

HIGLEY UNIFIED SCHOOL DISTRICT INSTRUCTIONAL ALIGNMENT

7th Grade Math First Quarter

Unit 1: Operations with Rational Numbers (4 weeks)

Topic A: Rational Number Operations – Addition and Subtraction

In Topic A, students return to the number line to model the addition and subtraction of integers (7.NS.A.1). They use the number line and other strategies to demonstrate that an integer added to its opposite equals zero, representing the additive inverse (7.NS.A.1a, 7.NS.A.1b). Their findings are formalized as students develop rules for adding and subtracting integers, and they recognize that subtracting a number is the same as adding its opposite (7.NS.A.1c). Real-life situations are represented by the sums and differences of signed numbers. Students extend integer rules to include the rational numbers and use properties of operations to perform rational number calculations without the use of a calculator (7.NS.A.1d). Students solve real-life problems that involve the addition and subtraction of rational numbers (7.NS.A.3)

Big Idea:

- Rational numbers use the same properties as whole numbers.
- Rational numbers can be used to represent and solve real-life situation problems.
- Rational numbers can be represented with visuals (including distance models), language, and real-life contexts.
- A number line model can be used to represent the unique placement of any number in relation to other numbers.
- There are precise terms and sequence to describe operations with rational numbers.


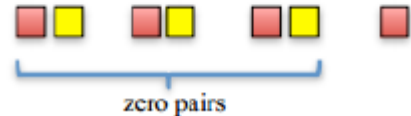

Essential Questions:



- How are positive and negative rational numbers used and applied in real-life and mathematical situations?
- What is the relationship between properties of operations and types of numbers?
- How can models be used to add and subtract integers?
- How is the sum of two rational numbers related to distance?

Vocabulary

rational numbers, integers, additive inverse, commutative property, associative property, distributive property, opposite, absolute value, distance, change

Grade	Domain	Standard	AZ College and Career Readiness Standards	Explanations & Examples	Resources
7	NS	1	<p>A. Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.</p> <p>Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.</p> <p>a. Describe situations in which opposite</p>	<p><u>Explanations:</u></p> <p>Students add and subtract rational numbers. Visual representations (number lines, chips/tiles, etc...) may be helpful as students begin this work; they become less necessary as students become more fluent with these operations. The expectation of the AZCCRS is to build on student understanding of number lines developed in 6th grade. Using both contextual and numerical problems, students explore what happens when negative and positive numbers are combined. By analyzing and solving problems, students are led to the generalization</p>	<p>Eureka Math: Module 2 Lessons 1-9</p> <p>Big Ideas: Sections 1.1, 1.2.1.3, 2.2, 2.3</p>

			<p>quantities combine to make 0. <i>For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.</i></p> <p>b. Understand $p + q$ as the number located a distance q from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.</p> <p>c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.</p> <p>d. Apply properties of operations as strategies to add and subtract rational numbers.</p> <p>7.MP.2. Reason abstractly and quantitatively. 7.MP.4. Model with mathematics. 7.MP.7. Look for and make use of structure.</p>	<p>of the rules for operations with integers.</p> <p>Distance is always positive. Change (elevation, temperature, bank account, etc...) may be positive or negative depending on whether it is increasing or decreasing.</p> <p>Looking for and making use of structure (MP.7) aids students' understanding of addition and subtraction of positive and negative rational numbers. Students also engage in MP.2 and MP.4 as they use multiple representations to add and subtract rational numbers.</p> <p>Examples:</p> <ul style="list-style-type: none"> Demonstrate $-4 + 3$ using the chip/tile model and state the answer. <p>Solution. First we lay out the appropriate tiles.</p>  <p>Collect together the zero pairs</p>  <p>Remove the zero pairs and what is left is the answer to the addition problem, which in this case is -1, or one red tile.</p> <ul style="list-style-type: none"> Demonstrate $2 - 7$ using the chip/tile model from two different perspectives. State the answer. <p>Solution. Let's examine the problem utilizing "take-away." We begin with 2 yellow tiles, add in 5 zero pairs, and "take away" 7 yellow tiles. Hence, $2 - 7 = -5$.</p>  <p>Yet, this method can be restructured. The process of</p>	
--	--	--	--	--	--

				<p>introducing 5 zero pairs and then removing 7 yellow tiles can be accomplished by inserting 7 red tiles. We already know a subtraction problem with whole numbers can be turned into an addition problem by finding a missing addend. This also applies to subtraction problems involving integers.</p> <p>We start with 2 yellow tiles and “add” in 7 red tiles. .</p> <p></p> <p>With the set of tiles, we create 2 zero pairs (equal to zero) and we are left with 5 red tiles. Hence, $2 - 7 = -5$.</p> <p></p> <p>This simplified method finds $2 - 7$ by finding $2 + (-7)$. Thus the method of subtraction replaces a subtraction problem with an equivalent addition problem - that is to say, adding the opposite.</p> <ul style="list-style-type: none"> • Use a number line to add $-5 + 7$. <p><i>Solution:</i> Students find -5 on the number line and move 7 in a positive direction (to the right). The stopping point of 2 is the sum of this expression. Students also add negative fractions and decimals and interpret solutions in given contexts.</p> <p>In 6th grade, students found the distance of horizontal and vertical segments on the coordinate plane. In 7th grade, students build on this understanding to recognize subtraction is finding the distance between two numbers on a number line.</p> <p>In the example, $7 - 5$, the difference is the distance between 7 and 5, or 2, in the direction of 5 to 7 (positive). Therefore the answer would be 2.</p> <ul style="list-style-type: none"> • Use a number line to subtract: $-6 - (-4)$ <p><i>Solution:</i> This problem is asking for the distance between -6 and -4. The</p>	
--	--	--	--	--	--

distance between -6 and -4 is 2 and the direction from -4 to -6 is left or negative. The answer would be -2. Note that this answer is the same as adding the opposite of -4: $-6 + 4 = -2$

- Use a number line to illustrate:

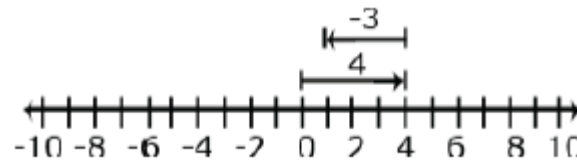
- $p - q$
- $p + (-q)$
- Is this equation true $p - q = p + (-q)$?

Students explore the above relationship when p is negative and q is positive and when both p and q are negative. Is this relationship always true?

- Morgan has \$4 and she needs to pay a friend \$3. How much will Morgan have after paying her friend?

Solution:

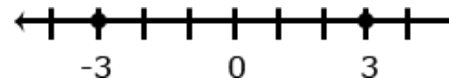
$$4 + (-3) = 1 \text{ or } (-3) + 4 = 1$$

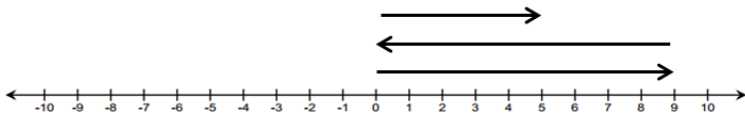
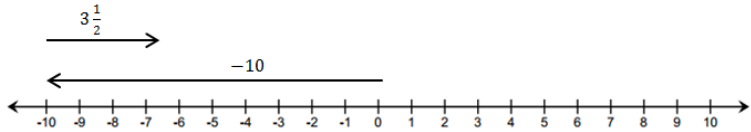


- Why are -3 and 3 opposites?

Solution:

-3 and 3 are opposites because they are equal distance from zero and therefore have the same absolute value and the sum of the number and its opposite is zero.



				<ul style="list-style-type: none"> Find the difference of $-6 - (-2\frac{1}{4})$ <p><i>Solution:</i> $-3\frac{3}{4}$</p> <ul style="list-style-type: none"> <p>Write a story problem that would model the sum of the arrows in the number diagram below.</p>  <p><i>Jill got on an elevator and went to the 9th floor. She accidentally pressed the down button and went back to the lobby. She pressed the button for the 5th floor and got off the elevator.</i></p> <p>Write an addition sentence, and find the sum using the diagram below.</p>  <p><i>Solution:</i></p> $\textcircled{-10} + 3\frac{1}{2} = -10 + \frac{7}{2} = -\frac{20}{2} + \frac{7}{2} = -\frac{13}{2} = -6\frac{1}{2}$ $\left \frac{-20}{2} \right = \frac{20}{2}$ $\left \frac{7}{2} \right = \frac{7}{2}$ 	
7	NS	3	<p>A. Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.</p> <p>Solve real-world and mathematical problems involving the four operations (addition and subtraction only) with rational numbers (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)</p>	<p>Explanation:</p> <p>Students use order of operations from 6th grade to write and solve problems with <u>all</u> rational numbers (decimals and fractions). In Topic A, the work with 7.NS.A.3 should focus on addition and subtraction of positive and negative rational numbers.</p> <p>Examples:</p> <ul style="list-style-type: none"> If the temperature outside was 73 degrees at 5:00 pm, but it fell 19 degrees by 10:00 pm, what is the temperature at 10:00 	<p>Eureka Math: Module 2 Lessons 1-9</p> <p>Big Ideas: Sections 1.1, 1.2.1.3, 2.2, 2.3</p>

				<p>pm? Write an equation and solve.</p> <p><i>Solution:</i></p> <p style="text-align: center;"> $\overset{73}{\text{73}}$ $\overset{-19}{-19}$ </p> <p>If the temperature outside was 73 degrees at 5:00 pm, but it fell 19 degrees by 10:00 pm, what is the temperature at 10:00 pm? Write an equation and solve.</p> <p> $73 + (-19)$ $73 = 73$ </p> <p> $73 - 19 = 54$ $-19 = 19$ </p> <p>54</p> <ul style="list-style-type: none"> A submarine was situated 800 feet below sea level. If it ascends 250 ft, what is its new position? Write an equation and solve. <p><i>Solution:</i></p> <p>$-800 + 250 = -550$ The submarine's new position is 550 ft below sea level.</p>	
--	--	--	--	---	--

7th Grade Math First Quarter

Unit 1: Rational Numbers

Topic B: Rational Number Operations – Multiplication and Division

Students develop the rules for multiplying and dividing signed numbers in Topic B. They use the properties of operations and their previous understanding of multiplication as repeated addition to represent the multiplication of a negative number as repeated subtraction (**7.NS.A.2a**). Students make analogies to real-life scenarios to understand that the product of two negative numbers is a positive number. From earlier grades, they recognize division as the inverse process of multiplication. Thus, signed number rules for division are consistent with those for multiplication, provided a divisor is not zero (**7.NS.A.2b**). Students represent the division of two integers as a fraction, extending product and quotient rules to all rational numbers. They realize that any rational number in fractional form can be represented as a decimal that either terminates in 0s or repeats (**7.NS.A.2d**). Students recognize that the context of a situation often determines the most appropriate form of a rational number, and they use long division, place value, and equivalent fractions to fluently convert between these fraction and decimal forms. Topic B concludes with students multiplying and dividing rational numbers using the properties of operations (**7.NS.A.2c**).

Big Idea:

- Rational numbers use the same properties as whole numbers.
- Rational numbers can be used to represent and solve real-life situation problems.
- Rational numbers can be represented with visuals (including distance models), language, and real-life contexts.
- There are precise terms and sequence to describe operations with rational numbers.

Essential Questions:

- How are rational numbers used and applied in real-life and mathematical situations?
- What is the relationship between properties of operations and types of numbers?
- What is a rational number?
- What does a positive/negative number times a /positive negative number mean? (ask all combinations)
- Is the quotient of two integers always an integer?

Vocabulary

rational numbers, integers, additive inverse, commutative property, associative property, distributive property, counterexample, terminating decimal, repeating decimal, non-terminating decimal

Grade	Domain	Standard	AZ College and Career Readiness Standards	Explanations & Examples	Resources
7	NS	2	A. Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. <p>a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the</p>	Explanations: <p>Students understand that multiplication and division of integers is an extension of multiplication and division of whole numbers. Students recognize that when division of rational numbers is represented with a fraction bar, each number can have a negative sign.</p> <p>In Grade 7 the awareness of rational and irrational numbers is initiated by observing the result of changing fractions to decimals. Students</p>	Eureka Math: Module 2 Lessons 10-16 Big Ideas: Sections 1.1, 1.4, 1.5, 2.1, 2.4

			<p>distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.</p> <p>b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.</p> <p>c. Apply properties of operations as strategies to multiply and divide rational numbers.</p> <p>7.MP.2. Reason abstractly and quantitatively.</p> <p>7.MP.4. Model with mathematics.</p> <p>7.MP.7. Look for and make use of structure.</p>	<p>should be provided with families of fractions, such as, sevenths, ninths, thirds, etc. to convert to decimals using long division. The equivalents can be grouped and named (terminating or repeating). Students should begin to see why these patterns occur. Knowing the formal vocabulary <i>rational</i> and <i>irrational</i> is not expected. Using long division from elementary school, students understand the difference between terminating and repeating decimals. This understanding is foundational for the work with rational and irrational numbers in 8th grade.</p> <p>Students multiply and divide rational numbers. Visual representations (number lines, chips/tiles, etc...) may be helpful as students begin this work; they become less necessary as students become more fluent with these operations. The expectation of the AZCCRS is to build on student understanding of number lines developed in 6th grade. Using both contextual and numerical problems, students explore what happens when negative and positive numbers are combined. By analyzing and solving problems, students are led to the generalization of the rules for operations with integers.</p> <p>When addressing 7.NS.A.2a, note that students already know the distributive property from earlier grades. It was first introduced in grade 3. In grade 6, students applied the distributive property to generate equivalent expressions involving both numbers and variables (6.EE.A.3).</p> <p>As with Topic A, looking for and making use of structure (MP.7) aids students' understanding of multiplication and division of positive and negative rational numbers. Students also engage in MP.2 and MP.4 as they use multiple representations to multiply and divide rational numbers.</p> <p><u>Examples:</u></p> <ul style="list-style-type: none"> Which of the following fractions is equivalent to $-4/5$? Explain your reasoning. 	
--	--	--	---	---	--

a. $\frac{4}{-5}$ b. $\frac{-16}{20}$ c. $\frac{-4}{-5}$

- Examine the family of equations. What patterns do you see? Create a model and context for each of the products.

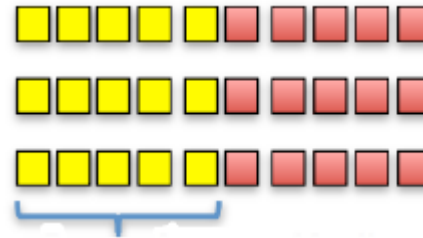
Equation	Number Line Model	Context
$2 \times 3 = 6$		Selling two posters at \$3.00 per poster
$2 \times -3 = -6$		Spending 3 dollars each on 2 posters
$-2 \times 3 = -6$		Owing 2 dollars to each of your three friends
$-2 \times -3 = 6$		Forgiving 3 debts of \$2.00 each

- Demonstrate $-3(5)$ using the chip/tile model. State the answer.

Solution:

The first factor, 3, is negative, which indicated that we should “remove (or take away) 3 groups of 5 yellow tiles.” In order to remove tiles from a set that has a value of 0, we must add some tiles with a value of 0 to the set. We can accomplish this

by adding in an equal number of yellow and red tiles to our set. Once done, we remove 3 groups of 5 yellow tiles.

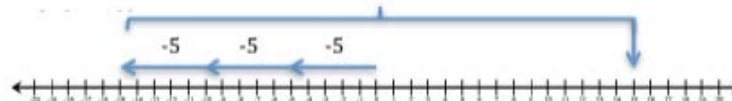


Removing 3 groups of 5 yellow tiles results in 15 red tiles. Therefore, $-3(5) = -15$.

- $-3(-5) = ?$ Provide justification for your answer.

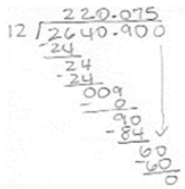
Solution: (answers will vary)

-3 times -5 can be interpreted as “the opposite of 3 groups of negative 5”. The number line below demonstrates that the answer is 15.



- Using long division, express the following fractions as decimals. Which result in terminating decimals; which result in repeating decimals? Why?

$$\frac{-5}{6}, \quad \frac{3}{8}, \quad \frac{1}{-25}, \quad \frac{2}{9}$$

				<p>•</p> <p>In one year, Melinda's parents spend \$2,640.90 on cable and internet service. If they spend the same amount each month, what is the resulting monthly change in the family's income?</p> <p>$-2,640.90 \div 12 = -220.08$. The average change to their income is $-\\$220.08$.</p>  <p>•</p> <p>In the space below, create a word problem that involves integer multiplication. Write an equation to model the situation.</p> <p>Both times we went to the fair, I borrowed \$3 from my older brother. $-3 \bullet 2 = -6$</p> <p>Now change the word problem by replacing the integers with non-integer rational numbers (fractions or decimals), and write the new equation.</p> <p>Both times we went to the fair, I borrowed \$3.50 from my older brother. $-3.50 \bullet 2 = -7.00$</p> <p>Was the process used to solve the second problem different from the process used to solve the first? Explain.</p> <p>No, the process was the same. Both times I had a positive number multiplied by a negative number, so the product is a negative number. The process, multiplication, is represented as repeated addition: $-3.50 + (-3.50) = -7.00$.</p>	
7	NS	3	<p>A. Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.</p> <p>Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)</p> <p>7.MP.1. Make sense of problems and persevere in solving them.</p> <p>7.MP.2. Reason abstractly and quantitatively.</p>	<p>Explanation:</p> <p>Students use order of operations from 6th grade to write and solve problems with <u>all</u> rational numbers (decimals and fractions).</p> <p>Examples:</p> <ul style="list-style-type: none"> Calculate: $[-10(-0.9)] - [(-10) \bullet 0.11]$ <p>Solution: 10.1</p> <ul style="list-style-type: none"> Jim's cell phone bill is automatically deducting \$32 from his bank account every month. How much will the deductions total for the year? 	<p>Eureka Math: Module 2 Lessons 10-16</p> <p>Big Ideas: Sections 1.1- 1.5, 2.2- 2.4</p>

		<p>7.MP.5. Use appropriate tools strategically.</p> <p>7.MP.6. Attend to precision.</p> <p>7.MP.7. Look for and make use of structure.</p> <p>7.MP.8. Look for and express regularity in repeated reasoning.</p>	<p><i>Solution:</i></p> $-32 + (-32) + (-32) + (-32) + (-32) + (-32) + (-32) + (-32) + (-32) + (-32) + (-32) + (-32) = 12(-32)$ <ul style="list-style-type: none"> It took a submarine 20 seconds to drop to 100 feet below sea level from the surface. What was the rate of the descent? <p><i>Solution:</i></p> $\frac{-100 \text{ feet}}{20 \text{ seconds}} = \frac{-5 \text{ feet}}{1 \text{ second}} = -5 \text{ ft/sec}$ <ul style="list-style-type: none"> A newspaper reports these changes in the price of a stock over four days: $\frac{-1}{8}, \frac{-5}{8}, \frac{3}{8}, \frac{-9}{8}.$ <p>What is the average daily change?</p> <p><i>Solution:</i></p> <p>The sum is $\frac{-12}{8}$; dividing by 4 will give a daily average of $\frac{-3}{8}$</p> <ul style="list-style-type: none"> Find an efficient strategy to evaluate the expression and complete the necessary work. <p><i>Methods will vary.</i></p> <div style="display: flex; align-items: center;"> <div style="flex: 1;"> $\begin{aligned} &4 \times \frac{1}{3} \times (-8) \times 9 \times \left(-\frac{1}{2}\right) \\ &4 \times \left[\left(-\frac{1}{2}\right) \times (-8)\right] \times 9 \times \frac{1}{3} \\ &4 \times 4 \times \left[9 \times \frac{1}{3}\right] \\ &4 \times 4 \times [3] \\ &4 \times 12 \\ &48 \end{aligned}$ </div> <div style="flex: 1; padding-left: 20px;"> <p><i>Commutative multiplication</i></p> <p><i>Associative Property</i></p> <p><i>Associative Property</i></p> <p><i>Associative Property</i></p> </div> </div>	
--	--	--	--	--

				<ul style="list-style-type: none"> <p>Multiply the expression using the distributive property.</p> $9 \times \left(-3\frac{1}{2}\right)$ $9 \times \left(-3 + \left(-\frac{1}{2}\right)\right)$ $\underbrace{(9 \times (-3))} + \underbrace{\left(9 \times \left(-\frac{1}{2}\right)\right)}$ $-27 + \left(-4\frac{1}{2}\right)$ $-31\frac{1}{2}$ 	
--	--	--	--	---	--

7th Grade Math First Quarter

Unit 2: Ratios and Proportional Relationships (4 weeks)

Topic A: Proportional Relationships

In Unit 2, students build upon their Grade 6 reasoning about ratios, rates, and unit rates (**6.RP.1**, **6.RP.2**, **6.RP.3**) to formally define proportional relationships and the constant of proportionality (**7.RP.2**). In Topic A, students examine situations carefully to determine if they are describing a proportional relationship. Their analysis is applied to relationships given in tables, graphs, and verbal descriptions (**7.RP.2a**).

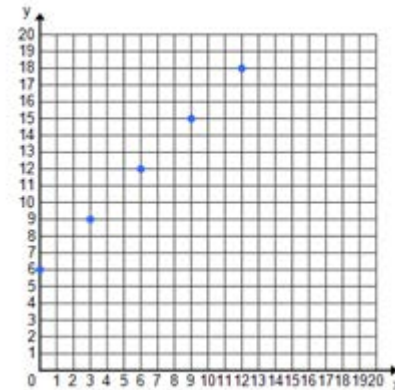
Big Idea:			<ul style="list-style-type: none"> Rates, ratios, and proportional relationships express how quantities change in relationship to each other. Rates, ratios, and proportional relationships can be represented in multiple ways. Rates, ratios, and proportional relationships can be applied to problem solving situations. 		
Essential Questions:			<ul style="list-style-type: none"> How do rates, ratios, and proportional relationships apply to our world? When and why do you use proportional comparisons? How does comparing quantities describe the relationship between them? How can you determine if a relationship is proportional? How are proportional quantities represented in a graph? 		
Vocabulary			proportional to, proportional relationship, ratio, rate, unit rate, equivalent ratio, ratio table, associated rate, origin, coordinate plane		
Grade	Domain	Standard	AZ College and Career Readiness Standards	Explanations & Examples	Resources
7	RP	2a	A. Analyze proportional relationships and use them to solve real-world and mathematical problems. Recognize and represent proportional relationships between quantities. <ol style="list-style-type: none"> Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. 	<u>Explanations:</u> Students' understanding of the multiplicative reasoning used with proportions continues from 6th grade. Students determine if two quantities are in a proportional relationship from a table or by graphing on a coordinate plane. Fractions and decimals could be used with this standard. Students may use a content web site and/or interactive white board to create tables and graphs of proportional or non-proportional relationships. Students model with mathematics (MP.4) and attend to precision (MP.6) as they look for and express repeated reasoning (MP.8) by	Eureka Math: Module 1 Lessons 1-6 Big Ideas: Section: 5.2, extension 5.2, 5.6

		<p>7.MP.3. Construct viable arguments and critique the reasoning of others.</p> <p>7.MP.4. Model with mathematics.</p> <p>7.MP.6. Attend to precision.</p> <p>7.MP.8. Look for and express regularity in repeated reasoning.</p>	<p>generating various representations of proportional relationships.</p> <p>Note: This standard focuses on the representations of proportions. Solving proportions is addressed in 7.RP.3.</p> <p><u>Examples:</u></p> <ul style="list-style-type: none">The table below gives the price for different numbers of books. Do the numbers in the table represent a proportional relationship? <table><tr><th>Number of Books</th><th>Price</th></tr><tr><td>1</td><td>3</td></tr><tr><td>3</td><td>9</td></tr><tr><td>4</td><td>12</td></tr><tr><td>7</td><td>18</td></tr></table> <p><i>Solution:</i></p> <p>Students can examine the numbers to determine that the price is the number of books multiplied by 3, except for 7 books. The row with seven books for \$18 is not proportional to the other amounts in the table; therefore, the table does not represent a proportional relationship.</p> <p>Students graph relationships to determine if two quantities are in a proportional relationship and to interpret the ordered pairs. If the amounts from the table above are graphed (number of books, price), the pairs (1, 3), (3, 9), and (4, 12) will form a straight line through the origin (0 books, 0 dollars), indicating that these pairs are in a proportional relationship. However, the ordered pair (7, 18) would not be on the line, indicating that it is not proportional to the other pairs. Therefore, the table does not represent a proportional relationship between the number of books and the price.</p> <ul style="list-style-type: none">Graph the following data. Are x and y proportional? Explain	Number of Books	Price	1	3	3	9	4	12	7	18	
Number of Books	Price													
1	3													
3	9													
4	12													
7	18													

your reasoning.

x	y
0	6
3	9
6	12
9	15
12	18

Solution:



There is not a proportional relationship between x and y . Although the points lie on the same line, the line does not go through the origin.

7th Grade Math First Quarter

Unit 2: Ratios and Proportional Relationships

Topic B: Unit Rate and the Constant of Proportionality

In Topic B, students learn that the unit rate of a collection of equivalent ratios is called the *constant of proportionality* and can be used to represent proportional relationships with equations of the form $y = kx$, where k is the constant of proportionality (**7.RP.2b**, **7.RP.2c**, **7.EE.4a**). Students relate the equation of a proportional relationship to ratio tables and to graphs and interpret the points on the graph within the context of the situation (**7.RP.2d**).

Big Idea:

- Rates, ratios, and proportional relationships express how quantities change in relationship to each other.
- Rates, ratios, and proportional relationships can be represented in multiple ways.
- Rates, ratios, and proportional relationships can be applied to problem solving situations.

Essential Questions:

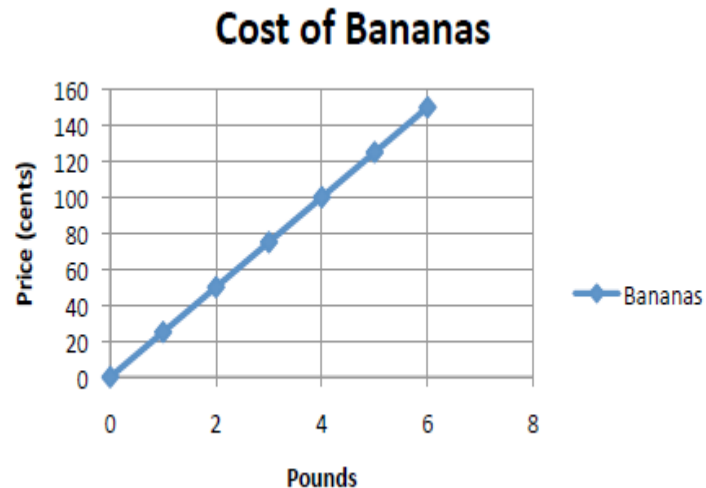
- How do rates, ratios, and proportional relationships apply to our world?
- When and why do I use proportional comparisons?
- How does comparing quantities describe the relationship between them?
- How do graphs illustrate proportional relationships?
- What does the unit rate represent?
- How can an equation be used to represent a proportional relationship?

Vocabulary

Proportional to, proportional relationship, constant of proportionality, ratio, rate of change, unit rate, equivalent ratio, ratio table, independent variable, dependent variable

Grade	Domain	Standard	AZ College and Career Readiness Standards	Explanations & Examples	Resources
7	RP	2bcd	<p>A. Analyze proportional relationships and use them to solve real-world and mathematical problems.</p> <p>Recognize and represent proportional relationships between quantities.</p> <p>b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.</p> <p>c. Represent proportional relationships by</p>	<p><u>Explanation:</u></p> <p>Students identify the constant of proportionality (unit rate) from tables, graphs, equations and verbal descriptions of proportional relationships. Fractions and decimals could be used with this standard.</p> <p>Graphing proportional relationships represented in a table helps students recognize that the graph is a line through the origin (0,0) with a constant of proportionality equal to the rate of change (slope) of the line.</p> <p>Students model with mathematics (MP.4) and attend to precision (MP.6) as they look for and express repeated reasoning (MP.8) by</p>	<p>Eureka Math: Module 1 Lessons 7-10</p> <p>Big Ideas: Sections: Extension 5.2, 5.3–5.6</p>

		<p>equations. <i>For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.</i></p> <p>d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.</p> <p>7.MP.1. Make sense of problems and persevere in solving them.</p> <p>7.MP.2. Reason abstractly and quantitatively.</p> <p>7.MP.3. Construct viable arguments and critique the reasoning of others.</p> <p>7.MP.4. Model with mathematics.</p> <p>7.MP.5. Use appropriate tools strategically.</p> <p>7.MP.6. Attend to precision.</p> <p>7.MP.7. Look for and make use of structure.</p> <p>7.MP.8. Look for and express regularity in repeated reasoning.</p>	<p>generating various representations of proportional relationships and use those representations to identify and describe constants of proportionality.</p> <p>Students write equations from context and identify the coefficient as the unit rate (rate of change, slope) which is also the constant of proportionality.</p> <p>Note: This standard focuses on the representations of proportions. Solving proportions is addressed in 7.RP.3.</p> <p><u>Examples:</u></p> <ul style="list-style-type: none">Why are the two quantities in the table proportional? What is the constant of proportionality? Explain the meaning of each data point and write an equation that represents the data. <table><tr><th>Number of Books</th><th>Price</th></tr><tr><td>1</td><td>3</td></tr><tr><td>3</td><td>9</td></tr><tr><td>4</td><td>12</td></tr><tr><td>7</td><td>21</td></tr></table> <p>The ordered pair $(4, 12)$ means that 4 books cost \$12, $(3, 9)$ means 3 books cost \$9, and $(7, 21)$ means 7 books cost \$21. The ordered pair $(1, 3)$ indicates that 1 book is \$3, which is the unit rate. The y-coordinate when $x = 1$ will be the unit rate. The constant of proportionality is the unit rate. The equation, $P=3b$, represents the relationship between the number of books, b, and the total price, P.</p> <ul style="list-style-type: none">The graph below represents the price of the bananas at one store. What is the constant of proportionality?	Number of Books	Price	1	3	3	9	4	12	7	21	
Number of Books	Price													
1	3													
3	9													
4	12													
7	21													



Solution:

From the graph, it can be determined that 4 pounds of bananas is \$1.00; therefore, 1 pound of bananas is \$0.25, which is the constant of proportionality for the graph. Note: Any point on the line will yield this constant of proportionality.

- The price of bananas at another store can be determined by the equation: $P = \$0.35n$, where P is the price and n is the number of pounds of bananas. What is the constant of proportionality (unit rate)?

Solution:

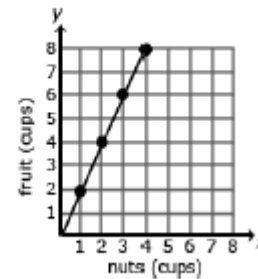
The constant of proportionality is the coefficient of x (or the independent variable). The constant of proportionality is 0.35.

- A student is making trail mix. Create a graph to determine if the quantities of nuts and fruit are proportional for each serving size listed in the table. If the quantities are

proportional, what is the constant of proportionality or unit rate that defines the relationship? Explain how the constant of proportionality was determined and how it relates to both the table and graph.

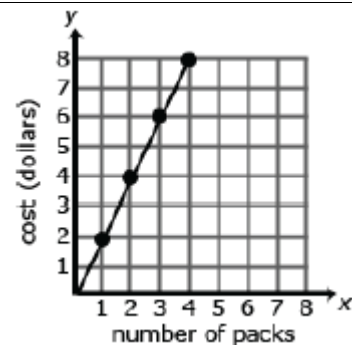
Serving Size	1	2	3	4
cups of nuts (x)	1	2	3	4
cups of fruit (y)	2	4	6	8

Solution:



The relationship is proportional. For each of the other serving sizes there are 2 cups of fruit for every 1 cup of nuts (2:1). The constant of proportionality is shown in the first column of the table and by the steepness (rate of change) of the line on the graph.

- The graph below represents the cost of gum packs as a unit rate of \$2 for every pack of gum. The unit rate is represented as \$2/pack. Represent the relationship using a table and an equation.



Solution:

Table:

Number of Packs of Gum (g)	Cost in Dollars (d)
0	0
1	2
2	4
3	6
4	8

Equation: $d = 2g$, where d is the cost in dollars and g is the packs of gum.

A common error is to reverse the position of the variables when writing equations. Students may find it useful to use variables specifically related to the quantities rather than using x and y . Constructing verbal models can also be helpful. A student might describe the situation as “the number of packs of gum times the cost for each pack is the total cost in dollars”. They can use this verbal model to construct the equation. Students can check their equation by substituting values and comparing their results to the table. The checking process helps student revise and recheck their model as necessary. The number of packs of gum times the cost for each pack is the total cost. ($g \times 2 = d$)

7th Grade Math First Quarter

Unit 2: Ratios and Proportional Relationships

Topic C: Ratios and Rates Involving Fractions

In Topic C, students extend their reasoning about ratios and proportional relationships to compute unit rates for ratios and rates specified by rational numbers, such as a speed of $\frac{1}{2}$ mile per $\frac{1}{4}$ hour (**7.RP.1**). Students apply their experience in the first two topics and their new understanding of unit rates for ratios and rates involving fractions to solve multistep ratio word problems (**7.RP.3**, **7.EE.4a**).

Big Idea:

- Rates, ratios, and proportional relationships express how quantities change in relationship to each other.
- Rates, ratios, and proportional relationships can be represented in multiple ways.
- Rates, ratios, and proportional relationships can be applied to problem solving situations.

Essential Questions:

- How do rates, ratios, percentages and proportional relationships apply to our world?
- When and why do I use proportional comparisons?
- How does comparing quantities describe the relationship between them?
- How do graphs illustrate proportional relationships?
- How can I use proportional relationships to solve ratio problems?

Vocabulary

Proportional to, proportional relationship, constant of proportionality, ratio, rate, unit rate, ratio table, complex fraction, commission, mark-down, mark-up, discount

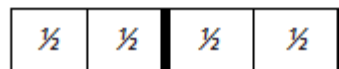
Grade	Domain	Standard	AZ College and Career Readiness Standards	Explanations & Examples	Resources
7	RP	1	<p>A. Analyze proportional relationships and use them to solve real-world and mathematical problems.</p> <p>Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{1/2}{1/4}$ miles per hour, equivalently 2 miles per hour.</p> <p>7.MP.2. Reason abstractly and quantitatively.</p>	<p><u>Explanation:</u></p> <p>Students continue to work with unit rates from 6th grade; however, the comparison now includes fractions compared to fractions. The comparison can be with like or different units. Fractions may be proper or improper.</p> <p>Students interpreted and computed quotients of fractions in 6th grade; solving word problems involving division of fractions by fractions (6.NS.A.1). Using visual representations to review this concept may be necessary.</p> <p>For example, review dividing fractions using a bar model.</p>	<p>Eureka Math: Module 1 Lessons 11-15</p> <p>Big Ideas: Sections: 5.1</p>

7.MP.6. Attend to precision.

•

$$1\frac{3}{4} \div \frac{1}{2}$$

1. Make $1\frac{3}{4}$ blocks, represented by green blocks.
2. Divide the blocks into groups of $\frac{1}{2}$.
3. The number of $\frac{1}{2}$ that are shaded are $3\frac{1}{2}$.



•

$$2\frac{1}{3} \div \frac{1}{6}$$

4. Make $2\frac{1}{3}$ blocks, represented by green blocks.
5. Divide the whole blocks into groups of 6th.
6. The number of $\frac{1}{6}$ blocks that are shaded is 14.



Examples:

				<ul style="list-style-type: none"> If $\frac{1}{2}$ gallon of paint covers $\frac{1}{6}$ of a wall, then how much paint is needed for the entire wall? <p><i>Solution:</i></p> $\frac{\frac{1}{2}(\text{gallon})}{\frac{1}{6}(\text{wall})} = \frac{\frac{1}{2} \times 6(\text{gallons})}{\frac{1}{6} \times 6(\text{wall})} = \frac{3\text{gallons}}{1\text{ wall}} = 3 \text{ gallons/wall}$	
7	RP	3	<p>A. Analyze proportional relationships and use them to solve real-world and mathematical problems.</p> <p>Use proportional relationships to solve multi-step ratio and percent problems. <i>Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.</i></p> <p>7.MP.1. Make sense of problems and persevere in solving them.</p> <p>7.MP.2. Reason abstractly and quantitatively.</p> <p>7.MP.3. Construct viable arguments and critique the reasoning of others.</p> <p>7.MP.4. Model with mathematics.</p> <p>7.MP.5. Use appropriate tools strategically.</p> <p>7.MP.6. Attend to precision.</p> <p>7.MP.7. Look for and make use of structure.</p> <p>7.MP.8. Look for and express regularity in repeated reasoning.</p>	<p><u>Explanation:</u></p> <p>In 6th grade, students used ratio tables and unit rates to solve problems. Students expand their understanding of proportional reasoning to solve problems that are more efficient to solve by setting up a proportion. Students use the properties of equality to solve the equation.</p> <p>Note: Only include one-step equations in this unit (6.EE.B.7). Students will learn how to solve multi-step equations in Unit 5.</p> <p><u>Examples:</u></p> <ul style="list-style-type: none"> Sally has a recipe that needs $\frac{3}{4}$ teaspoon of butter for every 2 cups of milk. If Sally increases the amount of milk to 3 cups of milk, how many teaspoons of butter are needed? <p><i>Solution:</i></p> <p>Using these numbers to find the unit rate may not be the most efficient method. Students can set up the following proportion to show the relationship between butter and milk.</p> $\frac{\frac{3}{4}\text{teaspoons of butter}}{2\text{ cups of milk}} = \frac{x\text{ teaspoons of butter}}{3\text{ cups of milk}}$ <p>Multiplying both sides of the equation by 6 cups of milk results in:</p>	<p>Eureka Math: Module 1 Lessons 11-15</p> <p>Big Ideas: Sections: 5.1, 5.3</p>

$$6 \text{ cups of milk} \times \frac{\frac{3}{4} \text{ teaspoons of butter}}{2 \text{ cups of milk}} = \frac{x \text{ teaspoons of butter}}{3 \text{ cups of milk}} \times 6 \text{ cups of milk}$$

$$\frac{9}{4} \text{ teaspoons of butter} = 2x \text{ teaspoons of butter}$$

Multiply both sides of the equation by $\frac{1}{2}$.

$$x = \frac{9}{8} \text{ teaspoons} = 1\frac{1}{8} \text{ teaspoons of butter.}$$

- A retail clothing store advertises the following sale: Shorts are $\frac{1}{2}$ off the original price; pants are $\frac{1}{3}$ off the original price, and shoes at $\frac{1}{4}$ off the original price (called the discount rate). If a pair of shoes costs \$40, what is the sale price?

Solution:

Method 1: Tape Diagram



$\frac{1}{4}$ off the original price is \$30.

Method 2: Subtracting $\frac{1}{4}$

of the price from the original price

$$40 - \frac{1}{4}(40) =$$

$$40 - 10 =$$

$$\$30$$

Method 3: Finding the fractional part
of the price being paid by subtracting

$\frac{1}{4}$ of the price from 1 whole

$$\left(1 - \frac{1}{4}\right) 40 =$$

$$\left(\frac{3}{4}\right) 40 =$$

$$\$30$$

- A used car sales person receives a commission of $\frac{1}{12}$ of the sales price of the car for each car he sells. What would the sales commission be on a car that sold for \$21,999?

Solution:

$$\text{Commission} = \$21,999 \left(\frac{1}{12}\right) = \$1833.25$$