7.4 Showing Triangles are Similar: SSS and SAS

Goal

Show that two triangles are similar using the SSS and SAS Similarity Theorems.

Key Words

- similar polygons p. 365

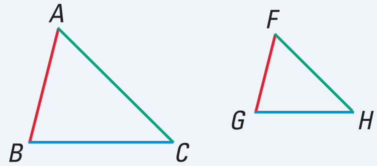
The triangles in the Navajo rug look similar. To show that they are similar, you can use the definition of similar polygons or the AA Similarity Postulate.

In this lesson, you will learn two new methods to show that two triangles are similar.

THEOREM 7.2

Side-Side-Side Similarity Theorem (SSS)

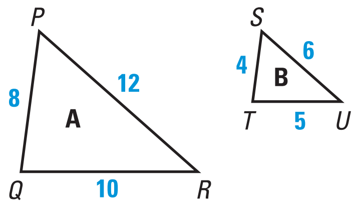
Words If the corresponding sides of two triangles are proportional, then the triangles are similar.



Symbols If $\frac{FG}{AB} = \frac{GH}{BC} = \frac{HF}{CA}$, then $\triangle ABC \sim \triangle FGH$.

EXAMPLE 1 Use the SSS Similarity Theorem

Determine whether the triangles are similar. If they are similar, write a similarity statement and find the scale factor of Triangle B to Triangle A.



Solution

Find the ratios of the corresponding sides.

$$\frac{SU}{PR} = \frac{6}{12} = \frac{6 \div 6}{12 \div 6} = \frac{1}{2}$$

$$\frac{UT}{RQ} = \frac{5}{10} = \frac{5 \div 5}{10 \div 5} = \frac{1}{2}$$

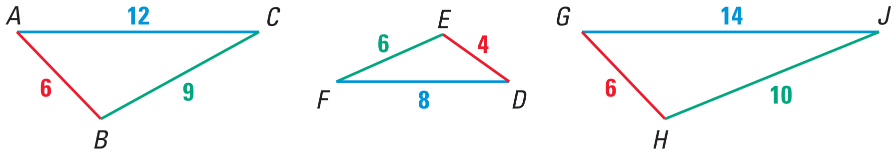
$$\frac{TS}{QP} = \frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$$

All three ratios are equal. So, the corresponding sides of the triangles are proportional.

ANSWER ▶ By the SSS Similarity Theorem, $\triangle PQR \sim \triangle STU$. The scale factor of Triangle B to Triangle A is $\frac{1}{2}$.

EXAMPLE 2 Use the SSS Similarity Theorem

Is either $\triangle DEF$ or $\triangle GHJ$ similar to $\triangle ABC$?



Solution

1 Look at the ratios of corresponding sides in $\triangle ABC$ and $\triangle DEF$.

Shortest sides	Longest sides	Remaining sides
$\frac{DE}{AB} = \frac{4}{6} = \frac{2}{3}$	$\frac{FD}{CA} = \frac{8}{12} = \frac{2}{3}$	$\frac{EF}{BC} = \frac{6}{9} = \frac{2}{3}$

ANSWER ▶ Because all of the ratios are equal, $\triangle ABC \sim \triangle DEF$.

2 Look at the ratios of corresponding sides in $\triangle ABC$ and $\triangle GHJ$.

Shortest sides	Longest sides	Remaining sides
$\frac{GH}{AB} = \frac{6}{6} = \frac{1}{1}$	$\frac{JG}{CA} = \frac{14}{12} = \frac{7}{6}$	$\frac{HJ}{BC} = \frac{10}{9}$

ANSWER ▶ Because the ratios are not equal, $\triangle ABC$ and $\triangle GHJ$ are not similar.

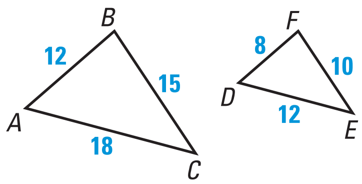
Student Help

STUDY TIP
When using the SSS Similarity Theorem, compare the shortest sides, the longest sides, and then the remaining sides.

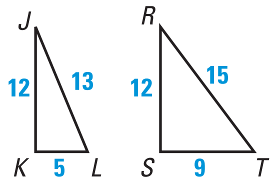
Checkpoint ✓ Use the SSS Similarity Theorem

Determine whether the triangles are similar. If they are similar, write a similarity statement.

1.



2.



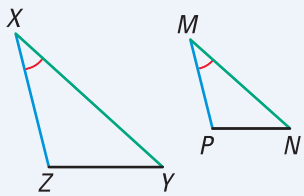
Student Help

LOOK BACK
To review included angles, see p. 242.

THEOREM 7.3

Side-Angle-Side Similarity Theorem (SAS)

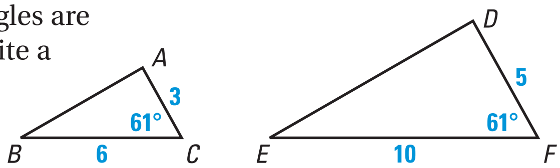
Words If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides that include these angles are proportional, then the triangles are similar.



Symbols If $\angle X \cong \angle M$ and $\frac{PM}{ZX} = \frac{MN}{XY}$, then $\triangle XYZ \sim \triangle MNP$.

EXAMPLE 3 Use the SAS Similarity Theorem

Determine whether the triangles are similar. If they are similar, write a similarity statement.



Solution

$\angle C$ and $\angle F$ both measure 61°, so $\angle C \cong \angle F$.

Compare the ratios of the side lengths that include $\angle C$ and $\angle F$.

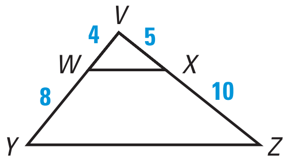
Shorter sides $\frac{DF}{AC} = \frac{5}{3}$ **Longer sides** $\frac{FE}{CB} = \frac{10}{6} = \frac{5}{3}$

The lengths of the sides that include $\angle C$ and $\angle F$ are proportional.

ANSWER ▶ By the SAS Similarity Theorem, $\triangle ABC \sim \triangle DEF$.

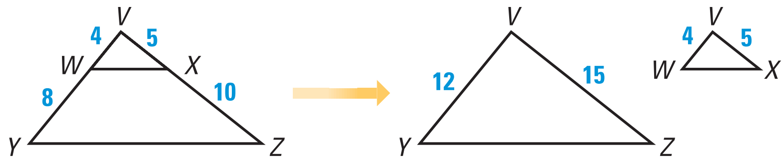
EXAMPLE 4 Similarity in Overlapping Triangles

Show that $\triangle VYZ \sim \triangle VWX$.



Solution

Separate the triangles, $\triangle VYZ$ and $\triangle VWX$, and label the side lengths.



$\angle V \cong \angle V$ by the Reflexive Property of Congruence.

Shorter sides

$$\frac{VW}{VY} = \frac{4}{4+8} = \frac{4}{12} = \frac{1}{3}$$

Longer sides

$$\frac{XV}{ZV} = \frac{5}{5+10} = \frac{5}{15} = \frac{1}{3}$$

The lengths of the sides that include $\angle V$ are proportional.

ANSWER ▶ By the SAS Similarity Theorem, $\triangle VYZ \sim \triangle VWX$.

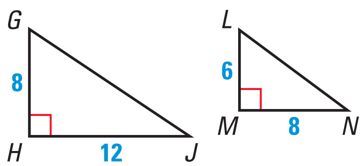
Student Help

VISUAL STRATEGY
Redraw overlapping triangles as two separate triangles, as shown on p. 356.

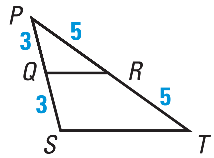
Checkpoint ✓ Use the SAS Similarity Theorem

Determine whether the triangles are similar. If they are similar, write a similarity statement. Explain your reasoning.

3.



4.



7.4 Exercises

Guided Practice

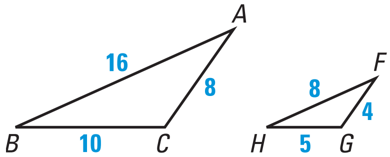
Vocabulary Check

1. If two sides of a triangle are proportional to two sides of another triangle, can you conclude that the triangles are similar?

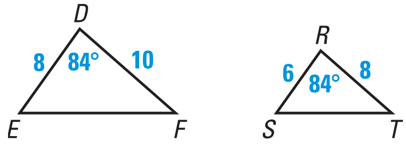
Skill Check

In Exercises 2 and 3, determine whether the triangles are similar. If they are similar, write a similarity statement.

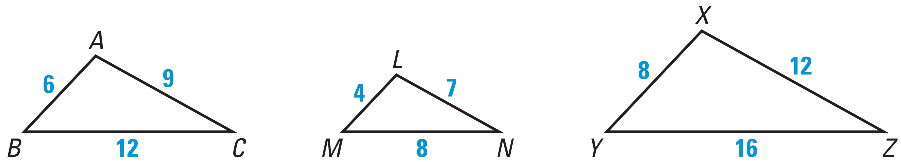
2.



3.



4. Is either $\triangle LMN$ or $\triangle XYZ$ similar to $\triangle ABC$? Explain.



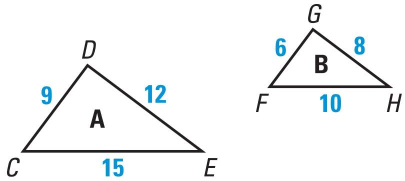
Practice and Applications

Extra Practice

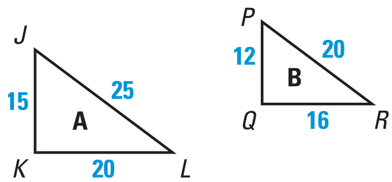
See p. 688.

SSS Similarity Theorem **Determine whether the two triangles are similar. If they are similar, write a similarity statement and find the scale factor of Triangle B to Triangle A.**

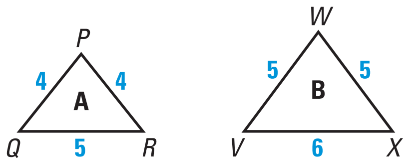
5.



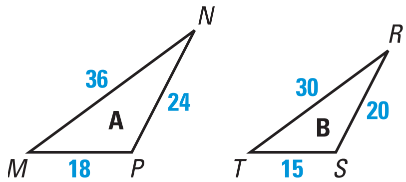
6.



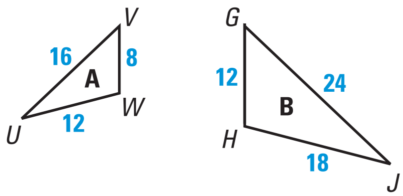
7.



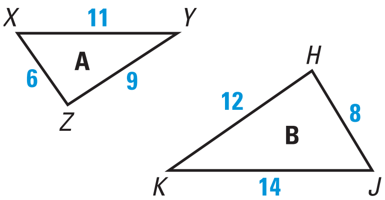
8.



9.



10.

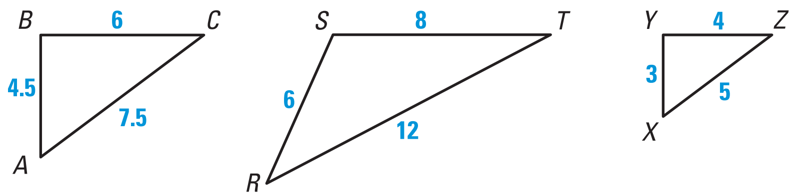


Homework Help

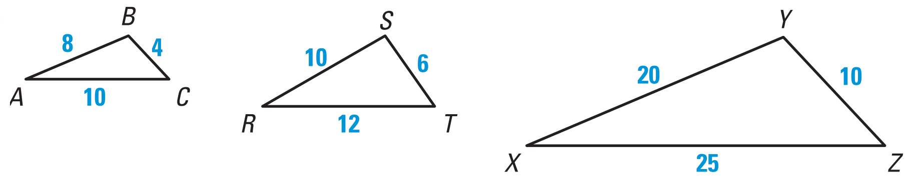
Example 1: Exs. 5–10, 21–26
Example 2: Exs. 11–13
Example 3: Exs. 14–18, 21–26
Example 4: Exs. 19, 20, 26–29

SSS Similarity Theorem **Is either $\triangle RST$ or $\triangle XYZ$ similar to $\triangle ABC$? Explain your reasoning.**

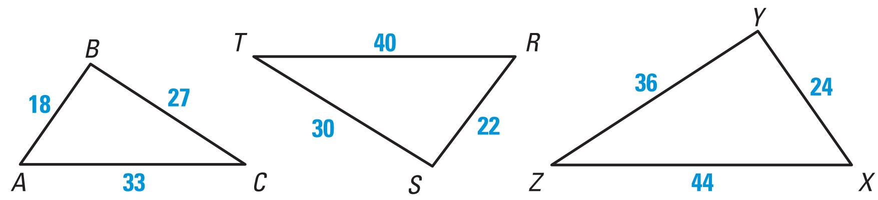
11.



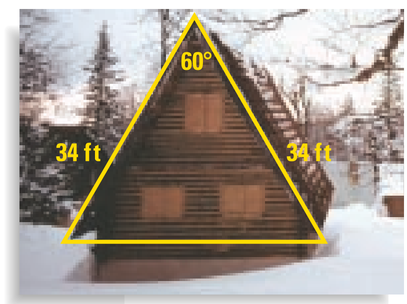
12.

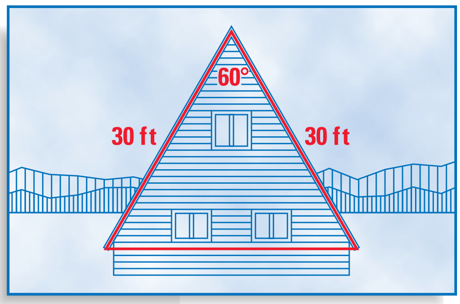


13.



14. **A-Frame Building** Suppose you are constructing an A-frame home that is modeled after a ski lodge. The ski lodge and home are shown below. Are the triangles similar? Explain your reasoning.

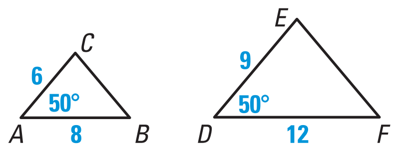




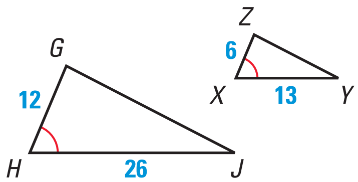
Student Help
CLASSZONE.COM
HOMEWORK HELP
Extra help with problem solving in Exs. 15–18 is at classzone.com

SAS Similarity Theorem **Determine whether the two triangles are similar. If they are similar, write a similarity statement.**

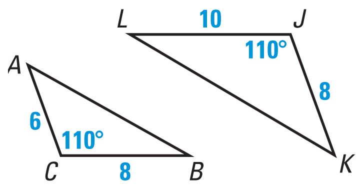
15.



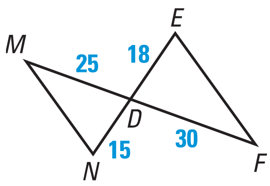
16.



17.



18.

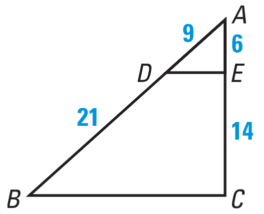


Student Help

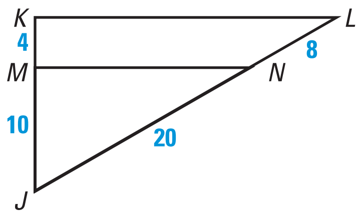
VISUAL STRATEGY
Redraw overlapping triangles as two separate triangles, as shown on p. 356.

Overlapping Triangles Show that the overlapping triangles are similar. Then write a similarity statement.

19.

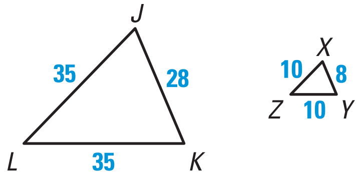


20.

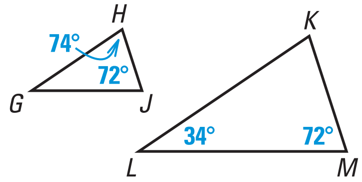


Determining Similarity Determine whether the triangles are similar. If they are similar, state the similarity and the postulate or theorem that justifies your answer.

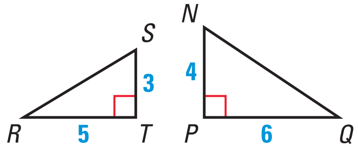
21.



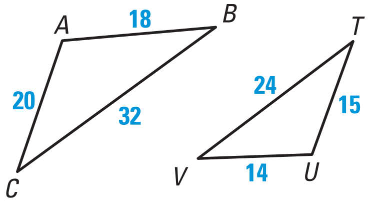
22.



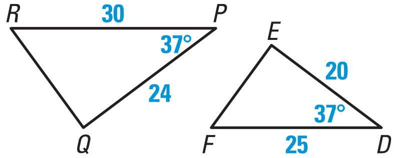
23.



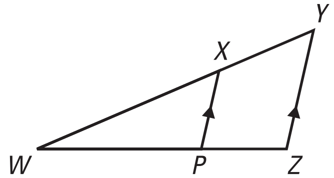
24.



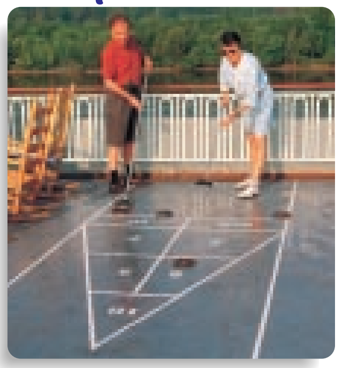
25.



26.

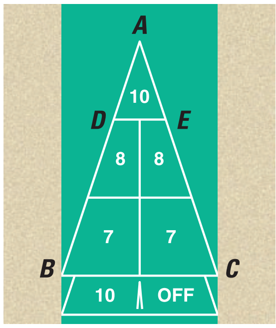


Link to Sports



SHUFFLEBOARD is played on a long flat court. Players earn points by using sticks called *cues* to push circular disks onto a scoring area at the opposite end of the court.

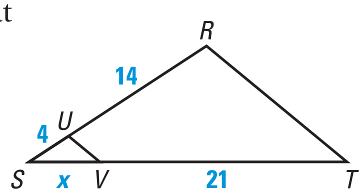
Shuffleboard In the portion of a shuffleboard court shown, $\frac{AD}{AB} = \frac{DE}{BC}$.



27. What piece of information do you need in order to show that $\triangle ADE \sim \triangle ABC$ using the SSS Similarity Theorem?

28. What piece of information do you need in order to show that $\triangle ADE \sim \triangle ABC$ using the SAS Similarity Theorem?

29. ***You be the Judge*** Jon claims that $\triangle SUV$ is similar to $\triangle SRT$ when $x = 6$. Dave believes that the triangles are similar when $x = 5$. Who is right? Explain your reasoning.



Technology **In Exercises 30 and 31, use geometry software to complete the steps below.**

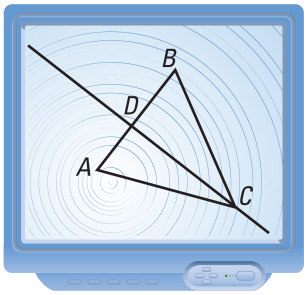
1 Draw $\triangle ABC$.

2 Construct a line perpendicular to $\overline{AB}$ through C. Label the intersection D.

3 Measure $\overline{CA}$, $\overline{CD}$, $\overline{CB}$, and $\overline{BD}$.

4 Calculate the ratios $\frac{CA}{CD}$ and $\frac{CB}{BD}$.

5 Drag point C until $\frac{CA}{CD} = \frac{CB}{BD}$.



30. For what measure of $\angle ACB$ are $\triangle ABC$ and $\triangle CBD$ similar?

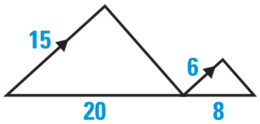
31. What theorem supports your answer to Exercise 30?

Standardized Test Practice

32. Multiple Choice Which method can be used to show that the two triangles at the right are similar?

Ⓐ AA　　Ⓑ SSS

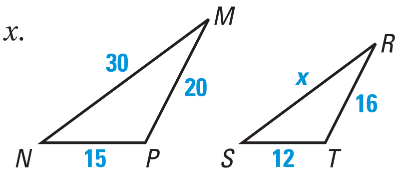
Ⓒ SAS　　Ⓓ Cannot be shown



33. Multiple Choice In the diagram, $\triangle MNP \sim \triangle RST$. Find the value of x.

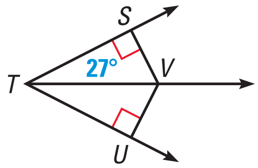
Ⓕ 20　　Ⓖ 24

Ⓗ 30　　Ⓙ 32



Mixed Review

Using Bisectors **In the diagram below, $\overrightarrow{TV}$ bisects $\angle STU$.** *(Lesson 5.6)*



34. $\overline{ST} \cong$ __?__

35. $\angle VTU \cong$ __?__

36. $m\angle STU =$ __?__

37. Is $\angle TVS$ congruent to $\angle TVU$? Explain your reasoning.

Solving Proportions **Solve the proportion.** *(Lesson 7.1)*

38. $\frac{b}{12} = \frac{5}{6}$　　**39.** $\frac{24}{y} = \frac{4}{9}$　　**40.** $\frac{5}{8} = \frac{c}{56}$　　**41.** $\frac{5}{2} = \frac{60}{a}$

Algebra Skills

Writing Decimals as Fractions **Write the decimal as a fraction in simplest form.** *(Skills Review, p. 657)*

42. 0.4　　**43.** 0.25　　**44.** 0.64　　**45.** 0.88

46. 0.26　　**47.** 0.55　　**48.** 0.7　　**49.** 0.34