

7th Grade Mathematics (Accelerated)

Proportionality and Linear Relationships

Unit 2 Pacing Calendar



ORANGE PUBLIC SCHOOLS
OFFICE OF CURRICULUM AND INSTRUCTION
OFFICE OF MATHEMATICS

From the Common Core State Standards:

Traditional Pathway Accelerated 7th Grade

In **Accelerated 7th Grade**, instructional time should focus on four critical areas: (1) Rational Numbers and Exponents; (2) Proportionality and Linear Relationships; (3) Introduction to Sampling Inference; (4) Creating, Comparing, and Analyzing Geometric Figures

1. Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percent as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems. They extend their mastery of the properties of operations to develop an understanding of integer exponents, and to work with numbers written in scientific notation.

2. Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x -coordinate changes by an amount A , the output or y -coordinate changes by the amount $m \times A$. Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation.

3. Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences

4. Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity, they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms. Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

A STORY OF UNITS

	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN
K										
1										
2										
3										
4										
5										
6										
Acc 7	Rational Numbers & Exponents			Proportionality & Linear Relationships			Sampling and Inference		Geometry	



Rational Numbers and Exponents: Operations with rational numbers, learn of irrational numbers, expressions and equations work with radical and integer exponents



Proportionality and Linear Relationships: Analyze proportional relationships, generate equivalent expressions using properties of operations, and understand connections between proportional relationships and



Sampling and Inference: Use random sampling, draw inferences, investigate chance processes, develop, use, and evaluate probability models



Geometry: construct geometrical figures, understand congruence and similarity using physical models, and solve real-life problems involving angle measure, area, surface area and volume

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Unit Overview

In this unit, students will

- Students compute unit rates associated with ratios of quantities measured in different units.
- Students use the context of the problem to recall the meaning of value of a ratio, equivalent ratios, rate and unit rate, relating them to the context of the experience.
- Students understand that two quantities are proportional to each other when a constant (number) exists such that each measure in the first quantity multiplied by this constant gives the corresponding measure in the second quantity.
- Students study examples of relationships that are not proportional in addition to those that are.
- Students decide whether two quantities are proportional to each other by graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- Students find and interpret the constant of proportionality within the contexts of problems.
- Students use the constant of proportionality to represent proportional relationships by equations in real world contexts as they relate the equations to a corresponding ratio table and/or graphical representation.
- Students write mathematical statements using symbols to represent numbers.
- Students know that written statements can be written as more than one correct mathematical sentence.
- Understand ratios, rates, and percents
- Understand proportionality in tables, graphs, and equations
- Recognize that constant growth in a table, graph, or equation is related to proportional situations
- Write an equation to represent the pattern in a table or graph of proportionally related variables
- Relate the unit rate and constant of proportionality to an equation, graph, or table describing a proportional situation
- Develop and use strategies for solving problems that require proportional reasoning

- Recognize situations in which proportional reasoning is appropriate to solve the problem
- Students know the properties of linear and nonlinear expressions in x
- Students transcribe and identify expressions as linear or nonlinear.
- Students know that a linear equation is a statement of equality between two expressions.
- Use various strategies to solve for an unknown in a proportion, including scaling, rate tables, percent bars, unit rates, and equivalent ratios
- Set up and solve proportions that arise from real-world applications, such as finding discounts and markups and converting measurement units

Pacing Guide & Calendar

Activity	New Jersey State Learning Standards (NJSLs)	Estimated Time
Grade 7 MIF Chapter 4 Pretest	7.EE.4	1 Block
Grade 7 Module 1 (EngageNY) Lesson 1-5	7.RP.1, 7.RP.3	5 Blocks
Grade 7 Module 1 (EngageNY) Lesson 6-10	7.RP.1, 7.RP.2, 7.RP.3	5 Blocks
Unit 2 Performance Task 1	7.RP.2, 7.RP.3	½ Block
Grade 7 Module 1 (EngageNY) Lesson 11-15	7.RP.1, 7.RP.2, 7.RP.3	5 Blocks
Unit 2 Assessment 1	7.RP.1, 7.RP.2, 7.RP.3	½ Block
Grade 7 Chapter 4 (MIF) Lesson 1-5	7.EE.1, 7.EE.2, 7.EE.3, 7.EE.4	4 Blocks
Grade 7 Module 3 (EngageNY) Lesson 7-15	7.EE.3, 7.EE.4, 7.G.5	8 Blocks
Unit 2 Assessment 2	7.EE.1, 7.EE.2, 7.EE.3, 7.EE.4	1 Block
Grade 8 Module 4 (EngageNY) Lesson 1-9	8.EE.5, 8.EE.6, 8.EE.7	8 Blocks
Unit 2 Performance Task 2	7.EE.3	½ Block
Grade 8 Module 4 (EngageNY) Lesson 10-13	8.EE.5, 8.EE.6, 8.EE.7	3 Blocks
Unit 2 Performance Task 3	8.EE.5	½ Block
Grade 8 Module 4 (EngageNY) Lesson 15-22	8.EE.5, 8.EE.6, 8.EE.7	7 Blocks
Unit 2 Assessment 3	8.EE.5, 8.EE.6, 8.EE.7	1 Block
Total Time		50 Blocks

Major Work Supporting Content Additional Contents

Pacing Calendar: Please complete the pacing calendar based on the suggested pacing (*see Pacing Guide on page 4*).

Math in Focus Chapter 4: In this chapter, students will learn how to solve algebraic equations and inequalities

EngageNY Grade 7 Module 1: Ratios and Proportional Relationship (Topic A - C): Students explore multiple representations of proportional relationships by looking at tables, graphs, equations, and verbal descriptions. Students extend their understanding about ratios and proportional relationships to compute unit rates for ratios and rates specified by rational numbers.

EngageNY Grade 7 Module 3: Expressions and Equations (Topic B): This module consolidates and expands upon students' understanding of equivalent expressions as they apply the properties of operations to write expressions in both standard form and in factored form. Students use the number line to understand the properties of inequality and recognize when to preserve the inequality and when to reverse the inequality when solving problems leading to inequalities. They interpret solutions within the context of problems.

EngageNY Grade 8 Module 4: Linear Equations (Topic A – C): Students extend what they already know about unit rates and proportional relationships to linear equations and their graphs. Students understand the connections between proportional relationships, lines, and linear equations in this module. Students learn to apply the skills they acquired in Grades 6 and 7, with respect to symbolic notation and properties of equality to transcribe and solve equations in one variable and then in two variables.

NOVEMBER

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

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DECEMBER

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

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JANUARY

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

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FEBRUARY

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28			

PARCC Assessments Evidence Statements

NJSLS	Evidence Statement	Clarification	Math Practices	Calculator?
<u>7.RP.1</u>	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction miles per hour, equivalently 2 miles per hour.	i) Tasks have a context	2, 6, 4	Yes
<u>7.RP.2a</u>	Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.	i) Tasks have “thin context” or no context. ii) Tasks may offer opportunities for students to investigate a relationship by constructing graphs or tables; however, students can opt not to use these tools. iii) Tasks are not limited to ratios of whole numbers	2,5	Yes
<u>7.RP.2b</u>	Recognize and represent proportional relationships between quantities. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.	i) Pool should contain tasks with and without context. ii) Tasks sample equally across the listed representations (graphs, equations, diagrams, and verbal descriptions).	2,5,8	No
<u>7.RP.2c</u>	Recognize and represent proportional relationships between quantities. c. Represent proportional relationships by equations. <i>For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between total the total cost and the number of items can be expressed as $t = pn$.</i>	i) Tasks have a context	2, 8	No
<u>7.RP.2d</u>	Recognize and represent proportional relationships between quantities. d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.	i) Tasks require students to interpret a point (x, y) on the graph of a proportional relationship in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.	2, 4	No
<u>7.RP.3-1</u>	Use proportional relationships to solve multi-step ratio problems.	i) Tasks will include proportional relationships that only involve positive numbers.	1, 2, 6	Yes
<u>7.RP.3-2</u>	Use proportional relationships to solve multi-step percent problems. <i>Examples: simple interest, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.</i>	-	1, 2, 5, 6	Yes
<u>7.G.1</u>	Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	i) Tasks may or may not have context	2, 5	Yes

Accelerated 7th Unit 2: Proportionality and Linear Relationships

<u>7.EE.1</u>	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients	i) Tasks are not limited to integer coefficients. ii) Tasks may involve issues of strategy, e.g., by providing a factored expression such as $y(3+x+k)$ and a fully expanded expression $3y + xy + ky$, and requiring students to produce or identify a new expression equivalent to both (such as $y(3+x) + yk$).	7	No
<u>7.EE.2</u>	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. <i>For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."</i>	-	7	No
<u>7.EE.3</u>	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. <i>For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional $\frac{1}{10}$ of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.</i>	-	5	Yes
<u>7.EE.4a-1</u>	Use variables to represent quantities in a real world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers.	i) Comparison of an algebraic solution to an arithmetic solution is not assessed here; for this aspect of 7.EE.4a, see 7.C.5.	1,2,6,7	No
<u>7.EE.4a.2</u>	Use variables to represent quantities in a real world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Fluently solve equations of the form $px + q = r$ and $p(x+q) = r$, where p , q , and r are specific rational numbers.	i) Each task requires students to solve two equations (one of each of the given two forms). Only the answer is required. ii) Comparison of an algebraic solution to an arithmetic solution is not assessed here; for this aspect of 7.EE.4a, see 7.C.5.	6,7	No
<u>7.EE.4b</u>	Use variables to represent quantities in a real world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. b. Solve word problems leading to	i) Tasks may involve $<$, $>$, \leq or \geq .	1,2,5,6,7	No

Accelerated 7th Unit 2: Proportionality and Linear Relationships

	inequalities of the form $px + q > r$ or $px + q < r$, where p , q and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. <i>For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.</i>			
<u>8.EE.5-1</u>	Graph proportional relationships, interpreting the unit rate as the slope of the graph.	Tasks may or may not contain context.	1, 5	Yes
<u>8.EE.5-2</u>	Compare two different proportional relationships represented in different ways. <i>For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has a greater speed.</i>	Tasks may or may not contain context.	7	Yes
<u>8.EE.6-1</u>	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane.	ii) Tasks do not have a context. ii) Given a non-vertical line in the coordinate plane, tasks might for example require students to choose two pairs of points and record the rise, run, and slope relative to each pair and verify that they are the same. iii) For the explain aspect of 8.EE.6, see 8.C.5.1.	2, 7	Yes
<u>8.EE.7b</u>	Solve linear equations in one variable. b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms	i) Tasks do not have a context.	6,7	No

Connections to the Mathematical Practices

1	Make sense of problems and persevere in solving them
	<ul style="list-style-type: none"> – Students make sense of ratio and unit rates in real-world contexts. They persevere by selecting and using appropriate representations for the given contexts.
2	Reason abstractly and quantitatively
	<ul style="list-style-type: none"> – Students will reason about the value of the rational number in relation to the models that are created to represent them.
3	Construct viable arguments and critique the reasoning of others
	<ul style="list-style-type: none"> – Students use arguments to justify their reasoning when creating and solving proportions used in real-world contexts.
4	Model with mathematics
	<ul style="list-style-type: none"> - Students create models using tape diagrams, double number lines, manipulatives, tables and graphs to represent real-world and mathematical situations involving ratios and proportions. For example, students will examine the relationships between slopes of lines and ratio tables in the context of given situations
5	Use appropriate tools strategically
	<ul style="list-style-type: none"> – Students use visual representations such as the coordinate plane to show the constant of proportionality.
6	Attend to precision
	<ul style="list-style-type: none"> - Students attend to the ratio and rate language studied in grade 6 to represent and solve problems involving rates and ratios.
7	Look for and make use of structure
	<ul style="list-style-type: none"> - Students look for patterns that exist in ratio tables in order to make connections between the constant of proportionality in a table with the slope of a graph.
8	Look for and express regularity in repeated reasoning
	<ul style="list-style-type: none"> – Students formally begin to make connections between covariance, rates, and representations showing the relationships between quantities. – Students might discover that points on the graph will always match the unit rate of the problem at hand.

Vocabulary

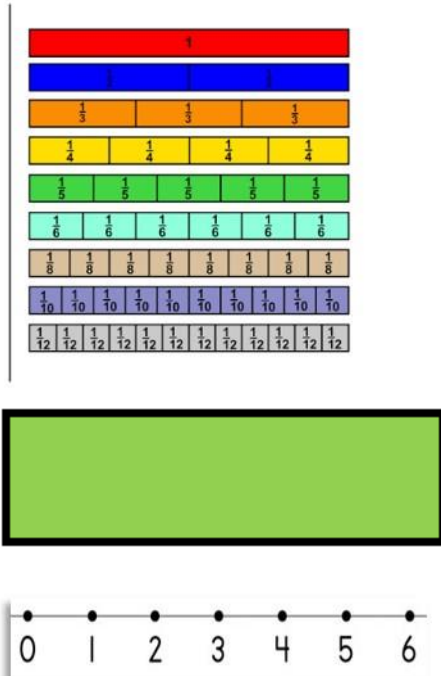
Term	Definition
<i>Constant of Proportionality</i>	Constant value of the ratio of proportional quantities x and y . Written as $y = kx$, k is the constant of proportionality when the graph passes through the origin. Constant of proportionality can never be zero.
<i>Corresponding Angles</i>	Corresponding angles have the same relative position in similar figures.
<i>Corresponding Sides</i>	Corresponding sides have the same relative position in similar figures.
<i>Directly Proportional</i>	If $y = kx$, then y is said to be directly proportional to x
<i>Equivalent Fractions</i>	Two fractions that have the same value but have different numerators and denominators; equivalent fractions simplify to the same fraction.
<i>Fraction</i>	A number expressed in the form a/b where a is a whole number and b is a positive whole number
<i>Inversely Proportional</i>	y is <i>inversely proportional</i> to x if $y = k/x$.
<i>Multiplicative Inverse</i>	Two numbers whose product is 1. Example $(3/4)$ and $(4/3)$ are multiplicative inverses of one another because $(3/4) \times (4/3) = (4/3) \times (3/4) = 1$
<i>Origin</i>	The point of intersection of the vertical and horizontal axes of a Cartesian Grid
<i>Percent rate of change</i>	A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by $(5/50) = 10\%$ per year
<i>Proportion</i>	An equation stating that two ratios are equivalent
<i>Proportional Relationship</i>	Two quantities are said to have a proportional relationship if they vary in such a way that one of the quantities is a constant multiple of the other, or equivalently if they have a constant ratio.
<i>Ratio</i>	A comparison of two numbers using division. The ratio of a to b (where $b \neq 0$) can be written as a to b , as (a/b) , or as $a:b$.
<i>Scale Factor</i>	A ratio between two sets of measurements

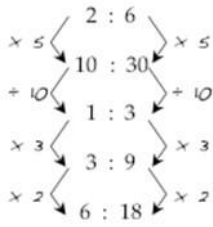
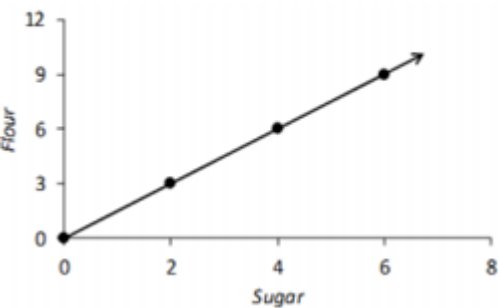
<i>Algebraic Expression</i>	An expression consisting of at least one variable and also consisting of numbers and operations
<i>Coefficient</i>	The number part of a term that includes a variable. For example, 3 is the coefficient of the term $3x$.
<i>Constant</i>	A quantity having a fixed value that does not change or vary, such as a number. For example, 5 is the constant of $x + 5$.
<i>Equation</i>	A mathematical sentence formed by setting two expressions equal.
<i>Inequality</i>	<i>A mathematical sentence formed by placing inequality symbol between two expressions</i>
<i>Term</i>	A number, a variable, or product and a number and variable
<i>Numerical Expression</i>	An expression consisting of numbers and operations
<i>Variable</i>	A symbol, usually a letter, which is used to represent one or more numbers
<i>Unit Rate</i>	Ratio in which the second term, or denominator, is 1

Potential Student Misconceptions

- Students will often confuse the terms 'ratio' and 'proportion', and need a clear understanding of when each is appropriate.
- Students see their work with ratios and proportion as an additive operation, replacing the multiplicative concepts with repeated additions
- Students may the concept "part/whole" of a fraction yet have difficulty with the concept of fraction equivalence, confuse quantity notions with proportionality, possess limited views of fractions as numbers, and have cognitive difficulty relating fractions to division
- Students may write the ratio with reversed values.
- Students may write the ratios without using same units.
- Students may not ensure there is a single unit in the denominator when finding the unit rate.
- Students may set up the proportion incorrectly.
- Not checking to see if solution results in a true proportion
- Changes in scale also change the values on the graph, with students unaware that the real change is their perception of the graph or the amount of the graph visible on the screen
- Many of the misconceptions when dealing with expressions stem from the misunderstanding/reading of the expression. For example, knowing the operations that are being referenced with notation like x^3 , $4x$, $3(x + 2y)$ is critical. The fact that x^3 means $(x)(x)(x)$ which is x times x times x , not $3x$ or 3 times x ; $4x$ means 4 times x or $x + x + x + x$, not forty-something.
- When collecting like terms, students fail to relate their knowledge of the addition of constants to the collection of variables.
- Students also misunderstand a "1" in front of a lone variable like a or x or p . For example, not realizing that $4a + a$ is $5a$.

Teaching Multiple Representations

CONCRETE REPRESENTATIONS												
<p>Partitioning with manipulatives</p> <p>Bar model</p> <p>Paper strips</p> <p>Number line</p>												
PICTORIAL REPRESENTATIONS												
<p>Fraction Strips (Tape Diagrams)</p> <p>Double Line Diagrams</p> <p>Table representation</p> <p>Graphical representation</p>	<div style="text-align: center;"> <p>Original Price</p> <table border="1" style="margin: auto;"> <tr> <td>20%</td> <td>20%</td> <td>20%</td> <td>20%</td> <td>20%</td> </tr> </table> <p>Sale Price \$5</p> </div> <hr/> <div style="display: flex; justify-content: space-between;"> <div style="background-color: #6aa84f; color: white; padding: 5px; border-radius: 5px;">Core Lesson</div> <div style="background-color: #fff9c4; padding: 5px; border: 1px solid #ccc;"> Books to DVDs 3 to 2 15 books ? DVDS </div> </div> <div style="margin-top: 20px;"> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 10%;"></td> <td style="text-align: center;">+3 +3 +3 +3</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">Books</td> <td style="border-bottom: 1px solid black; text-align: center;">3 6 9 12</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">DVDs</td> <td style="border-bottom: 1px solid black; text-align: center;">2 4 6 8</td> </tr> </table> </div>	20%	20%	20%	20%	20%		+3 +3 +3 +3	Books	3 6 9 12	DVDs	2 4 6 8
20%	20%	20%	20%	20%								
	+3 +3 +3 +3											
Books	3 6 9 12											
DVDs	2 4 6 8											

<p>Equivalent ratios</p> <p>Ratio Tables</p> <p>Coordinate Plane</p>	 <p><u>Ratio Table</u></p> <table border="1" data-bbox="820 546 982 787"> <thead> <tr> <th>Sugar</th> <th>Flour</th> </tr> </thead> <tbody> <tr> <td>2</td> <td>3</td> </tr> <tr> <td>4</td> <td>6</td> </tr> <tr> <td>6</td> <td>9</td> </tr> </tbody> </table> <p><u>Coordinate Plane</u></p> 	Sugar	Flour	2	3	4	6	6	9
Sugar	Flour								
2	3								
4	6								
6	9								
ABSTRACT REPRESENTATIONS									
<p>Scale Factor (within and between)</p> <p>Iteration</p> <p>Algorithm</p> <p>Part/Whole Relationships</p> <p>Part/Part Relationships</p> <p>Finding the Unit Rate/Constant of Proportionality</p> <p>Simplifying Rates</p>	<p>Setting up a Proportion</p> <p>Creating an Equation</p> <p>Finding the constant of proportionality</p> <p>Algorithm for Scale Factor: Image/Actual figure Actual Figure/Image $a/b = c/d$</p>								

Unit 1 Assessment Framework

Unit 2 Assessment Framework				
Assessment	NJSLS	Estimated Time	Format	Graded ?
Grade 7 Chapter 4 Pretest <i>Math in Focus</i>	7.EE.4	½ Block	Individual	Yes (No Weight)
Unit 2 Assessment 1 (After EngageNY Gr. 7 Module 1) <i>District Assessment</i>	7.RP.1, 7.RP.2b 7.RP.2a, 7.RP.2 7.RP.2d, 7.RP.3	½ Block	Individual	Yes
Unit 2 Assessment 2 (After EngageNY Gr. 7 Module 3) <i>District Assessment</i>	7.EE.1, 7.EE.2 7.EE.3, 7.EE.4 7.EE.4b	1 Block	Individual	Yes
Unit 2 Assessment 3 (Conclusion of Unit) <i>District Assessment</i>	8.EE.5, 8.EE.6, 8.EE.7	1 Block	Individual	Yes
Grade 7 Chapter 4 Test (Optional) <i>Math in Focus</i>	7.EE.4	½ Block	Individual	Yes, if administered
Mid- Module Assessment Gr. 7 Module 1 (Optional) <i>EngageNY</i>	7.RP.1, 7.RP.2, 7.RP.3	Teacher Discretion	Teacher Discretion	Optional
Mid- Module Assessment Gr. 7 Module 3 (Optional) <i>EngageNY</i>	7.EE.1, 7.EE.2 7.EE.3, 7.EE.4	Teacher Discretion	Teacher Discretion	Optional
Mid- Module Assessment Gr. 8 Module 4 (Optional) <i>EngageNY</i>	8.EE.5, 8.EE.6, 8.EE.7	Teacher Discretion	Teacher Discretion	Optional
End of Module Assessment Gr. 7 Module 1 (Optional) <i>EngageNY</i>	7.RP.1, 7.RP.2, 7.RP.3	Teacher Discretion	Teacher Discretion	Optional
End of Module Assessment Gr. 7 Module 3 (Optional) <i>EngageNY</i>	7.EE.1, 7.EE.2 7.EE.3, 7.EE.4	Teacher Discretion	Teacher Discretion	Optional
End of Module Assessment Gr.8 Module 4 (Optional) <i>EngageNY</i>	8.EE.5, 8.EE.6, 8.EE.7	Teacher Discretion	Teacher Discretion	Optional

Unit 2 Performance Assessment Framework				
Assessment	NJSLS	Estimated Time	Format	Graded ?
Unit 1 Performance Task 1 (Mid-December) <i>Thunder and Lightning</i>	7.RP.2	½ Block	Individual w/ Interview Opportunity	Yes; Rubric
Unit 1 Performance Task 2 (Late January) <i>Buying Protein Bars and Magazines</i>	7.RP.3	½ Block	Individual w/ Interview Opportunity	Yes: rubric
Unit 1 Performance Task 3 (Early February) <i>Who has the Best Job?</i>	8.EE.5	½ Block	Individual w/ Interview Opportunity	Yes; Rubric
Unit 1 Performance Task Option 1 (optional)	7.RP.3	Teacher Discretion	Teacher Discretion	Yes, if administered

Name _____

Block _____

Date _____

Thunder and Lightning (7.RP.2)

Alyssa sees a lightning bolt in the sky and counts four seconds until she hears the thunder.

a. There are 5280 feet in a mile and about 3.28 feet in a meter. Given that sound travels about 343 meters per second, is the lightning strike within one mile of Alyssa?

b. What is the speed of sound in miles per hour?

Solution**Solution**

a. Since the thunder from the lightning strike travels at about 343 meters per second, in 4 seconds the sound will have travelled about

$$4 \text{ seconds} \times 343 \frac{\text{meters}}{\text{second}} = 1372 \text{ meters.}$$

Since there are about 3.28 feet in a meter this is

$$1372 \text{ meters} \times 3.28 \frac{\text{feet}}{\text{meter}} \approx 4500 \text{ feet.}$$

This is less than a mile.

b. To convert from meters per second to miles per hour we have to convert both the distances and the times. We will do these separately and then put all of the calculations together to find the speed of the thunder in miles per hour. First we convert 343 meters to miles:

$$\begin{aligned} 343 \text{ meters} &= 343 \text{ meters} \times 3.28 \frac{\text{feet}}{\text{meter}} \times \frac{1 \text{ mile}}{5280 \text{ feet}} \\ &= \frac{343 \times 3.28}{5280} \text{ miles} \\ &\approx 0.213 \text{ miles.} \end{aligned}$$

Next we convert seconds to hours:

$$\begin{aligned} 1 \text{ second} &= 1 \text{ second} \times \frac{1 \text{ minute}}{60 \text{ seconds}} \times \frac{1 \text{ hour}}{60 \text{ minutes}} \\ &= \frac{1}{3600} \text{ hours.} \end{aligned}$$

Equivalently, we have 1 hour = 3600 seconds. When we combine all of our unit conversions, we use the complete expression $\frac{343 \times 3.28}{5280}$ miles instead of the rounded 0.213 miles since the rounding should take place after all operations have been performed. Using our calculations we find

$$\begin{aligned} 343 \frac{\text{meters}}{\text{second}} &\approx \frac{343 \times 3.28}{5280} \frac{\text{miles}}{\text{second}} \\ &= \frac{343 \times 3.28}{5280} \frac{\text{miles}}{\text{second}} \times 3600 \frac{\text{seconds}}{\text{hour}} \\ &\approx 767 \frac{\text{miles}}{\text{hour}} \end{aligned}$$

Unit 2 Performance Task 1 PLD Rubric

SOLUTION

A) $4 \text{ secs} \times 343 \frac{m}{s} = 1372 \text{ m}$ and $1372 \text{ m} \times 3.28 \frac{ft}{m} = 4500 \text{ ft.}$ and this is less than a mile

B) $343 \text{ m} \times 3.28 \frac{ft}{m} \times \frac{1}{5280} \frac{miles}{s} = 0.213 \text{ miles}$

Level 5: Distinguished Command	Level 4: Strong Command	Level 3: Moderate Command	Level 2: Partial Command	Level 1: No Command
<p>Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including:</p> <ul style="list-style-type: none"> • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with 	<p>Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including:</p> <ul style="list-style-type: none"> • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with minor conceptual error 	<p>Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including:</p> <ul style="list-style-type: none"> • a logical, but incomplete, progression of steps • minor calculation errors • partial justification of a conclusion • a logical, but incomplete, progression of steps 	<p>Constructs and communicates an incomplete response based on concrete referents provided in the prompt such as: diagrams, number line diagrams or coordinate plane diagrams, which may include:</p> <ul style="list-style-type: none"> • a faulty approach based on a conjecture and/or stated assumptions • An illogical and Incomplete progression of steps • major calculation errors • partial justification of a conclusion 	<p>The student shows no work or justification.</p>

Name _____

Block _____

Date _____

Buying Protein Bars and Magazines (7.RP.A.3)

Tom wants to buy some protein bars and magazines for a trip. He has decided to buy three times as many protein bars as magazines. Each protein bar costs \$0.70 and each magazine costs \$2.50. The sales tax rate on both types of items is 6½%. How many of each item can he buy if he has \$20.00 to spend?

Solution: Making a table

The table below shows the cost for the protein bars and magazines in a 3:1 ratio.

Number of magazines	1	2	3	4
Number of protein bars	3	6	9	12
Value of the magazines	\$2.50	\$5.00	\$7.50	\$10.00
Value of the protein bars	\$2.10	\$4.20	\$6.30	\$8.40
Value of both magazines and candy bars	\$4.60	\$9.20	\$13.80	\$17.40
Cost with tax	\$4.90	\$9.80	\$14.70	\$19.60

Looking at the last column of the table, we can see that Tom can buy 4 magazines and 12 protein bars for \$20 and that he cannot afford 5 magazines and 15 protein bars.

Unit 2 Performance Task 2 PLD Rubric

SOLUTION

- Student indicates that Tom can buy 4 magazines and 12 protein bars for \$20 and that he cannot afford 5 magazines and 15 protein bars and justifies the solution with reasoning.

Level 5: Distinguished Command	Level 4: Strong Command	Level 3: Moderate Command	Level 2: Partial Command	Level 1: No Command
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Name _____

Block _____

Date _____

Who Has the Best Job? (8.EE.5)

Kell works at an after-school program at an elementary school. The table below shows how much money he earned every day last week.

Time Worked	1.5 hours	2.5 hours	4 hours
Money Earned	\$12.60	\$21.00	\$33.60

Mariko has a job mowing lawns that pays \$7 per hour.

- Who would make more money for working 10 hours? Explain or show work.
- Draw a graph that represents y , the amount of money Kell would make for working x hours, assuming he made the same hourly rate he was making last week.
- Using the same coordinate axes, draw a graph that represents y , the amount of money Mariko would make for working x hours.
- How can you see who makes more per hour just by looking at the graphs? Explain.

Solution

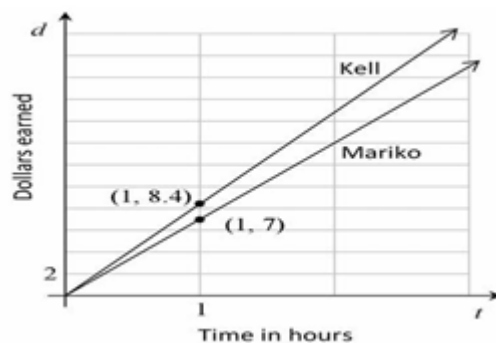
a. Mariko would make $7 \times 10 = 70$ dollars for working 10 hours. Kell's hourly rate can be found by dividing the money earned by the hours worked each day.

Time worked	1.5 hours	2.5 hours	4 hours
Money earned	\$12.60	\$21.00	\$33.60
Pay rate	\$8.40 per hour	\$8.40 per hour	\$8.40 per hour

If Kell works for 10 hours at this same rate, he will earn $8.4 \times 10 = 84$ dollars. So Kell will earn more money for working 10 hours.

Alternatively, we could reason proportionally without computing the unit rate. Since Mariko earned \$21.00 for 2.5 hours, she will earn four times as much for working four times as long ($10 = 4 \times 2.5$), for a total of $4 \times \$21 = \84 .

b) & c) See Below



d. You can see that Kell will make more per hour if you look at the points on the graph where $x = 1$ since this will tell you how much money each person will make for working 1 hour. You can also compare the slopes of the two graphs, which are equal to the hourly rates. See the figure below.

Unit 2 Performance Task 3 PLD Rubric

SOLUTION

- A. Student indicates that Kell would earn more money for working 10 hours. Student indicates that Kell would earn $(12.60) \times 4 + 33.60 = \84 and Mariko will earn $(7 \times 10 = \$70)$.
- B. Refer to Graph in Solutions Guide
- C. Refer to Graph in Solutions Guide
- D. Answers may vary. Student indicates looking at a point on the graph, for example $x = 1$, as it informs you who will earn more money for one hour of work. Students may also compare slopes of the two graphs, which are equivalent to the hourly rate

Level 5: Distinguished Command	Level 4: Strong Command	Level 3: Moderate Command	Level 2: Partial Command	Level 1: No Command
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Unit 2 Performance Task Option 1

How fast is Usain Bolt? (7.RP.A.3)

Jamaican sprinter Usain Bolt won the 100-meter sprint gold medal in the 2012 Summer Olympics. He ran the 100-meter race in 9.63 seconds. There are about 3.28 feet in a meter and 5280 feet in a mile. What was Usain Bolt's average speed for the 100-meter race in miles per hour?

21st Century Career Ready Practices

- CRP1. Act as a responsible and contributing citizen and employee.
- CRP2. Apply appropriate academic and technical skills.
- CRP3. Attend to personal health and financial well-being.
- CRP4. Communicate clearly and effectively and with reason.
- CRP5. Consider the environmental, social and economic impacts of decisions.
- CRP6. Demonstrate creativity and innovation.
- CRP7. Employ valid and reliable research strategies.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP9. Model integrity, ethical leadership and effective management.
- CRP10. Plan education and career paths aligned to personal goals.
- CRP11. Use technology to enhance productivity.
- CRP12. Work productively in teams while using cultural global competence.

For additional details see [21st Century Career Ready Practices](#) .

Extensions and Sources

Online Resources

<http://dashweb.pearsoncmg.com>

<http://www.illustrativemathematics.org/standards/k8>

- Performance tasks, scoring guides

<http://www.ixl.com/math/grade-7>

- Interactive, visually appealing fluency practice site that is objective descriptive

<https://www.khanacademy.org/math/>

- Interactive, tracks student points, objective descriptive videos, allows for hints

http://www.doe.k12.de.us/assessment/files/Math_Grade_7.pdf

- Common Core aligned assessment questions, including Next Generation Assessment Prototypes

<http://www.learnzillion.com>

- Videos organized by Common Core Standard presented with visual representations and student friendly language