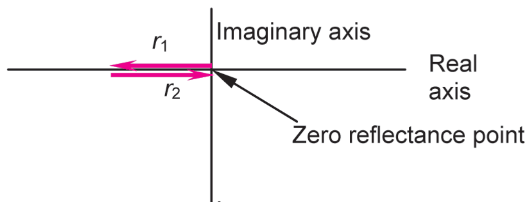
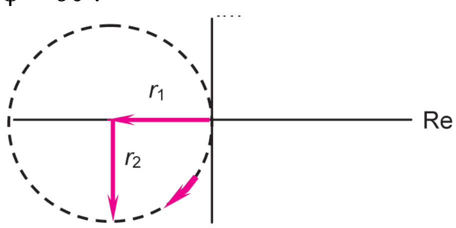
Reflectance as Vector Addition

The case of the soap bubble is illustrated from the viewpoint of vectors.

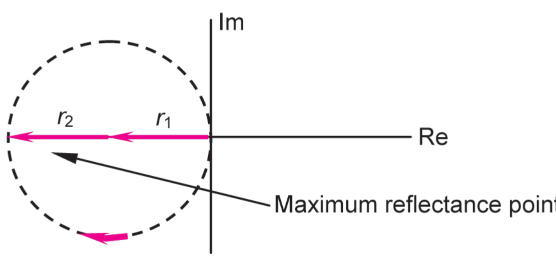
When $\varphi = 0$, $r_1 = -1/6$, and $r_2 = +1/6$.



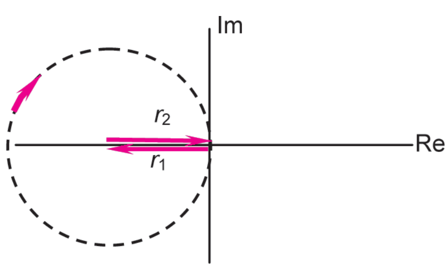
When $\varphi = 90°$:



When $\varphi = 180°$ (one QWOT):

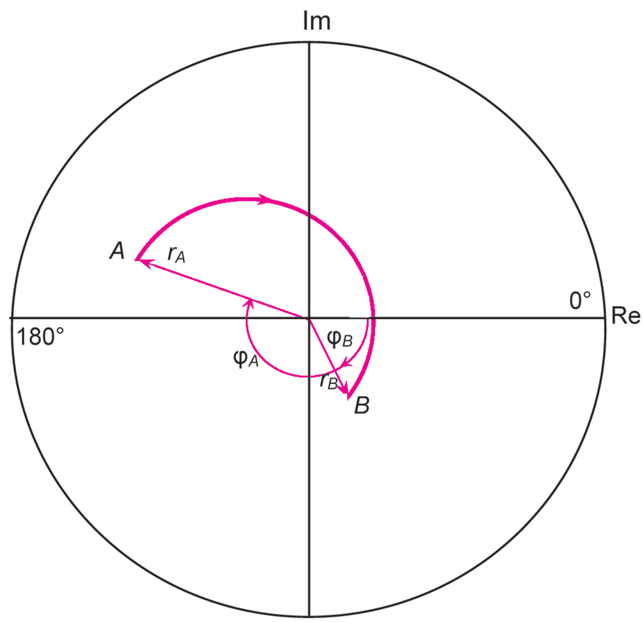


When $\varphi = 360°$ (two QWOTs or one half-wave OT):



Reflectance Amplitude Diagram

The **reflectance amplitude diagram**, often referred to as a **circle diagram**, follows from the foregoing vector diagrams (p.6).

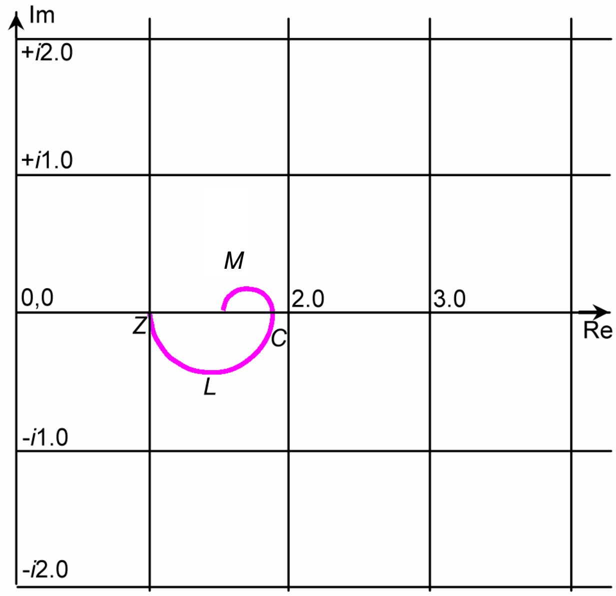


The outermost circle represents r = 1.0 and also $R = rr^*$ = 1.0 or R = 100% reflectance. The origin of the real and imaginary axes is where r = 0.0 or zero reflectance.

Any new layer starts from the point of the reflectance amplitude of whatever lies beneath it, whether that is a substrate or a stack of coating layers on a substrate. This point is represented by point A in the figure, where the starting reflectance amplitude is r_A and its phase is φ_A. After the addition of a given physical thickness (PT = d) and optical thickness (OT) of a layer of given index, the resulting reflectance reaches point B. The new reflectance amplitude is r_B and its phase is φ_B. Point B would then be the starting reflectance for the next layer.

Admittance Diagram

The **admittance** (Y) of a medium, as applied to optical thin films, is an electrical quantity, which is normalized to be equivalent to the index of refraction. When the effective index of a thin-film stack is plotted as a function of increasing thickness from the substrate to the end of the last layer, an **admittance diagram** is produced. This is a conformal mapping of the reflectance amplitude diagram. For many cases the diagrams look quite similar, except that they have been rotated 180° about a point. In the locus of homogeneous layers, nonabsorbing layers are circles.



The range of an admittance diagram is the semi-infinite plane from 0 to ∞ to the right on the real axis and $\pm i\infty$ on the imaginary axis. The two-layer case of a QWOT of M and L is plotted here for comparison. Note that point S is at 1.52 –i0.0, the index of the substrate; C is at 1.90132 – i0.0; and Z is at $Y = N_e = 1.0 - i0.0$, where $r = (1.0 - 1.0) / (1.0 + 1.0) = 0$.

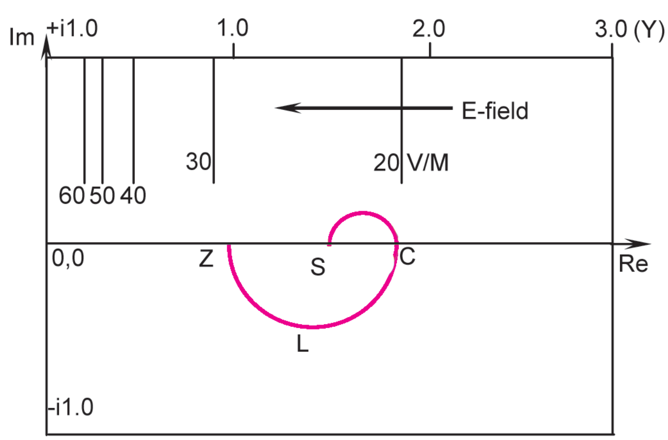
Electric Field in a Coating

The electric field within a coating layer is of importance when laser damage thresholds are considered and also when working with absorbing layers. In the latter case, the amount of energy absorbed in the layer depends upon the relative value of the electric field within that layer. One aspect of the laser damage issue seems to be that the interface between real deposited layers has some absorption and defects that are more vulnerable if the electric field is high at that interface.

The **relative volts/meter** is a function of the real value of the admittance and can be calculated as follows:

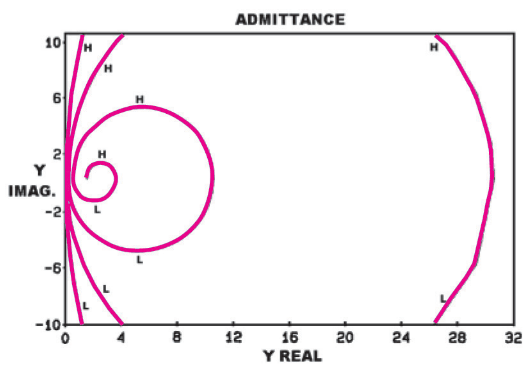
$$E = 27.46 / [\mathrm{Re}(Y)]^{0.5} .$$

This is very convenient to visualize on an **admittance diagram**. The closer the locus of the coating gets to $Y = 0$, the higher the electric field, and therefore the more vulnerable the coating is to high energy flux. A laser stack is sometimes designed with non-QWOT layers that do not terminate toward the left on the real axis. Instead, the locus stops short of, or continues past, the crossing of the real axis until it is further away from the risks of high electric field at $Y = 0$.

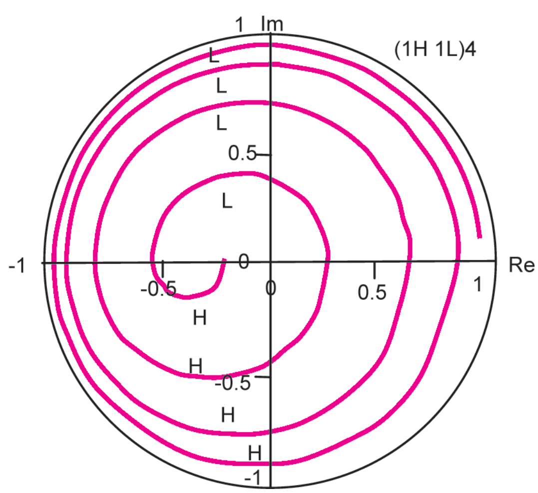


Admittance versus Reflectance Amplitude Diagrams

The admittance diagram is generally as useful as the **reflectance amplitude** or **circle diagram**, particularly in the realm of low reflectance (and thereby low admittance). However, the admittance diagram is not as useful for high reflectors such as a stack like (1H 1L)4, as can be seen in the figure below. For high reflectors of many layers, the admittance tends to go off of whatever scale is chosen for the plot, whereas the reflectance amplitude diagram is constrained to the unit circle.



Reflectance Amplitude Diagram



Triangle Diagram

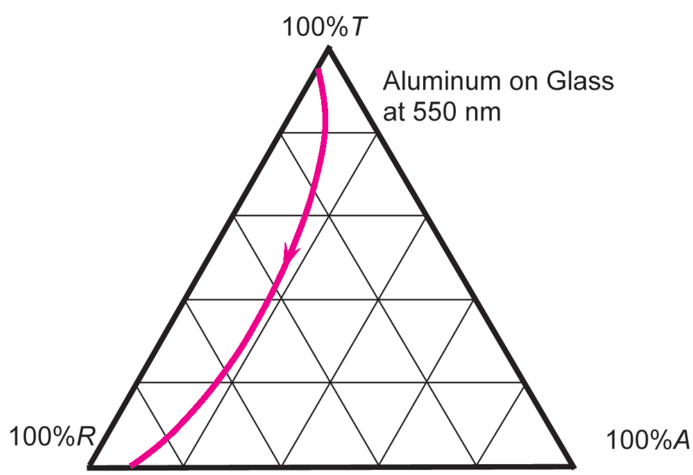
The energy falling on an optical thin film will be either reflected (R), transmitted (T), or absorbed (A). Scattering is ignored for the purposes of this section. It can then be stated that

$$R + T + A = 1 .$$

A convenient way to visualize the properties of materials with absorption, such as metals, is using a **triangle diagram**. When there is no absorption (A), the transmittance (T) is simply $1 - R$ and the diagram is not particularly useful.

This figure shows the path in R, T, and A for a coating of aluminum on glass. It starts at 96% T, 4% R, and 0% A on the bare glass surface. As the aluminum thickness increases, the locus moves downward. This happens to pass through a point of about 40% T, 43% R, and 17% A. When the film becomes opaque, it is at a point of 0% T, 90% R, and 10% A.

These triangle diagrams are useful when a material has a significant value of k, the imaginary index, as in metals and semiconductors.



Single-Layer Antireflection Coating

The most common **single-layer antireflection coating** (SLAR) on glass is a QWOT of MgF_2. If the index of the glass is 1.52 and that of the MgF_2 is 1.38:

$$r_1 = (1.0 - 1.38)/(1.0 + 1.38) = -0.1597\,,$$

$$r_2 = (1.38 - 1.52)/(1.38 + 1.52) = -0.0483\,.$$

When the film is infinitesimally thin, $\varphi = 0$, so that

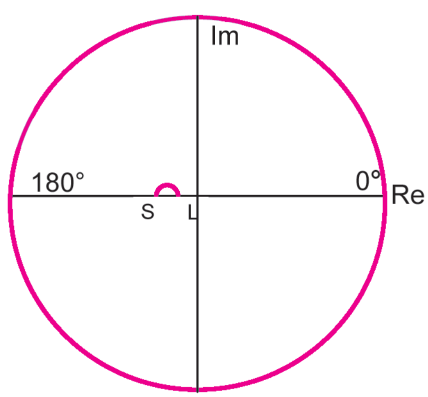
$$r = \left[(r_1 + r_2 * 1)/(1 + r_1 r_2 * 1)\right] = -0.2064\,;$$
$$R = 4.260\%\ (\text{same as bare substrate}).$$

When the thin film is one QWOT, at the wavelength under consideration, then $\varphi = 180°$:

$$r = \left[r_1 + r_2 * (-1)\right]/\left[1 + r_1 r_2 * (-1)\right] = -0.1123\,;$$
$$R = 1.260\%\,.$$

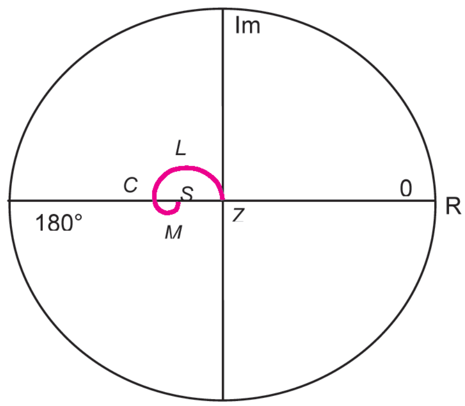
Thus, this SLAR reduces the reflection of the glass surface from 4.26% to 1.26% (only at the QWOT wavelength, and 0° AOI; at other wavelengths and angles, R is modified).

The locus of this coating as seen on a reflectance amplitude or circle diagram as it grows from zero to one QWOT is a semicircle moving clockwise from point S to L.



Two-Layer AR Amplitude Diagram Example

In this example (in air), the substrate is of index 1.52 and its **reflectance amplitude** point is at S. The first QWOT layer is of index $M = 1.70$. The second QWOT layer is of index $L = 1.38$. This two-layer coating results in zero reflectance at the origin point, Z, for this design wavelength (only).



The supporting calculations are

$r_{\text{substrate}} = (1.0 - 1.52)/(1.0 + 1.52) = -0.20635; R_S = 4.528\%$.

After the deposition of the first layer:

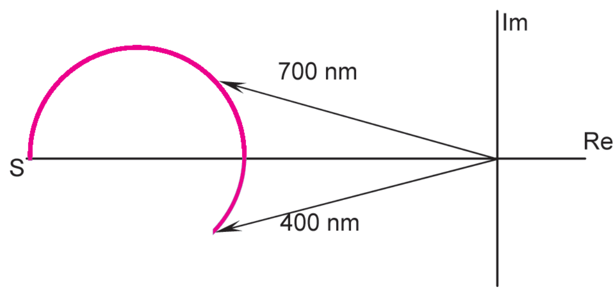
$$r_1 = (1.0 - 1.70)/(1.0 + 1.70) = -0.25926$$
$$r_2 = (1.70 - 1.52)/(1.70 + 1.52) = +0.05590; \quad \varphi = 180°$$
$$r_C = \{[-0.25926(-)(+0.05590)]/[1.0 + (-) - 0.25926 * (+0.05590)]\}$$
$$= -0.310658; n_{\text{effective}} = 1.90132; \quad R_C = 9.651\%$$

After the deposition of the second layer:

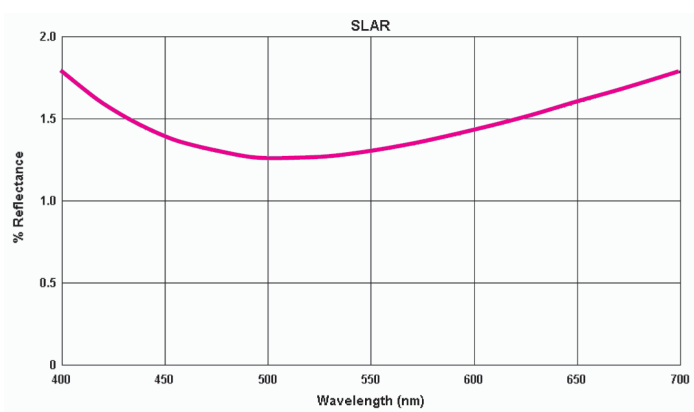
$$r_1 = (1.0 - 1.38)/(1.0 + 1.38) = -0.15966$$
$$r_2 = (1.38 - 1.90132)/(1.38 + 1.90132) = -0.15888; \varphi = 180°$$
$$r_Z = \{[-0.15966(-)(-0.15888)]/[(1.0 + (-) - 0.15966) * (-0.15888)]\}$$
$$= -0.00080; \quad R_C = 0.000064\%!$$

Wavelength Effects

If the SLAR were designed to be just one QWOT at a 510-nm wavelength, that same physical thickness (PT) would be 0.729 QWOT at a wavelength of 700 nm, and 1.275 QWOT at 400 nm. This means that the semicircle shown previously (p. 12) would be less than a semicircle for 700 nm and more than one for 400 nm. The 400 and 700 nm terminations of the layer would be further from the origin than QWOT at 510 nm, and therefore would have a higher reflectance. Specifically, the reflectance would be 1.799% at 400 nm and 1.786% at 700 nm, as compared to 1.260% for 510 nm.

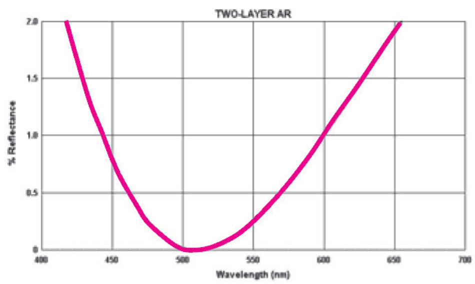


The resulting reflectance versus wavelength would appear as the figure below.

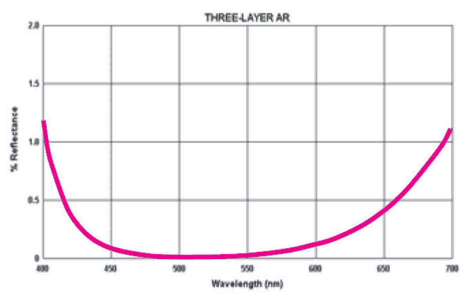


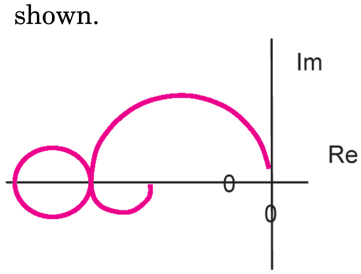
Wavelength Effects (cont.)

The two-layer AR design shown previously had one QWOT of index 1.70 and one QWOT of index 1.38 to bring the reflectance to essentially zero at the design wavelength. If this was designed for 510 nm, both layers would be 1.275× thicker at 400 nm and 0.729× thinner at 700 nm than one QWOT. The **wavelength effects** would be even more exaggerated than the SLAR case as shown.



Coatings such as these are referred to as "V-coatings" because of the V shape of the reflectance spectrum. However, if a layer of index 2.2 and thickness of two QWOTs were added between the two layers, the result is shown.

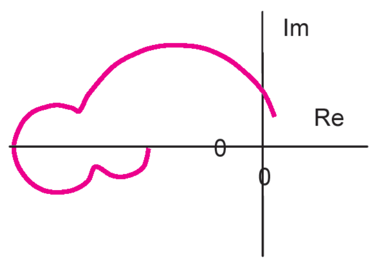




This additional half- wave OT layer is referred to as an **achromatizing layer** because it minimizes the changes in reflectance with color or wavelength. This might best be understood in the framework of the reflectance amplitude or circle diagrams at different colors/wavelengths. At the design wavelength of **510 nm**, the first QWOT of index 1.7 moves the locus from the substrate to the left and on the negative real axis. The first QWOT of index 2.2 moves the locus from this point again to the left and on the negative real axis.

Wavelength Effects (cont.)

The second QWOT of index 2.2 moves clockwise back to the beginning of the 2.2 layer. The half-wave layer is called an **absentee layer** because it does not change the reflectance, and therefore acts as though it were absent. The last layer of index 1.38 moves the locus from this point to the origin of the coordinates, $r = 0$.



At a wavelength of **650 nm**, the layers are not as thick as a QWOT, and therefore are not full semicircles. All of the layers are too short, but the two QWOTs of the 2.2 index layer compensate for this by opening in such a way as to push the end point of the last layer toward the origin and closer to $r = 0$.

At a wavelength of **450 nm** the layers are thicker than a QWOT, and therefore are more than full semicircles on such a diagram. All of the layers are too long, but the two QWOTs of the 2.2 index layer compensate for this by closing in such a way as to pull the end point of the last layer toward the origin.