

6th Grade Texas Mathematics: Unpacked Content

What is the purpose of this document?

To increase student achievement by ensuring educators understand specifically what the new standards mean a student must know, and be able to do. This document may also be used to facilitate discussion among teachers and curriculum staff and to encourage coherence in the sequence, pacing, and units of study for grade-level curricula. This document, along with on-going professional development, is one of many resources used to understand and teach the new math standards.

What is in the document?

Descriptions of what each standard means a student will know, understand, and be able to do. The “unpacking” of the standards done in this document is an effort to answer a simple question “What does this standard mean that a student must know and be able to do?” and to ensure the description is helpful, specific and comprehensive for educators.

At A Glance:

New to 6th Grade:

- Classifying whole numbers, integers, and rational numbers using visual representations such as a Venn diagram (Grade 7)
- Describe relationships between sets of numbers
- Absolute value
- Added use of number line
- Locate, compare and order integers and rational numbers with and without use of number line to include real world examples (Grade 8)
- Extend representations for division to include fraction notation such as a/b is the same as $a \div b$ where $b \neq 0$
- Introduction of operations with integers (Grade 7)
- Mastery of operations with integers; fluently (Grade 8)
- Multiplication and division of fractions and decimals (Grade 7)
- Changed Patterns, relationships and algebraic reasoning, expressions, equations and relationships to proportionality
- Compare two rules verbally, numerically, graphically, and symbolically in the form of $y = ax$ or $y = x + a$ in order to differentiate between additive and multiplicative relationships.
- Inequalities are added
- Apply qualitative and quantitative reasoning to solve prediction and comparison of real-world problems involving ratios and rates.
- Represent benchmark fractions and percents such as 1%, 10%, 25%, 33 $\frac{1}{3}$ % and multiples of these values using 10 by 10 grids, strip diagrams, number lines and numbers.
- Identify independent and dependent quantities from tables and graphs (Algebra I)
- Graphing relationships (Grade 8)
- Order of operations with exponents; generate equivalent numerical expressions
- Distinguish between expressions and equations verbally, numerically, and algebraically(Grade 7)
- Determine equivalency of two expressions, using concrete models, pictorial models, and algebraic representations.
- Commutative, associative, distributive, inverse and identity properties (Algebra I)
- Extended triangle relationships to include side lengths-specially their relationship to angle measures and whether a set of side lengths can form a triangle.
- Model area formulas for parallelograms, trapezoids, and triangles by decomposing and rearranging parts of these shapes.
- Write one variable, one step equations and inequalities to represent constraints or conditions within problems (Grade 7 & Algebra I)
- Represent solutions for one variable, one step equations and inequalities on number lines.
- Writing word problems from equations (Grade 7)
- Added inequalities

Moved from 6th Grade:

- Identification of factors and multiples (Grade 5)
- Moved the addition and subtraction of fractions and decimals (Grade 5)
- Moved the use of estimation to solve problems (Grade 5)
- Deleted the relationships in quadrilaterals
- Use of angle measurements to classify angles as acute, obtuse, or right (Grade 4)
- Measure angles (Grade 4)
- Describe the relationship between radius, diameter, and circumference of a circle (Grade 7)
- Probability (moved to 7th grade)

Instructional Implications for 2013-14:

Gaps for students:

- *7.1A Compare and order integers and positive rational numbers, compare and order integers and positive rational numbers
- *8.1A Compare and order rational numbers in various forms including integers, percents, and positive and negative fractions and decimals
- *7.2A Represent multiplication and division situations involving fractions and decimals with models, pictures, words, and numbers
- *7.2 B FLUENTLY use addition, subtraction, multiplication, and division to solve problems involving fractions and decimals
- *7.2 C Use models, such as pictorial models, and number lines, to add, subtract, multiply, and divide integers and connect the actions to algorithms
- *7.2 E Simplify numerical expressions involving order of operations and exponents
- *7.3A Estimate and find solutions to application problems involving percent
- *7.3 B Estimate and find solutions to application problems involving proportional relationships such as similarity scaling, unit costs, and related measurement units
- *7.5A Use concrete and pictorial models to solve equations and use symbols to record the actions
- *7.5 B Formulate problem situations when given a simple equation and formulate an equation when given a problem situation
- *7.11A Select and use an appropriate representation for presenting and displaying different graphical representations of the same data including line plot, line graph, bar graph, stem and leaf plot, circle graph, and Venn diagram, and justify the selection
- *8.12 C Same as 7.11 A box and whisker plots, histograms, and Venn diagrams with and without the use of technology
- *7.11 B Use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness

*8.1 B Select and use appropriate forms of rational numbers to solve real life problems including those involving proportional relationships

*8.2 B Use appropriate operations to solve problems involving rational numbers in problem situations

*8.4 A Generate a different representation of data given another representation of data (such as a table, graph, equation, or verbal description)

*8.7 D Locate and name points on a coordinate plane using ordered pairs of rational numbers

*8.12 A Use variability (range, including interquartile range (IQR) and select the appropriate measure of central tendency to describe a set of data and justify the choice for a particular situation.

*A 1.A Describe independent and dependent quantities in functional relationships

*A 4.B Use the commutative, associative, and distributive properties to simplify algebraic expression

*All the Personal Finance literacy components

Professional Learning Implications for 2013-14:

- Resources for teaching the newly introduced concepts. (textbook)
- Teachers will need to identify the gaps that will need to be addressed in the 2013-14 school year.
- Embed the process standards into instruction and application
- Identify academic vocabulary
- PD and resources regarding Personal Financial Literacy
- Initial learning of the teachers' grade level TEKS (teachers unpacking the TEKS at their grade level)
- Vertical study of the strands to know how the TEKS align and progress from 5th through 7th grade

Grade 6th Primary Focal Areas:

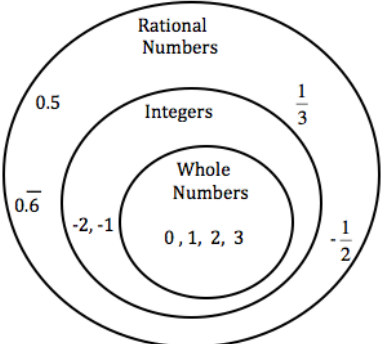
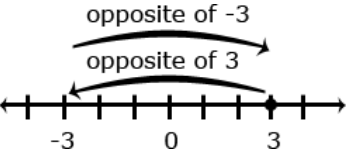
The Primary Focal Areas are designed to bring focus to the standards at each grade by describing the big ideas that educators can use to build their curriculum and to guide instruction.

The primary focal areas in Grade 6 are number and operations; proportionality; expressions, equations, and relationships; and measurement and data. Students use concepts, algorithms, and properties of rational numbers to explore mathematical relationships and to describe increasingly complex situations. Students use concepts of proportionality to explore, develop, and communicate mathematical relationships. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other. Students connect verbal, numeric, graphic, and symbolic representations of relationships, including equations and inequalities. Students use geometric properties and relationships, as well as spatial reasoning, to model and analyze situations and solve problems. Students communicate information about geometric figures or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, and reasoning to draw conclusions, evaluate arguments, and make recommendations. While the use of all types of technology is important, the emphasis on algebra readiness skills necessitates the implementation of graphing technology.

Mathematical process standards:

The student is expected to:

- (A) Apply mathematics to problems arising in everyday life, society, and the workplace
- (B) Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution
- (C) Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems
- (D) Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate
- (E) Create and use representations to organize, record, and communicate mathematical ideas
- (F) Analyze mathematical relationships to connect and communicate mathematical ideas
- (G) Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication

<p>Number and Operations: TEKS: 6.2</p>	<p>The student applies mathematical process standards to represent and compare whole numbers and understand relationships related to place value.</p>
<p>6.2(A) Classify whole numbers, integers, and rational numbers using a visual representation such as Venn diagram to describe relationships between sets of numbers</p>	
<p>6.2(B) Identify a number, its opposite, and its absolute value</p>	<p>In 6th grade, students extend the number line to represent all rational numbers and recognize that number lines may be either horizontal or vertical (i.e. thermometer) which facilitates the movement from number lines to coordinate grids. Students recognize that a number and its opposite are equidistance from zero (reflections about the zero). The opposite sign (–) shifts the number to the opposite side of 0. For example, – 4 could be read as “the opposite of 4” which would be negative 4. In the example, – (–6.4) would be read as “the opposite of the opposite of 6.4” which would be 6.4. On a number line model, the number is represented by an arrow drawn from zero to the location of the number on the number line; the absolute value is the length of this arrow.</p>  <p>Students recognize the distance from zero as the absolute value or magnitude of a rational number. Students need multiple experiences to understand the relationships between numbers, absolute value, and statements about order.</p> <p>Students understand absolute value as the distance from zero and recognize the symbols </p>

as representing absolute value.

Example 1: Which numbers have an absolute value of 7

Solution: 7 and -7 since both numbers have a distance of 7 units from 0 on the number line.

Example 2:

- Use an integer to represent 25 feet below sea level
- Use an integer to represent 25 feet above sea level.
- What would 0 (zero) represent in the scenario above?

Example 3:

What is the $|-3\frac{1}{2}|$

Solution: $3\frac{1}{2}$

In real-world contexts, the absolute value can be used to describe size or magnitude. For example, for an ocean depth of 900 feet, write $|-900| = 900$ to describe the distance below sea level.

When working with positive numbers, the absolute value (distance from zero) of the number and the value of the number is the same; therefore, ordering is not problematic. However, negative numbers have a distinction that students need to understand. As the negative number increases (moves to the left on a number line), the value of the number decreases. For example, -24 is less than -14 because -24 is located to the left of -14 on the number line. However, absolute value is the distance from zero. In terms of absolute value (or distance) the absolute value of -24 is greater than the absolute value of -14 . For negative numbers, as the absolute value increases, the value of the negative number decreases.

6.2(C) Locate, compare, and order integers and rational numbers using a number line

In working with number line models, students internalize the order of the numbers; larger numbers on the right (horizontal) of the number line and smaller numbers to the left (horizontal) of the number line. They use the order to correctly locate integers and other rational numbers on the number line. By placing two numbers on the same number line, they are able to write inequalities and make statements about the relationships between two numbers.

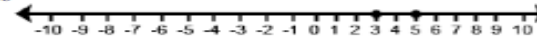
Students will use common models to represent and compare integers including number line models, temperature models and the profit- loss model.

The number line can also be viewed as a thermometer where each point of on the number line is a specific temperature. In the profit-loss model, a positive number corresponds to profit and the negative number corresponds to a loss.

Each of these models is useful for examining values but can also be used in performing operations with integers.

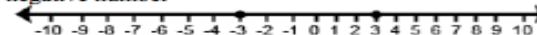
In working with number line models, students internalize the order of numbers; larger numbers on the right (horizontal) or top (vertical) of the number line and smaller numbers to the left (horizontal) or bottom (vertical) of the number line. They use the order to correctly locate integers and other rational numbers on the number line. By placing two numbers on the same number line, they are able to write inequalities and make statements about the relationships between two numbers.

Case 1: Two positive numbers



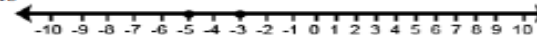
$5 > 3$
5 is greater than 3
3 is less than 5

Case 2: One positive and one negative number



$3 > -3$
positive 3 is greater than negative 3
negative 3 is less than positive 3

Case 3: Two negative numbers



$-3 > -5$
negative 3 is greater than negative 5
negative 5 is less than negative 3

Case 1: Two positive numbers

$$5 > 3$$

5 is greater than 3

3 is less than 5

Case 2: One positive and one negative number

$$3 > -3$$

positive 3 is greater than negative 3

negative 3 is less than positive 3

Case 3: Two negative numbers

$$-3 > -5$$

negative 3 is greater than negative 5

negative 5 is less than negative 3

Example 1: Write a statement to compare $-4\frac{1}{2}$ and -2 . Explain your answer.

Solution:

$-4\frac{1}{2} < -2$ because $-4\frac{1}{2}$ is located to the left of -2 on the number line

Example 2:

Students place where the following numbers would be on a number line:

$-4.5, 2, 3.2, -3\frac{3}{5}, 0.2, -2, \frac{11}{2}$

Based on number line placement, numbers can be placed in order.

Solution:

The numbers in order from least to greatest are:

$-4.5, -3\frac{3}{5}, -2, 0.2, 2, 3.2, \frac{11}{2}$

6.2(D) Order a set of rational numbers arising from mathematical and real-world contexts

Students write statements using $<$ or $>$ to compare rational number in context. However, explanations should reference the context rather than “less than” or “greater than”.

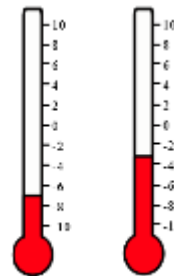
Example 1:

The balance in Sue’s checkbook was $-\$12.55$. The balance in John’s checkbook was $-\$10.45$. Write an inequality to show the relationship between these amounts. Who owes more?

Solution: $-12.55 < -10.45$, Sue owes more than John. The interpretation could also be “John owes less than Sue”.

Example 2:

One of the thermometers shows -3°C and the other shows -7°C . Which thermometer shows which temperature? Which is the colder temperature? How much colder? Write an inequality to show the relationship between the temperatures and explain how the model shows this relationship.



Solution:

The thermometer on the left is -7 ; right is -3 The left thermometer is colder by 4 degrees
Either $-7 < -3$ or $-3 > -7$

Although 6.NS.7a is limited to two numbers, this part of the standard expands the ordering of rational numbers to more than two numbers in context.

Example 3:

A meteorologist recorded temperatures in four cities around the world.
List these cities in order from coldest temperature to warmest temperature:

Albany: 5°

Anchorage: -6°

Buffalo: -7°

Junea: -9°

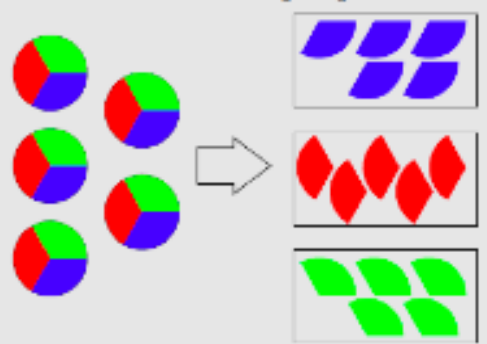
Reno: 12°

Solution: Junea, Buffalo, Anchorage, Albany, Reno

6.2(E) Extend representations for division to include fraction notation such as a/b represents the same number as $a \div b$ where $b \neq 0$

Students should connect fractions with division, understanding that $5 \div 3 = \frac{5}{3}$. Students should explain this by working with their understanding of division as equal sharing.

How to share 5 objects equally among 3 shares:
 $5 \div 3 = 5 \times \frac{1}{3} = \frac{5}{3}$



If you divide 5 objects equally among 3 shares, each of the 5 objects should contribute $\frac{1}{3}$ of itself to each share. Thus each share consists of 5 pieces, each of which is $\frac{1}{3}$ of an object, and so each share is $5 \times \frac{1}{3} = \frac{5}{3}$ of an object.

Students should also create story contexts to represent problems involving division of whole numbers.

Example 1:

If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? This can be solved in two ways.

First, they might partition each pound among the 9 people, so that each person gets $50 \times \frac{1}{9}$
 $= \frac{50}{9}$ pounds.

Second, they might use the equation $9 \times 5 = 45$ to see that each person can be given 5 pounds, with 5 pounds remaining. Partitioning the remainder gives $5 \frac{5}{9}$ pounds for each person.

Students need ample experiences to explore the concept that a fraction is a way to represent the division of two quantities.

Students are expected to demonstrate their understanding using concrete materials, drawing models, and explaining their thinking when working with fractions in multiple contexts.

They read $\frac{3}{5}$ as “three fifths” and after many experiences with sharing problems, learn that $\frac{3}{5}$ can also be interpreted as “3 divided by 5.”

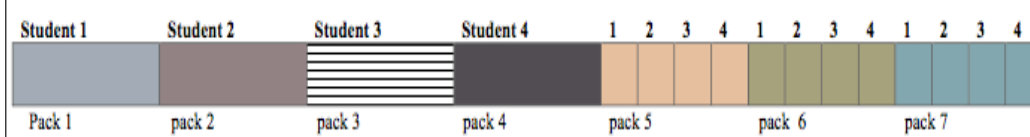
Example 2:

Ten team members are sharing 3 boxes of cookies. How much of a box will each student get? When working this problem a student should recognize that the 3 boxes are being divided into 10 groups, so s/he is seeing the solution to the following equation, $10 \times n = 3$ (10 groups of some amount is 3 boxes) which can also be written as $n = 3 \div 10$. Using models or diagram, they divide each box into 10 groups, resulting in each team member getting $\frac{3}{10}$ of a box.

Two after school clubs are having pizza parties. For the Math Club, the teacher will order 3 pizzas for every 5 students. For the student council, the teacher will order 5 pizzas for every 8 students. Since you are in both groups, you need to decide which party to attend. How much pizza would you get at each party? If you want to have the most pizza, which party should you attend?

The six fifth grade classrooms have a total of 27 boxes of pencils. How many boxes will each classroom receive? Students may recognize this as a whole number division problem but should also express this equal sharing problem as $\frac{27}{6}$. They explain that each classroom gets $\frac{27}{6}$ boxes of pencils and can further determine that each classroom get $4\frac{3}{6}$ or $4\frac{1}{2}$ boxes of pencils.

Example 3: Your teacher gives 7 packs of paper to your group of 4 students. If you share the paper equally, how much paper does each student get?



Each student receives 1 whole pack of paper and $\frac{1}{4}$ of the each of the 3 packs of paper. So each student gets $1\frac{1}{4}$ packs of paper.

**Number and Operations:
TEKS: 6.3**

The student applies mathematical process standards to represent addition, subtraction, multiplication, and division while solving problems and justifying solutions.

6.3(A) Recognize that dividing by a rational number and multiplying by its reciprocal result in equivalent values

This concept must be introduced and explored through models and visuals. (example: dividing by 2 and multiplying by $\frac{1}{2}$, dividing by 4 and multiplying by $\frac{1}{4}$, dividing by 3 and multiplying by $\frac{1}{3}$.)

6.3(B) Determine, with and without computation, whether a quantity is increased or decreased when multiplied by a fraction, including values greater than or less than one

This standard asks students to examine how numbers change when we multiply by fractions. Students should have ample opportunities to examine both cases in the standard: a) when multiplying by a fraction greater than 1, the number increases and b) when multiplying by a fraction less than one, the number decreases.

Example 1:

Mrs. Bennett is planting two flower beds. The first flower bed is 5 meters long and $\frac{6}{5}$ meters wide. The second flower bed is 5 meters long and $\frac{5}{6}$ meters wide. How do the areas of these two flower beds compare? Is the value of the area larger or smaller than 5 square meters? Draw pictures to prove your answer.

Example 2:

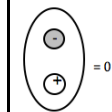
$2\frac{3}{8} \times 8$ must be more than 8 because 2 groups of 8 is 16 and $2\frac{2}{3}$ is almost 3 groups of 8. So the answer must be close to, but less than 24.

$\frac{3}{4} = \frac{5}{5} \times \frac{3}{4}$ because multiplying $\frac{3}{4}$ by $\frac{5}{5}$ is the same as multiplying by 1.

6.3(C) Represent integer operations with concrete models and connect the actions to with the models to standardized algorithms

Students add, subtract, multiply and divide integers. Visual representations such as color counters and number lines are necessary for students to gain understanding of operations with integers. When using color counters, student should be able to represent “zero pair” using color counters. The students will need to be able to match an integer equation to the model as well as match the model the integer equation.

6.3(D) Add, subtract, multiply, and divide integers fluently



Example 1:

Use a number line to add $-5 + 7$.

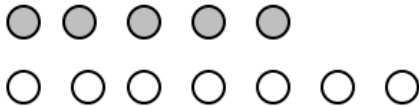
Solution 1: Number Line

Students find -5 on the number line and move 7 in a positive direction (to the right). The

stopping point of 2 is the sum of this expression.

In the example, $7 - 5$, the difference is the distance between 7 and 5, or 2, in the direction of 5 to 7 (positive). Therefore the answer would be 2.

Solution 2: Color Counters



Students should use color counters and combine the zero pairs. Once the zero pairs are combined, notice that there are 2 positive counters remaining. The answer is 2.

The equation is $-5 + 7 = 2$.

Example 2:

Use a number line to subtract: $-6 - (-4)$

Solution:

This problem is asking for the distance between -6 and -4. The distance between -6 and -4 is 2 and the direction from -4 to -6 is left or negative. The answer would be -2. Note that this answer is the same as adding the opposite of -4: $-6 + 4 = -2$

Example 3:

Use a number line to illustrate:

$$p - q \text{ ie. } 7 - 4$$

$$p + (-q) \text{ ie. } 7 + (-4)$$

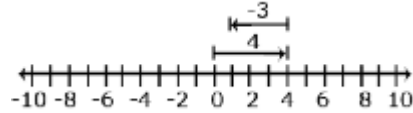
Is this equation true $p - q = p + (-q)$?

Students explore the above relationship when p is negative and q is positive and when both p and q are negative. Is this relationship always true?

Example 4:

Morgan has \$4 and she needs to pay a friend \$3. How much will Morgan have after paying her friend?

Solution: $4 + (-3) = 1$ or $(-3) + 4 = 1$



Using a number line, students should begin at 0 and follow the arrows to get the solution.

Students understand that multiplication and division of integers is an extension of multiplication and division of whole numbers. Students recognize that when division of rational numbers is represented with a fraction bar, each number can have a negative sign.

Example 5:

Which of the following expressions is equivalent to $-4 \div 5$? Explain your reasoning.

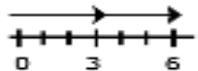
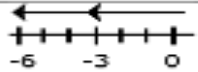
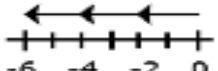
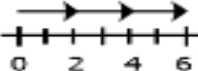
- a. $4 \div -5$ b. $-16 \div 20$ c. $-4 \div -5$

Solution:

A and B are correct. When dividing a positive and a negative, the answer will be negative.

Example 6:

Examine the family of equations in the table below. What patterns are evident? Create a model and context for each of the products. Write and model the family of equations related to 3×4

Equation	Number Line Model	Context
$2 \cdot 3 = 6$		Selling two packages of apples at \$3.00 per pack
$2 \cdot -3 = -6$		Spending 3 dollars each on 2 packages of apples
$-2 \cdot 3 = -6$		Owing 2 dollars to each of your three friends
$-2 \cdot -3 = 6$		Forgiving 3 debts of \$2.00 each

Using long division from elementary school, students understand the difference between terminating and repeating decimals.

Example 7:

Equation

Number Line Model Context

$2 \cdot 3 = 6$

Selling two packages of apples at \$3.00 per pack

$2 \cdot -3 = -6$

Spending 3 dollars each on 2 packages of apples

	<p>$-2 \cdot 3 = -6$</p> <p>Owing 2 dollars to each of your three friends</p> <p>$-2 \cdot -3 = 6$</p> <p>Forgiving 3 debts of \$2.00 each</p> <p>Using long division, express the following fractions as decimals. Which of the following fractions will result in terminating decimals; which will result in repeating decimals?</p> <p>Identify which fractions will terminate.</p>
6.3(E) Multiply and divide positive rational numbers fluently	<p>Provide ample opportunities in context to determine a need to multiply or divide to help student's computation/procedural fluency.</p> <p>Estimation is important to determine reasonableness of solutions.</p> <p>Problems in context will foster understanding of when to multiply or divide.</p> <p>Example 1:</p> <p>How many $1 \frac{1}{2}$ foot boards can be cut from a board that is $7 \frac{1}{2}$ feet long?</p> <p>What is the total length of 7 boards that are $1 \frac{1}{2}$ feet long?</p> <p>If the bill at a restaurant is \$9.50, how much will each person pay if 5 people share the bill?</p>

Proportionality: TEKS: 6.4	The student applies mathematical process standards to develop an understanding of proportional relationships in problem situations.
6.4(A) Compare two rules verbally, numerically, graphically, and symbolically in the form of $y = ax$ or $y = x + a$ in order to differentiate between additive and multiplicative relationships	<p>Students are expected to compare two rules through symbolic form, graphs or numerical.</p> <ul style="list-style-type: none"> • When creating tables, the convention is to use the first column (row) for the independent variable, and the second column (row) for the dependent variable. • When graphing, the convention is to use the x-coordinate to represent the independent variable and the y-coordinate to represent the dependent variable. • It is helpful to refer to coordinates as <i>ordered pairs</i> to remind students that order matters. Remind students that the first coordinate represents the distance from 0 on the horizontal axis, while the second coordinate represents the distance from 0 on the vertical axis. • It is likely that in previous graphing experiences, students have used the same scale for both axes. Asking students to graph an equation such as $y = 365x$, where x represents number of years and y represents total number of days will help students understand that it is not always practical to use the same scale for both axes. However, it is essential that students understand the necessity of using the same interval for each unit on an axis. • When writing rules, the convention is to use x to represent the independent variable and y to represent the dependent variable. • Students that interpret $y = 2x$ to mean that x is twice as large as y will benefit from a discussion about equality. Remind students that since $y = 2x$, then y and $2x$ have the same value. If $y = 4$, then $2x$ must be 4. Therefore, $x = 2$. • Remind students that order matters when writing function rules that involve subtraction and division, because those operations are not commutative. • Function sense comes from looking for visual and number patterns and predicting outcomes from applying a rule. It helps students relate pictorial, symbolic, verbal, and concrete representations of a pattern and so develop multiple perspectives (Which is better - doubling \$2 every year, or adding \$50 every year?)

x	y
1	2
2	4
3	6
4	8

y results from *x*
being **multiplied**
by 2, or $y = 2x$

x	y
1	3
2	4
3	5
4	6

y results from 2
being **added** to *x*,
or $y = x + 2$

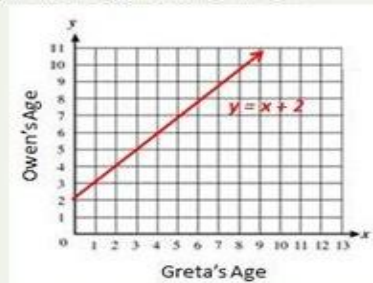
It is also important that students have multiple opportunities using a variety of representations to explore additive and multiplicative relationships. The chart below shows an example.

Situation:

Owen was 2 years old when his sister Greta was born.

Table representation:

Greta's age (x)	0	1	2	3	4
Owen's age (y)	2	3	4	5	6

Graphical representation:**Algebraic representation:**

$$y = x + 2$$

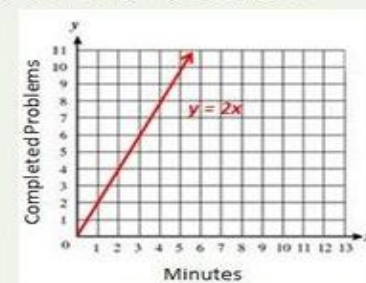
additive relationship

Situation:

On average, it takes 2 minutes to complete each math problem.

Table representation:

Minutes (x)	1	2	3	4
Completed problems(y)	2	4	6	8

Graphical representation:**Algebraic representation:**

$$y = 2x$$

multiplicative relationship

the relationship y, x , or

$$\frac{y}{x} \text{ is } \frac{2}{1} = \frac{4}{2} = \frac{6}{3} \text{. } v \text{ is always}$$

- Students misinterpret whether a relationship is additive or multiplicative given a table or graph.
- Students interpret $y = 2x$ to mean that x is twice as large as y .
- Students misrepresent relationships involving subtraction or division (e.g., $x - 3$ or $3 - x$; $3 \div x$ or $x \div 3$).

rules, you are expressing the *relationship* between two variables.

6.4(B) Apply qualitative and quantitative reasoning to solve prediction and comparison of real-world problems involving ratios and rates

Proportional reasoning involves both a qualitative and quantitative process and is critical to mathematical thinking.

In simple proportion-related problems, qualitative analysis may involve inferring in what direction the value or intensive quantity of a rate will change (decrease, stay the same, or increase) when the numerator and/or the denominator of the rate increases, stays the same, or decreases. The rate could be speed, package size, unit price, the concentration of a mixture, or any of the other types of rates commonly encountered in proportion-related problems. Qualitative problems answer “Which is better?”

Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. Quantitative problems answer “Which is more/less?”

6.4(C) Give examples of ratios as multiplicative comparisons of two quantities describing the same attribute

Ratios and rates can be used in ratio tables and graphs to solve problems. Previously, students have used additive reasoning in tables to solve problems. To begin the shift to proportional reasoning, students need to begin using multiplicative reasoning. To aid in the development of proportional reasoning the cross-product algorithm is NOT expected at this level. When working with ratio tables and graphs, whole number measurements are the expectation for this standard.

Example 1:

At Books Unlimited, 3 paperback books cost \$18. What would 7 books cost? How many books could be purchased with \$54.

Solution: To find the price of 1 book, divide \$18 by 3. One book costs \$6. To find the price of 7 books, multiply \$6 (the cost of one book times 7 to get \$42. To find the number of books that can be purchased with \$54, multiply \$6 times 9 to get \$54 and then multiply 1 book times 9 to get 9 books. Students use ratios, unit rates and multiplicative reasoning to solve problems in various contexts, including measurement, prices, and geometry. Notice in the table below, a multiplicative relationship exists between the numbers both horizontally (times 6) and vertically (ie. $1 \cdot 7 = 7$; $6 \cdot 7 = 42$). Red numbers indicate solutions.

Number of Books (n)	Cost (C)
1	6
3	18
7	42
9	54

Students use tables to compare ratios. Another bookstore offers paperback books at the prices below. Which bookstore has the best buy? Explain your answer.

Number of Books (n)	Cost (C)
4	20
8	40

To help understand the multiplicative relationship between the number of books and cost, students write equations to express the cost of any number of books. Writing equations is foundational for work in 7th grade. For example, the equation for the first table would be $C = 6n$, while the equation for the second bookstore is $C = 5n$. The numbers in the table can be expressed as ordered pairs (number of books, cost) and plotted on a coordinate plane.

Example 2:

Ratios can also be used in problem solving by thinking about the total amount for each ratio unit.

The ratio of cups of orange juice concentrate to cups of water in punch is 1: 3. If James made 32 cups of punch, how many cups of orange did he need?

Solution: Students recognize that the total ratio would produce 4 cups of punch. To get 32 cups, the ratio would need to be duplicated 8 times, resulting in 8 cups of orange juice concentrate.

Example 3:

Using the information in the table, find the number of yards in 24 feet.

Feet	3	6	9	15	24
Yards	1	2	3	5	?

Solution:

There are several strategies that students could use to determine the solution to this problem:

- Add quantities from the table to total 24 feet (9 feet and 15 feet); therefore the number of yards in 24 feet must be 8 yards (3 yards and 5 yards).
- Use multiplication to find 24 feet: 1) 3 feet x 8 = 24 feet; therefore 1 yard x 8 = 8 yards, or 2) 6 feet x 4 = 24 feet; therefore 2 yards x 4 = 8 yards.

Example 4:

Compare the number of black circles to white circles. If the ratio remains the same, how many black circles will there be if there are 60 white circles?



Black	4	40	20	60	?
White	3	30	15	45	60

Solution:

There are several strategies that students could use to determine the solution to this problem

- Add quantities from the table to total 60 white circles (15 + 45). Use the corresponding numbers to determine the number of black circles (20 + 60) to get 80 black circles.
- Use multiplication to find 60 white circles (one possibility 30 x 2). Use the corresponding numbers and operations to determine the number of black circles (40 x 2) to get 80 black circles.

6.4(D) Give examples of rates as the comparison by division of two quantities having different attributes, including rates as quotients

Students will write the ratio as a fraction and create a unit rate through division. Ratio tables can also be used.

Example 1:

120 miles traveled for 6 hour. How many miles are traveled per hour?

Solution:

120 miles ÷ 6 hours is 20 miles per hour.

Example 2:

I bought 5 sodas for \$6.50. What is the cost per soda?

Solution:

$\$6.50 \div 5 =$ Each soda will cost \$1.30.

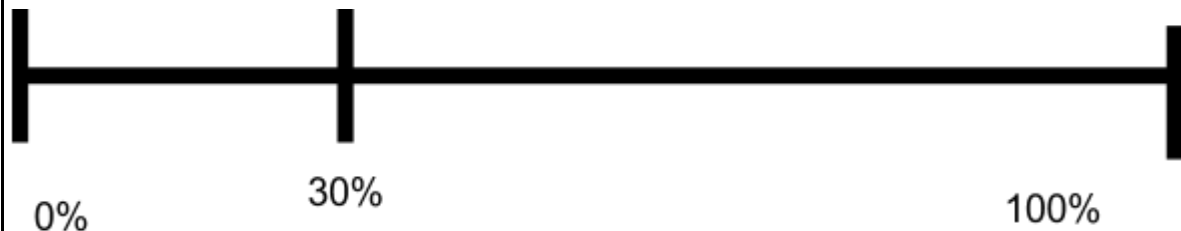
Misconceptions: Students will set up the rate incorrectly. Stress the first word stated is the numerator. Second word stated is the denominator.

6.4(E) Represent ratios and percents with concrete models, fractions, and decimals

The types of models that can be use for these types of problems are percent bars, hundred grids and other models.

Example 1:

If 30% of the students in Mrs. Rutherford's class like chocolate ice cream, then how many students are in Mrs. Rutherford's class if 6 like chocolate ice cream?



6 (grouping symbol around 3 Squares)

(Solution: 20)

Example 2:

A credit card company charges 17% interest fee on any charges not paid at the end of the month. Make a ratio table to show how much the interest would be for several amounts. If the bill totals \$450 for this month, how much interest would you have to be paid on the balance?

Solution:

Charges	\$1	\$50	\$100	\$200	\$450
Interest	\$0.17	\$8.50	\$17	\$34	?

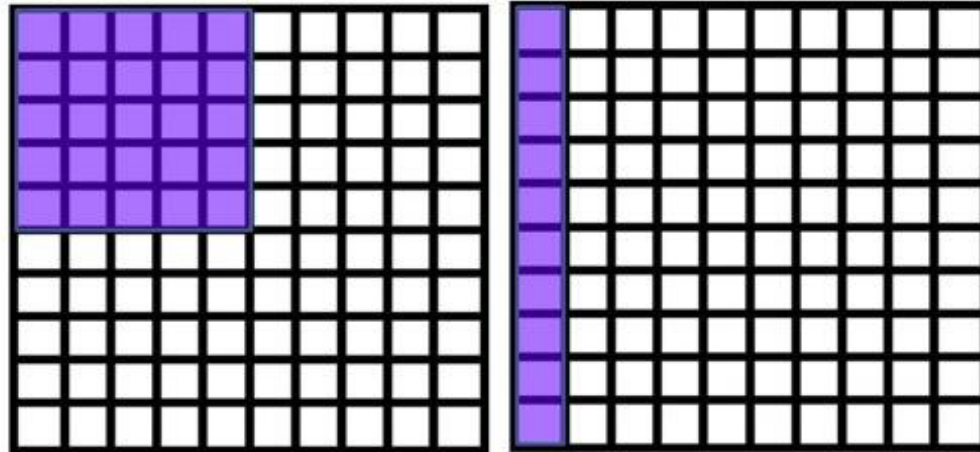
One possible solution is to multiply 1 by 450 to get 450 and then multiply 0.17 by 450 to get \$76.50.

6.4(F) Represent benchmark fractions and percents such as 1%, 10%, 25%, $33\frac{1}{3}\%$, and multiples of these values using 10 by 10 grids, strip diagrams, number lines, and numbers

By repeated exposure in multiple situations and representations, students should be developing a quick recall of common benchmark fractions and percents.

- 15% is 15 per one hundred or 0.15;
- 150% is 150 per one hundred or 1.5;
- 0.15% is 15 hundredths per one hundred or 0.0015;
- 0.015% is 15 thousandths per one hundred or 0.00015.

- Students have typically been taught about fractions, decimals, and percents in isolation. A conscious effort needs to be spent identifying numbers in a multitude of ways simultaneously to encourage flexible thinking. Students will benefit from using 100-grids as visual representations to connect fractions, decimals, and percent equivalencies.



$$\frac{1}{4} = 0.25 = 25\%$$

$$\frac{1}{10} = 0.10 = 10\%$$

Misconceptions:

- Students might not use benchmark numbers like 0, $\frac{1}{2}$, and 1 to compare fractions because their understanding reflects only part-whole situations and they do not think of the fractions as numbers.
- When looking at a 100-grid containing $28\frac{1}{2}$ colored squares, students may not understand how to write $\frac{28.5}{100}$ as 0.285 since $\frac{1}{2}$ of $\frac{1}{100}$ is 0.005.

6.4(G) Generate equivalent forms of fractions, decimals, and percents using real-world problems, including problems that involve money

Students need to be able to problem solve with fractions, decimals and percents. Use money with real-world problems.

Example1: Bill had a coupon for 25% off at Best Buy. Write 25% as a fraction and a decimal.

$$\text{Solution: } 25\% = 0.25 = \frac{25}{100} = \frac{1}{4}$$

Example 2: Larry, Moe, and Curley went out to lunch. If Larry paid 30% of the bill, including tax and tip, and Moe paid 0.2 of the bill, what fraction of the bill did Curley pay?

Solution: There are several ways to solve this problem)

Some solutions are:

Convert all the numbers to either all percents, decimals or fractions (since the question is asking for a fraction) and subtract from 1 whole, then convert to a fraction (as needed)

$$\begin{array}{l} c \\ 0.3 + 0.2 = 0.5 \quad \text{or} \quad 30\% + 20\% = 50\% \\ 1 - 0.5 = 0.5 \quad \quad \quad 100\% - 50\% = 50\% \\ 0.5 = \frac{5}{10} = \frac{1}{2} \quad \quad \quad 50\% = \frac{50}{100} = \frac{1}{2} \end{array}$$

6.4(H) Convert units within a measurement system, including the use of proportions and unit rates

A ratio can be used to compare measures of two different types, such as inches per foot, milliliters per liter and centimeters per inch. Students recognize that a conversion factor is a fraction equal to 1 since the numerator and denominator describe the same quantity. For example, $\frac{12 \text{ inches}}{1 \text{ foot}}$ is a conversion factor since the numerator and

denominator equal the same amount. Since the ratio is equivalent to 1, the identity property of multiplication allows an amount to be multiplied by the ratio. Also, the value of the ratio can also be expressed as $\frac{1 \text{ foot}}{12 \text{ inches}}$ allowing for the conversion ratios to be expressed in a format so that units will “cancel”.

Students use ratios as conversion factors and the identity property for multiplication to convert ratio units.

Example 1:

How many centimeters are in 7 feet, given that 1 inch \approx 2.54 cm.

Solution:

$$7 \text{ feet} \times \frac{12 \text{ inches}}{1 \text{ foot}} \times \frac{2.54 \text{ cm}}{1 \text{ inch}} = 7 \text{ feet} \times \frac{12 \text{ inches}}{1 \text{ foot}} \times \frac{2.54 \text{ cm}}{1 \text{ inch}} = 7 \times 12 \times 2.54 \text{ cm} = 213.36 \text{ cm}$$

Proportionality:
TEKS: 6.5

The student applies mathematical process standards to solve problems involving proportional relationships.

6.5(A) Represent mathematical and real-world problems involving ratios and rates using scale factors, tables, graphs, and proportions

Students determine if two quantities are in a proportional relationship from a table. Fractions and decimals could be used with this standard.

Note: This standard focuses on the representations of proportions. Ratios tables are helpful for conversions.

Example 1: The table below gives the price for different numbers of books. Do the numbers in the table represent a proportional relationship?

Number of Books	Price
1	3
3	9
4	12
7	18

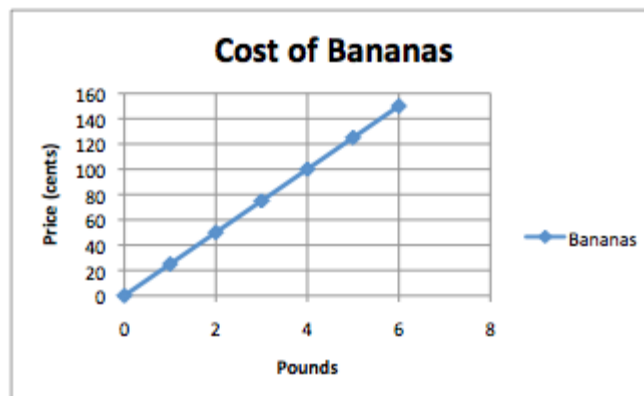
Solution:

Students can examine the numbers to determine that the price is the number of books multiplied by 3, except for 7 books. The row with seven books for \$18 is not proportional to the other amounts in the table; therefore, the table does not represent a proportional relationship.

Students graph relationships to determine if two quantities are in a proportional relationship and to interpret the ordered pairs. If the amounts from the table above are graphed (number of books, price), the pairs (1, 3), (3, 9), and (4, 12) will form a straight line through the origin (0 books, 0 dollars), indicating that these pairs are in a proportional relationship. The ordered pair (4, 12) means that 4 books cost \$12. However, the ordered pair (7, 18) would not be on the line, indicating that it is not proportional to the other pairs.

The ordered pair (1, 3) indicates that 1 book is \$3, which is the unit rate. The y-coordinate when $x = 1$ will be the unit rate. The constant of proportionality is the unit rate. Students identify this amount from tables (see example above), graphs, equations and verbal descriptions of proportional relationships.

Example 2: The graph below represents the price of the bananas at one store. What is the constant of proportionality?



Solution:

From the graph, it can be determined that 4 pounds of bananas is \$1.00; therefore, 1 pound of bananas is \$0.25, which is the constant of proportionality for the graph. Note: Any point on the line will yield this constant of proportionality.

Students write equations from context and identify the coefficient as the unit rate which is also the constant of proportionality.

Example 3:

The price of bananas at another store can be determined by the equation: $P = \$0.35n$, where P is the price and n is the number of pounds of bananas. What is the constant of proportionality (unit rate)?

Solution: The constant of proportionality is the coefficient of x (or the independent variable). The constant of proportionality is 0.35.

Example 4:

A student is making trail mix. Create a graph to determine if the quantities of nuts and fruit are proportional for each serving size listed in the table. If the quantities are proportional, what is the constant of proportionality or unit rate that defines the relationship? Explain how the constant of proportionality was determined and how it relates to both the table and graph.

Serving Size	1	2	3	4
cups of nuts (x)	1	2	3	4
cups of fruit (y)	2	4	6	8

	<p>Solution:</p> <p>The relationship is proportional. For each of the other serving sizes there are 2 cups of fruit for every 1 cup of nuts (2:1).</p> <p>The constant of proportionality is shown in the first column of the table and by the steepness (rate of change) of the line on the graph.</p> <p>Example 5:</p> <p>The graph below represents the cost of gum packs as a unit rate of \$2 dollars for every pack of gum. The unit rate is represented as \$2/pack. Represent the relationship using a table and an equation.</p> <p>Equation: $d = 2g$, where d is the cost in dollars and g is the packs of gum</p> <p>A common error is to reverse the position of the variables when writing equations. Students may find it useful to use variables specifically related to the quantities rather than using x and y. Constructing verbal models can also be helpful. A student might describe the situation as “the number of packs of gum times the cost for each pack is the total cost in dollars”. They can use this verbal model to construct the equation. Students can check their equation by substituting values and comparing their results to the table. The checking process helps student revise and recheck their model as necessary. The number of packs of gum times the cost for each pack is the total cost. ($g \times 2 = d$)</p>
<p>6.5(B) solve real-world problems to find the whole, given a part and the percent, to find the part given the whole and the percent, and to find the percent given the part and the whole including the use of concrete and pictorial models; and</p>	<p>This is the students' first introduction to percents. Percentages are a rate per 100. Models, such as percent bars or 10x10 grids should be used to model percents.</p> <p>Students use ratios to identify percents.</p> <p>Example1:</p> <p>What percent is 12 out of 25?</p> <p>Solution: One possible solution is to set up a ratio table.</p> <p>Multiply 25 by 4 to get 100. Multiplying 12 by 4 will give 48, meaning that 12 out of 25 is equivalent to 48 out of 100 or 48%</p>

Part	Whole
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--	--

12	25
----	----

?	100
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Students use percentages to find the part when given the percent, by recognizing that the whole is being divided into 100 parts and then taking a part of them (the percent).

Example 2:

What is 40% of 30?

Solution: There are several methods to solve this problem. One possible solution using rates is to use a 10x10 grid to represent the whole amount (or 30). If the 30 is divided into 100 parts, the rate for one block is 0.3. Forty percent would be 40 of the blocks or 40×0.3 , which equals 12.

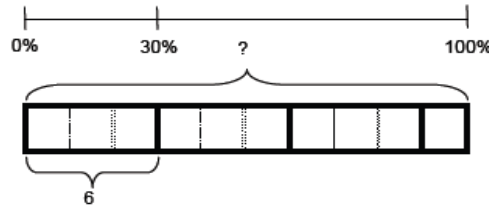
See the weblink below for more information.

<http://illuminations.nctm.org/LessonDetail.aspx?id=L249>

Students also determine the whole amount given a part and the percent.

Example 3:

If 30% of the students in Mrs. Rutherford's class like chocolate ice cream, the how many students are in Mrs. Rutherford's class if 6 like chocolate ice cream?



(Solution: 20)

Example 4:

A credit card company chares 17% interest fee on any charges not paid at the end of the month. Make a ratio table to show how much the interest would be for several amounts. If the bill totals \$450 for this month, how much interest would you have to be paid on the balance?

Solution:

Charges	\$1	\$50	\$100	\$200	\$450
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Interest	$\frac{0.17}{1}$	\$8.50	\$17	\$34	?
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One possible solution is to multiply 1 by 450 to get 450 and then multiply 0.17 by 450 to get \$76.50.

6.5(C) Use equivalent fractions, decimals, and percents to show equal parts of the same whole.

Students need many opportunities to express rational numbers in meaningful contexts.

	<p>Example: A baseball player's batting average is 0.625. What does the batting average mean? Explain the batting average in terms of a fraction, ratio, and percent.</p> <p>Solution:</p> <ul style="list-style-type: none"> • The player hit the ball $\frac{5}{8}$ of the time he was at bat; • The player hit the ball 62.5% of the time; or • The player has a ratio of 5 hits to 8 batting attempts (5:8).
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<p>Expressions, Equations and Relationships: TEKS: 6.6</p>	<p>The student applies mathematical process standards to use multiple representations to describe algebraic relationships.</p>
<p>6.6(A) Identify independent and dependent quantities from tables and graphs</p>	<p>The purpose of this standard is for students to understand the relationship between two variables, which begins with the distinction between dependent and independent variables. The independent variable is the variable that can be changed; the dependent variable is the variable that is affected by the change in the independent variable. Students recognize that the independent variable is graphed on the x-axis; the dependent variable is graphed on the y-axis.</p> <p>Students recognize that not all data should be graphed with a line. Data that is discrete would be graphed with coordinates only. Discrete data is data that would not be represented with fractional parts such as people, tents, records, etc. For example, a graph illustrating the cost per person would be graphed with points since part of a person would not be considered. A line is drawn when both variables could be represented with fraction parts.</p> <p>Students are expected to recognize and explain the impact on the dependent variable when the independent variable changes (As the x variable increases, how does the y- variable change?) Relationships should be proportional with the line passing through the origin. Additionally, students should be able to write an equation from a word problem and understand how the coefficient of the dependent variable is related to the graph and/or a table of values.</p>
<p>6.6(B) Write an equation that represents the relationship between independent and dependent quantities from a table</p>	<p>The purpose of this standard is for students to understand the relationship between two variables, which begins with the distinction between dependent and independent variables. The independent variable is the variable that can be changed; the dependent variable is the</p>

variable that is affected by the change in the independent variable. Students recognize that the independent variable is graphed on the x-axis; the dependent variable is graphed on the y-axis.

Students recognize that not all data should be graphed with a line. Data that is discrete would be graphed with coordinates only. Discrete data is data that would not be represented with fractional parts such as people, tents, records, etc. For example, a graph illustrating the cost per person would be graphed with points since part of a person would not be considered. A line is drawn when both variables could be represented with fractional parts. Students are expected to recognize and explain the impact on the dependent variable when the independent variable changes (As the x variable increases, how does they variable change?) *Relationships should be proportional with the line passing through the origin.* Additionally, students should be able to write an equation from a word problem and understand how the coefficient of the dependent variable is related to the graph and /or a table of values.

Students can use many forms to represent relationships between quantities. Multiple representations include describing the relationship using language, a table, an equation, or a graph. Translating between multiple representations helps students understand that each form represents the same relationship and provides a different perspective.

Example 1:

What is the relationship between the two variables? Write an expression that illustrates the relationship.

X	1	2	3	4
Y	2.5	5	7.5	10

Solution:
 $Y = 2.5x$

6.6(C) Represent a given situation using verbal descriptions, tables, graphs, and equations in the form

$$y = kx \text{ or } y = x + b.$$

Students can use many forms to represent relationships between quantities. Multiple representations include describing the relationship using language, a table, an equation, or a graph. Translating between multiple representations helps students understand that each form represents the same relationship and provides a different perspective.

For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.

<p>Expressions, Equations, and Relationships TEKS: 6.7</p>	<p>The student applies mathematical process standards to develop concepts of expressions and equations.</p>
<p>6.7(A) Generate equivalent numerical expressions using order of operations, including whole number positive exponents and prime factorization</p>	<p>Not only are students evaluating expressions, but they are also having to generate/create expressions showing that they understand the Order of Operation Rules. Have students justify each step used to solve expressions using the order of operations (including exponents) and work backwards. Verify each solution with a calculator.</p> <ul style="list-style-type: none"> • $3 + 6(2 + 4^2) \div 3$ • $2^3 \cdot 8 \div (5 - 1) - 16$ • $(6 - 2) \left(\frac{33}{3}\right) + 8 - 15$ <p>Example:</p> <p>$100 = 2^2 \times 5^2$</p>
<p>6.7(B) Distinguish between expressions and equations verbally, numerically, and algebraically</p>	<p>Students need to know that:</p> <ul style="list-style-type: none"> • Expressions do not have an equal sign. • Equations do have an equal sign. • Expression verbally in words: Jim is 6 years older than Sam. • Equation verbally in words: Jim 6 years older than Sam. The sum of their ages is 12. • Numerical expression $3 + 4$ • Numerical Equation: $3 + 4 = 7$ • Algebraic Expression: $6 + x$ • Algebraic Equation: $6 + X = 10$

6.7(C) Determine if two expressions are equivalent using concrete models, pictorial models, and algebraic representations; and

Students demonstrate an understanding of like terms as quantities being added or subtracted with the same variables and exponents. For example 9 and 3 are like terms that can be combined to equal 12. However $9x$ and 3 are not like terms because 9 is being multiplied by a variable. This concept can be illustrated by substituting in a value for x . For example, $9x-3x=6x$ not 6. Choosing a value for x , such as 2, can prove non-equivalence.

$$9x-3x=6x \text{ where } x = 2$$

$$9(2) - 3(2)=6(2) \quad \text{however} \quad 9(2)-3=?=6$$

$$18 - 6=12 \quad 18-6=6$$

$$12=12 \quad 12 \neq 6$$

or

$$3x = xxx \quad \text{and} \quad 4x = xxxx$$

$$3x + 4x = 7x$$

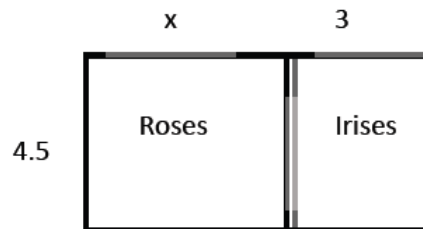
$$xxx+xxxx = xxxxxxx$$

where as $3x + 4$ does not equal 7 or even $7x$, since these terms are not like terms or common terms, they cannot be combined.

Students can also generate equivalent expressions using the associative, commutative, and distributive properties. They can prove that the expressions are equivalent by simplifying each expression into the same form.

Example 1:

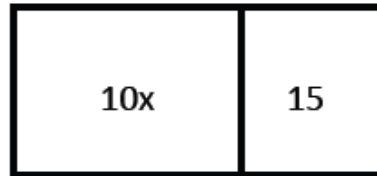
Given that the width is 4.5 units and the length can be represented by $x+2$, the area of the flowers below can be expressed as $4.5(X+3)$ or $4.5x+13.5$.



When given an expression representing area, students need to find the factors.

Example 2:

The expression $10x + 15$ can represent the area of the figure below. Students find the greatest common factor (5) to represent the width and then use the distributive property to find the length ($2x + 3$). The factors (dimensions) of this figure would be $5(2x+3)$.

**Example 3:**

Students use their understanding of multiplication to interpret $3(2+x)$ as 3 groups ($2+x$). They use a model to represent x , and make an array to show the meaning of $3(2+x)$. They can explain why it makes sense that $3(2+x)$ is equal to $6+3x$.

An array with 3 columns and $x+2$ in each column:



Students interpret y as referring to one y . Thus, they can reason that one y plus one y plus one y must be $3y$. They also use the distributive property, the multiplicative identity property of 1, and the commutative property for multiplication to prove that $y + y + y = 3y$

Solution:

$$y+y+y$$

$$y \cdot 1 + y \cdot 1 + y \cdot 1 \quad \text{Multiplicative Identity}$$

$$y \cdot (1+1+1) \quad \text{Distributive Property}$$

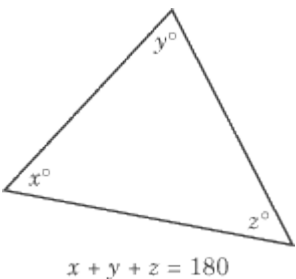
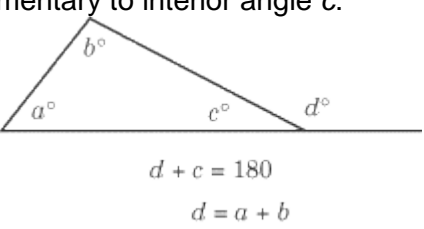
	$y \cdot 3$ $3y$ Commutative Property
6.7(D) Generate equivalent expressions using operations the inverse, identity, commutative, associative, and distributive properties	<p>Models should be used when introducing the concepts of these properties.</p> <p>For example, apply the distributive property to the expression $3(2+x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x+3y)$; apply properties of operations to $y+y+y$ to produce the equivalent expression $3y$.</p> <p>Students use the distributive property to write equivalent expressions. Using their understanding of area models from elementary, students illustrate the distributive property with variables.</p> <p>Properties are introduced throughout elementary grades; however, there has not been an emphasis on recognizing and naming the property. In 6th grade students are able to use the properties and identify by name as used when justifying solution methods (example 1)</p> <p>Example1: (This is an example, students will not be required to show proofs at this grade level)</p> <p>Prove that $y + y + y = 3y$</p> <p><i>Solution:</i></p> $y + y + y$ $y \cdot 1 + y \cdot 1 + y \cdot 1$ $y \cdot (1 + 1 + 1)$ $y \cdot 3$ $3y$ <p><i>Multiplicative Identity</i></p> <p><i>Distributive Property</i></p> <p><i>Commutative Property</i></p> <p>Example 2:</p>

Are the expressions equivalent? Explain your answer.

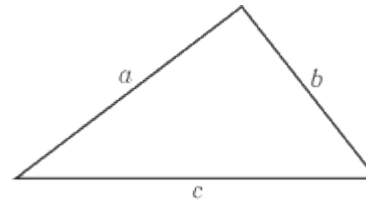
$$4n + 8 \quad 4(n + 2) \quad 3n + 8 + n \quad 2 + 2n + n + 6 + n$$

Solution:

Expression	Simplifying the Expression	Explanation
$4n + 8$	$4n + 8$	Already in simplest form
$4(n + 2)$	$4(n + 2)$ $4n + 8$	Distributive Property
$3n + 8 + n$	$3n + 8 + n$ $3n + n + 8$ $4n + 8$	Combined like terms
$2 + 2n + n + 6 + n$	$2n + n + n + 2 + 6$ $4n + 8$	Combined like terms

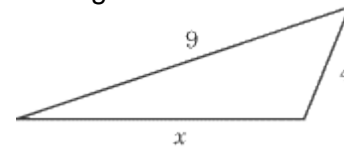
<p>Expressions, equations, and relationships: TEKS: 6.8</p>	<p>The student applies mathematical process standards to use geometry to represent relationships and solve problems.</p>
<p>6.8(A) Extend previous knowledge of triangles and their properties to include the sum of angles of a triangle, the relationship between the lengths of sides and measures of angles in a triangle, and determining when three lengths form a triangle</p>	<p>Students need to know the basic properties of a triangle. There are four main rules of triangles:</p> <p>1. Sum of the Interior Angles If you were stranded on a desert island and had to take the Math IIC test, this is the one rule about triangles you should bring along: the sum of the measures of the interior angles is 180°. Now, if you know the measures of two of a triangle's angles, you will be able to find the third. Helpful rule, don't you think</p>  <p>2. Measure of an Exterior Angle An exterior angle of a triangle is the angle formed by extending one of the sides of the triangle past a vertex (the point at which two sides meet). An exterior angle is always supplementary to the interior angle with which it shares a vertex and equal in measure to the sum of the measures of the remote interior angles. Take a look at the figure below, in which d, the exterior angle, is supplementary to interior angle c:</p>  <p>It doesn't matter which side of a triangle you extend to create an exterior angle; the exterior angle will always be supplementary to the interior angle with which it shares a vertex and therefore (because of the 180° rule) equal to the sum of the remote interior angles.</p> <p>3. Triangle Inequality The third important property of triangles is the triangle inequality rule, which states: the length of a side of a triangle is less than the sum of the lengths of the other two sides and greater</p>

than the difference of the lengths of the other two sides.
Observe the figure below:



From the triangle inequality, we know that $c - b < a < c + b$. The exact length of side a depends on the measure of the angle created by sides b and c . If this angle is large (close to 180°), then a will be large (close to $b + c$). If this angle is small (close to 0°), then a will be small (close to $b - c$).

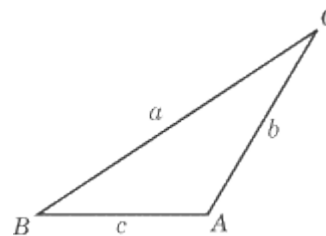
For an example, take a look at this triangle:



Using the triangle inequality, we can tell that $9 - 4 < x < 9 + 4$, or $5 < x < 13$. The exact value of x depends on the measure of the angle opposite side x .

4. Proportionality of Triangles

This brings us to the last basic property of triangles, which has to do with the relationships between the angles of a triangle and the lengths of the triangle's sides. In every triangle, the longest side is opposite the largest angle and the shortest side is opposite the smallest angle.

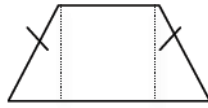


In this figure, side a is clearly the longest side and $\angle A$ is the largest angle. Conversely, side c is the shortest side and $\angle C$ is the smallest angle. It follows, therefore, that $c < b < a$ and $C < B < A$. This proportionality of side lengths and angle measures holds true for all triangles.

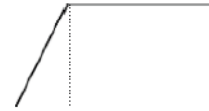
	<p>Example 1:</p> <p>Will three sides of any length create a triangle? Explain how you know which will work. Possibilities to examine are:</p> <ul style="list-style-type: none">a. 13 cm, 5cm, and 6 cmb. 3cm, 3cm, and 3cmc. 3cm, 7 cm , 6 cm <p>Solution:</p> <p>“A” above will not work; “B” and “C” will work. Students recognize that the sum of the two smaller sides must be larger than the third side.</p> <p>Example 2:</p> <p>Can a triangle have more than one obtuse angle? Explain you reasoning.</p> <p>Example 3:</p> <p>If give 2 angles determine the third based on triangle sum.</p> <p>Example 4:</p> <p>An isosceles triangle has only one 80° angle. What angle possibilities are there for the other angles?</p>
--	--

<p>6.8(B) Model area formulas for parallelograms, trapezoids, and triangles by decomposing and rearranging parts of these shapes.</p>	<p>Finding the area of triangles is introduced in relationship to the area of rectangles – a rectangle can be decomposed into two congruent triangles. Therefore, the area of the triangle is – the area of the rectangle. The area of a rectangle can be found by multiplying base X height; therefore, the area of the triangle is – bh or $\frac{1}{2}bh$.</p> <p>The following site helps students to discover the area formula of triangles.</p> <p>http://illuminations.nctm.org/LessonDetail.aspx?ID=L577</p>
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Students decompose shapes into rectangles and triangles to determine the area. For example, a trapezoid can be decomposed into triangles and rectangles (see figures below). Using the trapezoid's dimensions, the area of the individual triangle(s) and rectangle can be found and then added together. Special quadrilaterals include rectangles, squares, parallelograms, trapezoids, rhombi, and kites.



Isosceles trapezoid

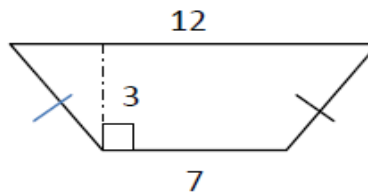


Right trapezoid

Note: Students recognize the marks on the isosceles trapezoid indicating the two sides have equal measure.

Example:

Find the area of the trapezoid shown below using the formulas for rectangles and triangles.



Solution:

The trapezoid could be decomposed into a rectangle with a length of 7 units and a height of 3 units. The area of the rectangle would be 21 units².

The triangles on each side would have the same area. The height of the triangles is 3 units. After taking away the middle rectangle's base length, there is a total of 5 units remaining for both of the side triangles. The base length of each triangle is half of 5. The base of each triangle is 2.5 units. The area of one triangle would be $\frac{1}{2}(2.5 \text{ units})(3 \text{ units})$ or 3.75 units².

6.8(C) Write equations that represent problems related to the area of rectangles, parallelograms, trapezoids, and triangles and volume of right rectangular prisms where dimensions are positive rational numbers; and

Find the equation that you could use to solve a given shape. This could be a combination or a breaking down of shapes.

	<p>Example: A trapezoid formula could be the area of a parallelogram + the area of a triangle.</p> <p>Example: The area of a triangle could be found by find the area of a rectangle divided by 2.</p> <p>This could also be used to find a missing value in an equation. <u>Example</u> $A \div l = w$</p>
<p>6.8(D) Determine solutions for problems involving the area of rectangles, parallelograms, trapezoids, and triangles and volume of right rectangular prisms where dimensions are positive rational numbers</p>	<p>Students will use models and formulas to find the area and volume of rectangles, parallelograms, trapezoids, triangles and right rectangular prisms.</p> <p>Example: Find the area of a right triangle with a base length of three units, a height of four units, and a hypotenuse of 5.</p> <p>Solution: Students understand that the hypotenuse is the longest side of a right triangle. The base and height would form the 90° angle and would be used to find the area using: $A = \frac{1}{2}bh$ $A = \frac{1}{2}(3 \text{ units})(4 \text{ units})$ $A = \frac{1}{2} - 12 \text{ units}^2$ $A = 6 \text{ units}^2$</p>
<p>Expressions, equations, and relationships: TEKS: 6.9</p>	<p>The student applies mathematical process standards to use equations and inequalities to represent situations.</p>
<p>6.9(A) Write one-variable, one-step equations and inequalities to represent constraints or conditions within problems</p>	<p>Understanding solving an equation or inequality is a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. Students use various processes to identify the value(s) that when substituted for the variable will make the equation true.</p> <p>Example 1:</p>

Joey had 26 papers in his desk. His teacher gave him some more and now he has 100. How many papers did his teacher give him?

This situation can be represented by the equation $26 + n = 100$ where n is the number of papers the teacher gives to Joey. This equation can be stated as “some number was added to 26 and the result was 100.” Students ask themselves “What number was added to 26 to get 100?” to help them determine the value of the variable that makes the equation true. Students could use several different strategies to find a solution to the problem:

- Reasoning: $26 + 70$ is 96 and $96 + 4$ is 100, so the number added to 26 to get 100 is 74.
- Use knowledge of fact families to write related equations:
 $n + 26 = 100$, $100 - n = 26$, $100 - 26 = n$. Select the equation that helps to find n easily.
- Use knowledge of inverse operations: Since subtraction “undoes” addition then subtract 26 from 100 to get the numerical value of n
- Scale model: There are 26 blocks on the left side of the scale and 100 blocks on the right side of the scale. All the blocks are the same size. 74 blocks need to be added to the left side of the scale to make the scale balance.
- Bar Model: Each bar represents one of the values. Students use this visual representation to demonstrate that 26 and the unknown value together make 100.

100	
26	n

Solution:

Students recognize the value of 74 would make a true statement if substituted for the variable.

$$26 + n = 100$$

$$26 + 74 = 100$$

$$100 = 100$$

Example 2:

The equation $0.44s = 11$ where s represents the number of stamps in a booklet. The booklet

of stamps costs 11 dollars and each stamp costs 44 cents. How many stamps are in the booklet? Explain the strategies used to determine the answer. Show that the solution is correct using substitution.

Solution:
 There are 25 stamps in the booklet. I got my answer by dividing 11 by 0.44 to determine how many groups of 0.44 were in 11.
 By substituting 25 in for s and then multiplying, I get 11.
 $0.44(25) = 11$
 $11 = 11$

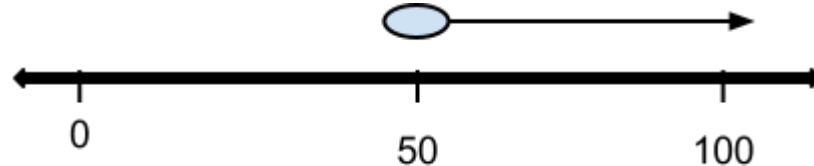
Example 3:
 Twelve is less than 3 times another number can be shown by the inequality $12 < 3n$. What numbers could possibly make this a true statement?

Solution:
 Since $3 \cdot 4$ is equal to 12 I know the value must be greater than 4. Any value greater than 4 will make the inequality true. Possibilities are 4.13, 6, \dots , and 200. Given a set of values, students identify the values that make the inequality true.

6.9(B) Represent solutions for one-variable, one-step equations and inequalities on number lines; and

Students use the number line to represent equations and inequalities from various contextual and mathematical situations.

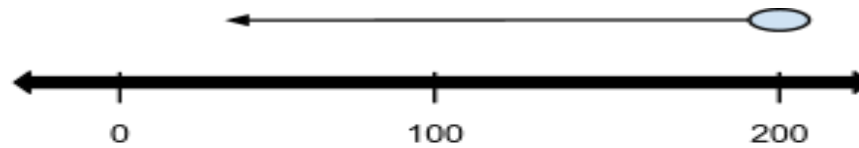
Example 1:
 Jonas spent more than \$50 at an amusement park. Write an inequality to represent the amount of money Jonas spent. What are some possible amounts of money Jonas could have spent? Represent the situation on a number line.



Solution:

Example 2:
 Less than \$200.00 was spent by the Flores family on groceries last month. Write an inequality to represent this amount and graph this inequality on a number line.

Solution: $200 > x$



6.9(C) Write corresponding real-world problems given one-variable, one-step equations or inequalities.

Many real-world situations are represented by equations and inequalities.

Example 1:

Equation: $12 = x + 3$

Possible Solution: Sam is three years older than his sister, Beth. If Sam is 12 years old, how old is Beth?

Example 2:

Inequality: $x \leq 4$.

Possible solution: The price of gas has been no more than \$ 4.00 for the last year.

Expressions, Equations, and Relationships:
TEKS: 6.10

The student applies mathematical process standards to use equations and inequalities to solve problems.

6.10(A) Model and solve one-variable, one-step equations and inequalities that represent problems, including geometric concepts

Students have used algebraic expressions to generate answers given values for the variable. This understanding is now expanded to equations where the value of the variable is unknown but the outcome is known. For example, in the expression, $6x + 3$, any value can be substituted for the x to generate a numerical answer; however, in the equation $6x + 3 = 6$, there is only one value that can be used to get a 6. Problems should be in context when possible and use only one variable.

Students write equations from real-world problems and then use inverse operations to solve one-step equations based on real world situations. Equations may include fractions and decimals with non-negative solutions.

Example 1:

Students recognize that dividing by 6 and multiplying by $\frac{1}{6}$ produces the same result. For example, $\frac{x}{6} = 9$ and $\frac{1}{6}x = 9$ will produce the same result.

Beginning experiences in solving equations require students to understand the meaning of the equation and the solution in the context of the problem.

Example 1:

Meagan spent \$56.58 on three pairs of jeans. If each pair of jeans costs the same amount, write an algebraic equation that represents this situation and solve to determine how much one pair of jeans cost.

\$56.58		
J	J	J

Sample Solution:

Students might say: "I created the bar model to show the cost of the three pairs of jeans. Each bar labeled J is the same size because each pair of jeans costs the same amount of money. The bar model represents the equation $3J = \$56.58$. To solve the problem, I need to divide the total cost of 56.58 between the three pairs of jeans. I know that it will be more than \$10 each because 10×3 is only 30 but less than \$20 each because 20×3 is 60. If I start with \$15 each, I am up to \$45. I have \$11.58 left. I then give each pair of jeans \$3. That's \$9 more dollars. I only have \$2.58 left. I continue until all the money is divided. I ended up giving each pair of jeans another \$0.86. Each pair of jeans costs \$18.86 ($15+3+0.86$). I double check that the jeans cost \$18.86 each because $\$18.86 \times 3$ is \$56.58."

Example 2:

Julie gets paid \$20 for babysitting. He spends \$1.99 on a package of trading cards and \$6.50 on lunch. Write and solve an equation to show how much money Julie has left.

20		
1.9	6.50	money left over (m)

Solution: $20 = 1.99 + 6.50 + x$, $x = \$11.51$

Example 3:

If the weight of each gray triangle below is 1 unit, write an equation for the balance scale.



Equation: _____

Identify the variables and terms in your equation:

variables: _____

terms: _____

What is the value of the rectangle? _____

Explain how you found your answer:

Example 4:

A rectangle is twice as long as wide. If the perimeter is 24 inches long, what are the lengths of each side?

Equation: _____

Identify the variables and terms in your equation:

variables: _____

terms: _____

What is the value of the width? _____ length? _____

Explain how you found your answer:

Many real-world situations are represented by inequalities. Students write inequalities to represent real work and mathematical situations. Presented with these situations, students must determine if a single value is required as a solution or if the situation allows for multiple solutions will be included. Students use the number line to represent inequalities from various contextual and mathematical situations. When working with inequalities, students will work with situations in which the solution is not limited to the set of positive whole numbers but includes positive rational numbers.

Example 1:

The class must raise at least \$100 to go on the field trip. They have collected \$20. Write an inequality to represent the amount of money, m , the class still needs to raise. Represent this inequality on a number line.

Solution:

The inequality $m \geq \$80$ represents this situation. Students recognize that possible values can include too many decimal values to name. Therefore, the values are represented on a number line by shading.

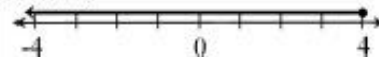


A number line diagram is drawn with an open circle when an inequality contains a $<$ or $>$ symbol to show solutions that are less than or greater than the number but not equal to the number. The circle is shaded, as in the example above, when the number is to be included. Students recognize that possible values can include fractions and decimals, which are represented on the number line by shading. Shading is extended through the arrow on a number line to show that an inequality has an infinite number of solutions.

Example 2:

Graph $x \leq 4$.

Solution:

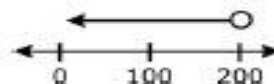


Example 3:

The Flores family spent less than \$200.00 last month on groceries. Write an inequality to represent this amount and graph this inequality on a number line.

Solution:

$200 > x$, where x is the amount spent on groceries.



6.10(B) Determine if the given value(s) make(s) one-variable, one-step equations or inequalities true

The skill of solving an equation must be developed *conceptually* before it is developed *procedurally*. Beginning experiences in solving equations should require students to understand the meaning of the equation as well as the question being asked. This means that students should be thinking about what numbers could possibly be a solution to the equation before solving the equation.

Example 1:

In the equation $x + 21 = 32$ students know that $21 + 9 = 30$ therefore the solution must be 2 more than 9 or 11, so $x = 11$.

Solving equations using reasoning and prior knowledge should be required of students to allow them to develop effective strategies such as using reasoning, fact families, and inverse operations. Students may use balance models in representing and solving equations and inequalities.

Students should begin by identifying values from a specified set that will make an equation true.

Example 2:

The equation $0.44s = 11$ where s represents the number of stamps in a booklet. The booklet of stamps costs 11 dollars and each stamp costs 44 cents. How many stamps are in the booklet? Explain the strategies used to determine the answer. Show that the solution is correct using substitution.

Solution:

There are 25 stamps in the booklet. I got my answer by dividing 11 by 0.44 to determine how many groups of 0.44 were in 11.

By substituting 25 in for s and then multiplying, I get 11.

$$0.44(25) = 11$$

$$11 = 11 \checkmark$$

Example 3:

Twelve is less than 3 times another number can be shown by the inequality $12 < 3n$. What numbers could possibly make this a true statement?

Solution:

Since $3 \cdot 4$ is equal to 12 I know the value must be greater than 4. Any value greater than 4 will make the

inequality true. Possibilities are 4.13, 6, $5\frac{3}{4}$, and 200. Given a set of values, students identify the values that make the inequality true.

Measurement and Data: TEKS: 6.11	The student applies mathematical process standards to use coordinate geometry to identify locations on a plane.
6.11(A) Graph points in all four quadrants using ordered pairs of rational numbers	<p>The x-axis and y-axis are extending to include negatives, so students begin to work with the Cartesian Coordinate system. Students worked with Quadrant I in elementary school. As the x-axis and y-axis are extending to include negatives, students begin to with the Cartesian Coordinate system. Students recognize the point where the x-axis and y-axis intersect as the origin. Students identify the four quadrants and are able to identify the quadrant for an ordered pair based on the signs of the coordinates. For example, students recognize that in Quadrant II, the signs of all ordered pairs would be $(-, +)$.</p> <p>Students understand the relationship between two ordered pairs differing only by signs as reflections across one or both axes. For example, in the ordered pairs $(-2, 4)$ and $(-2, -4)$, the y-coordinates differ only by signs, which represents a reflection across the x-axis. A change is the x-coordinates from $(-2, 4)$ to $(2, 4)$, represents a reflection across the y-axis. When the signs of both coordinates change, $[(2, -4)$ changes to $(-2, 4)]$, the ordered pair has been reflected across both axes.</p> <p>Students understand the relationship between two ordered pairs differing only by signs as reflections across one or both axes. For example, in the ordered pairs $(-2, 4)$ and $(-2, -4)$, the y-coordinates differ only by signs, which represents a reflection across the x-axis. A change is the x-coordinates from $(-2, 4)$ to $(2, 4)$, represents a reflection across the y-axis. When the signs of both coordinates change, $[(2, -4)$ changes to $(-2, 4)]$, the ordered pair has been reflected across both axes. Students are able to plot all rational numbers on a number line (either vertical or horizontal) or identify the values of given points on a number line.</p> <p>Example1: Identify where the following numbers would be on a number line (vertical or horizontal):</p> <p style="text-align: center;">— —</p>

**Measurement and Data:
TEKS: 6.12**

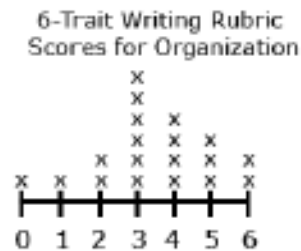
The student applies mathematical process standards to use numerical or graphical representations to analyze problems.

6.12(A) Represent numeric data graphically, including dot plots, stem-and-leaf plots, histograms, and box plots

Students display data graphically using number lines. Dot plots, stem-and-leaf plots, histograms and box plots are four graphs to be used. Students are expected to determine the appropriate graph as well as read data from graphs generated by others.

Dot plots are simple plots on a number line where each dot represents a piece of data in the data set. Dot plots are suitable for small to moderate size data sets and are useful for highlighting the distribution of the data including clusters, gaps, and outliers.

Example: Nineteen students completed a writing sample that was scored on organization. The scores for organization were 0,1,2,2,3,3,3,3,3,4,4,4,4,5,5,6,6,. Create a data display. What are some observations that can be made from the data display?



A histogram shows the distribution of continuous data using intervals on the number line. The height of each bar represents the number of data values in that interval. In most real data sets. There is a large amount of data and many numbers will be unique. A graph (such as a dot plot) that shows how many ones, how many twos, etc would not be meaningful; however, a histogram can be used. Students group the data into convenient ranges and use these intervals to generate a frequency table and histogram. Note that changing the size of the bin (interval) changes the appearance of the graph and the conclusions may vary from it.

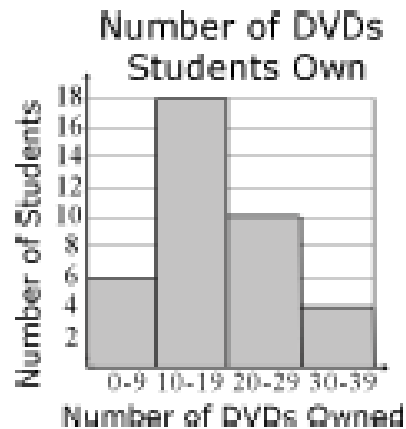
Example: Grade 6 students were collecting data for a math class project. They decided they would survey the other two grade 6 classes to determine how many DVD's each student owns. A total of 48 students were surveyed. The data are shown in the table below in no specific order. Create a data display. What are some observations that can be made from the data display?

11	21	5	12	10	31	19	13	23	33
10	11	25	14	34	15	14	29	8	5
22	26	23	12	27	4	25	15	7	

2	19	12	39	17	16	15	28	16	
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Solution:

A histogram using 5 intervals (bins) (0-9, 10-19,, ... 30-39) to organize the data is displayed below.



A stem-and-leaf plot summarizes the shape of a set of data and is used when there are large amounts of numbers to analyze. The data is arranged by place value. The digits in the largest place are referred to as the stem and the digits in the smallest place are referred to as the leaf (leaves). The leaves are displayed to the right of the stem. Examples: series of scores on sports teams, series of temperatures or rainfall over a period of time, series of classroom test scores.

A box plot shows the distribution of values in a data set by dividing the set into quartiles. It can be graphed either vertically or horizontally. The box plot is constructed from the five-number summary (minimum, lower quartile, median, upper quartile, and maximum). These values give a summary of the shape.

Example:

Ms. Wheeler asked each student in her class to write their age in months on a sticky note. The 28 students in the class brought their sticky note to the front of the room and posted them in order on the white board. The data set below is an example of a stem-and-leaf plot.

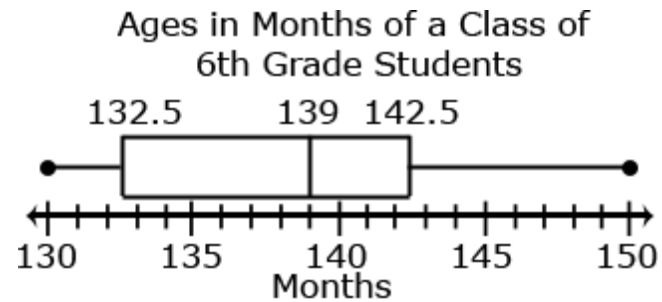
13|0,0,1,1,2,2,2,3,4,6,7,7,8,9,9,9

14 0,1,2,2,2,3,3,4,5,7,9

15|0

The box plot below shows that

- $\frac{1}{4}$ of the students in the class are from 130 to 132.5 months old
- $\frac{1}{4}$ of the students in the class are from 142.5 months to 150 months old
- $\frac{1}{2}$ of the class are from 132.5 to 142.5 months old
- The median class age is 139 months



6.12(B) Use the graphical representation of numeric data to describe the center, spread, and shape of the data distribution

The distribution is the arrangement of the values of a data set. Distribution can be described using center (median or mean), and spread. Data collected can be represented on graphs which will show the shape of the distribution of the data. Students examine the distribution of a data set and discuss the center, spread and overall shape with dot plots, histograms and box plots.

Example 1:

The dot plot shows the writing scores for a group of students on organization. Describe the data.



Solution: The values range from 0-6. There is a peak at 3. The median is 3, which means

	50% of the scores are greater than or equal to 3 and 50% are less than or equal to 3. The mean is 3.67. If all students scored the same, the score would be 3.68.
--	---

6.12(C) Summarize numeric data with numerical summaries, including the mean and median (measures of center) and the range and interquartile range (IQR) (measures of spread), and use these summaries to describe the center, spread, and shape of the data distribution; and

6.12(D) Summarize categorical data with numerical and graphical summaries, including the mode, the percent of values in each category (relative frequency table), and the percent bar graph, and use these summaries to describe the data distribution.

Students summarize numerical data by providing background information about the attribute being measured, methods and unit of measurement, the context of data collection activities (addressing random sampling), the number of observations and summary statistics. Summary statistics include quantitative measures of center (mean and median) and variability (interquartile range and mean absolute deviation) including extreme values (minimum and maximum), mean, median, mode, range, and quartiles.

Two of the most important features of the distribution of a quantitative variable can be described using numerical measures,

1. Its center, and
2. The spread of its values about the center

The numbers we use to describe the center of a distribution (i.e., the location where roughly half the values are below it and the other half above it) are:

1. The mean
2. The median

-- The mean of the distribution of a quantitative variable is the arithmetic average of its values.

The median is the "middle value". It is located after all the observations have been arranged in ascending order .

1. Arrange all observations in increasing (or ascending) order
2. If n is odd, then the median is the $(n+1)/2$ th observation
3. If n is even, the median is the mean of the two center observations

While measures of central tendency summarize a data set with a single 'typical' number, it is also useful to describe the 'spread' of the data with a single number. Describing how a data set is distributed can be accomplished through one of the measures of dispersion: variance, standard deviation, or interquartile range.

Measurement and Data:
TEKS: 6.13

6.13(A) Interpret numeric data summarized in dot plots, stem-and-leaf plots, histograms, and box plots; and

The student applies mathematical process standards to use numerical or graphical representations to solve problems.

Most of the students have between 10 and 19 DVDs as indicated by the peak on the graph. The data is pulled to the right since only a few students own more than 30 DVDs.

Example 3:

Ms. Wheeler asked each student in her class to write their age in months on a sticky note. The 28 students in the class brought their sticky note to the front of the room and posted them in order on the white board. The data set is listed below in order from least to greatest. Create a data display. What are some observations that can be made from the data display?

130	130	131	131	132	132	132	133	134	136
137	137	138	139	139	139	140	141	142	142
142	143	143	144	145	147	149	150		

Solution:

Five number summary

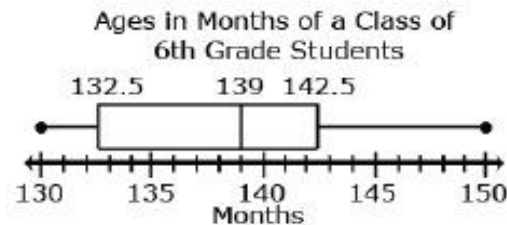
Minimum – 130 months

Quartile 1 (Q1) – $(132 + 133) \div 2 = 132.5$ months

Median (Q2) – 139 months

Quartile 3 (Q3) – $(142 + 143) \div 2 = 142.5$ months

Maximum – 150 months



This box plot shows that

- $\frac{1}{4}$ of the students in the class are from 130 to 132.5 months old
- $\frac{1}{4}$ of the students in the class are from 142.5 months to 150 months old
- $\frac{1}{2}$ of the class are from 132.5 to 142.5 months old
- The median class age is 139 months.

6.13(B) Distinguish between situations that yield data with and without variability

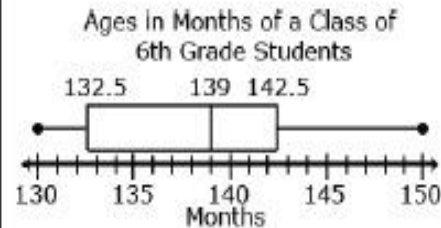
Measures of Variability

Measures of variability/variation can be described using the interquartile range or the Mean Absolute Deviation. The interquartile range (IQR) describes the variability between the middle 50% of a data set. It is found by subtracting the lower quartile from the upper quartile. It represents the length of the box in a box plot and is not affected by outliers.

Students find the IQR from a data set by finding the upper and lower quartiles and taking the difference or from reading a box plot.

Example 1:

What is the IQR of the data below:



Solution:

The first quartile is 132.5; the third quartile is 142.5. The IQR is 10 ($142.5 - 132.5$). This value indicates that the values of the middle 50% of the data vary by 10.

Mean Absolute Deviation (MAD) describes the variability of the data set by determining the absolute deviation (the distance) of each data piece from the mean and then finding the average of these deviations.

Both the interquartile range and the Mean Absolute Deviation are represented by a single numerical value. Higher values represent a greater variability in the data.

Example 2:

The following data set represents the size of 9 families:

3, 2, 4, 2, 9, 8, 2, 11, 4.

What is the MAD for this data set?

Solution:

The mean is 5. The MAD is the average variability of the data set. To find the MAD:

1. Find the deviation from the mean.
2. Find the absolute deviation for each of the values from step 1
3. Find the average of these absolute deviations.

The table below shows these calculations:

Data Value	Deviation from Mean	Absolute Deviation
3	-2	2
2	-3	3
4	-1	1
2	-3	3
9	4	4
8	3	3
2	-3	3
11	6	6
4	-1	1
MAD		26/9 = 2.89

This value indicates that on average family size varies 2.89 from the mean of 5.

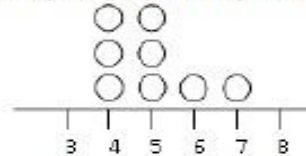
Students understand how the measures of center and measures of variability are represented by graphical displays.

Students describe the context of the data, using the shape of the data and are able to use this information to determine an appropriate measure of center and measure of variability. The measure of center that a student chooses to describe a data set will depend upon the shape of the data distribution and context of data collection. The mode is the value in the data set that occurs most frequently. The mode is the least frequently used as a measure of center because data sets may not have a mode, may have more than one mode, or the mode may not be descriptive of the data set. The mean is a very common measure of center computed by adding all the numbers in the set and dividing by the number of values. The mean can be affected greatly by a few data points that are very low or very high. In this case, the median or middle value of the data set might be more descriptive. In data sets that are symmetrically distributed, the mean and median will be very close to the same. In data sets that are skewed, the mean and median will be different, with the median frequently providing a better overall description of the data set.

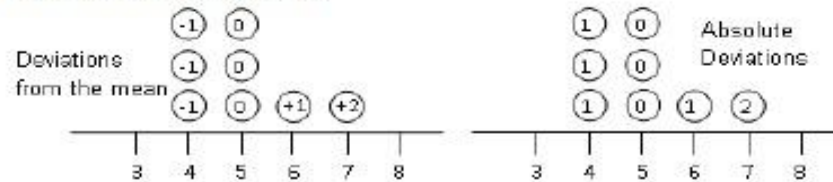
Understanding Mean Absolute Deviation

The use of mean absolute deviation in 6th grade is mainly exploratory. The intent is to build a deeper understanding of variability. Students would understand the mean distance between the pieces of data and the mean of the data set expresses the spread of the data set. Students can see that the larger the mean distance, the greater the variability. Comparisons can be made between different data sets.

In the previous data set, the names drawn were Carol, Mike, Maria, Luis, Monique, Sierra, John, and Karen. There were 3 names with 4 letters each, 3 names with 5 letters each, 1 name with 6 letters and 1 name with 7 letters. This data can be represented on a dot plot. The mean of the data set is 5.



To find the mean absolute deviation, students examine each of the data points and its difference from the mean. This analysis can be represented on the dot plot itself or in a table. Each of the names with 4 letters has one fewer letter than the mean, each of the names with 5 letters has zero difference in letters as compared to the mean, each of the names with 6 letters has one more letter than the mean, and each of the names with 7 letters has two more letters than the mean. The absolute deviations are the absolute value of each difference.



Statistics are numerical data relating to an aggregate of individuals; statistics is also the name for the science of collecting, analyzing and interpreting such data. A statistical question anticipates an answer that varies from one individual to the next and is written to account for the variability in the data. Data are the numbers produced in response to a statistical question. Data are frequently collected from surveys or other sources (i.e. documents).

Questions can result in a narrow or wide range of numerical values. For example, asking classmates "How old are the students in my class in years?" will result in less variability than asking "How old are the students in my class in months?"

Students might want to know about the fitness of the students at their school. Specifically, they want to know about the exercise habits of the students. So rather than asking "Do you exercise?" they should ask about the amount of exercise the students at their school get per week. A statistical question for this study could be: "How many hours per week on average do students at Jefferson Middle School exercise?"

To collect this information, students might design a survey question that anticipates variability by providing a variety of possible anticipated responses that have numerical answers, such as: 3 hours per week, 4 hours per week, and so on. Be sure that students ask questions that have specific numerical answers.

Personal Financial Literacy: TEKS: 6.14	The student applies mathematical process standards to develop an economic way of thinking and problem solving useful in one's life as a knowledgeable consumer and investor
6.14(A) Compare the features and costs of a checking account and a debit card offered by different local financial institutions	<p>http://ok.gov/sde/sites/ok.gov.sde/files/PFLTchrGuide_3.1.pdf</p> <p>The student will have the opportunity to research different financial institutions and compare the various services offered by each institution. Be sure to include the differences in rates and fee. Be sure to include Fees associated with each of the features. (ie. ATM fees for a non-network bank.)</p> <p>Sample Lessons: http://smartertexas.org/?page_id=914 http://www.msbankers.com/assets/1674/its_a_balancing_act-_teacher_guide.pdf</p>
6.14(B) Distinguish between debit cards and credit cards	<p>http://richmondfed.org/education/for_teachers/lesson_plans_and_classroom_activities/pdf/debit_credit_lesson.pdf</p> <p>Have students experience payments with debits cards and credit cards and financial differences in their commitments and financial costs.</p>

CARD TYPE	ADVANTAGE	DISADVANTAGE
DEBIT CARD	<ul style="list-style-type: none"> • You don't have to carry cash or a checkbook. • It's safe because only you know your PIN (personal identification number). • Without the PIN, no one else can access your money at an ATM. • If someone steals your card, you're only responsible for part of the purchases they make. 	<ul style="list-style-type: none"> • You must have enough money in your account to cover purchases. • You have to remember your PIN to access cash. • If you don't record your purchases, you may overdraw your account and have to pay a fine.
CREDIT CARD	<ul style="list-style-type: none"> • You can buy things before you have saved the entire purchase price. • It is a way to pay for emergency expenses. • If someone steals your card, you're only responsible for part of the purchase they make. • It is convenient and makes it easy to have a record of your expenses. 	<ul style="list-style-type: none"> • You may have to pay a yearly fee for the card. • You pay interest on the unpaid monthly balance. Because of the interest, you may pay more for items than if you paid cash.

Misconceptions: You can always use the debit card but students don't realize that they need to have money to back up the use of the card. The same can be true for the thought about a credit card, except you are borrow credit that will have to be paid back at a percentage penatlity where the debit card is usually subject to an overdraft charge.

Sample Lessons:

http://smartertexas.org/?page_id=914

http://www.msbankers.com/assets/1674/its_a_balancing_act-_teacher_guide.pdf

6.14(C) Balance a check register that includes deposits, withdrawals, and transfers	<p data-bbox="730 1305 1562 1370">Franklin Pierce schools: Navigation Lesson Plan Lesson:18 BALANCING A CHECKBOOK</p> <p data-bbox="730 1409 1923 1507">If you write checks or use a debit card, you cannot spend more money than you have in your account. If you do, your debit card will not work, and you may be charged large “overdraft” fees by the bank.</p>
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How do you keep track of the money in your account? It's a simple matter of addition and subtraction. You just have to stay organized.

Why don't you try it? Imagine that you have a checking account and use both checks and a debit card. Here are your transactions for the last half of January. Enter them into the "checkbook" below:

- Your beginning balance is \$150.
- On January 15th, you write check #1150 to your school for \$10 for two tickets to movie night.
- On January 16th, your neighbors pay you \$75 for dog-sitting and you deposit it in the bank.
- On January 17th, you use your debit card to pay \$80 for a new pair of shoes.
- On January 17th, you use your debit card to pay \$10 for your monthly music subscription.
- On January 20th, your parents pay you \$20 in allowance and you deposit it in the bank.
- On January 25th, you write check #1151 to your school for \$10 for a ticket to the school play.
- On January 28th, you use your debit card to pay \$15 for a movie ticket and popcorn.
- On January 29th, your parents pay you \$20 in allowance and you deposit it in the bank.
- On January 30th, you use your debit card for \$25 for pizza with your friends.

SAMPLE CHECKBOOK

Complete the checkbook below. The first several entries have been made for you:

			(-)	(+)	
Check or	Date	Description of Transaction	Amount of	Amount of	BALANCE
Receipt #			Withdrawal	Deposit	
					\$150.00

	1150	15-Jan	School movie night (2 tickets)	\$10.00		\$140.00
		16-Jan	Payment for dog-sitting		\$75.00	\$215.00

What is your checkbook balance on January 30th after your dinner with your friends? Do you still have money in your account?

Misconceptions: Students have a difficult time understanding the positive and negative numbers and how to work them in reverse. The perspective who is gaining and who is losing the amounts.

Sample Lessons:

http://smartertexas.org/?page_id=914

http://www.msbankers.com/assets/1674/its_a_balancing_act-_teacher_guide.pdf

6.14(D) Explain why it is important to establish a positive credit history

Having a good credit history is important to have a successful personal and financial future for yourself. A good credit history will help you buy a car, buy a home, etc.

Go to these links for a lesson example to show students the importance of having a good credit score. It also gives the ranges of scores and how different activities in their daily lives can affect these scores.

www.myfico.com.

<http://www.pbs.org/wgbh/pages/frontline/teach/meltdown/creditscore.pdf>

Sample Lessons:

http://smartertexas.org/?page_id=914

http://www.msbankers.com/assets/1674/its_a_balancing_act-_teacher_guide.pdf

6.14(E) Describe the information in a credit report and how long it is retained

The Fair Credit Reporting Act dictates the length of time that information remains on your credit report. Negative information, such as judgments, collection activity and charge-offs, can remain on your credit report up to seven years. Bankruptcies can be reported for 10 years.

Read more: How long is information retained on a credit report? | Answerbag
http://www.answerbag.com/q_view/2107042#ixzz2Hxy3ld2k

Sample Lessons:

http://smartertexas.org/?page_id=914

http://www.msbankers.com/assets/1674/its_a_balancing_act-teacher_guide.pdf

6.14(F) Describe the value of credit reports to borrowers and to lenders.

What's your Credit Score?

Unless you have the available cash, you will be financing your real estate purchase. That means applying and qualifying for a mortgage loan. In doing so, you will undoubtedly be faced with the realities of your credit score. But what is a credit score and what part does it play in your mortgage loan process? Here are the answers to these and other frequently asked questions about credit scores.



What is a credit score?

Whether you realize it or not, you have a credit score. Your score is a summary of your credit data expressed in the form of a three-digit number. That number essentially represents your credit rating. You might hear your credit score referred to as your FICO® score. This is because the computer model most commonly used as a basis to compute credit scores was developed by Fair Isaac Corporation, a company that provides analytic solutions, such as predictive models, that automate and improve decision strategies. Generic FICO® scores range from 300 to 850. In general, the higher your score, the lower risk you represent to lenders in terms of paying back your loan. On the reverse side, the lower your score, the

greater credit risk you are assumed to be.

How are credit scores computed?

Your credit score is the result of a sophisticated statistical analysis on the information in your credit report. Data elements drawn from your credit file are plugged into mathematical formulas that calculate your score. Those formulas and your score also take into account how your information compares to the credit track record of thousands, sometimes even millions, of other consumers with similar credit profiles.

According to Fair Isaac, the data that is used in the calculation of your credit score can be grouped into five categories. These categories are not equally important in the scoring process. Each is assigned a weight based on how strong a predictor it is of credit risk. These categories are presented below in order from greatest to least in terms of weight, with the first two categories being the most heavily weighted.

- Payment History
- Amounts Owed
- Length of Credit History
- New Credit
- Types of Credit Used

Each of the three major U.S. credit bureaus calculate credit scores. They use credit scoring software to do so. Even though each bureau has its own system and calculations, the statistical scoring models they use have been standardized so that the scores from each bureau are comparable.

How do credit scores affect mortgage loans?

Mortgage lenders recognize that there is a correlation between credit scores and risk of delinquency or default on mortgage loans. They therefore use credit scores to help them determine eligibility of borrowers as well as terms offered on the loan. Credit scores are not usually the sole determinant for approval or rejection, however they are a very important factor in that decision. If the mortgage loan is approved, credit scores can also impact the interest rate offered. Generally speaking the higher the credit risk, the higher the interest rates.

Does everyone have a credit score?

Not necessarily. Some consumers may not have enough information, or enough recent information, in their credit report for scoring models to generate an accurate score. Thus, if you have little or no credit history, you probably will not have a credit score.

Does your credit score change over time?

Yes it does. Your credit score is not a static number. Rather it is a fluid value that adjusts up or down based on the data that is in your credit file when your score is calculated. Since your credit data is continually updated as creditors provide new information based on your activity, your score changes to reflect that underlying information. How much your score changes will depend on your activity during that given time. If you are consistent in your credit behavior, your score will remain relatively stable. You are more likely to see a bigger change if you significantly modify your behavior.

How do you find out your credit score?

There is a fee associated with getting your credit score. You can access that information through each of the three national credit bureaus or the central credit report site sponsored by all three bureaus:

- Equifax (equifax.com)
- TransUnion (transunion.com)
- Experian (experian.com)
- Central credit report site is AnnualCreditReport.com

You can also purchase information on all three of your credit scores through myFICO.com, a division of Fair Isaac. This site also has a credit education section that provides additional detailed information about understanding and managing your credit score.

Official Site - Experian®

www.Experian.com

Your Credit Report & Score Direct From Experian. Easy to Read Online.

Sample Lessons:

http://smartertexas.org/?page_id=914

http://www.msbankers.com/assets/1674/its_a_balancing_act_-_teacher_guide.pdf

6.14(G) Explain various methods to pay for college, including through savings, grants, scholarships, student loans, and work study and

College costs a lot of money even for a single year. The lesson below will provide students an understanding of the different ways to pay for college.

http://www.getreadyforcollege.org/outreachMN/GCM/GCM_Lesson13.pdf

	<p>Sample Lessons: http://smartertexas.org/?page_id=914 http://www.msbankers.com/assets/1674/its_a_balancing_act-_teacher_guide.pdf</p>
<p>6.14(H) Compare the annual salary of several occupations requiring various levels of post-secondary education or vocational training and calculate the effects of the different annual salaries on lifetime income</p>	<p>Have a list of the top 5 current salaries and several top choices from students and a couple of jobs around town. Calculate the amount earned for a year. Show costs of items for the average daily life and how the salaries compare to the costs. What would be the financial earnings for a year. Sample Lessons: http://smartertexas.org/?page_id=914 http://www.msbankers.com/assets/1674/its_a_balancing_act-_teacher_guide.pdf</p>