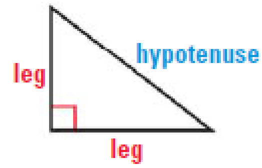
7.1 Apply the Pythagorean Theorem

Obj.: Find side lengths in right triangles.

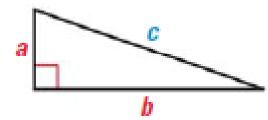
Key Vocabulary

- **Pythagorean triple -** A **Pythagorean triple** is a set of **three** positive integers *a*, *b*, and *c* that **satisfy** the equation $c^2 = a^2 + b^2$.
- **Right triangle –** A triangle with **one right angle**.
- **Leg of a right triangle -** In a right triangle, the sides **adjacent** to the **right angle** are called the **legs**.
- **Hypotenuse -** The side **opposite** the **right angle** is called the **hypotenuse** of the right triangle.



Pythagorean Theorem **Pyth. Th.**

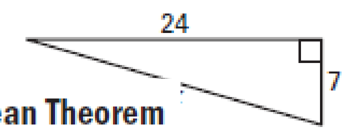
In a **right** triangle, the **square** of the length of the **hypotenuse** is equal to the **sum** of the squares of the lengths of the **legs**.



$c^2 = a^2 + b^2$

EXAMPLE 1 Find the length of a hypotenuse

Find the length of the hypotenuse of the right triangle.



Solution

$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$ **Pythagorean Theorem**

$x^2 = 7^2 + 24^2$ **Substitute.**

$x^2 = 49 + 576$ **Multiply.**

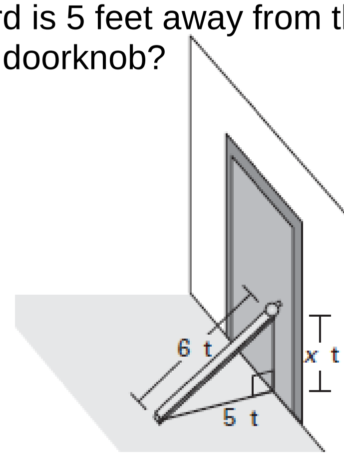
$x^2 = 625$ **Add.**

$x = 25$ **Find the positive square root.**

In the equation for the Pythagorean Theorem, "length of hypotenuse" and "length of leg" was shortened to "hypotenuse" and "leg".

EXAMPLE 2 Find the length of a leg

Door A 6 foot board rests under a doorknob and the base of the board is 5 feet away from the bottom of the door. Approximately how high above the ground is the doorknob?



Solution

$\left(\begin{matrix}\text{Length}\\\text{of board}\end{matrix}\right)^2 = \left(\begin{matrix}\text{Distance}\\\text{from door}\end{matrix}\right)^2 + \left(\begin{matrix}\text{Height of}\\\text{doorknob}\end{matrix}\right)^2$

$6^2 = 5^2 + x^2$ **Substitute.**

$36 = 25 + x^2$ **Multiply.**

$11 = x^2$ **Subtract 25 from each side.**

$\sqrt{11} = x$ **Find positive square root.**

$3.3 \approx x$ **Approximate with a calculator.**

The board is resting against the doorknob at about 3.3 feet above the ground.

In real-world applications, it is usually appropriate to use a calculator to approximate the square root of a number. Round your answer to the nearest tenth.

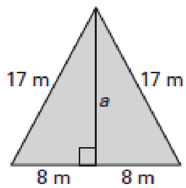
EXAMPLE 3 Find the area of an isosceles triangle

(7.1 cont.)

Find the area of the isosceles triangle with side lengths 16 meters, 17 meters, and 17 meters.

Solution

Step 1 **Draw** a sketch. By definition, the length of an altitude is the height of the triangle. In an isosceles triangle, the altitude to the base is also a perpendicular bisector. So, the altitude divides the triangle into two right triangles with the dimensions shown.



Step 2 **Use** the Pythagorean Theorem to find the height of the triangle.

$c^2 = a^2 + b^2$	**Pythagorean Theorem**
$17^2 = 8^2 + h^2$	**Substitute.**
$289 = 64 + h^2$	**Multiply.**
$225 = h^2$	**Subtract** 64 **from each side.**
$15 = h$	**Find the positive square root.**

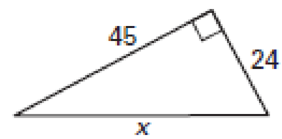
Step 3 **Find** the area.

$$\text{Area} = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(16)(15) = 120$$

The area of the triangle is 120 square meters.

EXAMPLE 4 Find length of a hypotenuse using two methods

Find the length of the hypotenuse of the right triangle.



Solution

Method 1: Use a Pythagorean triple.

A common Pythagorean triple is 8, 15, 17. Notice that if you multiply the lengths of the legs of the Pythagorean triple by 3, you get the lengths of the legs of this triangle: $8 \cdot 3 = 24$ and $15 \cdot 3 = 45$. So, the length of the hypotenuse is $17 \cdot 3 = 51$.

Method 2: Use the Pythagorean Theorem.

$x^2 = 24^2 + 45^2$	**Pythagorean Theorem**
$x^2 = 576 + 2025$	**Multiply.**
$x^2 = 2601$	**Add.**
$x = 51$	**Find the positive square root.**

KEY CONCEPT *For Your Notebook*

Common Pythagorean Triples and Some of Their Multiples

3, 4, 5	**5, 12, 13**	**8, 15, 17**	**7, 24, 25**
6, 8, 10	10, 24, 26	16, 30, 34	14, 48, 50
9, 12, 15	15, 36, 39	24, 45, 51	21, 72, 75
30, 40, 50	50, 120, 130	80, 150, 170	70, 240, 250
$3x, 4x, 5x$	$5x, 12x, 13x$	$8x, 15x, 17x$	$7x, 24x, 25x$

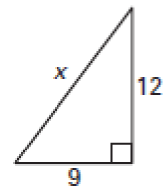
The most common Pythagorean triples are in bold. The other triples are the result of multiplying each integer in a bold face triple by the same factor.

7.1 Cont.

Checkpoint **Complete the following exercise.**

1. Find the length of the hypotenuse of the right triangle.

$x = 15$

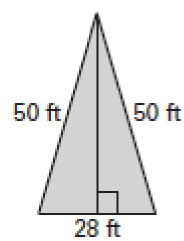


2. A 5 foot board rests under a doorknob and the base of the board is 3.5 feet away from the bottom of the door. Approximately how high above the ground is the doorknob?

about 3.6 feet

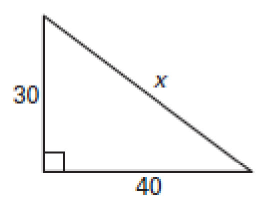
3. Find the area of the triangle.

672 ft^2



4. Use a Pythagorean triple to find the unknown side length of the right triangle.

50



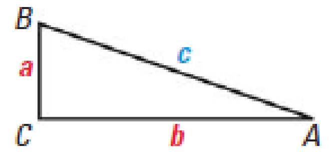
7.2 Use the Converse of the Pythagorean Theorem

Obj.: Use its converse to determine if a triangle is a right triangle.

Key Vocabulary

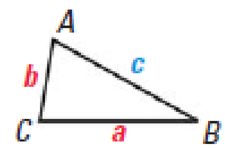
- **Acute triangle -** A triangle with three acute angle
- **Obtuse triangle -** A triangle with one obtuse angle

Converse of the Pythagorean Theorem **Conv. Pyth. Th.**



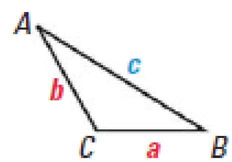
If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.
If $c^2 = a^2 + b^2$, then ΔABC is a right triangle.

Acute Triangle Theorem **Acute Δ Th.**



If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle *ABC* is an acute triangle.
If $c^2 < a^2 + b^2$, then the triangle *ABC* is acute.

Obtuse Triangle Theorem **Obtuse Δ Th.**

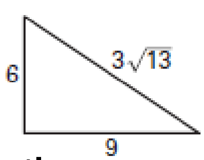


If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle *ABC* is an obtuse triangle.
If $c^2 > a^2 + b^2$, then triangle *ABC* is obtuse.

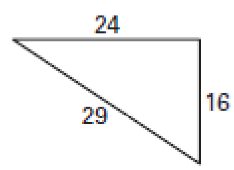
EXAMPLE 1 Verify right triangles

Tell whether the given triangle is a right triangle.

a.



b.



Solution

Let *c* represent the length of the longest side of the triangle. Check to see whether the side lengths satisfy the equation $c^2 = a^2 + b^2$.

a. $(3\sqrt{13})^2 \stackrel{?}{=} 6^2 + 9^2$

$9 \cdot 13 \stackrel{?}{=} 36 + 81$

$117 = 117$ ✓

The triangle is a right triangle.

b. $29^2 \stackrel{?}{=} 24^2 + 16^2$

$841 \stackrel{?}{=} 576 + 256$

$841 \neq 832$

The triangle is not a right triangle.

EXAMPLE 2 Classify triangles

Can segments with lengths of 2.8 feet, 3.2 feet, and 4.2 feet form a triangle? If so, would the triangle be *acute, right,* or *obtuse*?

Solution

The Triangle Inequality Theorem states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Step 1 **Use** the Triangle Inequality Theorem to check that the segments can make a triangle.

2.8 + 3.2 = 6	2.8 + 4.2 = 7	3.2 + 4.2 = 7.4
6 > 4.2	7 > 3.2	7.4 > 2.8

Step 2 **Classify** the triangle by comparing the square of the length of the longest side with the sum of squares of the lengths of the shorter sides.

$c^2 \ ? \ a^2 + b^2$	**Compare c^2 with $a^2 + b^2$.**
$4.2^2 \ ? \ 2.8^2 + 3.2^2$	**Substitute.**
$17.64 \ ? \ 7.84 + 10.24$	**Simplify.**
$17.64 < 18.08$	**c^2 is less than $a^2 + b^2$.**

The side lengths 2.8 feet, 3.2 feet, and 4.2 feet form an acute triangle.

EXAMPLE 3 Use the Converse of the Pythagorean Theorem

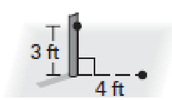
Lights You are helping install a light pole in a parking lot. When the pole is positioned properly, it is perpendicular to the pavement. How can you check that the pole is perpendicular using a tape measure?

Solution

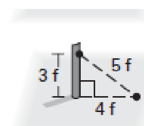
To show a line is perpendicular to a plane you must show that the line is perpendicular to two lines in the plane.

Think of the pole as a line and the pavement as a plane. Use a 3-4-5 right triangle and the Converse of the Pythagorean Theorem to show that the pole is perpendicular to different lines on the pavement.

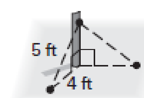
First mark 3 feet up the pole and mark on the pavement 4 feet from the pole.



Use the tape measure to check that the distance between the two marks is 5 feet. The pole makes a right angle with the line on the pavement.



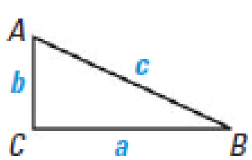
Finally, repeat the procedure to show that the pole is perpendicular to another line on the pavement.



CONCEPT SUMMARY *For Your Notebook*

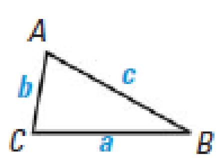
Methods for Classifying a Triangle by Angles Using its Side Lengths

Conv. Pyth. Th



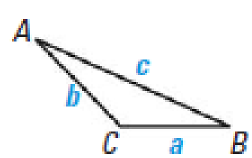
If $c^2 = a^2 + b^2$, then $m\angle C = 90°$ and $\triangle ABC$ is a right triangle.

Acute ∆Th.



If $c^2 < a^2 + b^2$, then $m\angle C < 90°$ and $\triangle ABC$ is an acute triangle.

Obtuse ∆Th.

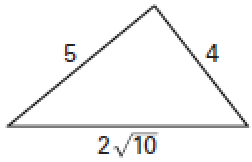


If $c^2 > a^2 + b^2$, then $m\angle C > 90°$ and $\triangle ABC$ is an obtuse triangle.

7.2 Cont.

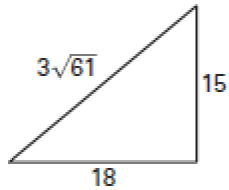
✔ ***Checkpoint*** **In Exercises 1 and 2, tell whether the triangle is a right triangle.**

1.



not a right triangle

2.



right triangle

3. Can segments with lengths of 6.1 inches, 9.4 inches, and 11.3 inches form a triangle? If so, would the triangle be *acute*, *right*, or *obtuse*?

Yes; obtuse

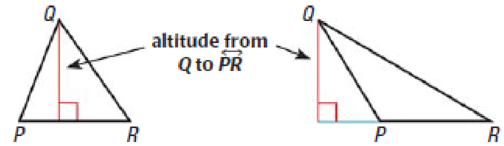
4. In Example 3, could you use triangles with side lengths 50 inches, 120 inches, and 130 inches to verify that you have perpendicular lines? *Explain.*

Yes; A triangle with side lengths 50 inches, 120 inches, and 130 inches is a right triangle. The right triangle shows that you have perpendicular lines.

7.3 Use Similar Right Triangles

Obj.: Use properties of the altitude of a right triangle.

Key Vocabulary

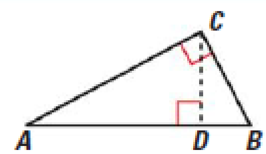


• **Altitude of a triangle -** An **altitude of a triangle** is the **perpendicular** segment from a **vertex** to the opposite side or to the line that contains the **opposite** side.

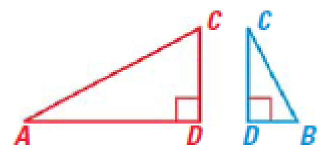
• **Geometric mean -** The **geometric mean** of **two** positive numbers *a* and *b* is the positive number **x** that satisfies $\frac{a}{x} = \frac{x}{b}$. So, $x^2 = ab$ and $x = \sqrt{ab}$.

• **Similar polygons -** Two polygons are **similar polygons** if corresponding **angles** are **congruent** and corresponding **side** lengths are **proportional**.

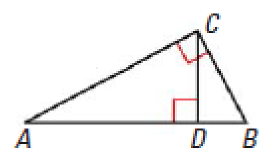
Alt. of rt. Δ→3~Δ



If the **altitude** is drawn to the **hypotenuse** of a **right** triangle, then the **two** triangles formed **are similar** to the **original** triangle and to each **other**.
ΔCBD ~ **ΔABC**, **ΔACD** ~ **ΔABC,** and **ΔCBD** ~ **ΔACD**.

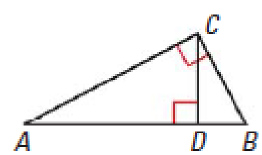


Geometric Mean (Altitude) Theorem
In a right triangle, the **altitude** from the **right angle** to the hypotenuse **divides** the **hypotenuse** into **two** segments.
The length of the **altitude** is the geometric **mean** of the lengths of the **two segments**.



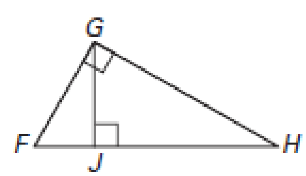
$$\frac{BD}{CD} = \frac{CD}{AD}$$

Geometric Mean (Leg) Theorem
In a right triangle, the **altitude** from the **right angle** to the hypotenuse **divides** the **hypotenuse** into two **segments**.
The length of **each leg** of the right triangle is the geometric **mean** of the **lengths** of the hypotenuse and the segment of the **hypotenuse** that is **adjacent to** the **leg**.

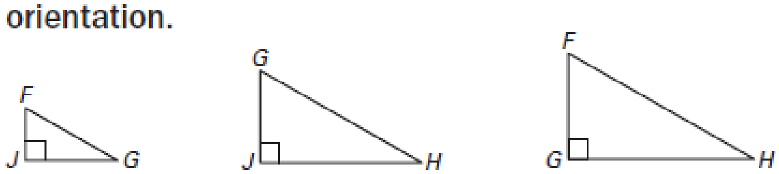


$$\frac{AB}{CB} = \frac{CB}{DB} \text{ and } \frac{AB}{AC} = \frac{AC}{AD}$$

EXAMPLE 1 Identify similar triangles
Identify the similar triangles in the diagram.



Solution Sketch the three similar right triangles so that the corresponding angles and sides have the same orientation.

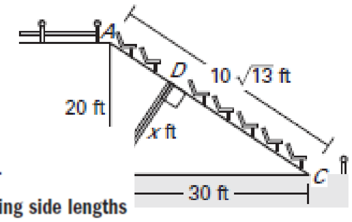


Δ *FJG* ~ Δ *GJH* ~ Δ *FGH*

EXAMPLE 2 Find the length of the altitude to the hypotenuse

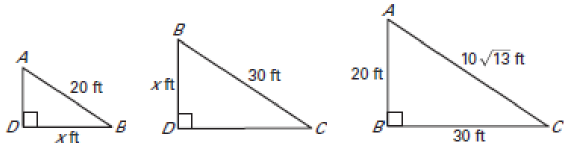
(7.3 cont.)

Stadium A cross section of a group of seats at a stadium shows a drainage pipe $\overline{BD}$ that leads from the seats to the inside of the stadium. What is the length of the pips?



Solution

Step 1 **Identify** the similar triangles and sketch them.



$\triangle$ ADB ~ $\triangle$ BDC ~ $\triangle$ ABC

Step 2 **Find** the value of x. Use the fact that $\triangle BDC \sim \triangle ABC$ to write a proportion.

$\frac{BD}{AB} = \frac{BC}{AC}$ — Corresponding side lengths of similar triangles are in proportion.

$\frac{x}{20} = \frac{30}{10\sqrt{13}}$ — Substitute.

$(10\sqrt{13})\,x = 20(30)$ — Cross Products Property

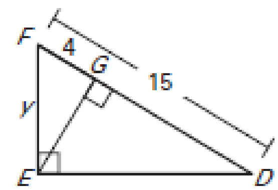
$x \approx 16.6$ — Approximate.

The length of the pipe is about 16.6 feet.

Notice that if you tried to write a proportion using $\triangle ADB$ and $\triangle BDC$, there would be two unknowns, so you would not be able to solve for x.

EXAMPLE 3 Use a geometric mean

Find the value of y. Write your answer in simplest radical form.



Solution

Write a proportion.

$$\frac{\text{length of hyp. of } \triangle FDE}{\text{length of hyp. of } \triangle FEG} = \frac{\text{length of shorter leg of } \triangle FDE}{\text{length of shorter leg of } \triangle FEG}$$

$\frac{15}{y} = \frac{y}{4}$ — Substitute.

$60 = y^2$ — Cross Products Property

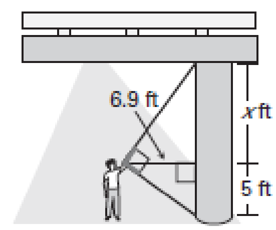
$\sqrt{60} = y$ — Take positive square roots.

$2\sqrt{15} = y$ — Simplify.

Notice that $\triangle FEG$ and $\triangle FDE$ both contain the side with length y, so these are the similar pair of triangles to use to solve for y.

EXAMPLE 4 Find a height using indirect measurement

Overpass To find the clearance under an overpass, you need to find the height of a concrete support beam. You use a cardboard square to line up the top and bottom of the beam. Your friend measures the vertical distance from the ground to your eye and the distance from you to the beam. Approximate the height of the beam.



Solution

By Theorem 7.6, you know that 6.9 is the geometric mean of x and 5.

$\frac{x}{6.9} = \frac{6.9}{5}$ — Write a proportion.

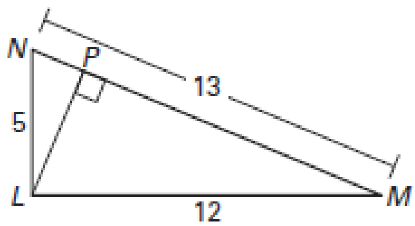
$x \approx 9.5$ — Solve for x.

So, the clearance under the overpass is $5 + x \approx 5 + 9.5 = 14.5$ feet.

7.3 Cont.

Checkpoint **Complete the following exercise.**

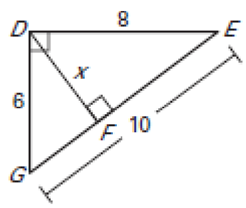
1. Identify the similar triangles in the diagram.



$\triangle NPL \sim \triangle LPM \sim \triangle NLM$

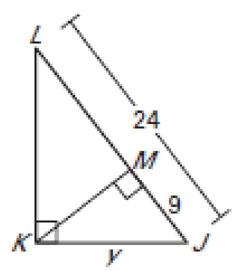
2. Identify the similar triangles. Then find the value of x.

$\triangle GFD \sim \triangle DFE \sim \triangle GDE$; $x = 4.8$



3. Find the value of y. Write your answer in simplest radical form.

$y = 6\sqrt{6}$



4. The distance from the ground to Larry's eyes is 4.5 feet. How far from the beam in Example 4 would he have to stand in order to measure its height?

about 6.7 feet

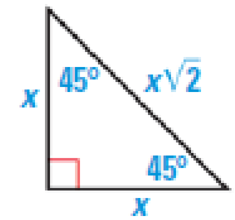
7.4 Special Right Triangles

Obj.: Use the relationships among the sides in special right triangles.

Key Vocabulary

• **Isosceles triangle -** A triangle with at least **two** **congruent** **sides**.

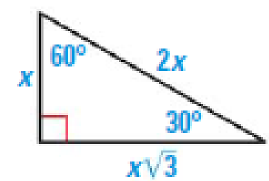
45°-45°-90° Triangle Theorem



In a **45°-45°-90°** triangle, the **hypotenuse** is $\sqrt{2}$ **times** as long as each **leg**.

hypotenuse = **leg $\cdot$ $\sqrt{2}$**

30°-60°-90° Triangle Theorem



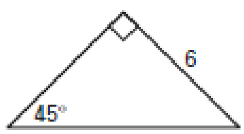
In a **30°-60°-90°** triangle, the **hypotenuse** is **twice** as long as the **shorter** leg, and the **longer** leg is **$\sqrt{3}$** times as long as the **shorter** leg.

hypotenuse = 2 ▪ **shorter** leg,   **longer** leg = shorter leg ▪ **$\sqrt{3}$**

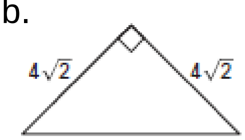
EXAMPLE 1 Find hypotenuse length in a 45°-45°-90° triangle

Find the length of the hypotenuse.

a.



b.



Remember the following properties of radicals:
$\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$;
$\sqrt{a \cdot a} = a$

Solution

a. By the Triangle Sum Theorem, the measure of the third angle must be 45°. Then the triangle is a 45°-45°-90° triangle, so by Theorem 7.8, the hypotenuse is $\sqrt{2}$ times as long as each leg.

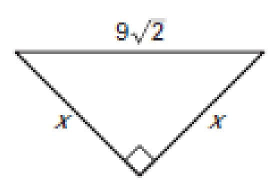
hypotenuse = leg $\cdot$ $\sqrt{2}$	**45°-45°-90° Triangle Theorem**
$= 6\sqrt{2}$	**Substitute.**

b. By the Base Angles Theorem and the Corollary to the Triangle Sum Theorem, the triangle is a 45°-45°-90° triangle.

hypotenuse = leg $\cdot$ $\sqrt{2}$	**45°-45°-90° Triangle Theorem**
$= 4\sqrt{2} \cdot \sqrt{2}$	**Substitute.**
$= 4 \cdot 2$	**Product of square roots**
$= 8$	**Simplify.**

EXAMPLE 2 Find leg lengths in a 45°-45°-90° triangle

Find the lengths of the legs in the triangle.



Solution

By the Base Angles Theorem and the Corollary to the Triangle Sum Theorem, the triangle is a 45°-45°-90° triangle.

hypotenuse = leg $\cdot$ $\sqrt{2}$	**45°-45°-90° Triangle Theorem**
$9\sqrt{2} = x \cdot \sqrt{2}$	**Substitute.**
$\dfrac{9\sqrt{2}}{\sqrt{2}} = \dfrac{x\sqrt{2}}{\sqrt{2}}$	**Divide each side by $\sqrt{2}$.**
$9 = x$	**Simplify.**

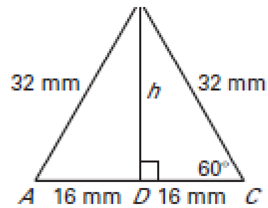
EXAMPLE 3 Find the height of an equilateral triangle

(7.4 cont.)

Music You make a guitar pick that resembles an equilateral triangle with side lengths of 32 millimeters. What is the approximate height of the pick?

Solution

Draw the equilateral triangle described. Its altitude forms the longer leg of two 30° - 60° -90° triangles. The length *h* of the altitude is approximately the height of the pick.



Remember that in an equilateral triangle, the altitude to a side is also the median to that side. So, altitude $\overline{BD}$ bisects $\overline{AC}$.

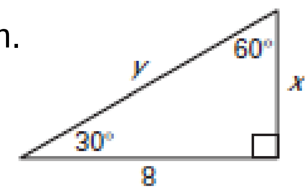
longer leg = shorter leg • $\sqrt{3}$

$h = 16 \cdot \sqrt{3} \approx 27.7$ mm

EXAMPLE 4 Find lengths in a 30°-60°-90° triangle

Find the values of *x* and *y*. Write your answer in simplest radical form.

Solution



Step 1 Find the value of *x*.

longer leg = shorter leg • $\sqrt{3}$

$8 = x\sqrt{3}$ Substitute.

$\frac{8}{\sqrt{3}} = x$ Divide each side by $\sqrt{3}$.

$\frac{8}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = x$ Multiply numerator and denominator by $\sqrt{3}$.

$\frac{8\sqrt{3}}{3} = x$ Multiply fractions.

Step 2 Find the value of *y*.

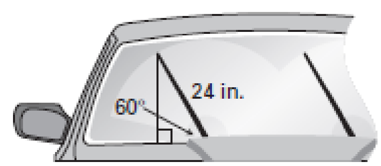
hypotenuse = 2 • shorter leg

$y = 2 \cdot \frac{8\sqrt{3}}{3} = \frac{16\sqrt{3}}{3}$

EXAMPLE 5 Find a height

Windshield wipers A car is turned off while the windshield wipers are moving. The 24 inch wipers stop, making a 60° angle with the bottom of the windshield. How far from the bottom of the windshield are the ends of the wipers?

Solution



The distance *d* is the length of the longer leg of a 30° - 60° -90° triangle.

The length of the hypotenuse is 24 inches.

hypotenuse = 2 • shorter leg — 30° - 60° -90° Triangle Theorem

$24 = 2 \cdot s$ — Substitute.

$12 = s$ — Divide each side by 2.

longer leg = shorter leg • $\sqrt{3}$ — 30° - 60° -90° Triangle Theorem

$d = 12\sqrt{3}$ — Substitute.

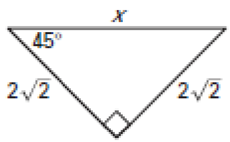
$d \approx 20.8$ — Approximate.

The ends of the wipers are about 20.8 inches from the bottom of the windshield.

7.4 Cont.

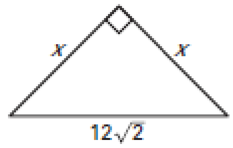
Checkpoint Find the value of the variable.

1.



$x = 4$

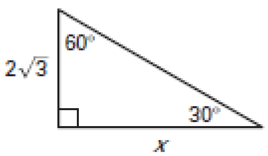
2.



$x = 12$

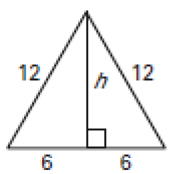
Checkpoint In Exercises 3 and 4, find the value of the variable.

3.



$x = 6$

4.



$h = 6\sqrt{3}$

5. In Example 5, how far from the bottom of the windshield are the ends of the wipers if they make a 30° angle with the bottom of the windshield?

12 inches

7.5 Apply the Tangent Ratio

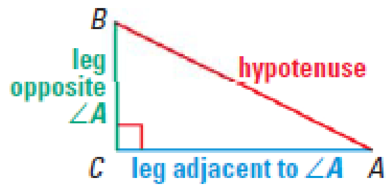
Obj.: Use the tangent ratio for indirect measurement.

Key Vocabulary

- **Trigonometric ratio -** A **trigonometric ratio** is a ratio of the lengths of two sides in a right triangle. You will use trigonometric ratios to find the measure of a side or an acute angle in a right triangle.
- **Tangent -** The ratio of the lengths of the legs in a right triangle is constant for a given angle measure. This ratio is called the **tangent** of the angle.

Tangent Ratio → tan

Let ΔABC be a right triangle with acute $\angle A$. The tangent of $\angle A$ (written as tan A) is defined as follows:

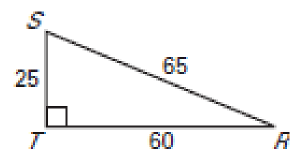


$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{BC}{AC}$$

$$\tan A = \frac{\text{opp.}}{\text{adj.}}$$

EXAMPLE 1 Find tangent ratios

Find tan S and tan R. Write each answer as a fraction and as a Decimal rounded to four places.



Solution

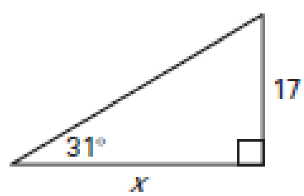
$$\tan S = \frac{\text{opp. } \angle S}{\text{adj. to } \angle S} = \frac{RT}{ST} = \frac{60}{25} = \frac{12}{5} = 2.4$$

$$\tan R = \frac{\text{opp. } \angle R}{\text{adj. to } \angle R} = \frac{ST}{RT} = \frac{25}{60} = \frac{5}{12} \approx 0.4167$$

Unless told otherwise, round values of trigonometric ratios to the ten-thousandths' place and round lengths to the tenths' place.

EXAMPLE 2 Find a leg length

Find the value of x.



Solution

Use the tangent of an acute angle to find a leg length.

$\tan 31° = \frac{\text{opp.}}{\text{adj.}}$ Write ratio for tangent of 31°.

$\tan 31° = \frac{17}{x}$ Substitute.

$x \cdot \tan 31° = 17$ Multiply each side by x.

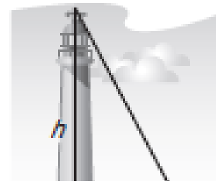
$x = \frac{17}{\tan 31°}$ Divide each side by tan 31°.

$x \approx \frac{17}{0.6009}$ Use a calculator to find tan 31°.

$x \approx 28.3$ Simplify.

EXAMPLE 3 Estimate height using tangent

Lighthouse Find the height *h* of the lighthouse to the nearest foot.



Solution

$\tan 62° = \frac{\text{opp.}}{\text{adj.}}$ Write ratio for tan 62°.

$\tan 62° = \frac{h}{100}$ Substitute.

$100 \cdot \tan 62° = h$ Multiply each side by 100.

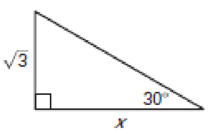
$188 \approx h$ Use a calculator and simplify.

EXAMPLE 4 Use a special right triangle to find a tangent

Use a special right triangle to find the tangent of a 30° angle.

Solution

Step 1 Choose $\sqrt{3}$ as the length of the shorter leg to simplify calculations. Use the 30°-60°-90° Triangle Theorem to find the length of the longer leg.



longer leg = shorter leg $\cdot \sqrt{3}$

$x = \sqrt{3} \cdot \sqrt{3} = 3$

Step 2 Find tan 30°.

$\tan 30° = \frac{\text{opp.}}{\text{adj.}}$ Write ratio for tangent of 30°.

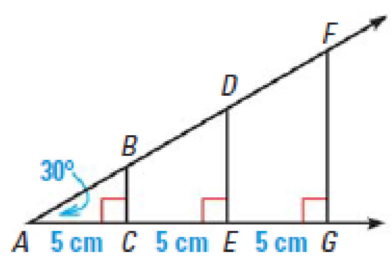
$\tan 30° = \frac{\sqrt{3}}{3}$ Substitute.

The tangent of any 30° angle is $\frac{\sqrt{3}}{3} \approx 0.5774$.

ACTIVITY RIGHT TRIANGLE RATIO

Materials: metric ruler, protractor, calculator

STEP 1 **Draw** a 30° angle and mark a point every 5 centimeters on a side as shown. Draw perpendicular segments through the 3 points.



STEP 2 **Measure** the legs of each right triangle. Copy and complete the table.

Triangle	Adjacent leg	Opposite leg	$\frac{\text{Opposite leg}}{\text{Adjacent leg}}$
$\triangle ABC$	5 cm	?	?
$\triangle ADE$	10 cm	?	?
$\triangle AFG$	15 cm	?	?

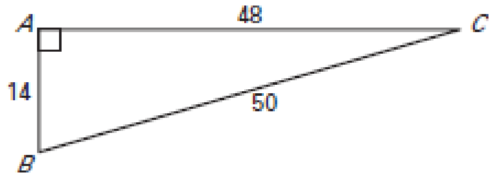
STEP 3 **Explain** why the proportions $\frac{BC}{DE} = \frac{AC}{AE}$ and $\frac{BC}{AC} = \frac{DE}{AE}$ are true.

STEP 4 **Make** a conjecture about the ratio of the lengths of the legs in a right triangle. Test your conjecture by using different acute angle measures.

7.5 Cont.

Checkpoint **Find tan *B* and tan *C*. Write each answer as a fraction and as a decimal rounded to four places.**

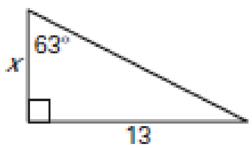
1.



$\tan B = \frac{24}{7} \approx 3.4286$, $\tan C = \frac{7}{24} \approx 0.2917$

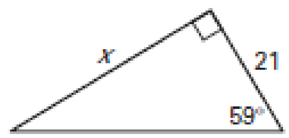
Checkpoint **In Exercises 2 and 3, find the value of *x*. Round to the nearest tenth.**

2.



$x \approx 6.6$

3.



$x \approx 34.9$

4. In Example 4, suppose the length of the shorter leg is 1 instead of $\sqrt{3}$. Show that the tangent of 30° is still equal to $\frac{\sqrt{3}}{3}$.

longer leg = shorter leg $\cdot \sqrt{3}$

$x = 1 \cdot \sqrt{3} = \sqrt{3}$

$\tan 30° = \frac{\text{opp.}}{\text{adj.}} = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

7.6 Apply the Sine and Cosine Ratios

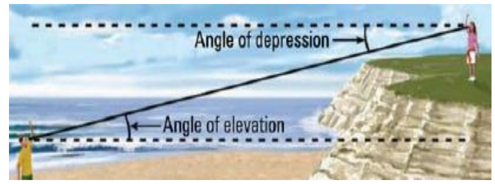
Obj.: Use the sine and cosine ratios.

Key Vocabulary

• **Sine, cosine -** The **sine** and **cosine** ratios are trigonometric ratios for acute angles that involve the lengths of a leg and the hypotenuse of a right triangle.

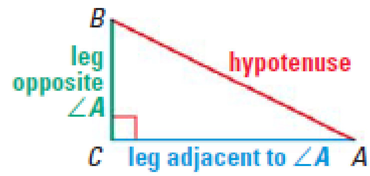
• **Angle of elevation -** If you look up at an object, the angle your line of sight makes with a horizontal line is called the **angle of** elevation.

• **Angle of depression -** If you look down at an object, the angle your line of sight makes with a horizontal line is called the **angle of** depression.



Sine and Cosine Ratios → sin & cos

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The sine of $\angle A$ and cosine of $\angle A$ (written sin A and cos A) are defined as follows:



$$\sin A = = \frac{\text{length of leg } \textbf{opposite } \angle A}{\text{length of } \textbf{hypotenuse}} = \frac{BC}{AB}$$

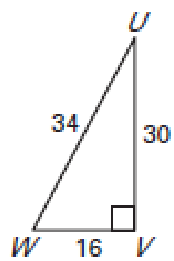
$$\sin A = \frac{\textbf{opp.}}{\textbf{hyp.}}$$

$$\cos A = = \frac{\text{length of leg } \textbf{adjacent } \angle A}{\text{length of } \textbf{hypotenuse}} = \frac{AC}{AB}$$

$$\cos A = \frac{\textbf{adj.}}{\textbf{hyp.}}$$

EXAMPLE 1 Find sine ratios

Find sin U and sin W. Write each answer as a fraction and as a decimal rounded to four places.



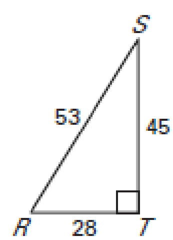
Solution

$$\sin U = \frac{\text{opp. } \angle U}{\text{hyp.}} = \frac{WV}{UW} = \frac{16}{34} = \frac{8}{17} \approx \underline{0.4706}$$

$$\sin W = \frac{\text{opp. } \angle W}{\text{hyp.}} = \frac{UV}{UW} = \frac{30}{34} = \frac{15}{17} \approx \underline{0.8824}$$

EXAMPLE 2 Find cosine ratios

Find cos S and cos R. Write each answer as a fraction and as a decimal rounded to four places.



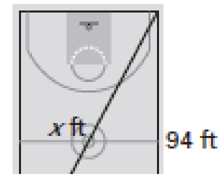
Solution

$$\cos S = \frac{\text{adj. to } \angle S}{\text{hyp.}} = \frac{ST}{SR} = \frac{45}{53} \approx \underline{0.8491}$$

$$\cos R = \frac{\text{adj. to } \angle R}{\text{hyp.}} = \frac{RT}{SR} = \frac{28}{53} \approx \underline{0.5283}$$

EXAMPLE 3 Use a trigonometric ratio to find a hypotenuse

Basketball You walk from one corner of a basketball court to the opposite corner. Write and solve a proportion using a trigonometric ratio to approximate the distance of the walk.



Solution

$\sin 62° = \frac{\text{opp.}}{\text{hyp.}}$ Write ratio for sine of 62°.

$\sin 62° = \frac{94}{x}$ Substitute.

$x \cdot \sin 62° = 94$ Multiply each side by x.

$x = \frac{94}{\sin 62°}$ Divide each side by sin 62°.

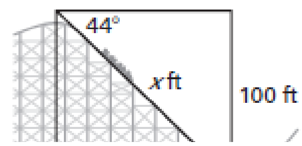
$x \approx \frac{94}{0.8829}$ Use a calculator to find sin 62°.

$x \approx 106.5$ Simplify.

The distance of the walk is about 106.5 feet.

EXAMPLE 4 Find a hypotenuse using an angle of depression

Roller Coaster You are at the top of a roller coaster 100 feet above the ground. The angle of depression is 44°. About how far do you ride down the hill?



Solution

$\sin 44° = \frac{\text{opp.}}{\text{hyp.}}$ Write ratio for sine of 44°.

$\sin 44° = \frac{100}{x}$ Substitute.

$x \cdot \sin 44° = 100$ Multiply each side by x.

$x = \frac{100}{\sin 44°}$ Divide each side by sin 44°.

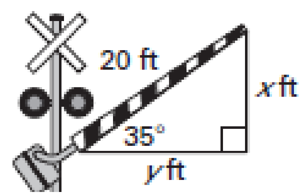
$x \approx \frac{100}{0.6947}$ Use a calculator to find sin 44°.

$x \approx 143.9$ Simplify.

ou ride about 144 feet down the hill.

EXAMPLE 5 Find leg lengths using an angle of elevation

Railroad A railroad crossing arm that is 20 feet long is stuck with an angle of elevation of 35°. Find the lengths x and y.



Solution

Step 1 Find x.

$\sin 35° = \frac{\text{opp.}}{\text{hyp.}}$ Write ratio for sine of 35°.

$\sin 35° = \frac{x}{20}$ Substitute.

$20 \cdot \sin 35° = x$ Multiply each side by 20.

$11.5 \approx x$ Use a calculator to simplify.

Step 2 Find y.

$\cos 35° = \frac{\text{adj.}}{\text{hyp.}}$ Write ratio for cosine of 35°.

$\cos 35° = \frac{y}{20}$ Substitute.

$20 \cdot \cos 35° = y$ Multiply each side by 20.

$16.4 \approx y$ Use a calculator to simplify.

EXAMPLE 6 Use a special right triangle to find a sine and cosine

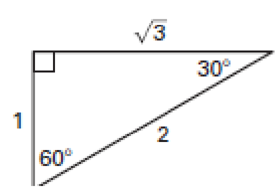
Use a special right triangle to find the sine and cosine of a 30° angle.

Solution

Use the 30°-60°-90° Triangle Theorem to draw a right triangle with side lengths of 1, $\sqrt{3}$, and 2. Then set up sine and cosine ratios for the 30° angle.

$\sin 30° = \frac{\text{opp.}}{\text{hyp.}} = \frac{1}{2} = 0.5000$

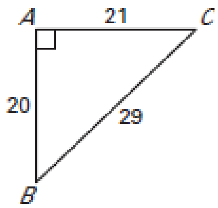
$\cos 30° = \frac{\text{adj.}}{\text{hyp.}} = \frac{\sqrt{3}}{2} \approx 0.8660$



7.6 Cont.

✔ *Checkpoint* Find sin *B*, sin *C*, cos *B*, and cos *C*. Write each answer as a fraction and as a decimal rounded to four places.

1.



$\sin B = \frac{21}{29} \approx 0.7241$, $\sin C = \frac{20}{29} \approx 0.6897$,

$\cos B = \frac{20}{29} \approx 0.6897$, $\cos C = \frac{21}{29} \approx 0.7241$

2. In Example 3, use the cosine ratio to approximate the width of the basketball court.

about 50 feet

3. Suppose the angle of depression in Example 4 is 72°. About how far would you ride down the hill?

about 105 feet

4. In Example 5, suppose the angle of elevation is 40°. What are the new lengths *x* and *y*?

$x \approx 12.9$, $y \approx 15.3$

5. Use a special right triangle to find the sine and cosine of a 60° angle.

$\sin 60° \approx 0.8660$

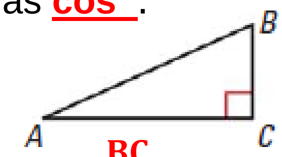
$\cos 60° = 0.5000$

7.7 Solve Right Triangles

Obj.: Use inverse tangent, sine, and cosine ratios.

Key Vocabulary

- **Solve a right triangle -** To **solve** **a right triangle** means to **find** the measures of **all** of its **sides** and **angles**.
- **Inverse tangent -** An inverse **trigonometric** ratio, abbreviated as **$\tan^{-1}$**.
- **Inverse sine -** An **inverse** trigonometric **ratio**, abbreviated as **$\sin^{-1}$**.
- **Inverse cosine -** An inverse trigonometric ratio, **abbreviated** as **$\cos^{-1}$**.



Inverse Trigonometric Ratios

Let $\angle A$ be an **acute** angle.

Inverse Tangent If **tan** $A = x$, then $\tan^{-1}x = m\angle A$. $\tan^{-1}\frac{BC}{AC} = m\angle A$

Inverse Sine If $\sin A = y$, then **$\sin^{-1}y$** $= m\angle A$. $\sin^{-1}\frac{BC}{AB} = m\angle A$

Inverse Cosine If $\cos A = z$, then $\cos^{-1}z =$ **$m\angle A$**. $\cos^{-1}\frac{BC}{AB} = m\angle A$

EXAMPLE 1 Use an inverse tangent to find an angle measure

Use a calculator to approximate the measure of ∠A to the nearest tenth of a degree.

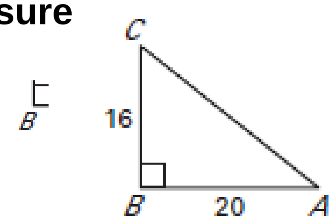
Solution

Because $\tan A = \frac{16}{20} = \frac{4}{5} = 0.8$,

$\tan^{-1} 0.8 = m\angle A$. Using a calculator,

$\tan^{-1} 0.8 \approx 38.65980825\ldots$.

So, the measure of $\angle A$ is approximately $38.7°$.



EXAMPLE 2 Use an inverse sine and an inverse cosine

Let ∠ *A* and ∠ *B* be acute angles in a right triangle. Use a calculator to approximate the measures of ∠ *A* and ∠ *B* to the nearest tenth of a degree.

a. $\sin A = 0.76$ **b.** $\cos B = 0.17$

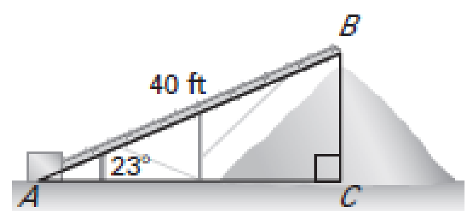
Solution

a. $m\angle A = \sin^{-1} 0.76 \approx 49.5°$

b. $m\angle B = \cos^{-1} 0.17 \approx 80.2°$

EXAMPLE 3 Solve a right triangle

Solve the right triangle. Round decimal answers to the nearest tenth.



Solution

Step 1 Find $m\angle B$ by using the Triangle Sum Theorem.

$180° = 90° + 23° + m\angle B$

$67° = m\angle B$

Step 2 Approximate BC using a sine ratio.

$\sin 23° = \frac{BC}{40}$ Write ratio for $\sin 23°$.

$40 \cdot \sin 23° = BC$ Multiply each side by 40.

$40 \cdot 0.3907 \approx BC$ Approximate $\sin 23°$.

$15.6 \approx BC$ Simplify and round answer.

Step 3 Approximate AC using a cosine ratio.

$\cos 23° = \frac{AC}{40}$ Write ratio for $\cos 23°$.

$40 \cdot \cos 23° = AC$ Multiply each side by 40.

$40 \cdot 0.9205 \approx AC$ Approximate $\cos 23°$.

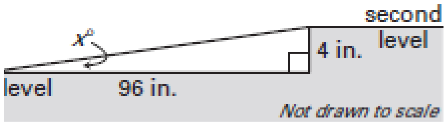
$36.8 \approx AC$ Simplify and round answer.

The angle measures are 23°, 67°, and 90°. The side lengths are 40 feet, about 15.6 feet, and about 36.8 feet.

EXAMPLE 4 Solve a real-world problem

Model Train You are building a track for a model train. You want the track to incline from the first level to the second level, 4 inches higher, in 96 inches. Is the angle of elevation less than 3^o?

Solution



Use the tangent and inverse tangent ratios to find the degree measure x of the incline.

$\tan x° = \frac{\text{opp.}}{\text{adj.}} = \frac{4}{96} \approx$ 0.0417

$x \approx \tan^{-1} 0.0417 \approx 2.4$

The incline is about 2.4°, so it is less than 3°.

Checkpoint Complete the following exercise.

1. In Example 1, use a calculator and an inverse tangent to approximate $m\angle C$ to the nearest tenth of a degree.

   $m\angle C \approx 51.3$

2. Find $m\angle D$ to the nearest tenth of a degree if $\sin D = 0.48$.

   $m\angle D \approx 28.7°$

3. Solve a right triangle that has a 50° angle and a 15 inch hypotenuse.

   Angles: 90°, 50°, and 40°; Side lengths: 15 in., about 9.6 in., and about 11.5 in.

4. In Example 4, suppose another incline rises 8 inches in 120 inches. Is the incline less than 3°?

   No, the incline is about 3.8°.