

What You'll Learn

You'll learn to identify the relationships between the sides and angles of a triangle.

Why It's Important

Surveying Triangle relationships are important in undersea surveying. See Example 2.

Florists often use triangles as guides in their flower arrangements. There are special relationships between the side measures and angle measures of each triangle. You will discover these relationships in the following activity.

Suppose in triangle ABC , the inequality $AC > BC$ holds true. Is there a similar relationship between the angles $\angle B$ and $\angle A$, which are across from those sides?

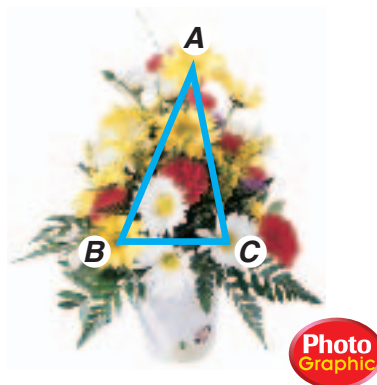


Photo Graphic

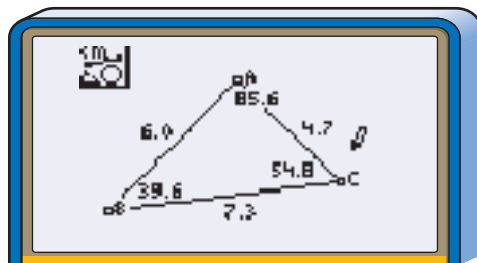
Graphing Calculator Tutorial
See pp. 782–785.

Graphing Calculator Exploration

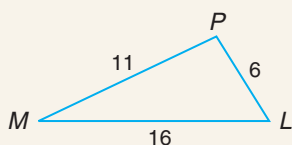
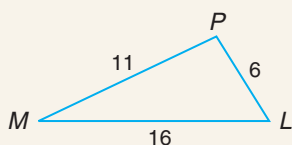
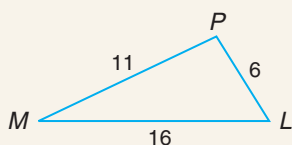
- Step 1** Use the Triangle tool on **F2** to draw and label $\triangle ABC$.
- Step 2** Select Measure from the **F5** menu. Then use the Distance & Length tool and the Angle tool on **F6** to display the measures of the sides and angles of $\triangle ABC$.

Try These

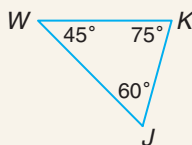
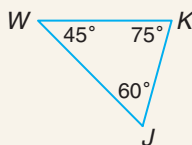
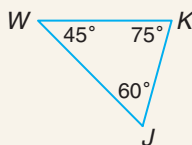
- Refer to the triangle drawn using the steps above.
 - What is the measure of the largest angle in your triangle?
 - What is the measure of the side opposite the largest angle?
 - What is the measure of the smallest angle in your triangle?
 - What is the measure of the side opposite the smallest angle?
- Drag vertex A to a different location.
 - What are the lengths of the longest and shortest sides of the new triangle?
 - What can you conclude about the measures of the angles of a triangle and the measures of the sides opposite these angles?
- Use the Perpendicular Bisector tool on **F3** to draw the perpendicular bisector of side AB . Drag vertex C very close to the perpendicular bisector. What do you observe about the measures of the sides and angles?



The observations you made in the previous activity suggest the following theorem.

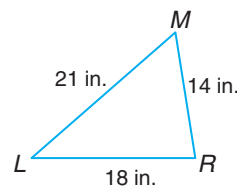
Theorem 7-6	Words: If the measures of three sides of a triangle are unequal, then the measures of the angles opposite those sides are unequal in the same order.					
	<table border="0"> <tr> <td style="vertical-align: top;">Model:</td> <td style="text-align: center;">  </td> <td style="vertical-align: top;">Symbols:</td> </tr> <tr> <td></td> <td></td> <td style="text-align: center;"> $PL < MP < LM$ $m\angle M < m\angle L < m\angle P$ </td> </tr> </table>	Model:		Symbols:		
Model:		Symbols:				
		$PL < MP < LM$ $m\angle M < m\angle L < m\angle P$				

The converse of Theorem 7-6 is also true.

Theorem 7-7	Words: If the measures of three angles of a triangle are unequal, then the measures of the sides opposite those angles are unequal in the same order.					
	<table border="0"> <tr> <td style="vertical-align: top;">Model:</td> <td style="text-align: center;">  </td> <td style="vertical-align: top;">Symbols:</td> </tr> <tr> <td></td> <td></td> <td style="text-align: center;"> $m\angle W < m\angle J < m\angle K$ $JK < KW < WJ$ </td> </tr> </table>	Model:		Symbols:		
Model:		Symbols:				
		$m\angle W < m\angle J < m\angle K$ $JK < KW < WJ$				

Example

1 In $\triangle LMR$, list the angles in order from least to greatest measure.



First, write the segment measures in order from least to greatest.

$$MR < RL < LM$$

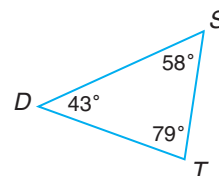
Then, use Theorem 7-6 to write the measures of the angles opposite those sides in the same order.

$$\begin{array}{ccc}
 \downarrow & \downarrow & \downarrow \\
 m\angle L & < & m\angle M < m\angle R
 \end{array}$$

The angles in order from least to greatest measure are $\angle L$, $\angle M$, and $\angle R$.

Your Turn

a. In $\triangle DST$, list the sides in order from least to greatest measure.



Example

Surveying Link



Undersea Robot Vehicle, *Oberon*

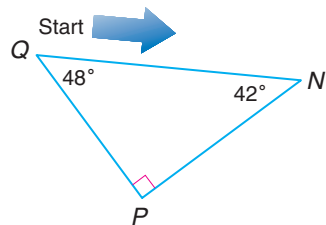
2

Scientists are developing automated robots for underwater surveying. These undersea vehicles will be guided along by sonar and cameras. If $\triangle NPQ$ represents the intended course for an undersea vehicle, which segment of the trip will be the longest?

First, write the angle measures in order from least to greatest.

Then, use Theorem 7-7 to write the measures of the sides opposite those angles in the same order.

So, \overline{QN} , the first segment of the course, will be the longest.

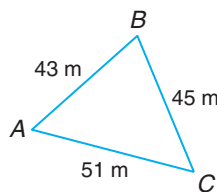


$$m\angle N < m\angle Q < m\angle P$$

$$PQ < NP < QN$$

Your Turn

b. If $\triangle ABC$ represents a course for an undersea vehicle, which turn will be the sharpest—that is, which angle has the least measure?



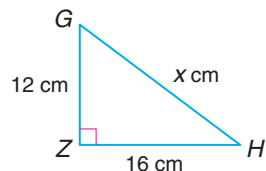
Example 2 illustrates an argument for the following theorem.

Theorem 7-8	Words: In a right triangle, the hypotenuse is the side with the greatest measure.	Symbols: $WY > YX$ $WY > XW$
	Model:	

Check for Understanding

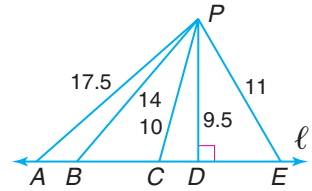
Communicating Mathematics

1. Name the angle opposite \overline{ZH} in $\triangle GHZ$.
2. Choose the correct value for x in $\triangle GHZ$ without using the Pythagorean Theorem: 14, 16, or 20. Explain how you made your choice.



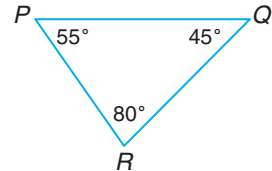
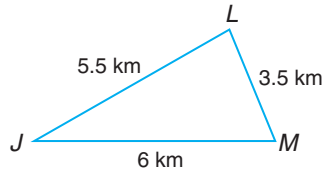
Exercises 1-2

3. **Writing Math** Identify the shortest segment from point P to line ℓ . Write a conjecture in your journal about the shortest segment from a point to a line.



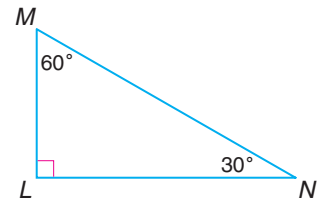
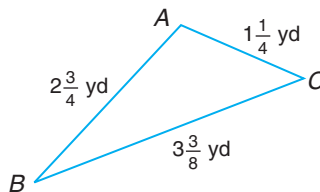
Guided Practice
Example 1

4. List the angles in order from least to greatest measure.
5. List the sides in order from least to greatest measure.



Example 2

6. Identify the angle with the greatest measure.
7. Identify the side with the greatest measure.



Lombard Street, San Francisco

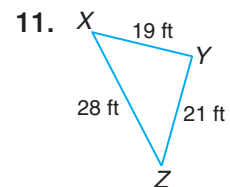
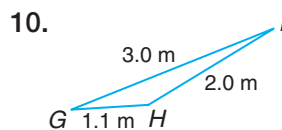
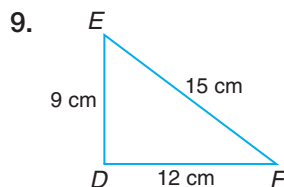
8. **Driving** The road sign indicates that a steep hill is ahead.
- Use a ruler to measure the sides of $\triangle STE$ to the nearest millimeter. Then list the sides in order from least to greatest measure.
 - List the angles in order from least to greatest measure. **Example 2**



Exercises

Practice

List the angles in order from least to greatest measure.



Homework Help

For Exercises

See Examples

9–11, 15–17,
22, 23

1

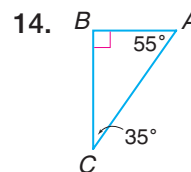
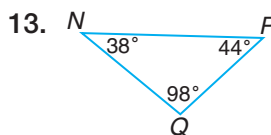
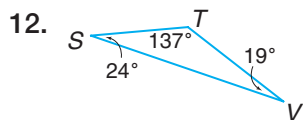
12–14, 18–20,
21, 24

2

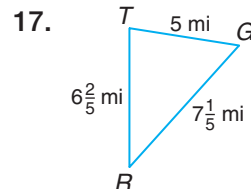
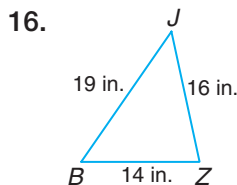
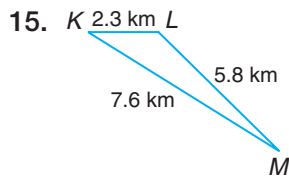
Extra Practice

See page 739.

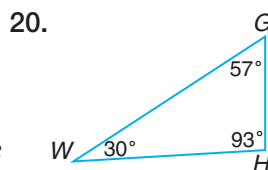
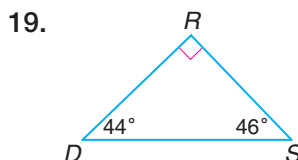
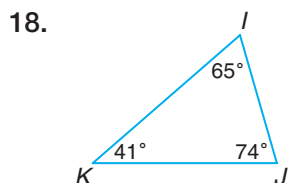
List the sides in order from least to greatest measure.



Identify the angle with the greatest measure.



Identify the side with the greatest measure.

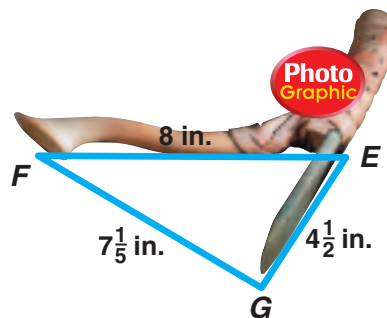


21. In $\triangle PRS$, $m\angle P = 30$, $m\angle R = 45$, and $m\angle S = 105$. Which side of $\triangle PRS$ has the greatest measure?

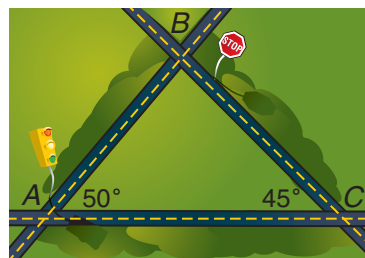
22. In $\triangle WQF$, $WQ > QF > FW$. Which angle of $\triangle WQF$ has the greatest measure?

Applications and Problem Solving

23. **Archaeology** Egyptian carpenters used a tool called an *adze* to smooth and shape wooden objects. Does $\angle E$, the angle the copper blade makes with the handle, have a measure less than or greater than the measure of $\angle G$, the angle the copper blade makes with the work surface? Explain.



24. **Maps** Two roads meet at an angle of 50° at point A . A third road from B to C makes an angle of 45° with the road from A to C . Which intersection, A or B , is closer to C ? Explain.



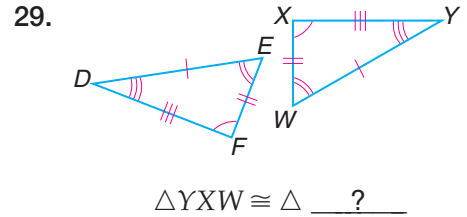
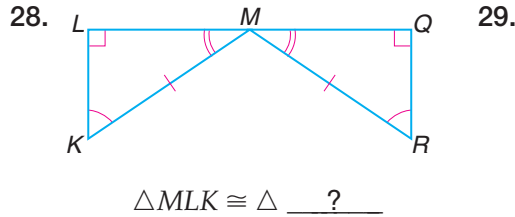
25. **Critical Thinking** In an obtuse triangle, why is the longest side opposite the obtuse angle?

Mixed Review

26. The measures of two interior angles of a triangle are 17 and 68. What is the measure of the exterior angle opposite these angles? (Lesson 7-2)

27. **Algebra** If $m\angle R = 48$ and $m\angle S = 2x - 10$, what values of x make $m\angle R \geq m\angle S$? (Lesson 7-1)

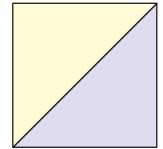
Complete each congruence statement. (Lesson 5-4)



Standardized Test Practice

- (A) (B) (C) (D)

30. **Short Response** Sketch at least three different quilt patterns that could be made using transformations of the basic square shown at the right. Identify each transformation. (Lesson 5-3)

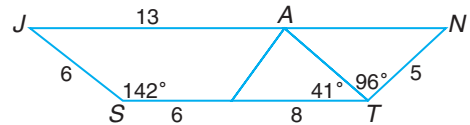


Quiz

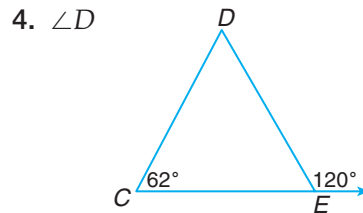
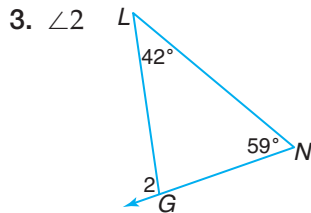
Lessons 7-1 through 7-3

▶ Replace each \bullet with $<$, $>$, or $=$ to make a true sentence. (Lesson 7-1)

1. $JA \bullet ST$ 2. $m\angle JST \bullet m\angle STN$



Find the measure of each angle. (Lesson 7-2)



5. **Geography** Perth, Darwin, and Sydney are three cities in Australia. Which two of the cities are the farthest apart? (Lesson 7-3)

