7-4 Similarity in Right Triangles

What You'll Learn

- To find and use relationships in similar right triangles

. . . And Why

To find a distance indirectly, as in Example 3

Check Skills You'll Need

GO for Help Lesson 7-1 and page 390

Algebra **Solve each proportion.**

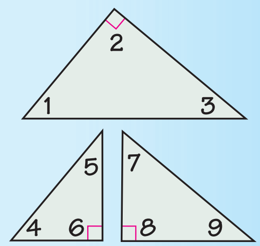
1. $\frac{x}{8} = \frac{18}{24}$ **6**
2. $\frac{2}{3} = \frac{x}{7}$ **$\frac{14}{3}$**
3. $\frac{15}{4} = \frac{18}{x}$ **$\frac{24}{5}$**
4. $\frac{51}{x} = \frac{17}{13}$ **39**
5. $\frac{4}{10} = \frac{x}{5}$ **2**
6. $\frac{3}{m} = \frac{9}{8}$ **$\frac{8}{3}$**
7. $\frac{w}{2} = \frac{20}{9}$ **$\frac{40}{9}$**
8. $\frac{9}{6} = \frac{27}{a}$ **18**
9. Draw a right triangle. Label the triangle $\triangle ABC$ with right angle $\angle C$. Draw the altitude to the hypotenuse. Label the altitude $\overline{CD}$. Name the two smaller right triangles that are formed. **See back of book.**

New Vocabulary • geometric mean

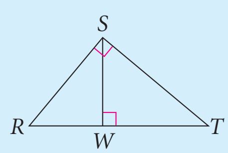
1 Using Similarity in Right Triangles

Hands-On Activity: Similarity in Right Triangles

- Draw one diagonal on a rectangular sheet of paper. Cut the paper on the diagonal to make two congruent right triangles.
- In one of the triangles, use paper folding to locate the altitude to the hypotenuse. Cut the triangle along the altitude to make two smaller right triangles.
- Label the angles of the three triangles as shown.
- Compare the angles of the three triangles by placing the angles on top of one another.



1. Which angles have the same measure as $\angle 1$? **$\angle 4$ and $\angle 7$**
2. Which angles have the same measure as $\angle 2$? **$\angle 6$ and $\angle 8$**
3. Which angles have the same measure as $\angle 3$? **$\angle 5$ and $\angle 9$**
4. Based on your results, what is true about the three triangles? **They are ~.**
5. Use the diagram at the right to complete the similarity statement. $\triangle RST \sim \triangle$ _?_ $\sim \triangle$ _?_ **RWS; SWT**



7-4

1. Plan

Objectives

1 To find and use relationships in similar right triangles

Examples

1 Finding the Geometric Mean
2 Applying Corollaries 1 and 2
3 Real-World Connection

Professional Development

Math Background

The geometric mean g of two positive numbers a and b has the algebraic formulation $g = \sqrt{ab}$, but the theorems in this lesson show how the mean can be viewed *geometrically.* The geometric mean of n positive numbers $a_1, a_2, a_3, \ldots, a_n$ is defined as $g = \sqrt[n]{a_1 a_2 a_3 \cdot \ldots \cdot a_n}$.

More Math Background: p. 364D

Lesson Planning and Resources

See p. 364E for a list of the resources that support this lesson.

PowerPoint

Bell Ringer Practice

Check Skills You'll Need
For intervention, direct students to:

Ratios and Proportions
Lesson 7-1: Example 3
Extra Skills, Word Problems, Proof Practice, Ch. 7

Medians and Altitudes
Lesson 5-3: Example 4
Extra Skills, Word Problems, Proof Practice, Ch. 5

Differentiated Instruction Solutions for All Learners

Special Needs L1
Have students copy $\triangle ABC$ three times. Students use color to indicate: (1) two pairs of $\cong$ angles in $\triangle ABC$ and $\triangle ACD$; (2) two pairs of $\cong$ angles in $\triangle ABC$ and $\triangle CBD$; and (3) two pairs of $\cong$ angles in $\triangle ACD$ and $\triangle CBD$.
learning style: visual

Below Level L2
Have students illustrate each theorem with a diagram and summarize each using proportions or similarity statements.
learning style: visual

2. Teach

Guided Instruction

Hands-On Activity
Suggest that students use single and double arcs indicating congruent acute angles to help them order the letters of the similar triangles correctly.

In discussing the proof of Theorem 7-3, have students refer to the right triangles they formed in the Hands-On Activity. Ask: *Which property allows you to conclude that the corresponding angles of the smaller triangles are congruent?* **Transitive Property of Congruence**

1 EXAMPLE Error Prevention

Check that students do not calculate the *arithmetic* mean, or average, when asked for the geometric mean. To help distinguish the two terms, have students calculate examples of each.

Visual Learners

Discuss as a class visual ways to remember Corollaries 1 and 2.

2 EXAMPLE

Students may have trouble remembering the corollaries. Remind them that they can use Theorem 7-3 to write similarity statements for three triangles and then derive the proportions from the similar triangles.

3 EXAMPLE

Have students explain why and how they can use Pythagorean triples to find *AB.*

In a right triangle, the altitude to the hypotenuse yields three similar triangles.

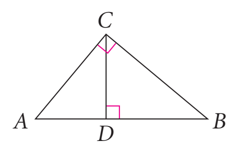
Key Concepts

Theorem 7-3

The altitude to the hypotenuse of a right triangle divides the triangle into two triangles that are similar to the original triangle and to each other.

Proof

Proof of Theorem 7-3



Given: Right triangle, $\triangle ABC$, with $\overline{CD}$ the altitude to the hypotenuse

Prove: $\triangle ABC \sim \triangle ACD \sim \triangle CBD$

Proof: Both smaller triangles are right triangles. Each also shares an angle with $\triangle ABC$. Thus each smaller triangle is similar to $\triangle ABC$ by the AA $\sim$ Postulate. Since both smaller triangles are similar to $\triangle ABC$, their corresponding angles are congruent. Thus they are similar to each other.

Vocabulary Tip

In the proportion $\frac{a}{b} = \frac{c}{d}$, b and c are the <u>means</u>.

Proportions in which the means are equal occur frequently in geometry. For any two positive numbers a and b, the **geometric mean** of a and b is the positive number x such that $\frac{a}{x} = \frac{x}{b}$. Note that $x = \sqrt{ab}$.

1 EXAMPLE Finding the Geometric Mean

Algebra Find the geometric mean of 4 and 18.

$\frac{4}{x} = \frac{x}{18}$	**Write a proportion.**
$x^2 = 72$	**Cross-Product Property**
$x = \sqrt{72}$	**Take the square root.**
$x = 6\sqrt{2}$	**Write in simplest radical form.**

The geometric mean of 4 and 18 is $6\sqrt{2}$.

Quick Check

1. Find the geometric mean of 15 and 20. **$10\sqrt{3}$**

Two important corollaries of Theorem 7-3 involve a geometric mean.

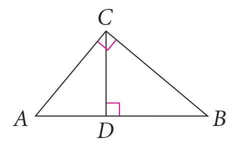
Key Concepts

Corollary — **Corollary 1 to Theorem 7-3**

The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the segments of the hypotenuse.

Proof

Proof of Corollary 1



Given: Right triangle, $\triangle ABC$, with $\overline{CD}$ the altitude to the hypotenuse

Prove: $\frac{AD}{CD} = \frac{CD}{DB}$

Proof: By Theorem 7-3, $\triangle ACD \sim \triangle CBD$. Since corresponding sides of similar triangles are proportional, $\frac{AD}{CD} = \frac{CD}{DB}$.

Differentiated Instruction Solutions for All Learners

Advanced Learners L4
After Example 1, challenge students to prove that the geometric mean of two numbers is always less than or equal to their arithmetic mean.
learning style: verbal

English Language Learners ELL
The name *geometric mean* has a precise definition. Remind students that b and c are the *means* in $\frac{a}{b} = \frac{c}{d}$; so x is the mean for $\frac{a}{x} = \frac{x}{d}$. The value of x in the proportion $\frac{a}{x} = \frac{x}{d}$ is $\sqrt{ab}$.
learning style: verbal

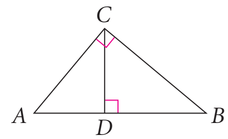
Key Concepts

Corollary | **Corollary 2 to Theorem 7-3**

The altitude to the hypotenuse of a right triangle separates the hypotenuse so that the length of each leg of the triangle is the geometric mean of the length of the adjacent hypotenuse segment and the length of the hypotenuse.

Proof

Proof of Corollary 2



Given: Right triangle, $\triangle ABC$, with $\overline{CD}$ the altitude to the hypotenuse

Prove: $\frac{AD}{AC} = \frac{AC}{AB}, \frac{DB}{CB} = \frac{CB}{AB}$

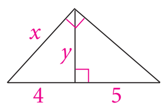
Proof: By Theorem 7-3, $\triangle ACD \sim \triangle ABC$. Their corresponding sides are proportional, so $\frac{AD}{AC} = \frac{AC}{AB}$. Similarly, $\triangle CBD \sim \triangle ABC$ and $\frac{DB}{CB} = \frac{CB}{AB}$.

Online active math

For: Similarity Activity
Use: Interactive Textbook, 7-4

2 EXAMPLE Applying Corollaries 1 and 2

Algebra Solve for x and y.

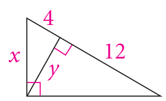


Use Corollary 2 to solve for x: | Use Corollary 1 to solve for y:

$\frac{4}{x} = \frac{x}{4 + 5}$	← **Write a proportion.** →	$\frac{4}{y} = \frac{y}{5}$
$x^2 = 36$	← **Cross-Product Property** →	$y^2 = 20$
$x = 6$	← **Take the square root.** →	$y = 2\sqrt{5}$

Quick Check

2. Solve for x and y.
$x = 8; y = 4\sqrt{3}$

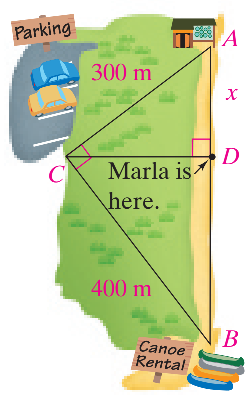


Real-World Connection

Paddling a canoe burns about 175 calories per hour.

3 EXAMPLE Real-World Connection

Recreation The 300-m path to the information center and the 400-m path to the canoe rental dock meet at a right angle at the parking lot. Marla walks straight from the parking lot to the lake as shown, where a sign tells her that she is 320 m from the dock. How far is Marla from the information center?



$\frac{x}{300} = \frac{300}{x + 320}$	**Corollary 2**
$x^2 + 320x = 90{,}000$	**Cross-Product Property**
$x^2 + 320x - 90{,}000 = 0$	**Standard Form Quadratic Equation**
$x = 180$	**Solve. Use a calculator (see p. 372).**

Marla is 180 m from the information center.

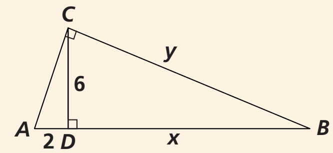
Quick Check

3. How far did Marla walk from the parking lot to the lake?
240 m

PowerPoint Additional Examples

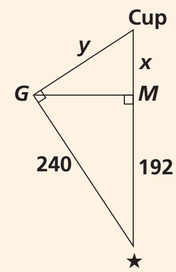
1. Find the geometric mean of 3 and 12. **6**

2. Solve for x and y.



$x = 18, y = 6\sqrt{10}$

3. At a golf course, Maria drove her ball 192 yd straight toward the cup. Her brother Gabriel drove his ball straight 240 yd, but not toward the cup. The diagram shows the results. Find x and y, their remaining distances from the cup.



$x = 108$ yd, $y = 180$ yd

Resources
- Daily Notetaking Guide 7-4 L3
- Daily Notetaking Guide 7-4—Adapted Instruction L1

Closure

Draw a right triangle with legs 8 cm and 15 cm long. Find each length.

a. hypotenuse **17**
b. altitude to the hypotenuse **$\frac{120}{17}$**
c. segments of the hypotenuse formed by the altitude **$\frac{64}{17}, \frac{225}{17}$**

3. Practice

Assignment Guide

1 A B 1-51
C Challenge 52-54

Test Prep 55-59
Mixed Review 60-67

Homework Quick Check
To check students' understanding of key skills and concepts, go over Exercises 8, 18, 23, 38, 39.

Exercises 15–20 Have students copy the diagrams and indicate congruent angles to help them identify the hypotenuses of the similar triangles.

Exercise 24 Ask: *What special type of right triangle must this be?* **isosceles**

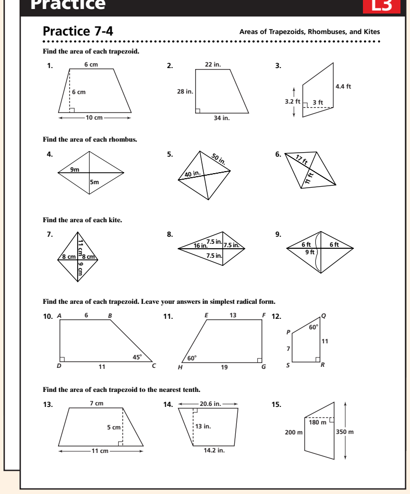
Connection to Algebra

Exercise 37 Suggest that students use x and $2x$ for the segments of the hypotenuse.

Exercise 38 Have students stand in place and try this measurement technique by using the corner of a 3 by 5 index card.

Differentiated Instruction Resources

GPS Guided Problem Solving	L3
Enrichment	L4
Reteaching	L2
Adapted Practice	L1
Practice	L3



EXERCISES

For more exercises, see *Extra Skill, Word Problem, and Proof Practice.*

Practice and Problem Solving

A Practice by Example

GO for Help

Example 1 (page 392)

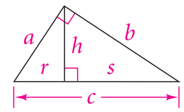
Algebra Find the geometric mean of each pair of numbers.

1. 4 and 9 **6**
2. 4 and 10 $2\sqrt{10}$
3. 4 and 12 $4\sqrt{3}$
4. 3 and 48 **12**
5. 7 and 56 $14\sqrt{2}$
6. 5 and 125 **25**
7. 9 and 24 $6\sqrt{6}$
8. 7 and 9 $3\sqrt{7}$

Example 2 (page 393)

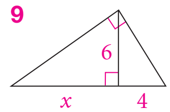
Algebra Refer to the figure to complete each proportion.

9. $\frac{r}{h} = \frac{h}{■}$ **s**
10. $\frac{c}{a} = \frac{a}{■}$ **r**
11. $\frac{■}{b} = \frac{b}{s}$ **c**
12. $\frac{r}{■} = \frac{■}{c}$ **a; a**
13. $\frac{r}{h} = \frac{■}{s}$ **h**
14. $\frac{s}{b} = \frac{■}{c}$ **b**

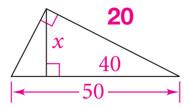


Algebra Solve for x.

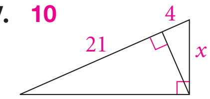
15. **9**



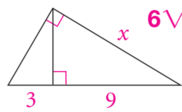
16. **20**



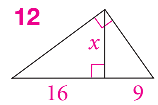
17. **10**



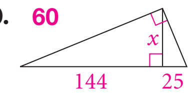
18. $6\sqrt{3}$



19. **12**

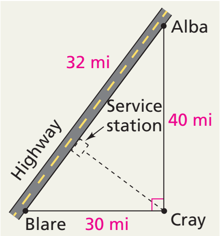


20. **60**



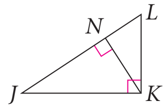
Example 3 (page 393)

21. a. **Civil Engineering** Study the plan at the right. A service station will be built on the highway, and a road will connect it with Cray. How far from Blare should the service station be located so that the proposed road will be perpendicular to the highway? **18 mi**
    b. How long will the new road be? **24 mi**

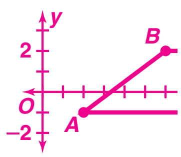


B Apply Your Skills

22. Complete:
$\triangle JKL \sim \triangle \underline{?} \sim \triangle \underline{?}$
KNL; JNK



24a.



b. They are =. Explanations may vary. Sample: The altitude and hyp. segments are ≅ sides of two isosc. △.

23. a. The altitude to the hypotenuse of a right triangle divides the hypotenuse into segments 2 cm and 8 cm long. Find the length h of the altitude. **4 cm**
    b. **Drawing** Use the value you found for h in part (a), along with the lengths 2 cm and 8 cm, to draw the right triangle accurately. **b-c. See margin.**
    c. **Writing** Explain how you drew the triangle in part (b).

24. a. **Open-Ended** Draw a right triangle so that the altitude from the right angle to the hypotenuse bisects the hypotenuse. **a-b. See left.**
    b. How does the length of the altitude compare with the lengths of the segments of the hypotenuse? Explain.

Vocabulary Tip

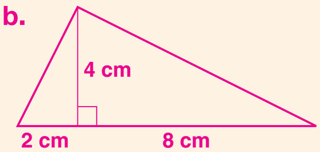
Respectively in Exercise 25 means you match the lists in the order named: $A(4, 2)$, $D(4, 6)$, $B(4, 15)$.

25. **Coordinate Geometry** $\overline{CD}$ is the altitude to the hypotenuse of right $\triangle ABC$. The coordinates of A, D, and B are (4, 2), (4, 6), and (4, 15), respectively. Find all possible coordinates of point C. **(10, 6), (−2, 6)**

Algebra Find the geometric mean of each pair of numbers.

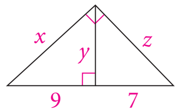
26. 3 and 16 $4\sqrt{3}$
27. 4 and 49 **14**
28. $\sqrt{8}$ and $\sqrt{2}$ **2**
29. $\sqrt{28}$ and $\sqrt{7}$ $\sqrt{14}$
30. $\frac{1}{2}$ and 2 **1**
31. 5 and 1.25 **2.5**
32. 1 and 1000 $10\sqrt{10}$
33. 11 and 1331 **121**

23. b.

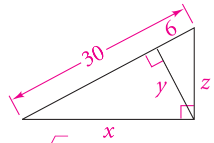


c. Answers may vary. Sample: Draw a 10-cm segment; 2 cm from one endpoint, construct a ⊥ of length 4 cm. Connect to form a △.

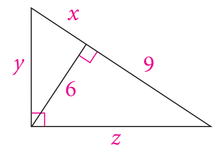
x^2 **Algebra** **Find the values of the variables.** 36. $x = 4; y = 2\sqrt{13}; z = 3\sqrt{13}$

34. 

$x = 12; y = 3\sqrt{7}; z = 4\sqrt{7}$

35. 

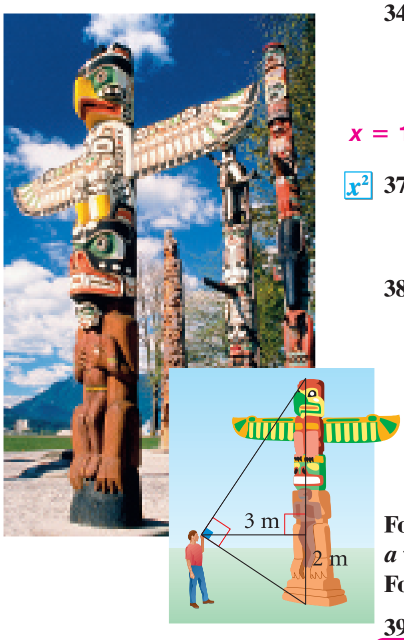
$x = 12\sqrt{5}; y = 12; z = 6\sqrt{5}$

36. 

x^2 **37. Algebra** The altitude to the hypotenuse of a right triangle divides the hypotenuse into segments with lengths in the ratio 1 : 2. The length of the altitude is 8. How long is the hypotenuse? $12\sqrt{2}$

38. Multiple Choice To estimate the height of a totem pole, Jorge uses a small square of plastic. He holds the square up to his eyes and walks backward from the pole. He stops when the bottom of the pole lines up with the bottom edge of the square and the top of the pole lines up with the top edge of the square. Jorge's eye level is about 2 m from the ground. He is about 3 m from the pole. Which is the best estimate for the height of the totem pole? **C**

Ⓐ 4.5 m Ⓑ 5 m Ⓒ 6.5 m Ⓓ 9 m



Exercise 38

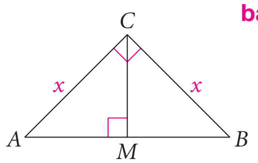
For a right triangle, denote lengths as follows: ℓ_1 and ℓ_2 the legs, h the hypotenuse, a the altitude, and h_1 and h_2 the hypotenuse segments determined by the altitude. For the two given measures, find the other four. Use simplest radical form. 39–46. See margin.

39. $h = 2, h_1 = 1$ (GPS) **40.** $h_1 = 4, h_2 = 9$ **41.** $a = 6, h_1 = 6$ **42.** $\ell_1 = 2, h_2 = 3$

43. $h = 13, \ell_2 = 12$ **44.** $\ell_1 = 4, h_1 = 3$ **45.** $a = 8, h_1 = 16$ **46.** $h_1 = 3, \ell_2 = 6\sqrt{3}$

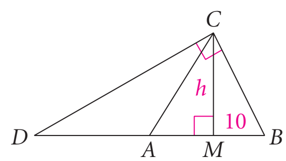
Proof **47. Given:** isosceles right $\triangle ABC$

Prove: $AB = x\sqrt{2}$ 47–48. See back of book.



48. Given: equilateral $\triangle ABC$

Prove: $h = 10\sqrt{3}$



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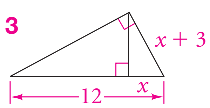
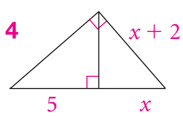
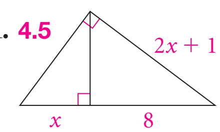
x^2 **Algebra** **Find the value of x.**

49. 3 

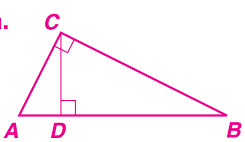
50. 4 

51. 4.5 

C Challenge

52. a. Lauren thinks she has found a new corollary: The product of the lengths of the two legs of a right triangle is equal to the product of the lengths of the hypotenuse and the altitude to the hypotenuse. Draw a figure for this corollary. Write the *Given* information and what you are to *Prove*.

b. Critical Thinking Is Lauren's corollary true? Explain. See left.

52a. 

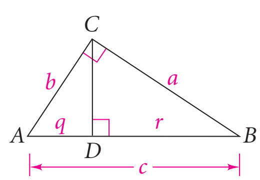
Given: rt. $\triangle ABC$ with alt. $\overline{CD}$;
Prove: $AC \cdot CB = AB \cdot CD$

52b. Yes; $\triangle ACB \sim \triangle CDB$, so $\frac{AC}{AB} = \frac{CD}{CB}$.

53. In the diagram $c = q + r$. Also, Corollary 2 to Theorem 7-3 gives you two more equations involving a, b, c, q, and r. The result is a system of three equations in five variables.

a. Reduce the system to one equation in three variables by eliminating q and r. a. Check students' work.

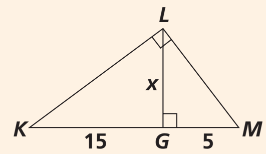
b. State in words what the one resulting equation tells you. The square of the hypotenuse equals the sum of the squares of the legs.



4. Assess & Reteach

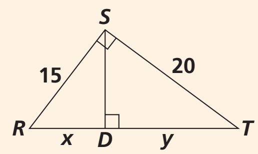
Lesson Quiz

1. Find the geometric mean of 32 and 2. 8
2. Find the geometric mean of 6 and 20. $2\sqrt{30}$
3. Solve for x.



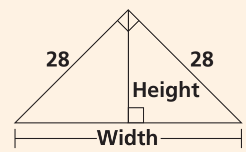
$5\sqrt{3}$

4. Solve for x and y.



$x = 9, y = 16$

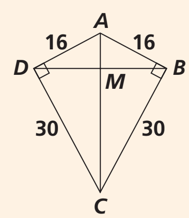
5. The roof of a house forms a right angle, with each side of the roof measuring 28 ft in length. Find the width and height of the roof.



width $= 28\sqrt{2}$ ft, height $= 14\sqrt{2}$ ft

Alternative Assessment

Have students work in pairs to calculate AM, BM, CM, and DM in kite $ABCD$.



$AM = \frac{128}{17}$, $BM = \frac{240}{17}$, $CM = \frac{450}{17}$, $DM = \frac{240}{17}$

39. $\ell_1 = \sqrt{2}, \ell_2 = \sqrt{2}, a = 1, h_2 = 1$

40. $\ell_1 = 2\sqrt{13}, \ell_2 = 3\sqrt{13}, h = 13, a = 6$

41. $\ell_1 = \ell_2 = 6\sqrt{2}, h = 12, h_2 = 6$

42. $\ell_2 = 2\sqrt{3}, h = 4, a = \sqrt{3}, h_1 = 1, h_2 = 3$

43. $\ell_1 = 5, a = \frac{60}{13}, h_1 = \frac{25}{13}, h_2 = \frac{144}{13}$

44. $\ell_2 = \frac{4\sqrt{7}}{3}, h = \frac{16}{3}, a = \sqrt{7}, h_2 = \frac{7}{3}$

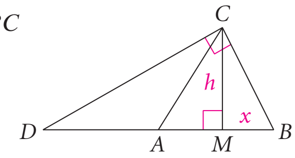
45. $\ell_1 = 8\sqrt{5}, \ell_2 = 4\sqrt{5}, h_2 = 4, h = 20$

46. $\ell_1 = 6, h = 12, a = 3\sqrt{3}, h_2 = 9$

Test Prep

Resources
For additional practice with a variety of test item formats:
- Standardized Test Prep, p. 411
- Test-Taking Strategies, p. 406
- Test-Taking Strategies with Transparencies

Proof **54. Given:** equilateral $\triangle ABC$
Prove: $h = x\sqrt{3}$



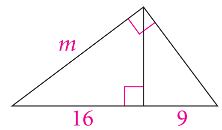
Test Prep

Multiple Choice

55. What is the geometric mean of 12 and 18? **D**
A. 1.5 **B.** $\sqrt{6}$ **C.** 15 **D.** $6\sqrt{6}$

56. What is the geometric mean of 2 and 36? **G**
F. 17 **G.** $6\sqrt{2}$ **H.** 38 **J.** $2\sqrt{6}$

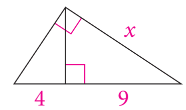
57. Solve for m. **C**
A. 7 **B.** 15
C. 20 **D.** 25



58. The altitude to the hypotenuse of a right triangle divides the hypotenuse into segments of lengths 5 and 15. What is the length of the altitude? **H**
F. 3 **G.** 10 **H.** $5\sqrt{3}$ **J.** $5\sqrt{5}$

Short Response

59. a. Explain how you could solve for x.
b. What is the value of x?
a–b. See margin.



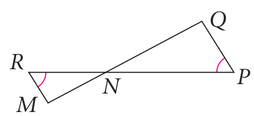
Mixed Review

GO for Help

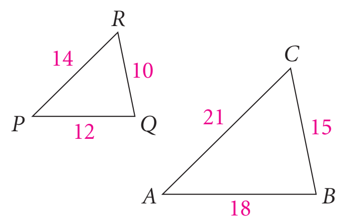
Lesson 7-3 If the triangles are similar, (a) write a similarity statement and (b) name the postulate or theorem you used. If the triangles are not similar, write *not similar*.

60a. $\triangle RNM \sim \triangle PNQ$
b. AA ~ Post.
61a. $\triangle PRQ \sim \triangle ACB$
b. SSS ~ Thm.

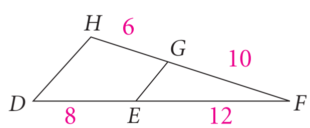
60.



61.

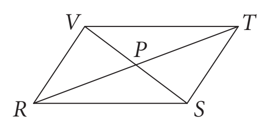


62.



not similar

Lesson 6-2 **Algebra** Find the values of x and y in $\square RSTV$.



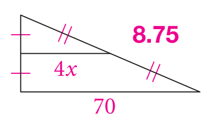
63. $RP = 2x, PT = y + 2, VP = y, PS = x + 3$ **$x = 5; y = 8$**

64. $RP = 4x, PT = 3y - 3, VP = 2x + 3, PS = y + 6$ **$x = 6; y = 9$**

65. $RV = 2x + 3, VT = 5x, TS = y + 5, SR = 4y - 1$ **$x = 3; y = 4$**

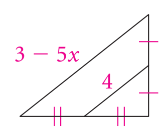
Lesson 5-1 Find the value of x.

66.



8.75

67.



−1

59. [2] a. Solve for x by making a prop. from similar rt. △s. The proportion is $\frac{x}{13} = \frac{9}{x}$. Now cross multiply and solve for x.
b. $3\sqrt{13}$
[1] incorrect proportion OR incorrect x value