7-1

1. Plan

Objectives

1 To write ratios and solve proportions

Examples

1 Real-World Connection
2 Properties of Proportions
3 Solving for a Variable
4 Real-World Connection

Professional Development

Math Background

The properties of proportions are variations of applying the Multiplication and Addition Properties of Equality. Thus the same units must be within each ratio or in comparable positions in a proportion.

More Math Background: p. 364C

Lesson Planning and Resources

See p. 364E for a list of the resources that support this lesson.

PowerPoint

Bell Ringer Practice

Check Skills You'll Need
For intervention, direct students to:

Simplifying Expressions
Skills Handbook, p. 756

Midsegments of Triangles
Lesson 5-1: Example 1
Extra Skills, Word Problems, Proof Practice, Ch. 5

ALGEBRA

7-1 Ratios and Proportions

What You'll Learn

- To write ratios and solve proportions

. . . And Why

To find dimensions from a scale drawing, as in Example 4

Check Skills You'll Need GO for Help Skills Handbook p. 756 and Lesson 5-1

Simplify each ratio.

1. $\frac{2}{4}$ $\frac{1}{2}$
2. $\frac{8}{12}$ $\frac{2}{3}$
3. $\frac{6}{8}$ $\frac{3}{4}$
4. $\frac{10}{10}$ 1
5. 20 : 30 $\frac{2}{3}$
6. 8 to 2 4
7. 2 to 8 $\frac{1}{4}$
8. 12 : 9 $\frac{4}{3}$
9. Draw a triangle. Then draw its three midsegments to form a smaller triangle. How do the lengths of the sides of the smaller triangle compare to the lengths of the sides of the larger triangle?
Each side of the smaller △ is $\frac{1}{2}$ the length of a side of the larger △.

New Vocabulary • ratio • proportion • extended proportion • Cross-Product Property • scale drawing • scale

1 Using Ratios and Proportions

Vocabulary Tip

You can read $a : b$ as the ratio a to b.

A **ratio** is a comparison of two quantities. You can write the ratio of a to b or $a : b$ as the quotient $\frac{a}{b}$ when $b \neq 0$. Unless otherwise stated, the terms and expressions appearing in ratios in this book are assumed to be nonzero.

1 EXAMPLE Real-World Connection

Photography A photo that is 8 in. wide and $5\frac{1}{3}$ in. high is enlarged to a poster that is 2 ft wide and $1\frac{1}{3}$ ft high. What is the ratio of the width of the photo to the width of the poster?

$$\frac{\text{width of photo}}{\text{width of poster}} = \frac{8 \text{ in.}}{2 \text{ ft}} = \frac{8 \text{ in.}}{24 \text{ in.}} = \frac{8}{24} = \frac{1}{3}$$

The ratio of the width of the photo to the width of the poster is 1 : 3 or $\frac{1}{3}$.

Quick Check

1. What is the ratio of the height of the photo to the height of the poster? 1 : 3

Differentiated Instruction Solutions for All Learners

Special Needs L1	Below Level L2
Have each student make a scale drawing of their bedroom as shown in Example 4. Students need to measure their room and use graph paper to make their drawings. Each drawing must include a scale. **learning style: tactile**	Have students make a list of equivalent fractions and test them for equivalence by applying the properties of proportions. **learning style: verbal**

Vocabulary Tip

You can read both $\frac{a}{b} = \frac{c}{d}$ and $a : b = c : d$ as a is to b as c is to d.

A **proportion** is a statement that two ratios are equal. You can write a proportion in these forms:

$$\frac{a}{b} = \frac{c}{d} \text{ and } a : b = c : d$$

When three or more ratios are equal, you can write an **extended proportion.** For example, you could write the following:

$$\frac{6}{24} = \frac{4}{16} = \frac{1}{4}$$

Two equations are equivalent when either can be deduced from the other using the Properties of Equality. Several equations are equivalent to a proportion. Some of them are important enough to be called Properties of Proportions.

Key Concepts

Property **Properties of Proportions**

$\frac{a}{b} = \frac{c}{d}$ is equivalent to (1) $ad = bc$ (2) $\frac{b}{a} = \frac{d}{c}$

(3) $\frac{a}{c} = \frac{b}{d}$ (4) $\frac{a+b}{b} = \frac{c+d}{d}$

Multiplying both sides of $\frac{a}{b} = \frac{c}{d}$ by bd results in the first property, called the **Cross-Product Property.** You may state this property as "The product of the extremes is equal to the product of the means."

means

$a : b = c : d$

extremes

$\frac{a}{b} = \frac{c}{d}$

$ad = bc$

2 EXAMPLE Properties of Proportions

Algebra If $\frac{x}{y} = \frac{5}{6}$, complete each statement.

a. $6x = ▪$
$6x = 5y$

b. $\frac{y}{x} = \frac{▪}{▪}$
$\frac{y}{x} = \frac{6}{5}$

c. $\frac{x}{5} = \frac{▪}{▪}$
$\frac{x}{5} = \frac{y}{6}$

d. $\frac{x+y}{y} = \frac{▪}{▪}$
$\frac{x+y}{y} = \frac{11}{6}$

Quick Check 2 **Critical Thinking** Write two proportions that are equivalent to $\frac{m}{4} = \frac{n}{11}$.
Answers may vary. Sample: $\frac{4}{m} = \frac{11}{n}$, $\frac{m+4}{4} = \frac{n+11}{11}$

You solve a proportion by finding the value of the variable.

3 EXAMPLE Solving for a Variable

Algebra Solve each proportion.

a. $\frac{x}{5} = \frac{12}{7}$

$7x = 5(12)$ ⟵ **Cross-Product Property** ⟶

$7x = 60$

$x = \frac{60}{7}$

b. $\frac{y+3}{8} = \frac{y}{4}$

$4(y + 3) = 8y$

$4y + 12 = 8y$

$12 = 4y$

$y = 3$

Quick Check 3 Solve each proportion.

a. $\frac{5}{z} = \frac{20}{3}$ **0.75**

b. $\frac{18}{n+6} = \frac{6}{n}$ **3**

2. Teach

Guided Instruction

1 EXAMPLE Math Tip

Students should understand that units of measurement must be the same.

2 EXAMPLE Teaching Tip

Replace the variables with numbers to verify that the proportions are equivalent.

4 EXAMPLE

Point out that the scale compares inches to feet, so each ratio has inches in the numerator and feet in the denominator.

PowerPoint

Additional Examples

1 A scale model of a car is 4 in. long. The actual car is 15 ft long. What is the ratio of the length of the model to the length of the car? **1 : 45**

2 Complete: If $\frac{a}{4} = \frac{12}{b}$, then $\frac{b}{12} = \frac{?}{?}$. $\frac{4}{a}$

3 Solve each proportion.

a. $\frac{2}{5} = \frac{n}{35}$ **14**

b. $\frac{x+1}{3} = \frac{x}{2}$ **2**

4 Two cities are $3\frac{1}{2}$ in. apart on a map with the scale 1 in. = 50 mi. Find the actual distance. **175 mi**

Resources

- Daily Notetaking Guide 7-1 L3
- Daily Notetaking Guide 7-1—Adapted Instruction L1

Closure

A baseball batting average is the ratio of hits to at-bats, expressed as a decimal. If a player with 540 at-bats has a batting average of 0.350, how many hits did the player make? **189 hits**

Advanced Learners L4

Ask: If $\frac{a}{b} = \frac{c}{d}$, does $\frac{a^2}{b^2}$ equal $\frac{c^2}{d^2}$? Does $\frac{a}{b}$ equal $\frac{a^2}{b^2}$?

Yes; yes, only if $a = b$ or $a = 0$

learning style: verbal

English Language Learners ELL

Point out that a proportion involves two ratios that are equal. Sometimes the term proportion is used incorrectly for a ratio, such as "the proportion of students who have cell phones is 3 out of 5."

learning style: verbal

3. Practice

Assignment Guide

1 A B 1-55
C Challenge 56-61

Test Prep 62-66
Mixed Review 67-77

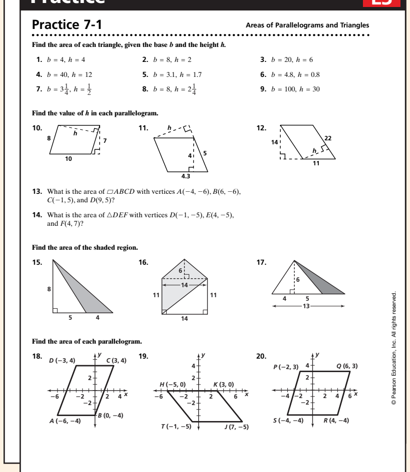
Homework Quick Check
To check students' understanding of key skills and concepts, go over Exercises 16, 22, 26, 34, 44.

Error Prevention!
Exercise 2 Ask: *Why is writing the ratio as $\frac{6}{185}$ incorrect?* **185 ft first must be converted to inches, or 6 in. to feet.**

Auditory Learners
Exercises 3–11 Have students explain their reasoning aloud to the class, citing the Property of Proportions.

Differentiated Instruction Resources

GPS Guided Problem Solving L3
Enrichment L4
Reteaching L2
Adapted Practice L1
Practice L3



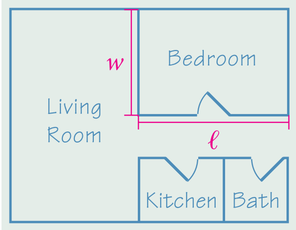
In a **scale drawing,** the **scale** compares each length in the drawing to the actual length. The lengths used in a scale can be in different units. A scale might be written as 1 in. to 100 mi, 1 in. = 12 ft, or 1 mm : 1 m. You can use proportions to find the actual dimensions represented in a scale drawing.

Test-Taking Tip
If you use a ruler and your answer does not match any answer choice, first check that you measured correctly.

4 EXAMPLE Real-World Connection

Multiple Choice Use a ruler to measure the length ℓ of the bedroom in the scale drawing. What is the length of the actual bedroom?

Ⓐ 10 ft Ⓑ 14 ft Ⓒ $18\frac{2}{7}$ ft Ⓓ $24\frac{2}{5}$ ft



Scale: 1 in. = 16 ft

An inch ruler shows that $\ell = \frac{7}{8}$ in.

$\frac{1}{16} = \frac{\frac{7}{8}}{\ell}$ ← $\frac{\text{drawing length (in.)}}{\text{actual length (ft)}}$

$\ell = 16\left(\frac{7}{8}\right)$ **Cross-Product Property**

$\ell = 14$

The actual bedroom is 14 ft long. The correct answer is B.

Quick Check

4 Find the width of the actual bedroom. **10 ft**

EXERCISES

For more exercises, see *Extra Skill, Word Problem, and Proof Practice.*

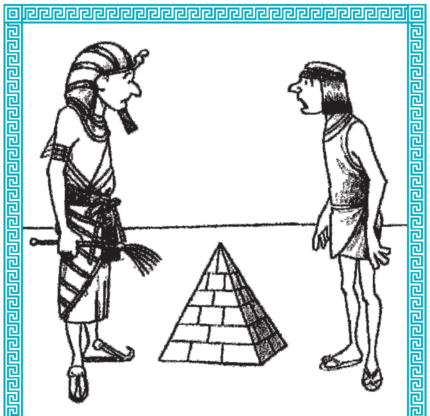
Practice and Problem Solving

A Practice by Example

Example 1 (page 366)

GO for Help

1. The base of the pyramid at the right is a square whose sides measure 0.675 m. The intent was for the sides to measure 675 m. What is the ratio of the length of a base side in the small pyramid to the length of a base side in the intended pyramid? **1 : 1000**


"We had a little problem with the decimal point."

2. **Models** The Leaning Tower of Pisa in Italy is about 185 ft tall. A model of the Leaning Tower is 6 in. tall. What is the ratio of the height of the model to the height of the real tower? **1 : 370**

Example 2 (page 367)

Algebra **If $\frac{a}{b} = \frac{3}{4}$, complete each statement.**

3. $4a = ■$ **$3b$**
4. $\frac{b}{a} = ■$ **$\frac{4}{3}$**
5. $\frac{a}{3} = ■$ **$\frac{b}{4}$**
6. $\frac{4}{3} = ■$ **$\frac{b}{a}$**
7. $\frac{4}{b} = ■$ **$\frac{3}{a}$**
8. $3b = ■$ **$4a$**
9. $\frac{a+b}{b} = ■$ **$\frac{7}{4}$**
10. $\frac{a}{a+b} = ■$ **$\frac{3}{7}$**
11. $\frac{a+3}{3} = ■$ **$\frac{b+4}{4}$**

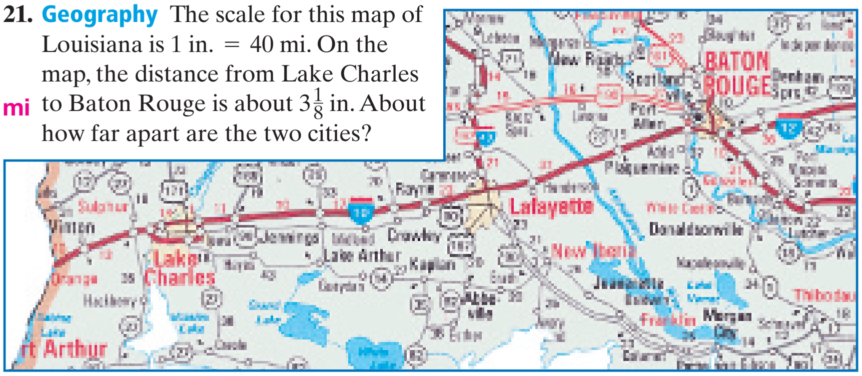
Example 3 (page 367)

x^2 **Algebra** **Solve each proportion.**

12. $\frac{x}{2} = \frac{8}{4}$ **4**
13. $\frac{9}{5} = \frac{3}{x}$ **$1\frac{2}{3}$**
14. $\frac{1}{3} = \frac{x}{12}$ **4**
15. $\frac{5}{x} = \frac{8}{11}$ **6.875**
16. $\frac{4}{x} = \frac{5}{9}$ **7.2**
17. $\frac{5}{6} = \frac{6}{x}$ **7.2**
18. $\frac{x+3}{3} = \frac{10+4}{4}$ **7.5**
19. $\frac{x+7}{7} = \frac{15}{5}$ **14**
20. $\frac{3}{5} = \frac{6}{x+3}$ **7**

Example 4 (page 368)

21. **Geography** The scale for this map of Louisiana is 1 in. = 40 mi. On the map, the distance from Lake Charles to Baton Rouge is about $3\frac{1}{8}$ in. About how far apart are the two cities? **125 mi**



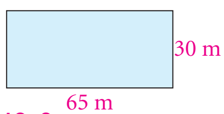
Map Reading **Measure the map distance. Then find the actual distance.**

22. Morgan City to Rayne **about 75 mi**
23. Vinton to New Roads **about 135 mi**
24. Kaplan to Plaquemine **about 67.5 mi**

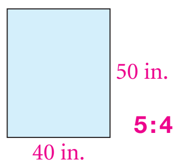
25. **Design** You want to make a scale drawing of your bedroom to help you arrange your furniture. You decide on a scale of 3 in. = 2 ft. Your bedroom is a 12 ft-by-15 ft rectangle. What should be its dimensions in your scale drawing? **18 in. by 22.5 in.**

B Apply Your Skills

For each rectangle, find the ratio of the longer side to the shorter side.

26.


13:6

27.


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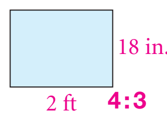
28.


4:3

29. **Multiple Choice** The diameter of a dinner plate is 1 ft. In a dollhouse set, the diameter of a dinner plate is $1\frac{1}{4}$ in. What is the ratio of the diameter of the dollhouse plate to the diameter of the full-size plate? **A**

Ⓐ $\frac{5}{48}$ Ⓑ $\frac{4}{5}$ Ⓒ $\frac{5}{4}$ Ⓓ $\frac{48}{5}$

Complete each statement.

30. If $\frac{x}{7} = \frac{y}{3}$, then $\frac{x}{y} = \blacksquare$. **$\frac{7}{3}$**
31. If $4m = 9n$, then $\frac{m}{n} = \blacksquare$. **$\frac{9}{4}$**
32. If $\frac{30}{t} = \frac{18}{r}$, then $\frac{t}{r} = \blacksquare$. **$\frac{30}{18}$**
33. If $\frac{a+5}{5} = \frac{b+2}{2}$, then $\frac{a}{5} = \blacksquare$. **$\frac{b}{2}$**

34. **Writing** Use a map in your classroom or a map from a textbook. Explain how to use a ruler and the scale of the map to approximate an actual distance. Give an example. (If you do not have access to another map, use the map above.) **Check students' work.**

GO Online Homework Help
Visit: PHSchool.com
Web Code: aue-0701

x^2 **Algebra** **Solve each proportion.**

35. $\frac{y}{10} = \frac{15}{25}$ **6**
36. $\frac{9}{24} = \frac{12}{n}$ **32**
37. $\frac{11}{14} = \frac{b}{21}$ **16.5**
38. $\frac{5}{x-3} = \frac{10}{x}$ **6**
39. $\frac{8}{n+4} = \frac{4}{n}$ **4**
40. $\frac{2b-1}{5} = \frac{b}{12}$ **$\frac{12}{19}$**
41. $\frac{x-3}{3} = \frac{2}{x+2}$ **4, −3**
42. $\frac{3-4x}{1+5x} = \frac{1}{2+3x}$ **$\frac{1}{2}, -\frac{5}{6}$**

Online lesson quiz, PHSchool.com, Web Code: aua-0701

Exercises 12–20 Remind students to check their answers by substituting the solution for x in the original proportion. Ask: *How can you tell whether your answer is correct?* **The proportion with the substituted value is true.**

Exercise 21 Provide students with copies of a map of your own state. Have them write and solve problems similar to the one in Exercise 21 and then exchange and solve one another's problems.

Exercise 29 Ask: *In what order should the ratio list the dinner plate sizes?* **dollhouse to full-size**

Exercise 50 The baskets on each end of a basketball court are 10 ft above the floor. Have students explain how they would represent baskets on their scale drawings.

Connection to Algebra

Exercises 56–58 You can do these as class exercises by having students provide steps and reasons in prooflike fashion. For each exercise, a possible first step is using the Cross-Product Property.

Exercises 59–61 If students need help getting started, suggest that they break each extended proportion into two proportions.

4. Assess & Reteach

PowerPoint

Lesson Quiz

1. A scale model of a boat is 9 in. long. The boat's actual length is 60 ft. Find the ratio of the length of the scale model to the length of the boat. **1 : 80**
2. Solve the proportion $\frac{10}{8} = \frac{15}{x}$. **12**
3. A map uses the scale 1 cm = 20 mi. A county is 90 mi wide. How wide is the county on the map? **$4\frac{1}{2}$ cm**

If $\frac{x}{y} = \frac{7}{11}$, complete each of the following.

4. $\frac{y}{x} = \frac{?}{?}$ **$\frac{11}{7}$**
5. $7y = ?$ **11x**
6. $\frac{x+y}{y} = \frac{?}{11}$ **18**

Alternative Assessment

Show the class a wall map of the continental United States, making sure that you cover the scale. Ask students to measure the width of the United States on the map from San Francisco, California, to Washington, D.C. Point out that the actual width of the United States is about 3000 mi. Have students use this information to estimate the scale on the map. When they finish their work, uncover the scale so that students can compare it with their answers.

Real-World Connection

This sandwich shop is on Museum Wharf in Boston, Massachusetts.

43. **Models** The sandwich shop at the left is 40 ft tall. The shop is an enlargement of an actual milk bottle. The scale used in construction is 5 ft = 2 cm. Find the height of the actual milk bottle. **16 cm**

44. **Geography** GPS Students at the University of Minnesota in Minneapolis built a model globe 42 ft in diameter using a scale of 1 : 1,000,000. About how tall is Mount Everest on the model? (Mount Everest is about 29,000 ft tall.) **0.348 in.**

Complete each extended proportion.

45. $\frac{8}{12} = \frac{6}{\square} = \frac{12}{\square}$ **9; 18**
46. $\frac{\square}{15} = \frac{15}{25} = \frac{\square}{20}$ **9; 12**
47. $\frac{14}{\square} = \frac{\square}{12} = \frac{35}{20}$ **8; 21**

Games **Choose a scale and make a scale drawing of the playing region.** **48–51. See margin.**

48. A pool table is 5 ft by 10 ft.
49. A bowling lane is 3.5 ft by 60 ft.
50. A basketball court is 92 ft by 50 ft.
51. A football field is 160 ft by 120 yd.

52. **Error Analysis** One rectangle has length 3 in. and width 4 ft. Another rectangle has length 3 ft and width 4 yd. Elaine claims that the two rectangles are similar because their corresponding angles are congruent and their corresponding sides are in proportion. Explain why Elaine's reasoning is incorrect. **Elaine did not convert units, and thought the ratios equaled 1.**

If $\frac{a}{b} = \frac{c}{d}$, complete each statement.

53. $\frac{a+b}{c+d} = \square$ **$\frac{b}{d}$ or $\frac{a}{c}$**
54. $\frac{a+c}{b+d} = \square$ **$\frac{c}{d}$ or $\frac{a}{b}$**
55. $\frac{a+2b}{b} = \square$ **$\frac{c+2d}{d}$**

Challenge **Algebra** **Justify the indicated property of proportions.** **56–58. See margin p. 421.**

56. Property (2): If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$.
57. Property (3): If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$.
58. Property (4): If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$.

Solve each extended proportion for x and y with $x > 0$ and $y > 0$.

59. $\frac{x}{6} = \frac{x+10}{18} = \frac{4x}{y}$ **$x = 5; y = 24$**
60. $\frac{x}{5} = \frac{9}{y} = \frac{y}{25}$ **$x = 3; y = 15$**
61. $\frac{1}{x} = \frac{4}{x+9} = \frac{7}{y}$ **$x = 3; y = 21$**

Test Prep

Multiple Choice

Solve each proportion.

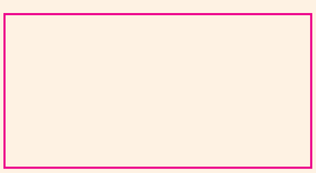
62. $\frac{21}{x} = \frac{7}{3}$ **C** A. 3 B. 7 C. 9 D. 14
63. $\frac{4}{x-1} = \frac{1}{x}$ **G** F. -3 G. $-\frac{1}{3}$ H. $\frac{1}{3}$ J. 3
64. $\frac{x}{x+6} = \frac{2}{3}$ **D** A. 4 B. 6 C. 8 D. 12
65. $\frac{3}{8} = \frac{x+3}{9}$ **H** F. $3\frac{3}{8}$ G. 3 H. $\frac{3}{8}$ J. $\frac{1}{3}$

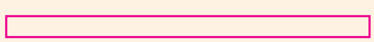
Short Response

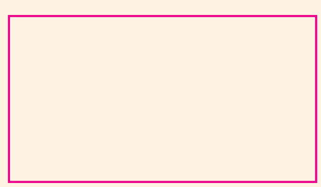
66. A map of Long Island has the scale 2.75 cm = 16 km. On the map, Target Rock is 23.2 cm from Lake Montauk.
 a. Write a proportion that you can solve to determine the actual distance from Target Rock to Lake Montauk. **a–b. See left.**
 b. Find the actual distance. Round your answer to the nearest kilometer.

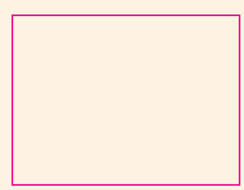
66a. $\frac{2.75}{16} = \frac{23.2}{x}$ (OR equivalent proportion)
b. 135 km

48–51. Answers may vary. Samples are given.

48.

Scale 1 in. = 5 ft

49.

Scale 1 cm = 10 ft

50.

Scale 1 in. = 46 ft

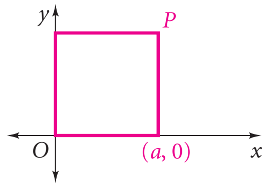
51.

Scale 1 cm = 32 ft

Mixed Review

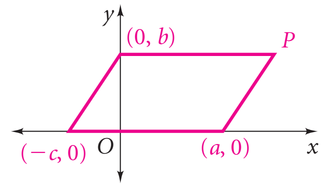
GO for Help

Lesson 6-7 **Give the coordinates for point P without using any new variables.**

67. square **(a, a)**



68. parallelogram **$(a + c, b)$**



Lesson 6-1 **Graph each quadrilateral $ABCD$. Classify $ABCD$ in as many ways as possible.** **69–72. See margin.**

69. $A(-1, -2), B(3, -2), C(1, 4), D(-3, 4)$

70. $A(2, -1), B(6, 2), C(8, 2), D(10, -1)$

71. $A(-7, 1), B(-5, 3), C(0, -2), D(-2, -4)$

72. $A(1, 1), B(-4, 4), C(1, 7), D(6, 4)$

Lesson 5-4 **In each exercise, identify two statements that contradict each other.**

73. I. $\triangle PQR$ is isosceles. **I and III**
II. $\triangle PQR$ is an obtuse triangle.
III. $\triangle PQR$ is scalene.

74. I. $\angle 1 \cong \angle 2$ **II and III**
II. $\angle 1$ and $\angle 2$ are complementary.
III. $m\angle 1 + m\angle 2 = 180$

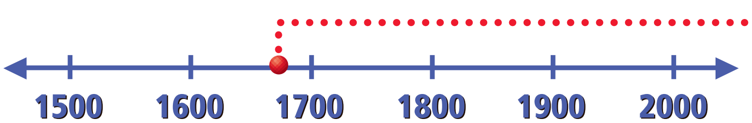
Write (a) the inverse and (b) the contrapositive of each statement. **75–77. See margin.**

75. If an angle is acute, then it has measure between 0 and 90.

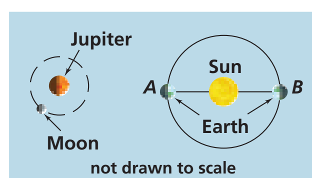
76. If two lines are parallel, then they are coplanar.

77. If two angles are complementary, then both angles are acute.

A Point in Time



In 1675, Danish astronomer Ole Römer used proportions to estimate the speed of light. He carefully measured the movements of Jupiter's moons. With Earth at point B, a moon emerged from behind Jupiter 16.6 min later than when Earth was at point A. He reasoned that it must have taken 16.6 min for the light to travel from point A to point B. Using proportions, Römer estimated the speed of light to be 150,000 mi/s. This estimate is about 81% of today's accepted value of 186,282 mi/s.



Go Online PHSchool.com **For:** Information about the speed of light **Web Code:** aue-2032

56. $\frac{a}{b} = \frac{c}{d}$ (Given); $ad = bc$ (Cross-Product Prop.); $bc = ad$ (Symm. Prop. of =); $\frac{bc}{ac} = \frac{ad}{ac}$ (Div. Prop. of =); $\frac{b}{a} = \frac{d}{c}$ (Simplify.)

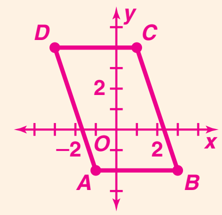
57. $\frac{a}{b} = \frac{c}{d}$ (Given); $ad = bc$ (Cross-Prod. Prop.); $\frac{ad}{cd} = \frac{bc}{cd}$ (Div. Prop. of =); $\frac{a}{c} = \frac{b}{d}$ (Simplify.)

58. $\frac{a}{b} = \frac{c}{d}$ (Given); $\frac{a}{b} + 1 = \frac{c}{d} + 1$ (Add. Prop. of =); $\frac{a}{b} + \frac{b}{b} = \frac{c}{d} + \frac{d}{d}$ (Subst.); $\frac{a+b}{b} = \frac{c+d}{d}$ (Simplify.)

Test Prep

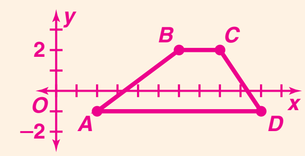
Resources
For additional practice with a variety of test item formats:
- Standardized Test Prep, p. 411
- Test-Taking Strategies, p. 406
- Test-Taking Strategies with Transparencies

69.



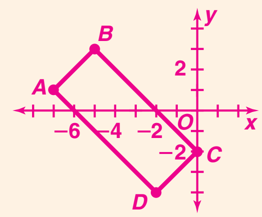
parallelogram

70.



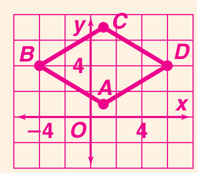
trapezoid

71.



rectangle, parallelogram

72.



rhombus, parallelogram

75. **a.** If an $\angle$ is not acute, then it does not have measure between 0 and 90.
b. If an $\angle$ does not have measure between 0 and 90, then it is not acute.

76. **a.** If two lines are not $\parallel$, then they are not coplanar.
b. If two lines are not coplanar, then they are not $\parallel$.

77. **a.** If two $\angle$s are not compl., then the $\angle$s are not both acute.
b. If two $\angle$s are not both acute, then they are not compl.