 5 miles to 2 hours	Eureka Math: M1 Lessons 16-23

		<p>Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. <i>For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $3/4$ cup of flour for each cup of sugar." "We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger."</i></p> <p>6.MP.2. Reason abstractly and quantitatively. 6.MP.6. Attend to precision.</p>	<p>corresponds to a rate of 2.5 miles per hour, where the <i>unit rate</i> is the numerical part of the rate, 2.5, and <i>miles per hour</i> is the newly formed unit of measurement of the rate (6.RP.2). Students solve unit rate problems involving unit pricing, constant speed, and constant rates of work (6.RP.3b).</p> <p>A rate is a ratio where two measurements are related to each other. When discussing measurement of different units, the word rate is used rather than ratio. Understanding rate, however, is complicated and there is no universally accepted definition. When using the term rate, contextual understanding is critical. Students need many opportunities to use models to demonstrate the relationships between quantities before they are expected to work with rates numerically.</p> <p>A unit rate compares a quantity in terms of one unit of another quantity. Students will often use unit rates to solve missing value problems. Cost per item or distance per time unit are common unit rates, however, students should be able to flexibly use unit rates to name the amount of either quantity in terms of the other quantity. Students will begin to notice that related unit rates are reciprocals as in the first example. It is not intended that this be taught as an algorithm or rule because at this level, students should primarily use reasoning to find these unit rates.</p> <p>In Grade 6, students are not expected to work with unit rates expressed as complex fractions. Both the numerator and denominator of the original ratio will be whole numbers.</p> <p>Solving problems using ratio reasoning and rates calls for careful attention to the referents for a given situation (MP.2).</p> <p>Example 1:</p> <ul style="list-style-type: none"> On a bicycle you can travel 20 miles in 4 hours. What are the unit rates in this situation, (the distance you can travel in 1 hour and the amount of time required to travel 1 mile)? <p>Solution: You can travel 5 miles in 1 hour written as $\frac{5 \text{ mi}}{1 \text{ hr}}$ and</p>	<p>Big Ideas: Section 5.3</p>
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				<p>it takes $\frac{1}{5}$ of an hour to travel each mile written as $\frac{1 \text{ hr}}{5 \text{ mi}}$.</p> <p>Students can represent the relationship between 20 miles and 4 hours.</p>  <p>A simple modeling clay recipe calls for 1 cup corn starch, 2 cups salt, and 2 cups boiling water. How many cups of corn starch are needed to mix with each cup of salt?</p> <p>Example 2:</p> <p>There are 2 cookies for 3 students. What is the amount of cookie each student would receive? (i.e. the unit rate)</p> <p><i>Solution:</i> This can be modeled as shown below to show that there is $\frac{2}{3}$ of a cookie for 1 student, so the unit rate is $\frac{2}{3}:1$.</p> 	
6	RP	3bd	<p>A. Understand ratio concepts and use ratio reasoning to solve problems.</p> <p>Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.</p>	<p>Explanation (RP.3b):</p> <p>A unit rate compares a quantity in terms of one unit of another quantity. Students will often use unit rates to solve missing value problems. Cost per item or distance per time unit are common unit rates, however, students should be able to flexibly use unit rates to name the amount of either quantity in terms of the other quantity. Students will begin to notice that related unit rates are reciprocals as in</p>	<p>Eureka Math: M1 Lessons 16-23</p> <p>Big Ideas: Section 5.3, 5.4, 5.7</p>

- b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?
- d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

6.MP.1. Make sense of problems and persevere in solving them.

6.MP.2. Reason abstractly and quantitatively.

6.MP.4. Model with mathematics

6.MP.5. Use appropriate tools strategically.

6.MP.7. Look for and make use of structure.

the first example. It is not intended that this be taught as an algorithm or rule because at this level, **students should primarily use reasoning** to find these unit rates.

In Grade 6, **students are not expected to work with unit rates expressed as complex fractions**. Both the numerator and denominator of the original ratio will be whole numbers.

Solving problems using ratio reasoning and rates calls for careful attention to the referents for a given situation (**MP.2**).

Students recognize the use of ratios, unit rate and multiplication in solving problems, which could allow for the use of fractions and decimals.

Students build further on their understanding of ratios and the value of a ratio as they come to understand that a ratio of 5 miles to 2 hours corresponds to a rate of 2.5 miles per hour, where the *unit rate* is the numerical part of the rate, 2.5, and *miles per hour* is the newly formed unit of measurement of the rate (**6.RP.2**). Students solve unit rate problems involving unit pricing, constant speed, and constant rates of work (**6.RP.3b**).

Example 1:

In trail mix, the ratio of cups of peanuts to cups of chocolate candies is 3 to 2. How many cups of chocolate candies would be needed for 9 cups of peanuts?

Peanuts	Chocolate
3	2

Solution:

One possible solution is for students to find the number of cups of chocolate candies for 1 cup of peanuts by dividing both sides of the table by 3, giving $\frac{2}{3}$ cup of chocolate for each cup of peanuts. To find the amount of chocolate needed for 9 cups of peanuts, students

multiply the unit rate by nine ($9 \cdot 2$), giving 6 cups of chocolate.

Example 2:

If steak costs \$2.25 per pound, how much does 0.8 pounds of steak cost? Explain how you determined your answer.

Solution:

The unit rate is \$2.25 per pound so multiply $\$2.25 \times 0.8$ to get \$1.80 per 0.8 lb of steak.

Explanation (RP.3d)

A ratio can be used to compare measures of two different types, such as inches per foot, milliliters per liter and centimeters per inch. Students recognize that a conversion factor is a fraction equal to 1 since the numerator and denominator describe the same quantity.

For example, $\frac{12 \text{ inches}}{1 \text{ foot}}$ is a conversion factor

since the numerator and denominator equal the same amount. Since the ratio is equivalent to 1, the identity property of multiplication allows an amount to be multiplied by the ratio. Also, the value of the ratio can also be expressed as

$\frac{1 \text{ foot}}{12 \text{ inches}}$ allowing for the conversion ratios to be expressed in a

format so that units will “cancel”. Students use ratios as conversion factors and the identity property for multiplication to convert ratio units.

Example 1:

How many centimeters are in 7 feet, given that 1 inch \approx 2.54 cm.

Solution:

$$7 \text{ feet} \times \frac{12 \text{ inches}}{1 \text{ foot}} \times \frac{2.54 \text{ cm}}{1 \text{ inch}} = 7 \text{ feet} \times \frac{12 \text{ inches}}{1 \text{ foot}} \times \frac{2.54 \text{ cm}}{1 \text{ inch}} = 7 \times 12 \times 2.54 \text{ cm} = 213.36 \text{ cm}$$

				Note: Conversion factors will be given. Conversions can occur both between and across the metric and English system. Estimates are not expected.	
6	EE	9	<p>C. Represent and analyze quantitative relationships between dependent and independent variables.</p> <p>Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. <i>For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.</i></p> <p>Connection to 6.RP.3</p> <p>6.MP.1. Make sense of problems and persevere in solving them.</p> <p>6.MP.2. Reason abstractly and quantitatively.</p> <p>6.MP.3. Construct viable arguments and critique the reasoning of others.</p> <p>6.MP.4. Model with mathematics.</p> <p>6.MP.7. Look for and make use of structure.</p> <p>6.MP.8. Look for and express regularity in repeated reasoning</p>	<p>Explanation:</p> <p>The purpose of this standard is for students to understand the relationship between two variables, which begins with the distinction between dependent and independent variables. The independent variable is the variable that can be changed; the dependent variable is the variable that is affected by the change in the independent variable. Students recognize that the independent variable is graphed on the x-axis; the dependent variable is graphed on the y-axis.</p> <p>Students recognize that not all data should be graphed with a line. Data that is discrete would be graphed with coordinates only. Discrete data is data that would not be represented with fractional parts such as people, tents, records, etc. For example, a graph illustrating the cost per person would be graphed with points since part of a person would not be considered. A line is drawn when both variables could be represented with fractional parts.</p> <p>Students are expected to recognize and explain the impact on the dependent variable when the independent variable changes (As the x variable increases, how does the y variable change?) Relationships should be proportional with the line passing through the origin. Additionally, students should be able to write an equation from a word problem and understand how the coefficient of the dependent variable is related to the graph and /or table of values.</p> <p>Students can use many forms to represent relationships between quantities. Multiple representations include describing the relationship using language, a table, an equation, or a graph. Translating between multiple representations helps students understand that each form represents the same relationship and provides a different perspective on the function.</p> <p>Students look for and express regularity in repeated reasoning (MP.8) as they generate algebraic models (MP.4) to represent relationships.</p>	<p>Eureka Math: M4 Lesson 31-32</p> <p>Big Ideas: Section 7.4</p>

Examples:

- A school is having a walk-a-thon for a fund raiser. Each student in the walk-a-thon must find sponsors to pledge \$2.00 for each mile the student walks. Sponsors want to know how much money they would owe given the total distance the students would walk.

Solution:

Table of Values:

Horizontal

Number of Miles Walked (Independent Variable known as x)	0	1	2	3	4	5	6
Total Dollars Sponsor Would Owe (Dependent Variable known as y)	0	2	4	6	8	10	12

Vertical

Miles	Dollars
0	0
1	2
2	4
3	6
4	8
5	10
6	12

The table helps students recognize the pattern in the function.

Let m = the number of miles walked

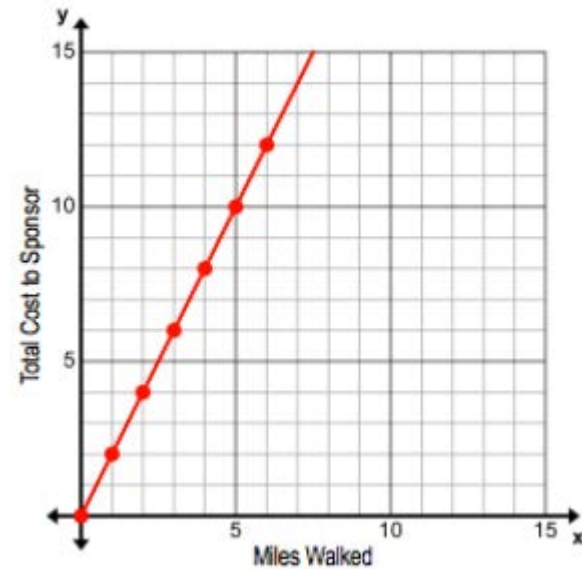
Let D = the total cost to the sponsor

$$D = 2m$$

Graph:

Students can graph the quantitative relationship on a coordinate plane. When graphing the data, the horizontal x-axis represents the independent variable of miles walked, and the vertical y-axis

represents the dependent variable of total dollars the sponsor owes. The graph gives the students a visual image that helps them describe the relationship between miles walked and total money owed. When connected, the points form a straight line, which means it is a linear function. The rate of change is constant meaning that for every mile walked, there is a two-dollar cost for the sponsor.



Let m = the number of miles walked

Let D = the total cost to the sponsor

$$D = 2m$$

When representing quantitative relationships on a graph it is important to discuss whether the plotted points should or should not be connected. When graphing things that cannot be broken into smaller parts, like number of cars and riders per car, the points should not be connected. When graphing

things that can be broken into smaller parts, like miles walked and dollars owed, the points should be connected. In other words, if it is reasonable within the context to have a value at any point on the line, the points should be connected. If it is not reasonable within the context to have a value at any point on the line, the points should not be connected.

Equation:

The students can translate the verbal statement to develop an equation that represents the quantitative relationship of the context.

The total sponsor cost equals miles walked times \$2.00.

Or

$$\text{Dollars} = \$2.00 \times \text{miles}$$

Or

$$D = 2m$$

Additional Examples:

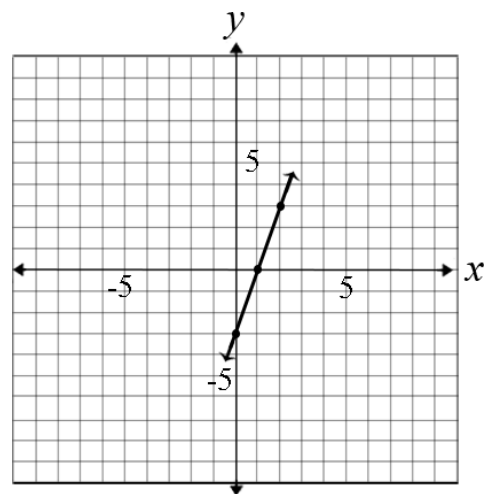
- What is the relationship between the two variables? Write an expression that illustrates the relationship.

X	1	2	3	4
Y	2.5	5	7.5	10

Solution:

$$y = 2.5x$$

- Use the graph below to describe the change in y as x increases by 1.



Solution:

As x increases by 1 y increases by 3.

6th Grade Math 3rd Quarter

Unit 4: Ratios and Unit Rates

Topic D: Percent

In the final topic of this unit, students are introduced to percent and find percent of a quantity as a *rate per 100*. Students understand that N percent of a quantity has the same value as $N/100$ of that quantity. Students express a fraction as a percent, and find a percent of a quantity in real-world contexts. Students learn to express a ratio using the language of percent and to solve percent problems by selecting from familiar representations, such as tape diagrams and double number lines, or a combination of both (**6.RP.3c**).

Big Idea:

- A percent is a quantity expressed as a rate per 100.
- A fraction can be expressed as a decimal and a percent.
- A decimal can be expressed as a fraction and a percent.
- A percent can be expressed as a fraction and a decimal.

Essential Questions:

- How is a percent represented as a quantity?
- What is the relationship between a fraction, decimal and percent?

Vocabulary

Rate, unit rate, unit price, percent, part-to-whole ratios, part-to-part ratios

Grade	Domain	Standard	AZ College and Career Readiness Standards	Explanations & Examples	Resources
6	RP	3c	<p>A. Understand ratio concepts and use ratio reasoning to solve problems.</p> <p>Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.</p> <p>c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means $30/100$ times the quantity); solve problems involving finding the whole, given a part and the percent.</p>	<p><u>Explanation:</u></p> <p>This is the students' first introduction to percents. Percentages are a rate per 100. Models, such as percent bars or 10×10 grids should be used to model percents. Students use ratios to identify percents.</p> <p>PERCENTAGES can be thought of as PART-TO-WHOLE RATIOS because 100 is the unit whole around which quantities are being compared.</p> <p>As students work with unit rates and interpret percent as a rate per 100, and as they analyze the relationships among the values, they look for and make use of structure (MP.7). As students become more</p>	<p>Eureka Math: M1 Lessons 24-29</p> <p>Big Ideas: Section 5.5, 5.6</p>

- 6.MP.1. Make sense of problems and persevere in solving them.
- 6.MP.2. Reason abstractly and quantitatively.
- 6.MP.4. Model with mathematics
- 6.MP.5. Use appropriate tools strategically.
- 6.MP.7. Look for and make use of structure.

sophisticated in their application of ratio reasoning, they learn when it is best to solve problems with ratios, their associated unit rates, or percents (**MP.5**). Solving problems using ratio reasoning and rates calls for careful attention to the referents for a given situation (**MP.2**).

Example 1:

What percent is 12 out of 25?

Solution: One possible solution method is to set up a ratio table: Multiply 25 by 4 to get 100. Multiplying 12 by 4 will give 48, meaning that 12 out of 25 is equivalent to 48 out of 100 or 48%.

Students use percentages to find the part when given the percent, by recognizing that the whole is being divided into 100 parts and then taking a part of them (the percent).

Part	Whole
12	25
?	100

Example 2:

What is 40% of 30?

Solution: There are several methods to solve this problem. One possible solution using rates is to use a 10 x 10 grid to represent the whole amount (or 30). If the 30 is divided into 100 parts, the rate for one block is 0.3. Forty percent would be 40 of the blocks, or 40 x 0.3, which equals 12.

See the web link below for more information.

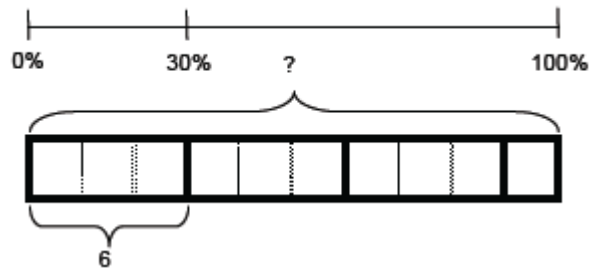
<http://illuminations.nctm.org/LessonDetail.aspx?id=L249>

Students also determine the whole amount, given a part and the percent.

Example 3:

If 30% of the students in Mrs. Rutherford's class like chocolate ice cream, then how many students are in Mrs.

Rutherford's class if 6 like chocolate ice cream?



(Solution: 20)

Example 4:

A credit card company charges 17% interest fee on any charges not paid at the end of the month. Make a ratio table to show how much the interest would be for several amounts. If the bill totals \$450 for this month, how much interest would you have to be paid on the balance?

Solution:

Charges	\$1	\$50	\$100	\$200	\$450
Interest	\$0.17	\$8.50	\$17	\$34	?

One possible solution is to multiply 1 by 450 to get 450 and then multiply 0.17 by 450 to get \$76.50.

Finding the Whole Given a Part and a Percent: The tape diagram (bar model) provides the imagery needed to help students conceptualize the whole in terms of the part and percent. The example below takes the approach of capturing the precise information from the story problem in the bar model. (Scaffolding for some students might include dividing the number into 10 sections with 10% intervals if they're having a difficult time locating the percent.)

There are several strategies students can use to find the unknown whole after creating this diagram. For example, the bar can be subdivided into 20% sections, which provides a tool for adding up (or multiplying) to find the total quantity.

A **double bar diagram** also helps students solve the problem pictorially.

Step 1: Identify the information Step 2: Fill in equivalent ratios to locate solution.

A **function or input/output table** can be used in a similar way.

Percentage	0%	20%	40%	60%	80%	100%
Part	0	14	28	42	56	70

6 **AZ.** **9**
NS
C. Apply and extend previous understandings of the system of rational numbers.
 Convert between expressions for positive rational numbers, including fractions, decimals, and percents.

Students need many opportunities to express rational numbers in meaningful contexts.

Example:

- A baseball player's batting average is 0.625. What does the

Eureka Math:
 M1 Lessons 24-29

Big Ideas:
 Section 5.5, 5.6

		<p>6.MP.2. Reason abstractly and quantitatively.</p> <p>6.MP.8. Look for and express regularity in repeated reasoning.</p>	<p>batting average mean? Explain the batting average in terms of a fraction, ratio, and percent.</p> <p><i>Solution:</i></p> <ul style="list-style-type: none">○ The player hit the ball $\frac{5}{8}$ of the time he was at bat;○ The player hit the ball 62.5% of the time; or○ The player has a ratio of 5 hits to 8 batting attempts (5:8).	
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6th Grade Math 3rd Quarter

Unit 5: Geometry (5 weeks)

Topic A: Area of Triangles, Quadrilaterals, and Polygons

Unit 5 is an opportunity to practice the material learned in Unit 4 in the context of geometry; students apply their newly acquired capabilities with expressions and equations to solve for unknowns in area, surface area, and volume problems. They find the area of triangles and other two-dimensional figures and use the formulas to find the volumes of right rectangular prisms with fractional edge lengths. Students use negative numbers in coordinates as they draw lines and polygons in the coordinate plane. They also find the lengths of sides of figures, joining points with the same first coordinate or the same second coordinate and apply these techniques to solve real-world and mathematical problems. In Topic A, students use composition and decomposition to determine the area of triangles, quadrilaterals, and other polygons. They determine that area is additive. Students learn through exploration that the area of a triangle is exactly half of the area of its corresponding rectangle.

Big Idea:

- Geometry and spatial sense offer ways to envision, to interpret and to reflect on the world around us.
- Area, volume and surface area are measurements that relate to each other and apply to objects and events in our real life experiences.
- Measurement is used to quantify attributes of shapes and objects in order to make sense of our world.

Essential Questions:

- How does what we measure influence how we measure?
- How can space be defined through numbers and measurement?
- How does investigating figures help us build our understanding of mathematics?
- What is the relationship with 2-dimensional shapes and our world?
- How can you use area formulas to find missing dimensions of plane figures?

Vocabulary

compose, decompose, composite figure, polygon, area, base, altitude, height, perpendicular, quadrilateral, rectangle, parallelogram, triangle, trapezoid, rhombus, right angle, kite

Grade	Domain	Standard	AZ College and Career Readiness Standards	Explanations & Examples	Resources
6	G	1	<p>A. Solve real-world and mathematical problems involving area, surface area, and volume.</p> <p>Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.</p> <p><i>6.MP.1. Make sense of problems and persevere in</i></p>	<p>Explanation:</p> <p>Students continue to understand that area is the number of squares needed to cover a plane figure. Students should know the formulas for rectangles and triangles. “Knowing the formula” does not mean memorization of the formula. To “know” means to have an understanding of <i>why</i> the formula works and how the formula relates to the measure (area) and the figure. This understanding should be for <i>all</i> students.</p>	<p>Eureka Math: M5 Lesson 1-6</p> <p>Big Ideas: Section 4.1, 4.2, 4.3, Extension 4.3</p>

solving them.

6.MP.2. Reason abstractly and quantitatively.

6.MP.3. Construct viable arguments and critique the reasoning of others.

6.MP.4. Model with mathematics.

6.MP.5. Use appropriate tools strategically.

6.MP.6. Attend to precision.

6.MP.7. Look for and make use of structure.

6.MP.8. Look for and express regularity in repeated reasoning.

Finding the area of triangles is introduced in relationship to the area of rectangles – a rectangle can be decomposed into two congruent triangles. Therefore, the area of the triangle is $\frac{1}{2}$ the area of the rectangle. The area of a rectangle can be found by multiplying base x height; therefore, the area of the triangle is $\frac{1}{2}bh$ or $(b \times h)/2$. The conceptual understanding of the area of a rectangle was developed in 3rd grade. By the end of 4th grade, students should be able to apply the area formula for rectangles in real world contexts.

Special quadrilaterals include rectangles, squares, parallelograms, trapezoids, rhombi, and kites. Students can use tools such as the Isometric Drawing Tool on NCTM’s Illuminations site to shift, rotate, color, decompose and view figures in 2D or 3D (<http://illuminations.nctm.org/ActivityDetail.aspx?ID=125>)

Students decompose shapes into rectangles and triangles to determine the area. For example, a trapezoid can be decomposed into triangles and rectangles (see figures below). Using the trapezoid’s dimensions, the area of the individual triangle(s) and rectangle can be found and then added together. Special quadrilaterals include rectangles, squares, parallelograms, trapezoids, rhombi, and kites.



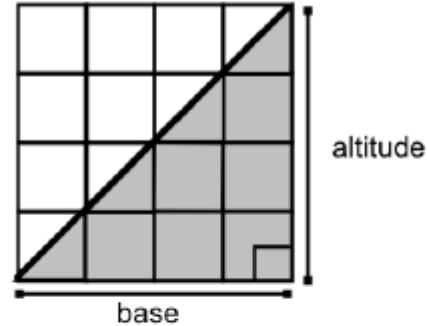
Isosceles trapezoid



Right trapezoid

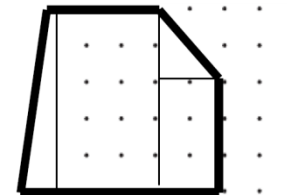
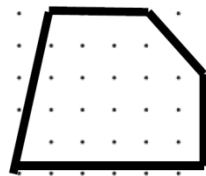
Note: Students recognize the marks on the isosceles trapezoid indicating the two sides have equal measure.

The standards in this unit require that students persevere in solving problems (**MP.1**) and model real-world scenarios with mathematical models, including equations (**MP.4**), with a degree of precision appropriate for the given situation (**MP.6**).



This is a good entry point for students to begin developing ideas about area of triangles. It is clear to see, to count, and to prove that the shaded area is half of the rectangle, $A = \frac{1}{2}(b \times h)$. However, at this stage, it is unproductive for students to think of area as 'counting'. (In fact, this strategy only works for right isosceles triangles; it becomes increasingly difficult for other triangles.)

A follow-up strategy is to place a composite shape on grid or dot paper. This aids in the decomposition of a shape into its foundational parts. Once the composite shape is decomposed, the area of each part can be determined and the sum of the area of each part is the total area.



Examples:

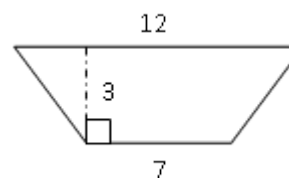
- Find the area of a triangle with a base length of three units and a height of four units.

Solution:

Students understand that the hypotenuse is the longest side of a right triangle. The base and height would form the 90° angle and would be used to find the area using:

$$\begin{aligned}A &= \frac{1}{2} bh \\A &= \frac{1}{2} (3 \text{ units})(4 \text{ units}) \\A &= \frac{1}{2} 12 \text{ units}^2 \\A &= 6 \text{ units}^2\end{aligned}$$

- Find the area of the trapezoid shown below using the formulas for rectangles and triangles.



Solution:

The trapezoid could be decomposed into a rectangle with a length of 7 units and a height of 3 units. The area of the rectangle would be 21 units.

The triangles on each side would have the same area. The height of the triangles is 3 units. After taking away the middle rectangle's base length, there is a total of 5 units remaining for both of the side triangles. The base length of each triangle is half of 5. The base of each triangle is 2.5 units. The area of one triangle would be $\frac{1}{2} (2.5 \text{ units})(3 \text{ units})$ or 3.75 units^2 .

Using this information, the area of the trapezoid would be:

$$\begin{array}{r}
 21 \text{ units}^2 \\
 3.75 \text{ units}^2 \\
 \hline
 +3.75 \text{ units}^2 \\
 28.5 \text{ units}^2
 \end{array}$$

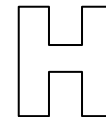
- A rectangle measures 3 inches by 4 inches. If the lengths of each side are doubled, what is the effect on the area?

Solution:

The new rectangle would have side lengths of 6 inches and 8 inches. The area of the original rectangle was 12 inches². The area of the new rectangle is 48 inches². The area increased 4 times (quadrupled).

Students may also create a drawing to show this visually.

- The sixth grade class at Hernandez School is building a giant wooden H for their school. The H will be 10 feet tall and 10 feet wide and the thickness of the block letter will be 2.5 feet.
 - How large will the H be if measured in square feet?
 - The truck that will be used to bring the wood from the lumber yard to the school can only hold a piece of wood that is 60 inches by 60 inches. What pieces of wood (how many pieces and what dimensions) are needed to complete the project?



Solution:

One solution is to recognize that, if filled in, the area would be 10 feet tall and 10 feet wide or 100 ft². The size of one piece removed is 5 feet by 3.75 feet or 18.75 ft². There are two of these

				<p>pieces. The area of the “H” would be $100 \text{ ft}^2 - 18.75 \text{ ft}^2 - 18.75 \text{ ft}^2$, which is 62.5 ft^2.</p> <p>A second solution would be to decompose the “H” into two tall rectangles measuring 10 ft by 2.5 ft and one smaller rectangle measuring 2.5 ft by 5 ft. The area of each tall rectangle would be 25 ft^2 and the area of the smaller rectangle would be 12.5 ft^2. Therefore the area of the “H” would be $25 \text{ ft}^2 + 25 \text{ ft}^2 + 12.5 \text{ ft}^2$ or 62.5 ft^2.</p> <p>2. Sixty inches is equal to 5 feet, so the dimensions of each piece of wood are 5ft by 5ft. Cut two pieces of wood in half to create four pieces 5 ft. by 2.5 ft. These pieces will make the two taller rectangles. A third piece would be cut to measure 5ft. by 2.5 ft. to create the middle piece.</p>	
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6th Grade Math 3rd Quarter

Unit 5: Geometry

Topic B: Polygons on the Coordinate Plane

In Unit 2, students used coordinates and absolute value to find distances between points on a coordinate plane (6.NS.C.8). In Topic B, students extend this learning by finding edge lengths of polygons (the distance between two vertices using absolute value) and draw polygons given coordinates (6.G.A.3). From these drawings, students determine the area of polygons on the coordinate plane by composing and decomposing into polygons with known area formulas. Students investigate and calculate the area of polygons on the coordinate plane and also calculate the perimeter. They note that finding perimeter is simply finding the sum of the polygon's edge lengths (or finding the sum of the distances between vertices). Topic B concludes with students determining distance, perimeter, and area on the coordinate plane in real-world contexts.

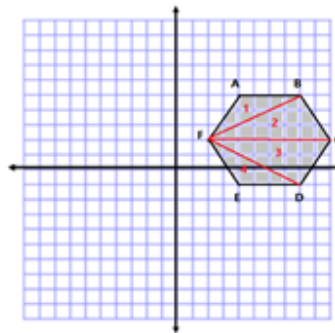
Big Idea:			<ul style="list-style-type: none"> • Geometry and spatial sense offer ways to envision, to interpret and to reflect on the world around us. • Area, volume and surface area are measurements that relate to each other and apply to objects and events in our real life experiences. • Measurement is used to quantify attributes of shapes and objects in order to make sense of our world. 		
Essential Questions:			<ul style="list-style-type: none"> • How does measurement help you solve problems in everyday life? • How does what we measure influence how we measure? • How can space be defined through numbers and measurement? • How does investigating figures help us build our understanding of mathematics? • What is the relationship with 2-dimensional shapes and our world? • How can you use area and perimeter formulas to find missing dimensions of plane figures? • How can a coordinate plane be used to solve measurement problems? 		
Vocabulary			Composite figure, polygon, coordinate plane, vertices, perimeter, area		
Grade	Domain	Standard	AZ College and Career Readiness Standards	Explanations & Examples	Resources
6	G	3	<p>A. Solve real-world and mathematical problems involving area, surface area, and volume.</p> <p>Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.</p>	<p>Explanation:</p> <p>Students are given the coordinates of polygons to draw in the coordinate plane. If both x-coordinates are the same (2, -1) and (2, 4), then students recognize that a vertical line has been created and the distance between these coordinates is the distance between -1 and 4, or 5. If both the y-coordinates are the same (-5, 4) and (2, 4), then students recognize that a horizontal line has been created and the distance between these coordinates is the distance between -5 and 2, or 7. Using this understanding, student solve real-world and mathematical problems, including finding the area and perimeter of</p>	<p>Eureka Math: M5 Lesson 7-10</p> <p>Big Ideas: Section 4.4</p>

		<p>Connections: <i>6.NS.8</i>; <i>6-8.RST.7</i></p> <p>6.MP.1. Make sense of problems and persevere in solving them.</p> <p>6.MP.2. Reason abstractly and quantitatively.</p> <p>6.MP.4. Model with mathematics.</p> <p>6.MP.5. Use appropriate tools strategically.</p> <p>6.MP.7. Look for and make use of structure.</p>	<p>geometric figures drawn on a coordinate plane.</p> <p>Examples:</p> <ul style="list-style-type: none"> • Four friends used Google Map to map out their neighborhood. They discovered that their houses form a rectangle. Use a coordinate grid to plot their houses; then answer the questions. <ul style="list-style-type: none"> ○ How many units away is Henry’s house from Ron’s house? ○ How many units away is Minnie’s house from Henry’s house? ○ Who lives closer to Kim – Ron or Minnie? How do you know? • On a map, the library is located at (-2, 2), the city hall building is located at (0,2), and the high school is located at (0,0). Represent the locations as points on a coordinate grid with a unit of 1 mile. <ul style="list-style-type: none"> ○ What is the distance from the library to the city hall building? The distance from the city hall building to the high school? How do you know? ○ What shape is formed by connecting the three locations? ○ The city council is planning to place a city park in this area. How large is the area of the planned park? 	
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Calculate the area of the polygon using two different methods. Write two expressions to represent the two methods and compare the structure of the expressions.

Answers will vary. The following are two possible methods. However, students could also break the shape into two triangles and a rectangle or another correct method.

Method One:



Area of triangle 1 and 4

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(4 \text{ units})(3 \text{ units})$$

$$A = \frac{1}{2}(12 \text{ units}^2)$$

$$A = 6 \text{ units}^2$$

Since there are 2, we have a total area of 12 units².

$$\text{Total area} = 12 \text{ units}^2 + 24 \text{ units}^2 = 36 \text{ units}^2$$

Area of triangle 2 and 3

$$A = \frac{1}{2}bh$$

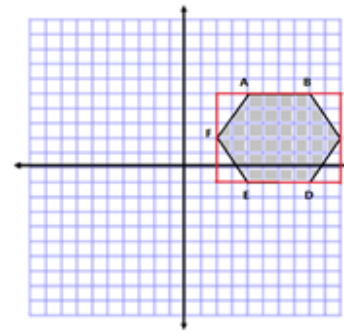
$$A = \frac{1}{2}(8 \text{ units})(3 \text{ units})$$

$$A = \frac{1}{2}(24 \text{ units}^2)$$

$$A = 12 \text{ units}^2$$

Since there are 2, we have a total area of 24 units².

Method Two:



$$A = l \cdot w$$

$$A = (8 \text{ units})(6 \text{ units})$$

$$A = 48 \text{ units}^2$$

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(2 \text{ units})(3 \text{ units})$$

$$A = 3 \text{ units}^2$$

There are 4 triangles of equivalent base and height.

$$4(3 \text{ units}^2) = 12 \text{ units}^2$$

$$\text{Total area} = 48 \text{ units}^2 - 12 \text{ units}^2$$

$$\text{Total area} = 36 \text{ units}^2$$

Expressions

$$2 \left[\frac{1}{2}(4)(3) \right] + 2 \left[\frac{1}{2}(8)(3) \right] \quad \text{or} \quad (8)(6) - 4 \left[\frac{1}{2}(2)(3) \right]$$

The first expression shows terms being added together because I separated the hexagon into smaller pieces and had to add their areas back together.

The second expression shows terms being subtracted because I made a larger outside shape, and then had to take away the extra pieces.