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FOR TEACHING & LEARNING

PMI[®]

Progressive Mathematics Initiative[®]

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NEW JERSEY CENTER
FOR TEACHING & LEARNING

7th Grade

Ratios and Proportions

2016-12-05

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Writing Ratios

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Ratios

What do you know about **ratios**?

When have you seen or used ratios?

Ratios

Ratio - A comparison of two numbers by division

Ratios can be written three different ways:

$$a \text{ to } b \quad a : b \quad \frac{a}{b}$$

Each is read, "the ratio of a to b."

Each ratio should be in simplest form.

Find the ratio of boys to girls in this class



Ratios Video

Click for a ratios video



Writing Ratios

There are 48 animals in the field. Twenty are cows and the rest are horses.

Write the ratio in three ways:

- a. The number of cows to the number of horses

- b. The number of horses to the number of animals in the field

Remember to write your ratios in simplest form!

- 1 There are 27 cupcakes. Nine are chocolate, 7 are vanilla and the rest are strawberry. What is the ratio of vanilla cupcakes to strawberry cupcakes?

A $7:9$

B $\frac{7}{27}$

C $\frac{7}{11}$

D $1:3$

Answer

Remember to write your ratios in simplest form!

- 2 There are 27 cupcakes. Nine are chocolate, 7 are vanilla and the rest are strawberry. What is the ratio of chocolate & strawberry cupcakes to vanilla & chocolate cupcakes?

A $\frac{20}{16}$

B $\frac{11}{7}$

C $\frac{5}{4}$

D $\frac{16}{20}$

Answer

Remember to write your ratios in simplest form!

3 There are 27 cupcakes. Nine are chocolate, 7 are vanilla and the rest are strawberry. What is the ratio of chocolate cupcakes to total cupcakes?

A $\frac{7}{9}$

B $\frac{7}{27}$

C $\frac{9}{27}$

D $\frac{1}{3}$

Answer

4 There are 27 cupcakes. Nine are chocolate, 7 are vanilla and the rest are strawberry. What is the ratio of total cupcakes to vanilla cupcakes?

A 27 to 9

B 7 to 27

C 27 to 7

D 11 to 27

Answer

Equivalent Ratios

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Equivalent Ratios

Equivalent ratios have the same value.

3 : 2 is equivalent to 6 : 4

1 to 3 is equivalent to 9 to 27

$\frac{5}{6}$ is equivalent to $\frac{35}{42}$

Equivalent Ratios

There are two ways to determine if ratios are equivalent.

1. Common Factor

$$\frac{4}{5} = \frac{12}{15}$$

$$\begin{array}{c} \text{x 3} \\ \overbrace{\frac{4}{5} = \frac{12}{15}} \\ \text{x 3} \end{array}$$

Since the numerator and denominator were multiplied by the same value, the ratios are equivalent.

Equivalent Ratios

2. Cross Products

$$\frac{4}{5} = \frac{12}{15}$$

Since the cross products are equal, the ratios are equivalent.

$$4 \times 15 = 5 \times 12$$
$$60 = 60$$

5 $\frac{4}{9}$ is equivalent to $\frac{8}{18}$

True

False

Answer

6 $\frac{5}{9}$ is equivalent to $\frac{30}{54}$

True

False

Answer

7

18:12 is equivalent to $\frac{9}{6}$, which is equivalent to $\frac{36}{24}$

True

False

Answer

8

1 : 7 is equivalent to $\frac{10}{70}$, which is equivalent to 5 to 65

True

False

Answer

Rates

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Rates Video

[Click for video](#)



Rates

Rate: a ratio of two quantities measured in different units

Examples of rates:

4 participants/2 teams

5 gallons/3 rooms

8 burgers/2 tomatoes

Unit Rates

Unit rate: Rate with a denominator of one
Often expressed with the word "per"

Examples of unit rates:

34 miles/gallon

2 cookies per person

62 words/minute

Finding a Unit Rate

Six friends have pizza together. The bill is \$63.
What is the cost per person?

Hint: Since the question asks for cost per person,
the cost should be first, or in the numerator.

click

Since unit rates always have a denominator of one,
rewrite the rate so that the denominator is one.

click

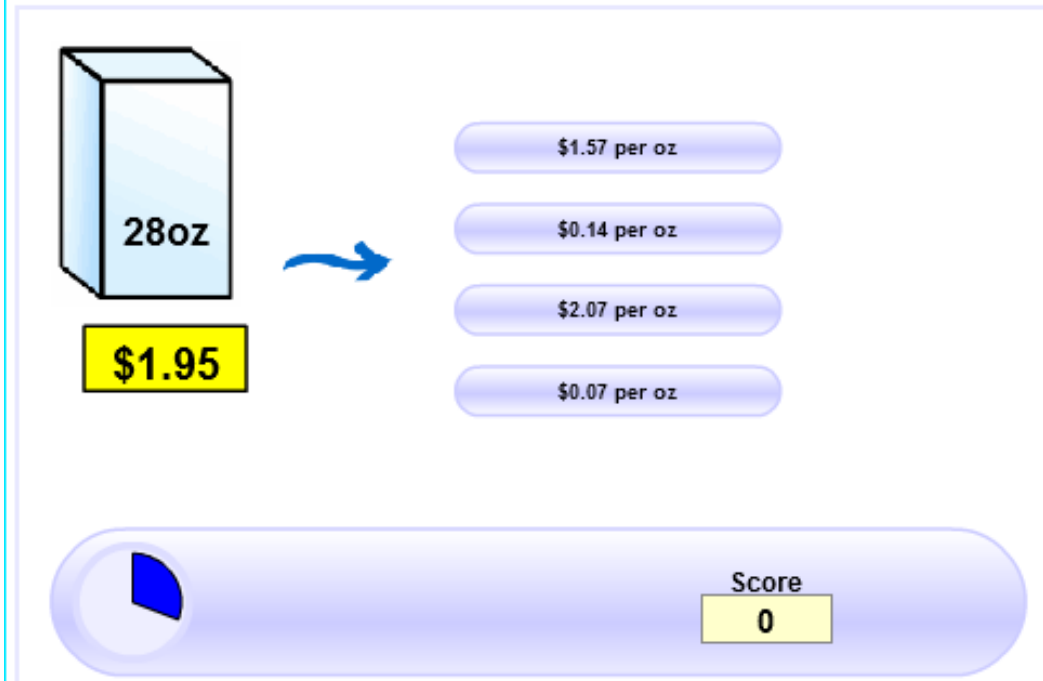
Click for Practice

Unit Price Game

Are you getting *Value For Money*?

The "[Unit Price](#)" tells you the cost per liter, per kilogram, per pound, etc, of what you want to buy.

It is a good way of comparing costs. To help you become expert at calculating Unit Prices we have this game for you:



The game interface shows a product box on the left with "28oz" written on it and a yellow price tag below it that says "\$1.95". A blue arrow points from the box to four blue buttons on the right, each containing a unit price option: "\$1.57 per oz", "\$0.14 per oz", "\$2.07 per oz", and "\$0.07 per oz". At the bottom, there is a blue progress bar with a blue quarter-circle on the left and a yellow score box on the right that says "Score 0".

9 Sixty cupcakes are at a party for twenty children. How many cupcakes per person?

Answer

10 John's car can travel 94.5 miles on 3 gallons of gas.
How many miles per gallon can the car travel?

Answer

11 The snake can slither 240 feet in half a day. How many feet can the snake move in an hour?

Answer

12 There are five chaperones at the dance of 100 students. How many students per chaperone are there?

Answer

13 The recipe calls for 6 cups of flour for every four eggs.
How many cups of flour are needed for one egg?

Answer

14 Sarah rode her bike $14\frac{1}{4}$ miles in $\frac{3}{4}$ hour.

What is Sarah's unit rate in miles per hour?

Answer

15 An airplane's altitude changed -378 feet over 7 minutes. What was the mean change of altitude in feet per minute?

Answer

From PARCC PBA sample test non-calculator #3



- 16 A $4\frac{1}{2}$ -ounce hamburger patty has $25\frac{1}{2}$ grams of protein, and 6 ounces of fish has 32 grams of protein. Determine the grams of protein per ounce for each type of food.

A hamburger patty has approximately _____ grams of protein per ounce. The fish has approximately _____ grams of protein per ounce.

A 0.2

F 0.2

B 4.5

G 5.3

C 5.7

H 6.0

D 21.0

I 26.0

E 25.5

J 32.0

Answer

From PARCC PBA sample test calculator #1



17 Rosy waxes $\frac{2}{3}$ of her car with $\frac{1}{4}$ bottle of car wax.

At this rate, what fraction of the bottle of car wax will Rosy use to wax her entire car?

Answer

From PARCC EOY sample test calculator #4



Compare Rates

We often use unit rates to easily compare rates.

Example:

Sebastian and Alexandra both work during the summer. Sebastian worked 26 hours one week and earned \$188.50 before taxes. Alexandra worked 19 hours and earned \$128.25 before taxes. Who earns more per hour at their job?

Sebastian

Alexandra

click

Compare Rates

Jim traveled 480 miles on a full tank of gas. His gas tank holds 15 gallons.

Tara traveled 540 miles on a full tank of gas. Her gas tank holds 18 gallons.

Which person's car gets better gas mileage?

Jim

Tara

click

18 Tahira and Brendan going running at the track. Tahira runs 3.5 miles in 28 minutes and Brendan runs 4 miles in 36 minutes. Who runs at a faster pace (miles per hour)?

Show your work!

A Tahira

B Brendan

Answer

- 19 Red apples cost \$3.40 for ten.
Green apples cost \$2.46 for six.
Which type of apple is cheaper per apple?

Show your work!

- A Tahira
B Brendan

Answer

20 Fruity Oats is \$2.40 for a 12 oz. box.
Snappy Rice is \$3.52 for a 16 oz. box.
Which cereal is cheaper per ounce?

Show your work!

- A Fruity Oats
- B Snappy Rice

Answer

21 Two families drive to their vacation spot. The Jones family drives 432 miles and used 16 gallons of gas. The Alvarez family drives 319 miles and uses 11 gallons of gas. Which family got more miles per gallon of gas?

Show your work!

A Jones Family

B Alvarez Family

Answer

22 Mariella typed 123 words in 3 minutes.
Enrique typed 155 words in 5 minutes.
Who typed more words per minute?

Show your work!

A Mariella

B Enrique

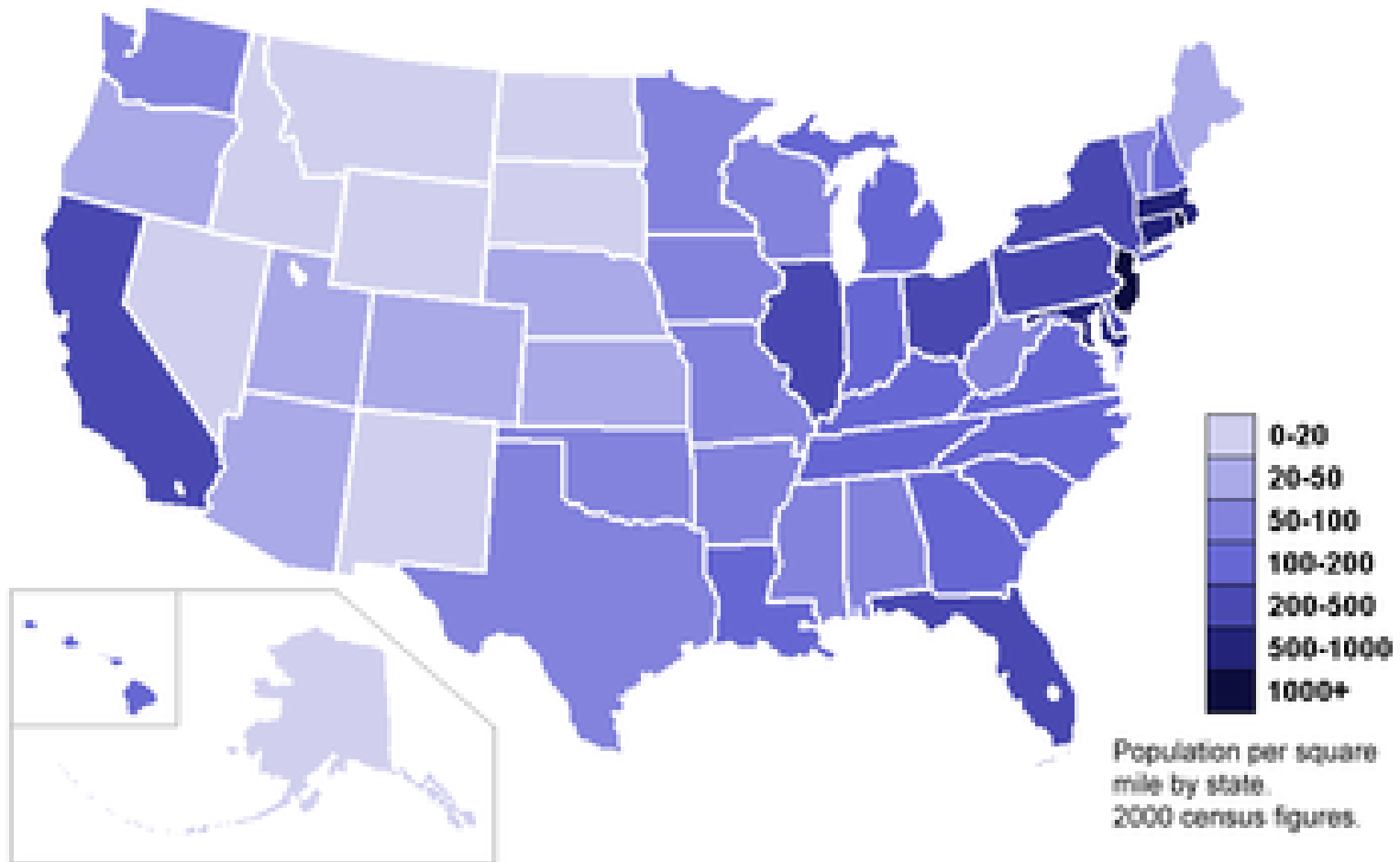
Answer

Population Density

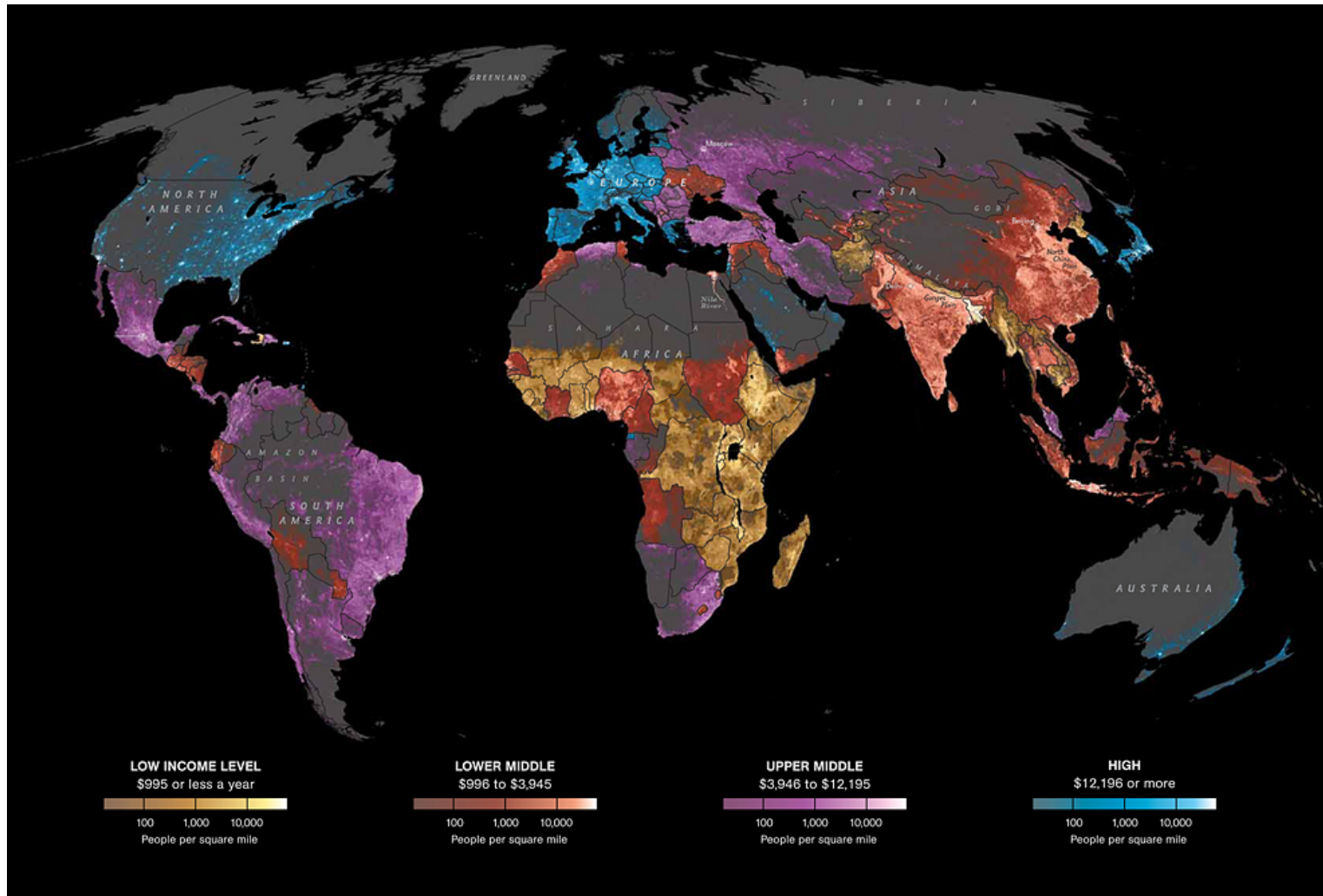
Population Density: A unit rate of people per square mile

This data is compiled by the US Census Bureau every 10 years and is used when determining the number of Representatives each state gets in the House of Representatives.

Population Density



Click for National Geographic Web Site



Population Density

To calculate population density:

- Find the population of the state.

NJ = 8,791,894 people

- Find the area of the state.

NJ = 7,790 square miles

- Divide

$$\frac{\textit{Population}}{\textit{Area}} = \frac{8,791,894}{7,790} = 1,129 \textit{ people per square mile}$$

Population Density

We know that New Jersey has a population density of 1,129 people per square mile. Use the links below to compare this data with two other states.

Population Density =

Click the map for population data

Population

Area

Click the table for area data



Rank	State Name	Area (Sq Miles)
1	Alaska	587,878
2	Texas	266,874
3	California	158,648
4	Montana	147,047
5	New Mexico	121,599
6	Arizona	114,007
7	Nevada	110,567
8	Colorado	104,100
9	Wyoming	97,818
10	Oregon	97,052
11	Utah	84,905
12	Minnesota	84,397
13	Idaho	83,574
14	Kansas	82,282
15	Nebraska	77,359
16	South Dakota	77,122

23 The population of Newark, NJ is 278,980 people in 24.14 square miles. What is its population density?



Answer

24 The population of Moorestown, NJ is 19,509 people in 15 square miles. What is its population density?



Moorestown, NJ

Answer

25 The population of Waco, TX is 124,009 people in 75.8 square miles. What is its population density?



Answer

26 The population of Argentina is 40,091,359 people and Argentina is 1,042,476 square miles. What is the population density?



Answer

27 The population of Argentina is 40,091,359 people and Argentina is 1,042,476 square miles. What is the population density?



San Luis, Argentina

Answer

Proportions

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Proportions

A **proportion** is an equation that states that two ratios are equivalent.

Example:

$$\frac{2}{3} = \frac{12}{18}$$

$$\frac{5}{9} = \frac{15}{27}$$




Proportions

If one of the numbers in a proportion is unknown, mental math can be used to find an equivalent ratio.

Example 1:


$$\frac{2}{3} = \frac{6}{x}$$

$\times 3$


$$\frac{2}{3} = \frac{6}{x}$$

$$\frac{2}{3} = \frac{6}{9}$$

$\times 3$



Hint: To find the value of x , multiply 3 by 3 also.


Proportions

If one of the numbers in a proportion is unknown, mental math can be used to find an equivalent ratio.

Example:

$$\frac{28}{32} = \frac{7}{x}$$

÷ 4


$$\frac{28}{32} = \frac{7}{x}$$

$$\frac{28}{32} = \frac{7}{8}$$

÷ 4

Hint: To find the value of x, divide 32 by 4 also.

28 Solve the proportion using equivalent ratios.

$$\frac{2}{5} = \frac{8}{x}$$

Answer

29 Solve the proportion using equivalent ratios.

$$\frac{4}{9} = \frac{x}{36}$$

Answer

30 Solve the proportion using equivalent ratios.

$$\frac{7}{2} = \frac{35}{x}$$

Answer

31 Solve the proportion using equivalent ratios.

$$\frac{x}{60} = \frac{4}{12}$$

Answer

32 Solve the proportion using equivalent ratios.

$$\frac{3}{x} = \frac{21}{28}$$

Answer

Proportion

In a proportion, the cross products are equal.

$$\frac{5}{2} = \frac{30}{12}$$

$$5 \cdot 12 = 2 \cdot 30$$

$$60 = 60$$

Cross Products

Proportions can also be solved using cross products.

$$\frac{4}{5} \times \frac{12}{x}$$

Cross multiply

$$4x = 5 \cdot 12$$

Solve for x

$$4x = 60$$

$$x = 15$$

Cross Products

Example 2

$$\frac{7}{8} \times \frac{x}{48}$$

Cross multiply

$$7 \cdot 48 = 8x$$

$$336 = 8x$$

Solve for x

$$42 = x$$

33 Use cross products to solve the proportion.

$$\frac{9}{51} = \frac{x}{17}$$

Answer

34 Use cross products to solve the proportion.

$$\frac{x}{12} = \frac{56}{96}$$

Answer

35 Use cross products to solve the proportion.

$$\frac{45}{18} = \frac{x}{6}$$

Answer

36 Use cross products to solve the proportion.

$$\frac{2}{15} = \frac{x}{60}$$

Answer

37 Use cross products to solve the proportion.

$$\frac{7}{x} = \frac{3}{21}$$

Answer

38 Today, Joelle walked 20 minutes at a rate of 3 miles per hour, and she ran 15 minutes at a rate of 6 miles per hour.

Part A

How many total miles did Joelle travel while walking and running?

Answer

From PARCC EOY sample test calculator #14



39 (Continued from previous slide.)

Part B

Tomorrow, Joelle wants to travel a total of 4 miles by walking and running. She plans to run for 20 minutes at a rate of 6 miles per hour.

How many minutes should she walk at a rate of 3 miles per hour to finish traveling the 4 miles?

Answer

From PARCC EOY sample test calculator #14



40 The directions on a bottle of vinegar say, "mix 1 cup of vinegar with 1 gallon of water to make a cleaning solution." The ratio of vinegar to water is 1 to 16.

Part A

How many cups of water should be mixed with $\frac{1}{4}$ cup of vinegar to make the cleaning solution?

Answer



41 (Continued from previous slide.)

Part B

How many fluid ounces of vinegar should be mixed with 80 ounces of water to make the cleaning solution?

Answer

From PARCC EOY sample test calculator #12



42 (Continued from previous slide.)

Part C

The bottle contains 1 quart of vinegar. What is the total number of quarts of cleaning solution that can be made using the entire bottle of vinegar?

Answer

From PARCC EOY sample test calculator #12



43 (Continued from previous slide.)

Part D

A spray bottle holds up to 1 cup of the cleaning solution.

When the spray bottle is full, what fraction of the cleaning solution is vinegar?

Answer

From PARCC EOY sample test calculator #12



Direct & Indirect Relationships in Tables & Graphs

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Proportional Relationships

You can determine if a relationship is proportional by looking at a table of values or the graph.

How?

Table

If all the ratios of numbers in the table are equivalent, the relationship is proportional.

Graph

If the graph of the numbers forms a straight line through the origin $(0,0)$, the relationship is proportional.

Tables & Proportions

On a field trip, every chaperone is assigned 12 students. Is the student to chaperone ratio proportional?

If you use a table to demonstrate, you would need several ratios to start.

Chaperones	1	2	3	4	5
Students	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>

Next, find the simplified ratios and compare them. Are they the same?

click to
reveal

Tables & Proportions

Try this:

The local pizza place sells a plain pie for \$10. Each topping costs an additional \$1.50. Is the cost of pizza proportional to the number of toppings purchased?

Toppings	1	2	3	4
Cost (\$)	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

click to reveal

44 Is the relationship shown in the table proportional?

Yes

No

Year	1	2	4	5
Income	\$22,000	\$44,000	\$88,000	\$110,000

Answer

45 Is the relationship shown in the table proportional?

Yes

No

x	2	5	6	9
y	7	17.5	21	34.5

Answer

46 Is the relationship shown in the table proportional?

Yes

No

x	1	2	6	9
y	5	11	31	46

Answer

47 Is the relationship shown in the table proportional?

Yes

No

x	1	2	4	7
y	4	8	16	35

Answer

48 Is the relationship shown in the table proportional?

Yes

No

x	2	4	6	8
y	-3	-10	-15	-20

Answer

Proportional Relationships

Remember:

Table

If all the ratios of numbers in the table are equivalent, the relationship is proportional.

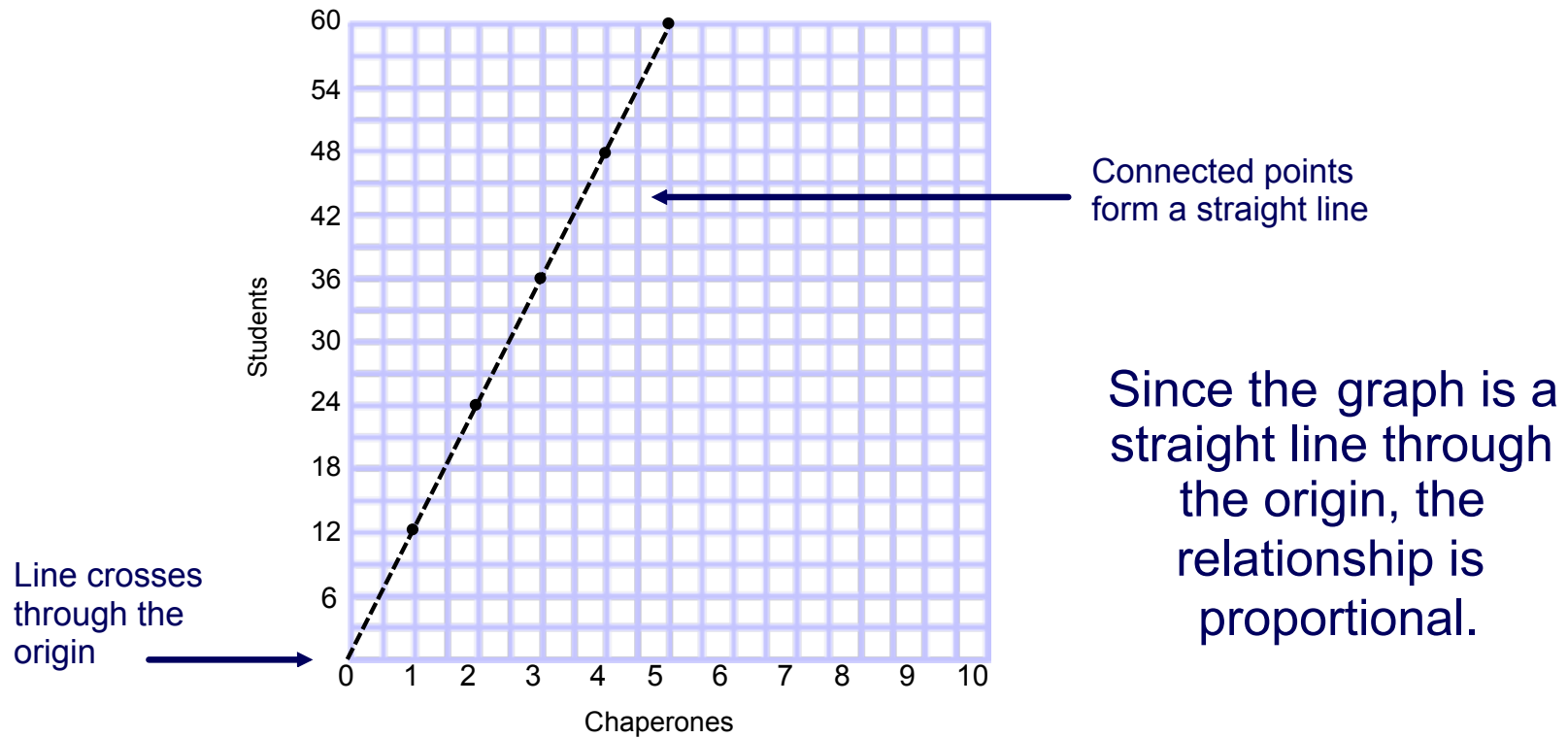
Graph

If the graph of the numbers forms a straight line through the origin $(0,0)$, the relationship is proportional.

Graphs & Proportions

On a field trip, every chaperone is assigned 12 students. Is the student to chaperone ratio proportional?

Chaperones	1	2	3	4	5
Students	12	24	36	48	60

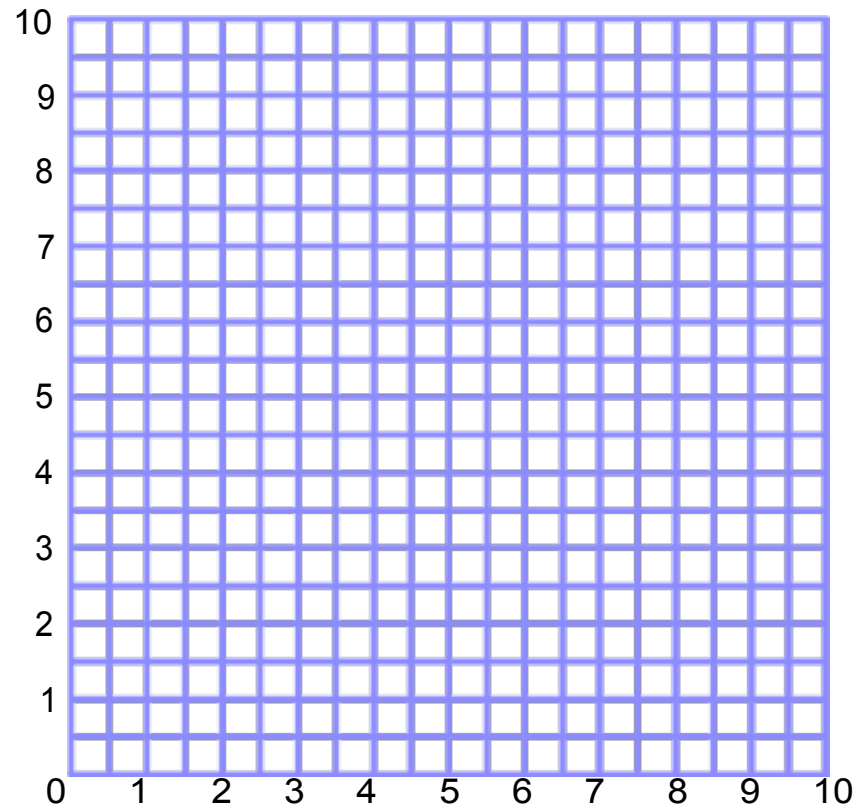


Graphs & Proportions

Draw a graph to represent the relationship. Is the relationship proportional?

Click for answer

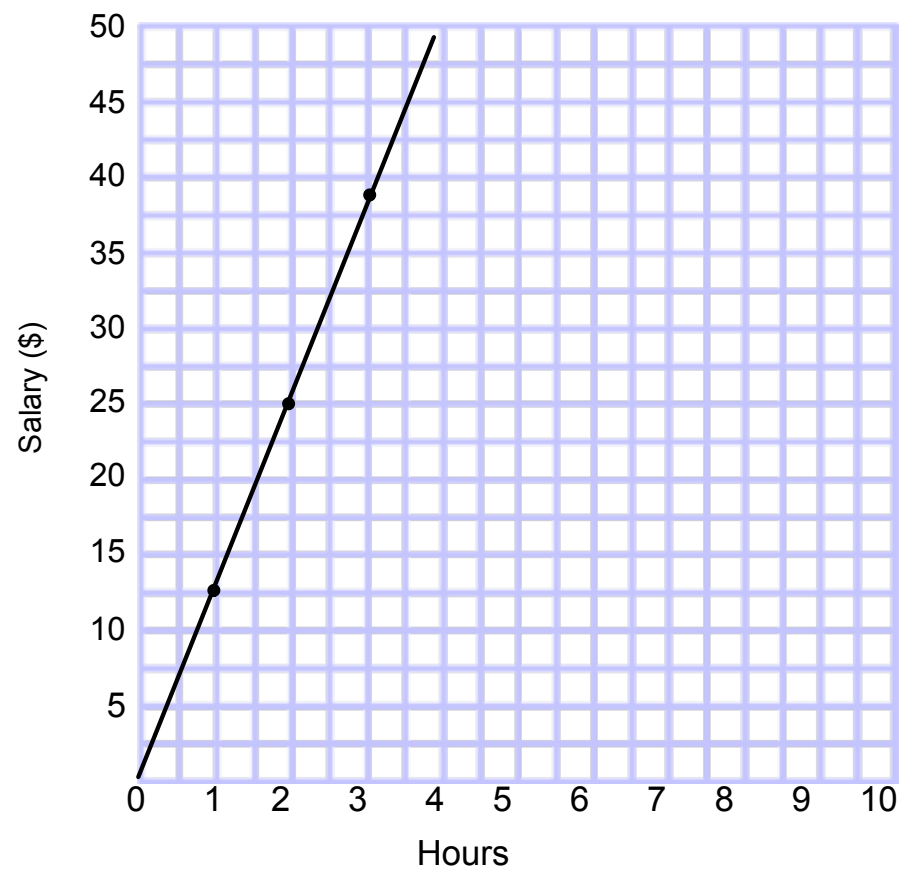
X	Y
1	5.5
2	7
3	8.5
4	10



49 Is the relationship shown on the graph proportional?

Yes

No

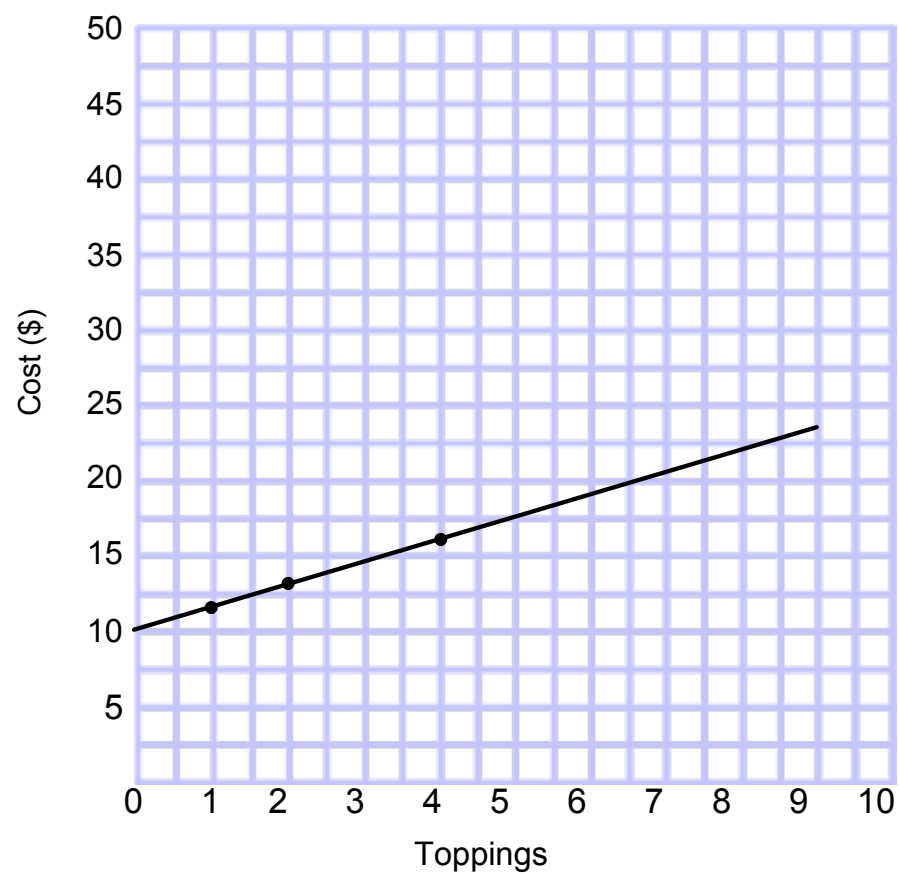


Answer

50 Is the relationship shown on the graph proportional?

Yes

No

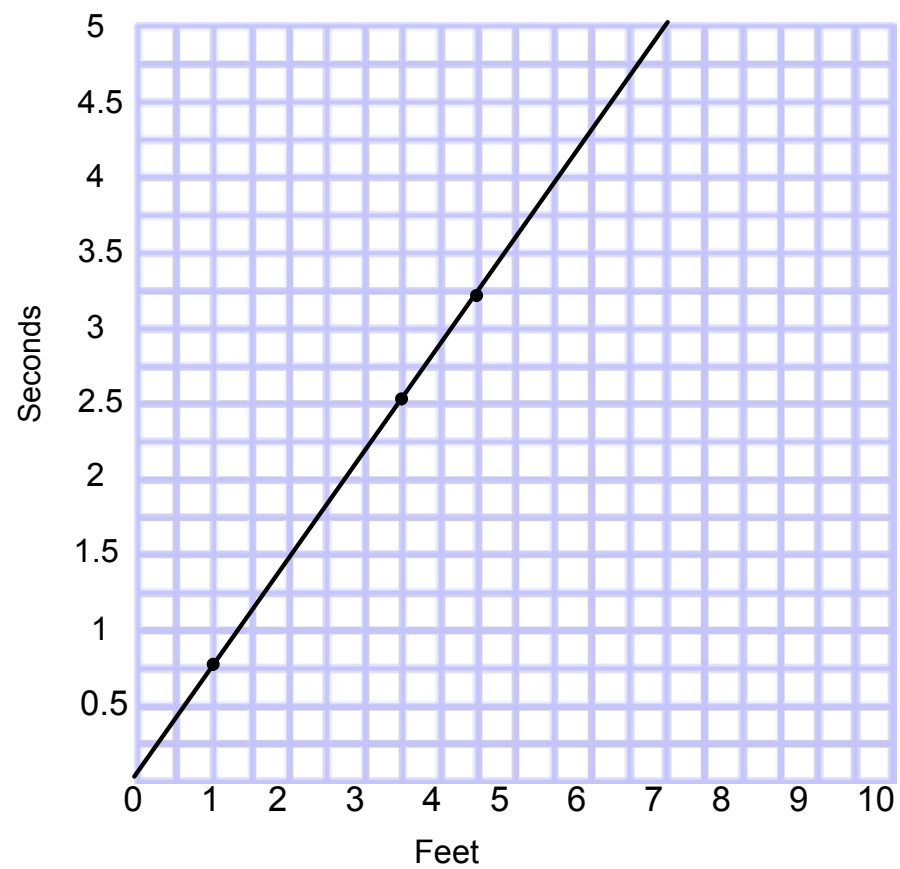


Answer

51 Is the relationship shown on the graph proportional?

Yes

No

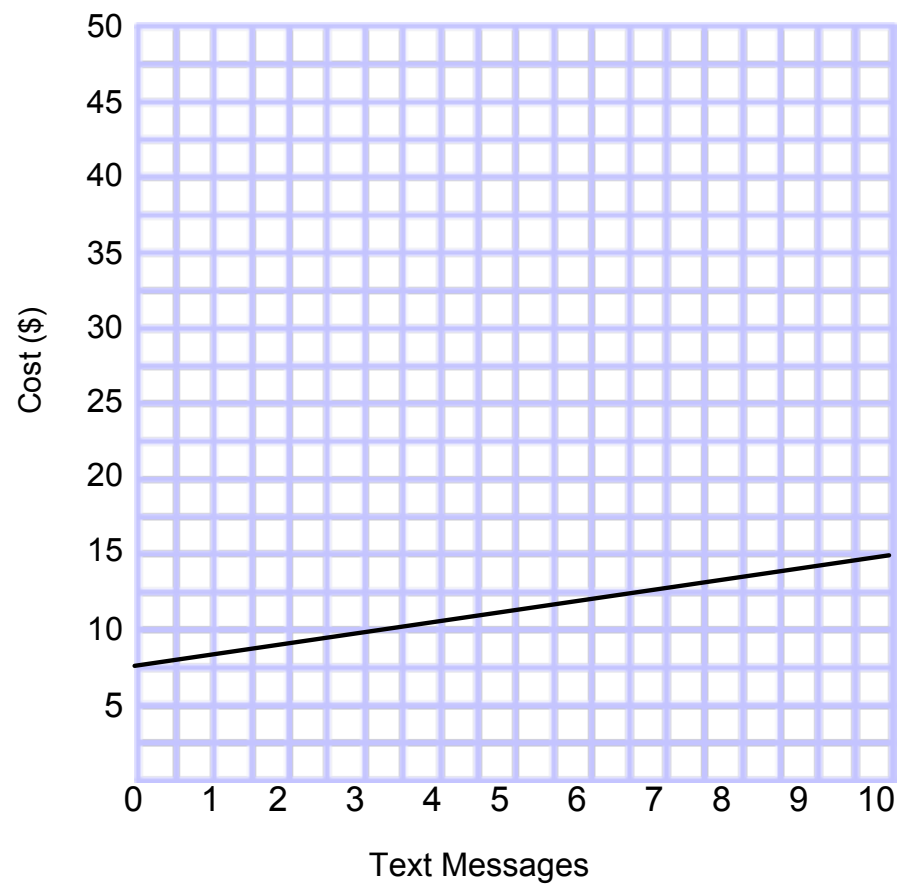


Answer

52 Is the relationship shown on the graph proportional?

Yes

No

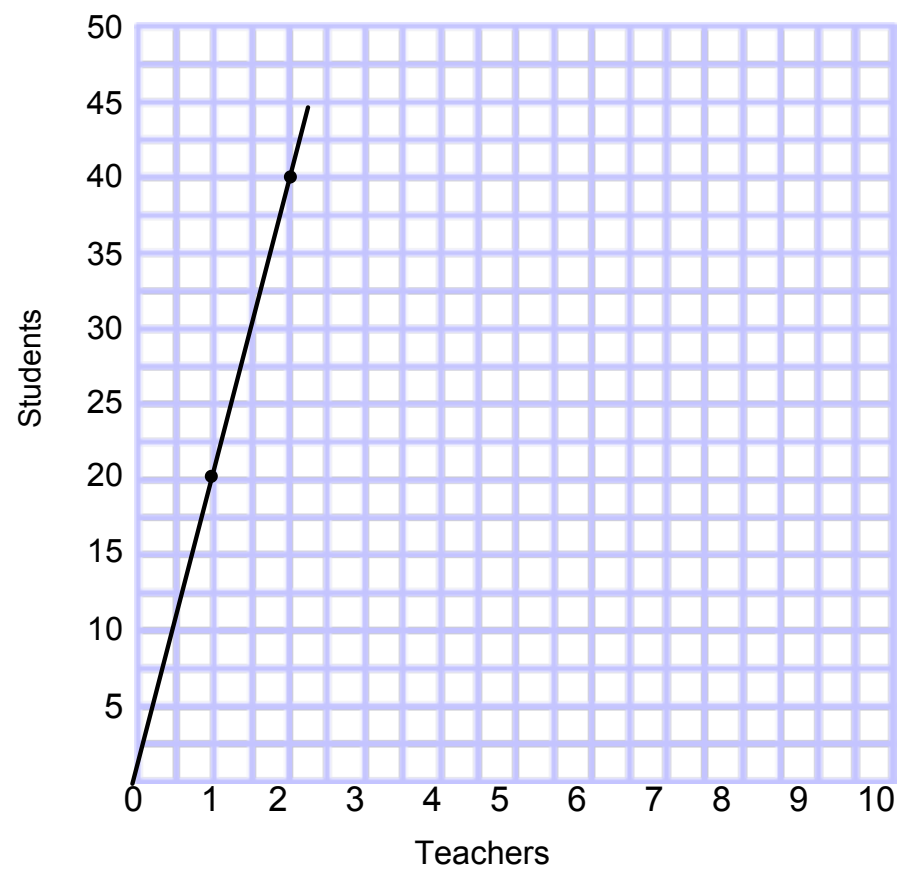


Answer

53 Is the relationship shown on the graph proportional?

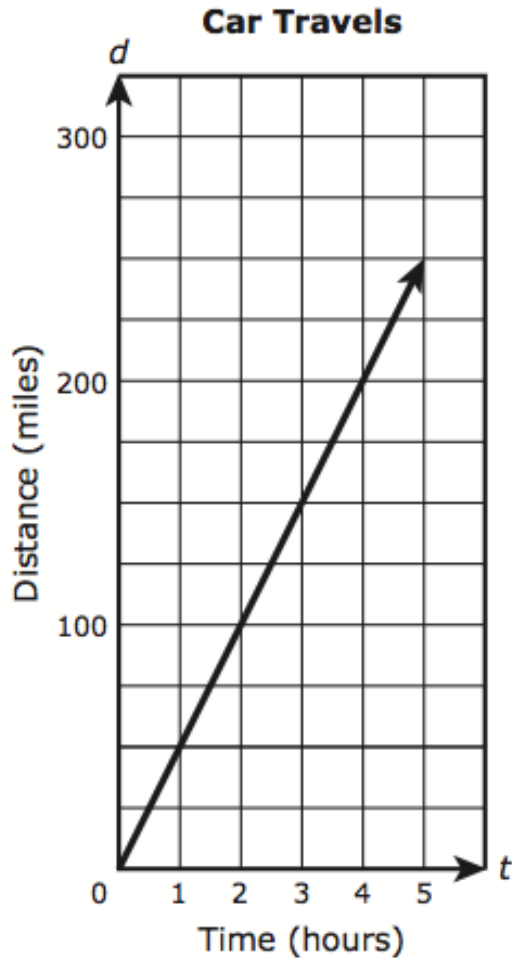
Yes

No



Answer

54 The graph shows the distance in miles, d , a car travels in t hours.



Part A

Explain why the graph does or does not represent a proportional relationship between the variables d and t .

Answer

From PARCC PBA sample test calculator #10



55 (Continued from previous slide.)

Part B

Two cars leave from the same city at the same time and drive in the same direction. The table shows the distances traveled by each car.

Two Cars Travel

Hours of Travel	Miles Traveled by Red Car	Miles Traveled by White Car
1	77	55
2	122	110
3	167	165
4	212	220
5	257	275

Answer

Determine whether the relationship between the number of hours traveled and the number of miles traveled is proportional for each car. (Use the table to explain how you determined your answers. Describe how the graph of the distance traveled by each car would support your answers.)

From PARCC PBA sample test calculator #10



Constant of Proportionality

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Constant of Proportionality

The **constant of proportionality** is a constant ratio (unit rate) in any proportional relationship.

We use the letter k to represent the constant of proportionality.

Equations:

$$y = kx \quad \text{or} \quad k = \frac{y}{x}$$

Constant of Proportionality

We can find the constant of proportionality from a table of values, equation and a graph.

In a table, simplify any one of the ratios.

Chaperones	1	2	3	4	5
Students	12	24	36	48	60

$$k = \frac{y}{x} = \frac{36}{3} = 12$$

Constant of Proportionality

Find the constant of proportionality:

Apples (lbs)	2	2.5	3	3.5	4
Cost (\$)	3.96	4.95	5.94	6.93	7.92

Click

Constant of Proportionality

Find the constant of proportionality:

X	Y
3	4.5
4	6
5	7.5
8	12
9	13.5

Click

56 Find the constant of proportionality.

X	Y
2	1.5
5	3.75
10	7.5
12	9

Answer

57 Find the constant of proportionality.

X	Y
2	2.5
3	3.75
4	5
9	11.25

Answer

58 Find the constant of proportionality.

X	Y
50	3
75	4.5
100	6
140	8.4

Answer

59 This table shows a proportional relationship between x and y .

x	y
2	1.25
4	2.5
6	3.75
10	6.25

What is the constant of proportionality between x and y ? Type your answer as a decimal.

Answer

From PARCC EOY sample test non-calculator #3



Constant of Proportionality

In an equation, write the equation in the form $y = kx$.

Examples:

$$y = 5$$

Click

$$y = \frac{1}{4}x$$

Click

$$y = 3.5x$$

Click

Constant of Proportionality

Find the constant of proportionality:

$$y = 3.6x$$

Click

$$y = 53x$$

Click

$$y = 0.38x$$

Click

$$y = \frac{3}{8}x$$

Click

$$y = \frac{2}{3}x$$

Click

$$y = 1.85x$$

Click

60 Find the constant of proportionality.

$$y = \frac{1}{9}x$$

Answer

61 Find the constant of proportionality.

$$y = 12.9x$$

Answer

62 Find the constant of proportionality.

$$y = 0.45x$$

Answer

63 Which equation has a constant of proportionality equal to 4?

A $4y = 4x$

B $4y = 12x$

C $3y = 4x$

D $3y = 12x$

Answer

From PARCC PBA sample test #1 non-calculator



- 64 A worker has to drive her car as part of her job. She receives money from her company to pay for the gas she uses. The table shows a proportional relationship between y , the amount of money that the worker received, and x , the number of work-related miles driven.

Mileage Rates

Distance Driven, x (miles)	Amount of Money Received, y (dollars)
25	12.75
35	17.85
40	20.40
50	25.50

Part A

Explain how to compute the amount of money the worker receives for any number of work-related miles. Based on your explanation, write an equation that can be used to determine the total amount of money, y , the worker received for driving x work-related miles.

Answer

From PARCC PBA sample test calculator #9



65 (Continued from previous slide.)

Part B

On Monday, the worker drove a total of 134 work-related and personal miles. She received \$32.13 for the work-related miles she drove on Monday. What percent of her total miles driven were work-related on Monday? Show or explain your work.

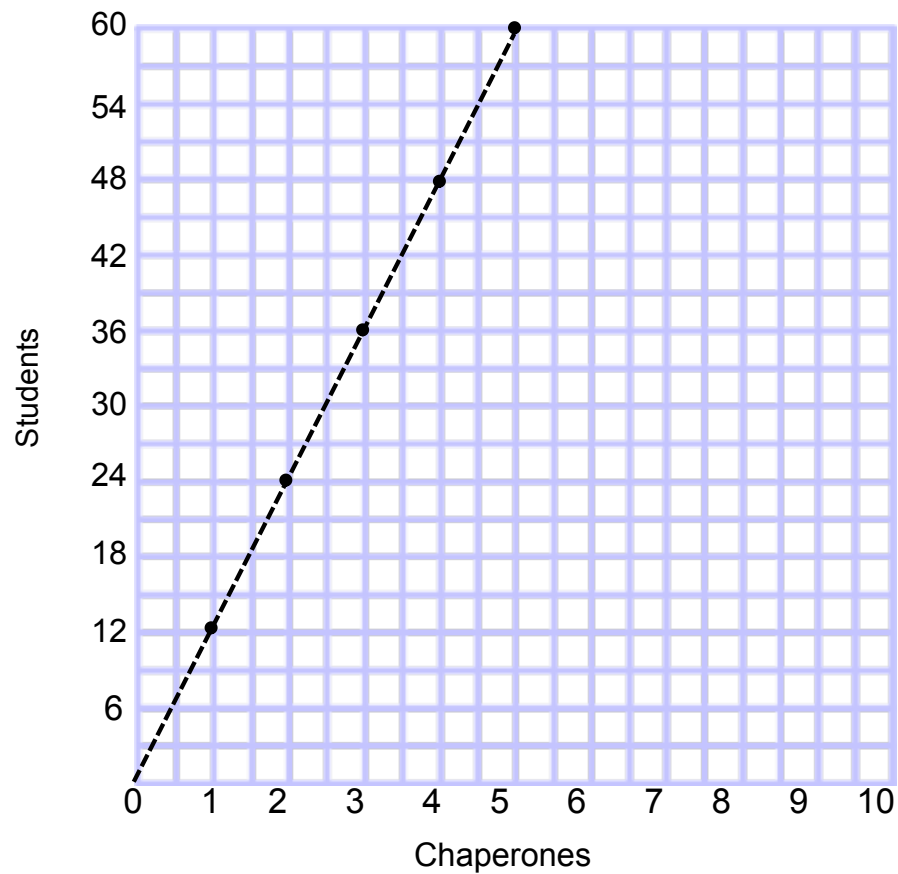
Answer

From PARCC PBA sample test calculator #9



Constant of Proportionality

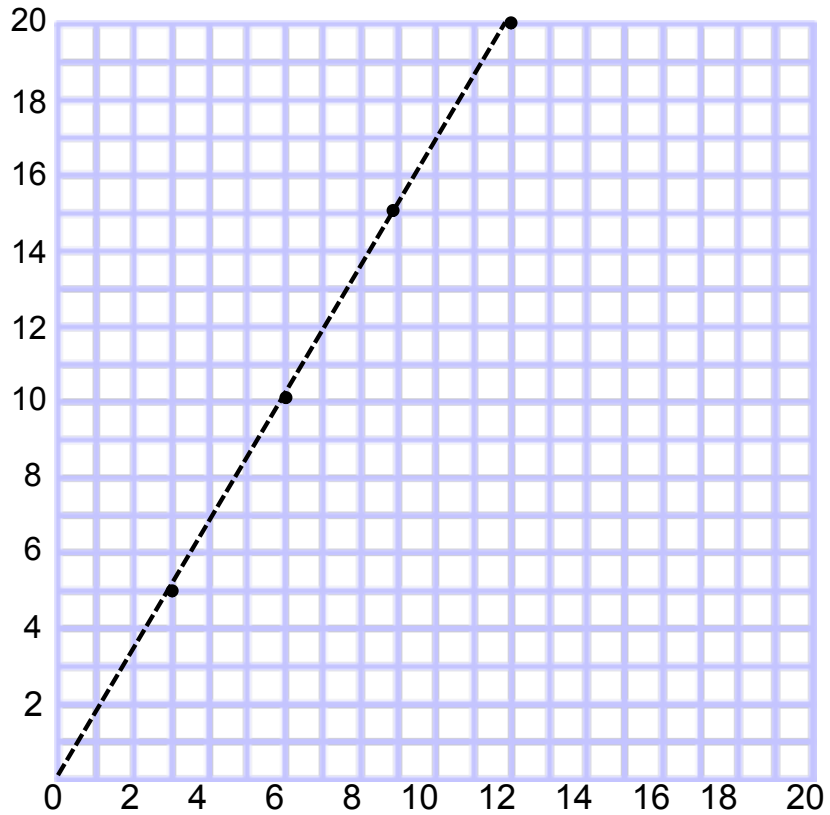
In a graph, choose a point (x, y) to find and simplify the ratio.



$$k = \frac{y}{x} = \frac{24}{2} = 12$$

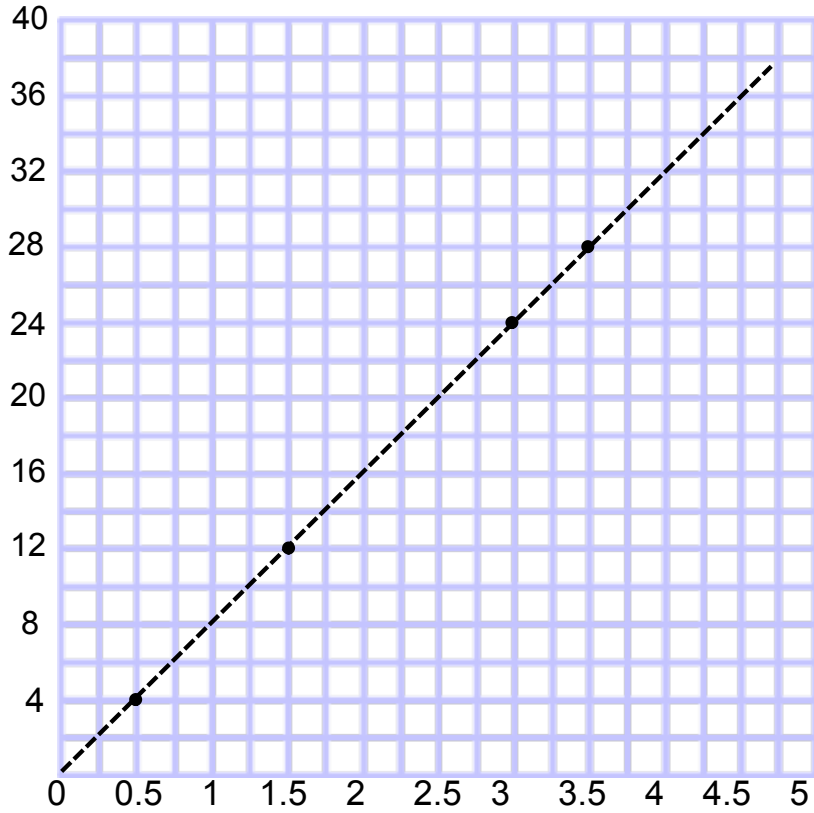
Constant of Proportionality

Find the constant of proportionality.



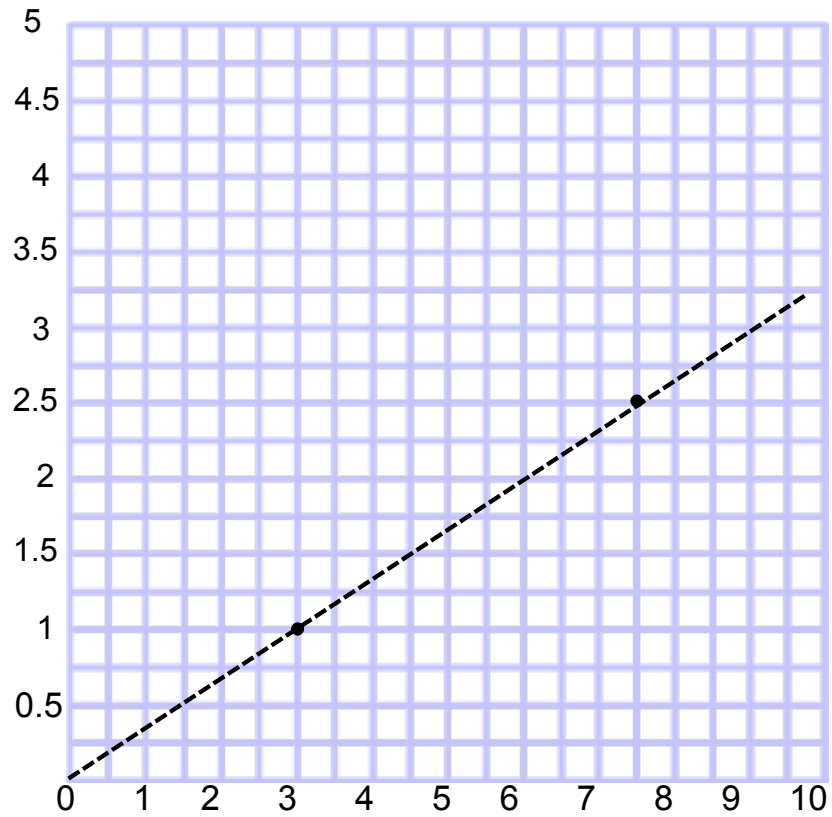
Click

66 Find the constant of proportionality.



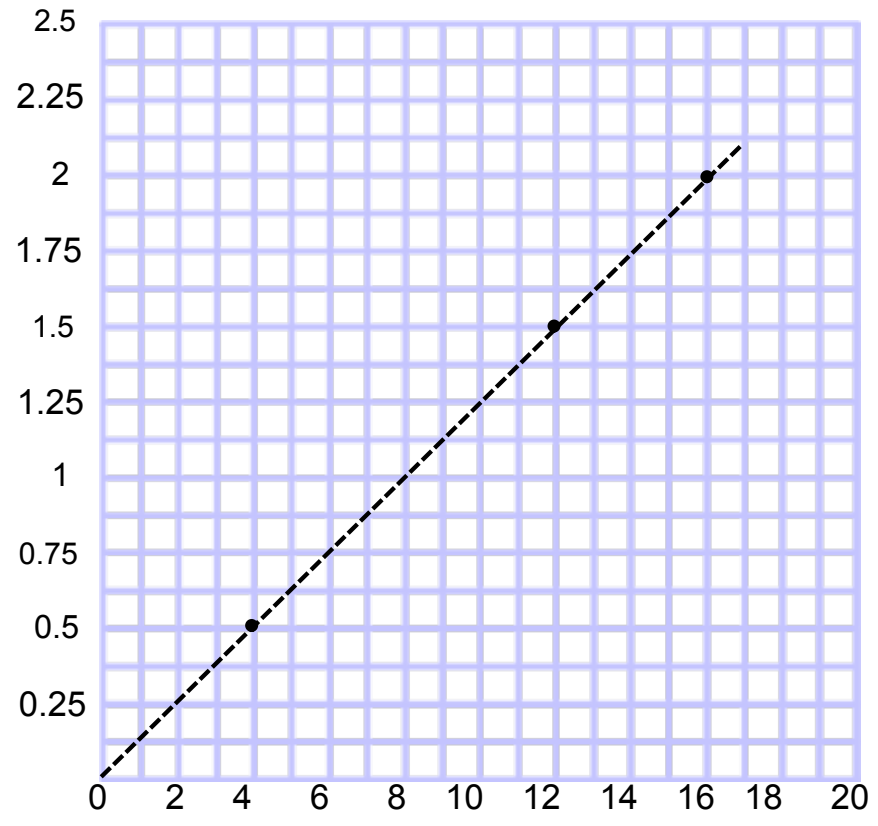
Answer

67 Find the constant of proportionality.



Answer

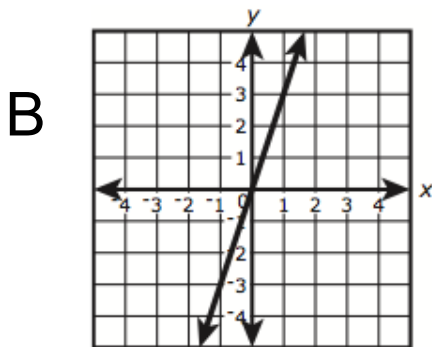
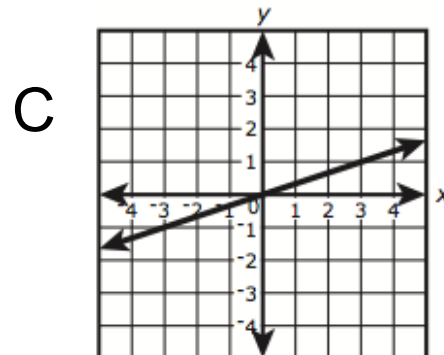
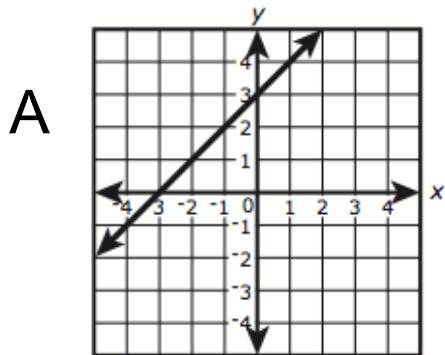
68 Find the constant of proportionality.



Answer

69 Which relationships have the same constant of proportionality between y and x as in the equation

$y = \frac{1}{3}x$? Select *each* correct answer.



D

x	-1.5	0	1.6	9.7
y	-4.5	0	4.8	29.1

E

x	-5.4	-2.7	1.5	2.4
y	-1.8	-0.9	0.5	0.8

Answer



Writing Equations For Proportions

**Return to Table
of Contents**

Writing Equations

The constant of proportionality and the unit rate are equivalent.

We can use the constant of proportionality to help write equations using proportional relationships.

By transforming the equation from: $k = \frac{y}{x}$ to $y = kx$, we can write an equation that can be applied to various situations.

*Remember: x is the independent variable and y is the dependent variable. This means that a change in x will effect y .

Writing Equations

EXAMPLE

You are buying Jersey Tomatoes for a cost of 2 pounds for \$3.98.
Write an equation to represent the proportional relationship.

- Let c = cost
 p = pounds
- Determine the unit rate:
 click
- Write an equation to relate the two quantities:
 click

Writing Equations

At the candy store, you purchase 5 lbs for \$22.45. Write an equation to represent the proportional relationship.

- Let c = cost
 p = pounds
- Determine the unit rate:

click

- Write an equation to relate the two quantities:

click

Writing Equations

Write an equation to represent the proportional relationship shown in the table.

Gallons	10	15	20	25
Miles	247	370.5	494	617.5

$$k = \frac{m}{g} = \frac{247}{10} = \frac{24.7}{1}$$

Let g = gallons
 m = miles

click

70 Write an equation that represents the proportional relationship.

The total cost (c) of grapes for \$1.40 per pound(p)

A $c = 1.4p$

B $p = 1.4c$

Answer

71 Write an equation that represents the proportional relationship.

Shirts	5	15	25	35
Cost	\$57.50	\$172.50	\$287.50	\$402.50

A $s = 11.5c$

B $c = 11.5s$

C $c = 0.09s$

D $s = 0.09c$

Answer

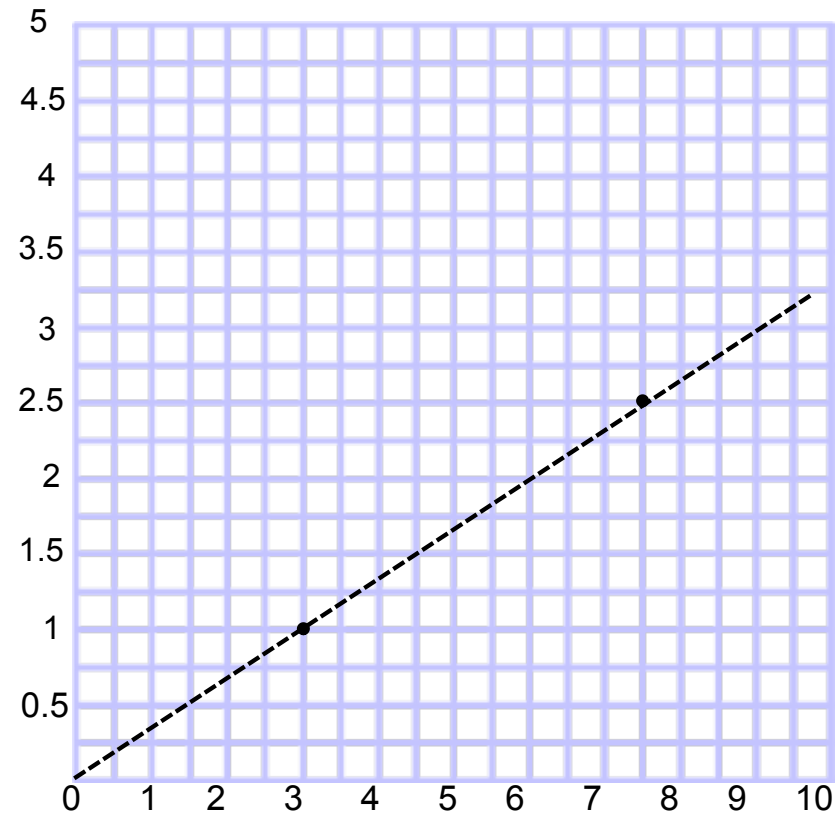
72 Write an equation that represents the proportional relationship.

A $y = \frac{1}{3}x$

B $y = 3x$

C $y = 2.5x$

D $y = 7.5x$



Answer

73 Write an equation that represents the proportional relationship.

You are ordering new menus for your restaurant. You pay \$362.50 for 50 menus.

A $c = 0.14m$

B $m = 7.25c$

C $m = 0.14c$

D $c = 7.25m$

Answer

74 Write an equation that represents the proportional relationship.

Days, d	2	3	4	5
Hours, h	17	25.5	34	42.5

A $d = 8.5h$

B $d = \frac{2}{17}h$

C $h = \frac{2}{17}d$

D $h = 8.5d$

Answer

75 The amount of money Jamie earns is proportional to the number of hours she works. Jamie earns \$62.50 working 5 hours.

Create an equation that models the relationship between m , the amount of money Jamie earns, in dollars, and h , the number of hours she works.

Drag and drop the appropriate number and variables into each box.

12.05 12.50 57.50 m h

$$\boxed{} = \boxed{} \cdot \boxed{}$$

Answer

From PARCC PBA sample test non-calculator #2



- 76 The number of parts produced by three different machines are shown in the table.

Numbers of Machine Parts

Minutes	Machine Q	Machine R	Machine S
1	9	8	6
3	18	24	18
9	27	72	52

Only one of the machines produces parts at a constant rate. Write an equation that can be used to represent y , the number of parts produced in x minutes, for that machine.

Answer

From PARCC PBA sample test non-calculator #5



77 Hayden mixed 6 cups of blue paint with 8 cups of yellow paint to make green paint.

Write an equation that shows the relationship between the number of cups of blue paint, b , and the number of cups of yellow paint, y , that are needed to create the same shade of green paint. The equation should be in the form $b = ky$.

Answer



Understanding Graphs of Proportions

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of Contents**

Graphs of Proportions

Remember, you can use a graph to determine if a relationship is proportional. How?

If the graph is a straight line going through the origin $(0, 0)$.

Once you determine that the relationship is proportional, you can calculate k , the constant of proportionality. Then, write an equation to represent the relationship.

What do these equations mean? Once we have determined the equation, we can understand what the graph was showing us visually.

Graphs of Proportions

The jitneys in Atlantic City charge passengers for rides. What amount do they charge per ride?

- Find a point on the graph

click

- Use the point to find the unit rate

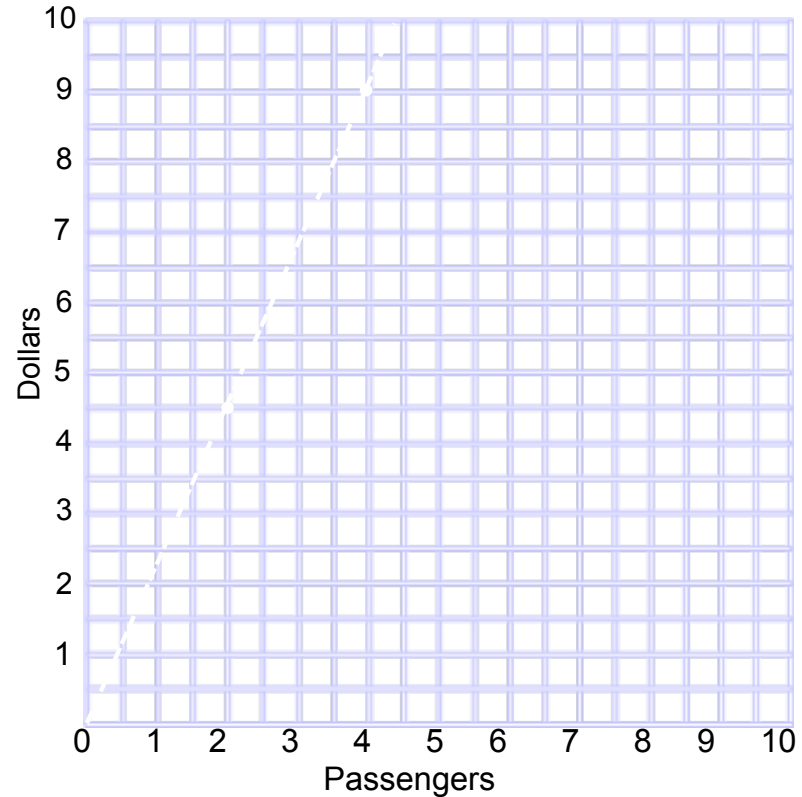
click

- What does the unit rate represent?

click

- What coordinate pair represents the unit rate?

click



- Does the line run through the unit rate?

click

Graphs of Proportions

Mark drives to work each day. His gas mileage is shown in the graph. What is the unit rate? What does it represent?

- Find a point on the graph

click

- Use the point to find the unit rate

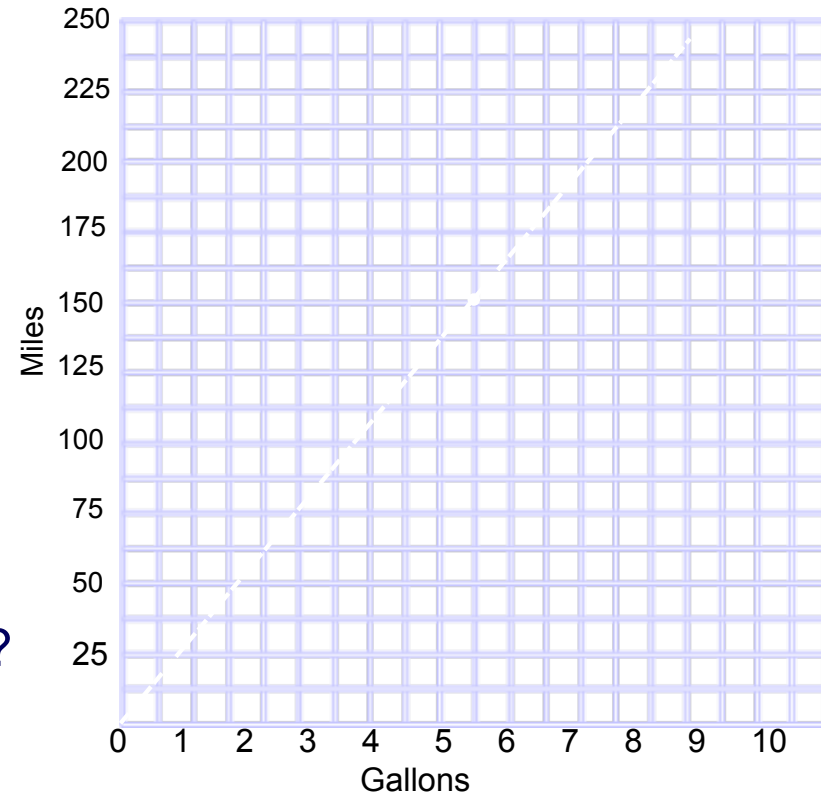
click

- What does the unit rate represent?

click

- What coordinate pair represents the unit rate?

click



- Does the line run through the unit rate?

click

Graphs of Proportions

Try This:

Jasmine gets paid for every dog that she walks according to the graph at the right. What does she earn per dog?

- Find a point on the graph

click

- Use the point to find the unit rate

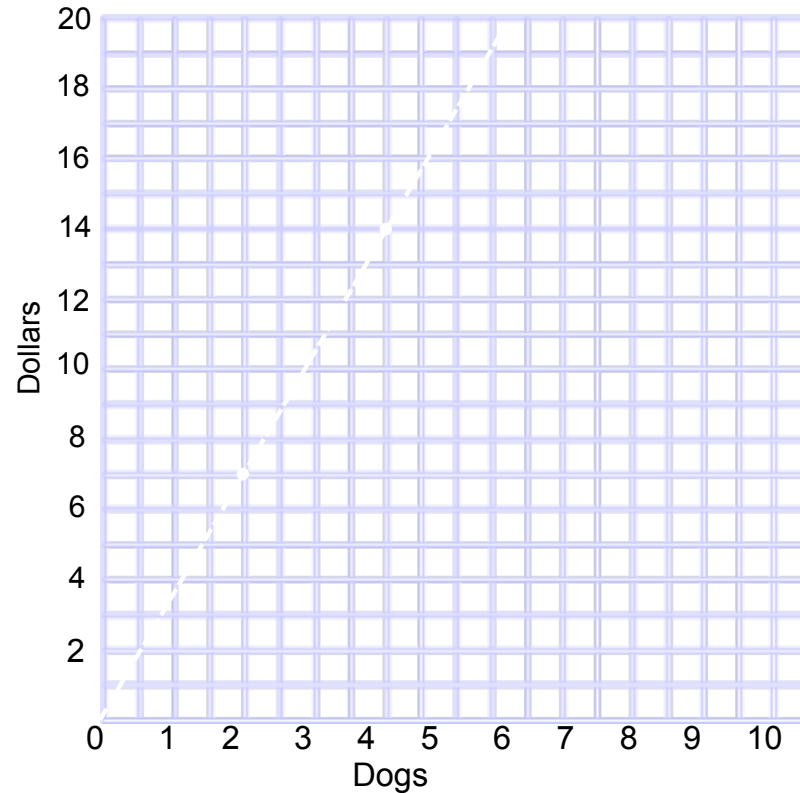
click

- What does the unit rate represent?

click

- What coordinate pair represents the unit rate?

click



- Does the line run through the unit rate?

click

Graphs of Proportions

Try This:

Mary drives the bus. Her rate is shown in the graph. What is the unit rate? What does it represent?

- Find a point on the graph

click

- Use the point to find the unit rate

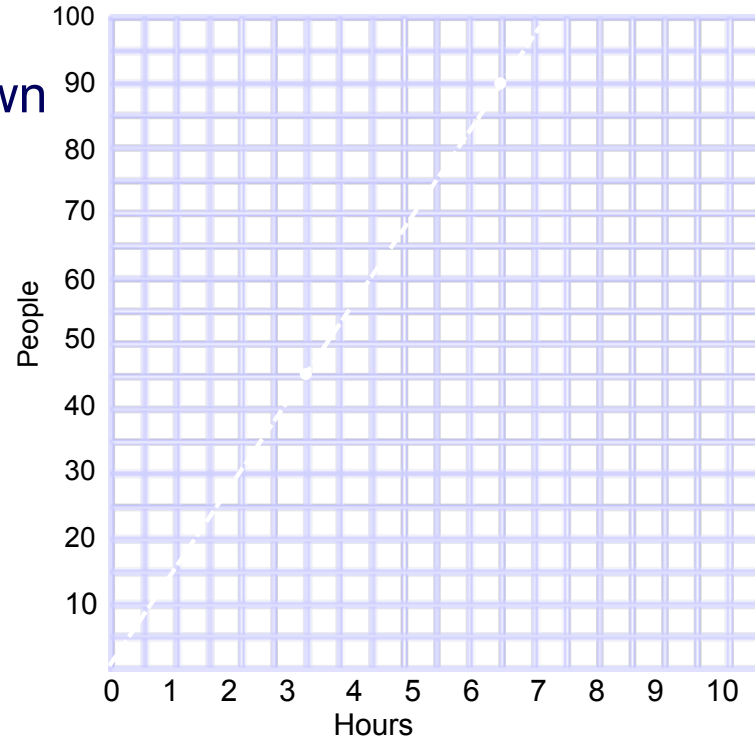
click

- What does the unit rate represent?

click

- What coordinate pair represents the unit rate?

click

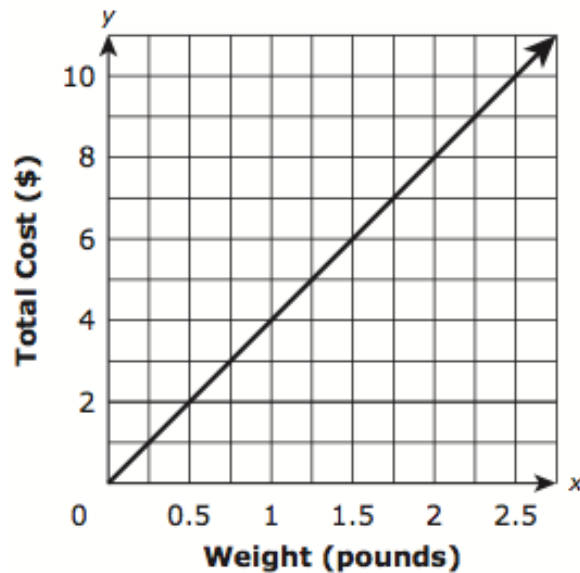


- Does the line run through the unit rate?

click

78 This graph shows the relationship between the pounds of cheese bought at a deli and the total cost, in dollars, for the cheese.

Select each statement about the graph that is true.



- A The point $(0,0)$ shows the cost is \$0.00 for 0 pounds of cheese.
- B The point $(0.25, 1)$ shows the cost is \$0.25 for 1 pound of cheese.
- C The point $(0.5,2)$ shows that 0.5 pound of cheese costs \$2.00.
- D The point $(1,4)$ shows the cost is \$4.00 for 1 pound of cheese.
- E The point $(2,8)$ shows that 8 pounds of cheese cost \$2.00.

Answer

From PARCC EOY sample test non-calculator #1



Problem Solving

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Problem Solving

Chocolates at the candy store cost \$6.00 per dozen. How much does one candy cost? Round your answer to the nearest cent.

Solution:

$$\frac{\$}{\text{candy}} \quad \frac{6.00}{12} = \frac{x}{1}$$

(Use equivalent rates
to set up a proportions)

$$6.00(1) = 12x$$

$$0.50 = x$$

\$0.50 per candy

Problem Solving

Example 2:

There are 3 books per student. There are 570 students.
How many books are there?

Set up the proportion:

$\frac{\text{Books}}{\text{Students}}$

$$\frac{3}{1} = \frac{\quad}{\quad} \quad \text{Where does the 570 go?}$$

$$\frac{3}{1} = \frac{x}{570}$$

$$3 \cdot 570 = 1x$$

$$1,710 = x$$

1,710 books

Problem Solving

Example 3:

The ratio of boys to girls is 4 to 5. There are 135 people on a team.
How many are girls?

Set up the proportion:

$\frac{\textit{Girls}}{\textit{People}}$

$$\frac{5}{9} =$$

$$\frac{5}{9} = \frac{x}{135}$$

$$5 \cdot 135 = 9x$$

$$675 = 9x$$

$$x = 75$$

75 girls

How did we determine this ratio?
Where does the 135 go?

79 Cereal costs \$3.99 for a one pound box. What is the price per ounce? Round your answer to the nearest penny.

Answer

80 Which is the better buy?

Brand A: \$2.19 for 12 ounces

Brand B: \$2.49 for 16 ounces

A Brand A

B Brand B

Answer

81 There are 4 girls for every 10 boys at the party. There are 56 girls at the party. How many boys are there?

Answer

82 The farmer has cows and chickens. He owns 5 chickens for every cow. He has a total of 96 animals. How many cows does he own?

Answer

83 The auditorium can hold 1 person for every 5 square feet. It is 1210 square feet. How many people can the auditorium hold?

Answer

84 The recipe for one serving calls for 4 oz of beef and 2 oz of bread crumbs. 50 people will be attending the dinner. How many ounces of bread crumbs should be purchased?

Answer

85 Mary received 4 votes for every vote that Jane received. 1250 people voted. How many votes did Jane receive?

Answer

86 To make the desired shade of pink paint, Brandy uses 3 oz. of red paint for each oz. of white paint. She needs one quart of pink paint. How many oz. of red paint will she need?
(1 quart = 32 ounces)

Answer

Making Sense of Your Answers

Sometimes your answer will be a decimal or fraction that may not make sense as an answer.

Double check:

- Reread the problem
- Does your answer make sense?
- Do you need to round your answer?
- If so, which way should you round your answer?

87 Cole earned a total of \$11 by selling 8 cups of lemonade. How many cups of lemonade does Cole need to sell in all to earn \$15? Assume the relationship is directly proportional.

Answer

88 Hayley learned a total of 13 appetizer recipes over the course of 3 weeks of culinary school. How many weeks does she need to complete to have learned 21 appetizers? Assume the relationship is directly proportional.

Answer

89 Kailyn took a total of 2 quizzes over the course of 5 days. After attending 16 days of school this quarter, how many quizzes will Kailyn have taken in total? Assume the relationship is directly proportional.

Answer

90 Brittany baked 18 cookies with 1 cup of flour. How many cups of flour does Brittany need in order to bake 27 cookies? Assume the relationship is directly proportional.

Answer

91 Shane caught a total of 10 fish over the course of 2 days on a family fishing trip. At the end of what day will Shane have caught his 22 fish? Assume the relationship is directly proportional.

Answer

92 In a sample of 50 randomly selected students at a school, 38 students eat breakfast every morning. There are 652 students in the school. Using these results, predict the number of students that eat breakfast.

- A 76
- B 123
- C 247
- D 496

Answer

- 93 Sal exercised by stretching and jogging 5 days last week.
- He stretched for a total of 25 minutes during the *week*.
 - He jogged for an equal number of minutes each of the 5 days.
 - He exercised for a total of 240 minutes.

Elena also exercised by stretching and jogging 5 days last week.

- She stretched for 15 minutes each day.
- She jogged for an equal number of minutes each of the 5 days.
- She exercised for a total of 300 minutes.

Determine the number of minutes Sal jogged each day last week and the number of minutes Elena jogged each day last week. Show your work or explain all the steps you used to determine your answers.

Answer

From PARCC PBA sample test calculator #11



Scale Drawings

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Scale Drawings

Scale drawings are used to represent objects that are either too large or too small for a life size drawing to be useful.

Examples:

A life size drawing of an ant or an atom would be too small to be useful.

A life size drawing of the state of New Jersey or the Solar System would be too large to be useful.

Scale Drawings

A scale is always provided with a scale drawing.

The **scale** is the ratio: $\frac{\textit{drawing}}{\textit{real life (actual)}}$

When solving a problem involving scale drawings you should:

- Write the scale as a ratio
- Write the second ratio by putting the provided information in the correct location (drawing on top & real life on the bottom)
- Solve the proportion

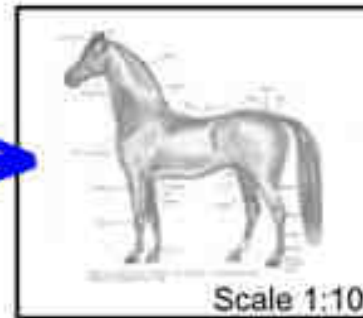
Scale Drawings

Example:

This drawing has a scale of "1:10", so anything drawn with the size of "1" would have a size of "10" in the real world, so a measurement of 150mm on the drawing would be 1500mm on the real horse.



Real Horse
1500 mm high



Drawn Horse
150 mm high

Scale Drawings

Example:

The distance between Philadelphia and San Francisco is 2,950 miles. You look on a map and see the scale is 1 inch : 100 miles. What is the distance between the two cities on the map?

$$\frac{\textit{drawing}}{\textit{actual}} = \frac{1}{100}$$

Writing the scale as a ratio

$$\frac{1}{100} = \frac{x}{2950}$$

$$100x = 2950$$

$$x = 29.5$$

29.5 inches on the map

Scale Drawings

Try This:

On a map, the distance between your town and Washington DC is 3.6 inches. The scale is 1 inch : 55 miles. What is the distance between the two cities?

Answer

94 On a map with a scale of $1 \text{ inch} = 100 \text{ miles}$, the distance between two cities is 7.55 inches. If a car travels 55 miles per hour, about how long will it take to get from one city to the other.

- A 13 hrs 45 min.
- B 14 hrs 30 min.
- C 12 hrs
- D 12 hrs 45 min.

Answer

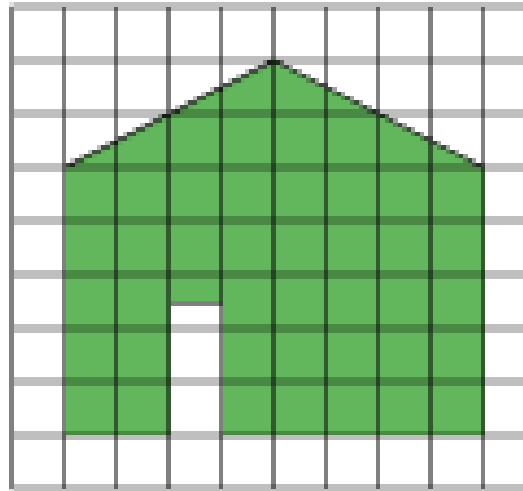
95 On a map, the scale is $\frac{1}{2}$ inch = 300 miles. Find the actual distance between two stores that are $5\frac{1}{2}$ inches apart on the map.

- A 3000 miles
- B 2,727 miles
- C 3,300 miles
- D 1,650 miles

Answer

96 The figure is a scale of the east side of a house. In the drawing, the side of each square represents 4 feet. Find the width and height of the door.

- A 4 ft by 9 ft
- B 4 ft by 12 ft
- C 4 ft by 8 ft
- D 4 ft by 10 ft



Answer

97 The distance between Moorestown, NJ and Duck, NC is 910 miles. What is the distance on a map with a scale of 1 inch to 110 miles?

Answer

98 The distance between Philadelphia and Las Vegas is 8.5 inches on a map with a scale 1.5 in : 500 miles. What is the distance in miles?

Answer

99 You are building a room that is 4.6 m long and 3.3 m wide. The scale on the architect's drawing is 1 cm : 2.5 m. What is the length of the room on the drawing?

Answer

100 You are building a room that is 4.6 m long and 3.3 m wide. The scale on the architect's drawing is 1 cm : 2.5 m. What is the width of the room on the drawing?

Answer

101 Find the length of a 72 inch wide wall on a scale drawing with a scale 1 inch : 2 feet.

Answer

102 You recently purchased a scale model of a car. The scale is 15 cm : 10 m. What is the length of the model car if the real car is 4 m?

Answer

103 You recently purchased a scale model of a car. The scale is 15 cm : 10 m. The length of the model's steering wheel is 1.25 cm. What is the actual length of the steering wheel?

Answer

104 The scale on a map shows that 5 centimeters = 2 kilometers.

Part A

What number of centimeters on the map represents an actual distance of 5 kilometers?

Answer



105 (Continued from previous slide.)

Part B

What is the actual number of kilometers that is represented by 2 centimeters on the map?

Answer

From PARCC EOY sample test calculator #2



Similar Figures

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Similar Figures

Two objects are **similar** if they are the same shape .

In similar objects:

- corresponding angles are congruent (the same)
- corresponding sides are proportional

Similar Figures

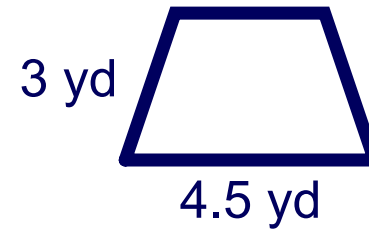
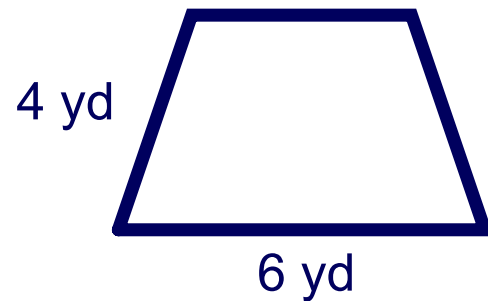
To check for similarity:

- Check to see that corresponding angles are congruent
- Check to see that corresponding sides are proportional
(Cross products are equal)

Similar Figures

Example:

Is the pair of polygons similar? Explain your answer.



$$\frac{4}{6} = \frac{3}{4.5}$$

$$4(4.5) = 6(3)$$

$$18 = 18$$

YES

or

$$\frac{4}{3} = \frac{6}{4.5}$$

$$4(4.5) = 6(3)$$

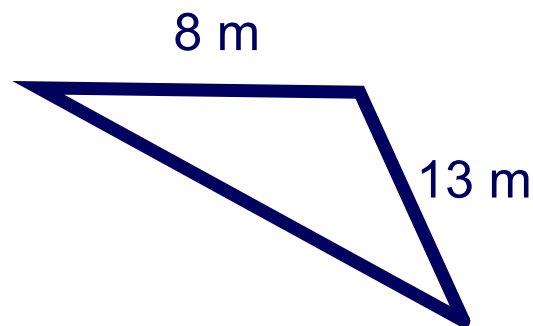
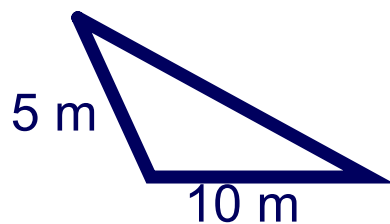
$$18 = 18$$

YES

Similar Figures

Example:

Is the pair of polygons similar? Explain your answer.



$$\frac{5}{10} = \frac{8}{13}$$

$$5(13) = 10(8)$$

$$65 = 80$$

NO

$$\frac{5}{10} = \frac{10}{13}$$

$$5(13) = 8(10)$$

$$65 = 80$$

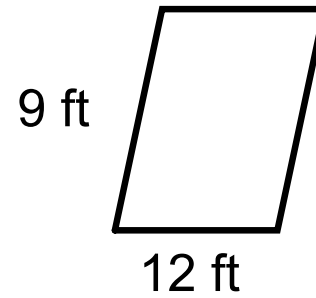
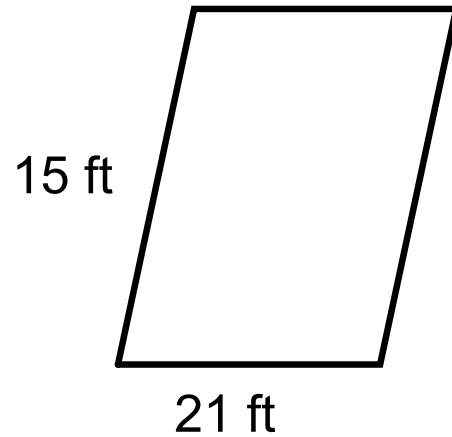
NO

or

106 Are the polygons similar? You must be able to justify your answer. (Shapes not drawn to scale.)

Yes

No

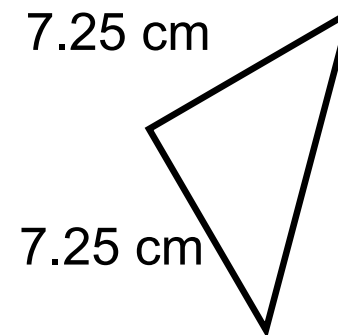
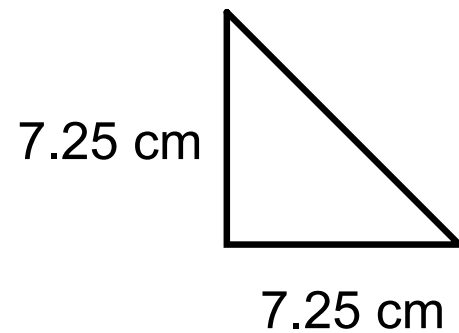


Answer

107 Are the polygons similar? You must be able to justify your answer. (Shapes not drawn to scale.)

Yes

No

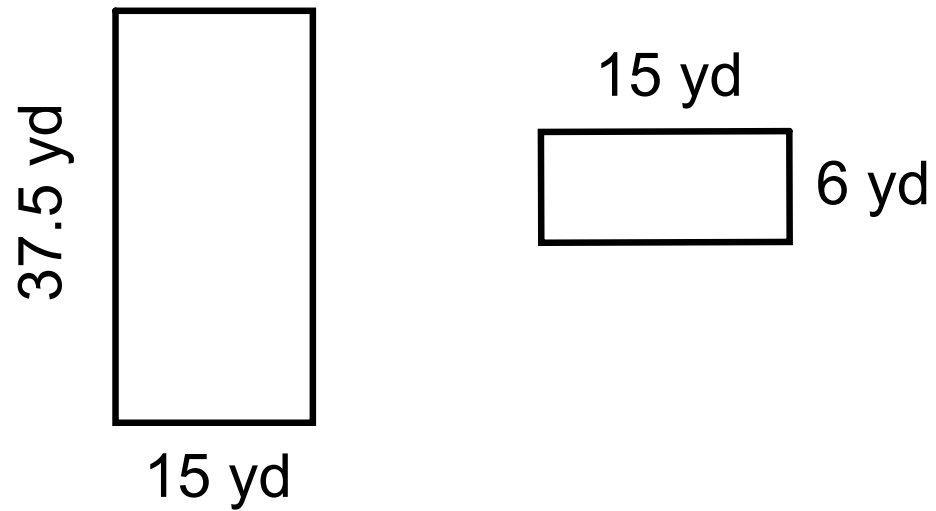


Answer

108 Are the polygons similar? You must be able to justify your answer. (Shapes not drawn to scale.)

Yes

No

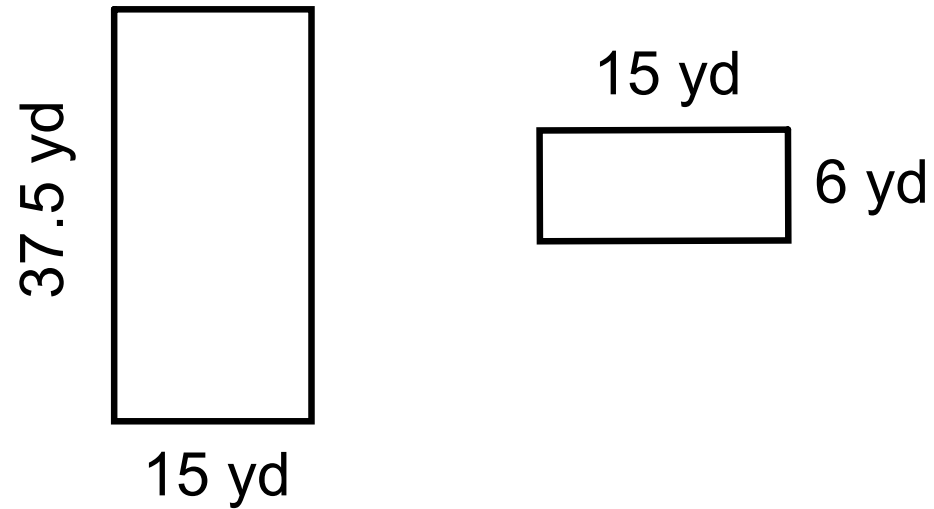


Answer

109 Are the polygons similar? You must be able to justify your answer. (Shapes not drawn to scale.)

Yes

No



Answer

110 A right triangle has legs measuring 4.5 meters and 1.5 meters.

The lengths of the legs of a second triangle are proportional to the lengths of the legs of the first triangle.

Which could be the lengths of the legs of the second triangle? Select *each* correct pair of lengths.

- A 6 m and 2 m
- B 8 m and 5 m
- C 7 m and 3.5 m
- D 10 m and 2.5 m
- E 11.25 m and 3.75 m

Answer

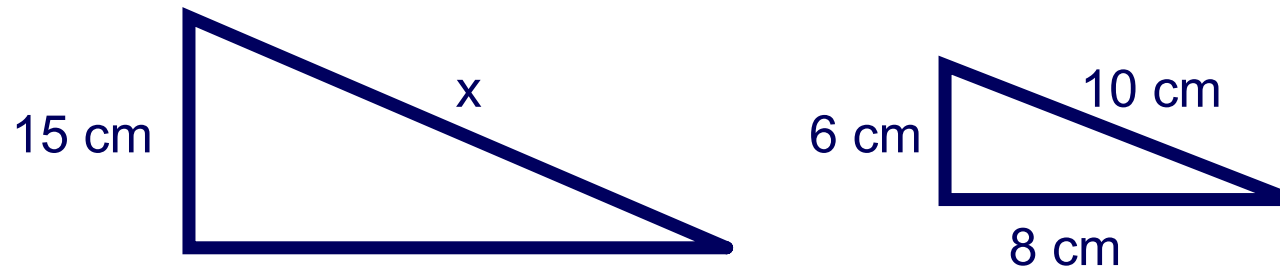
From PARCC PBA sample test calculator #2



Similar Figures

Example:

Find the value of x in the pair of similar polygons.



$$\frac{5}{x} = \frac{6}{10}$$

$$15(10) = 6x$$

$$150 = 6x$$

$$25\text{cm} = x$$

or

$$\frac{15}{6} = \frac{x}{10}$$

$$15(10) = 6x$$

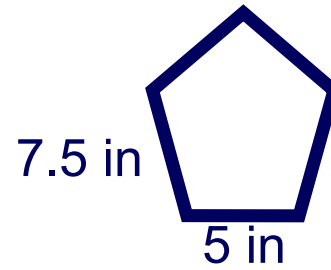
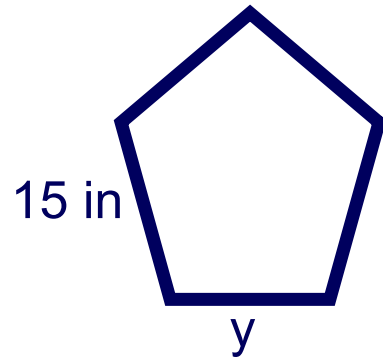
$$150 = 6x$$

$$25\text{cm} = x$$

Similar Figures

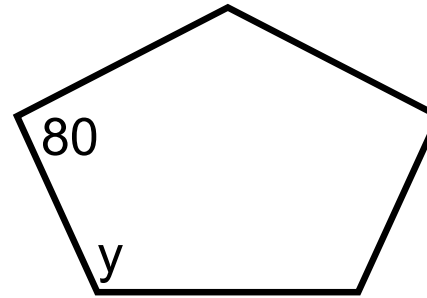
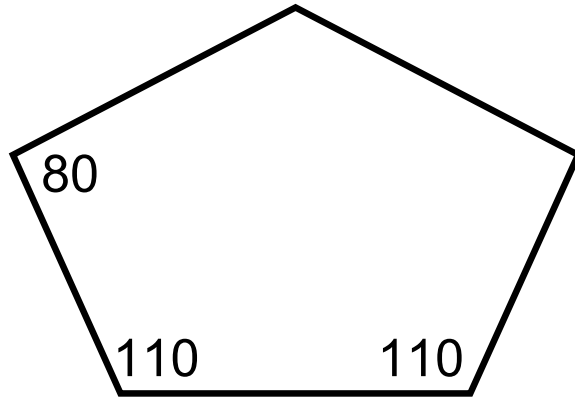
Try This:

Find the value of y in the pair of similar polygons.



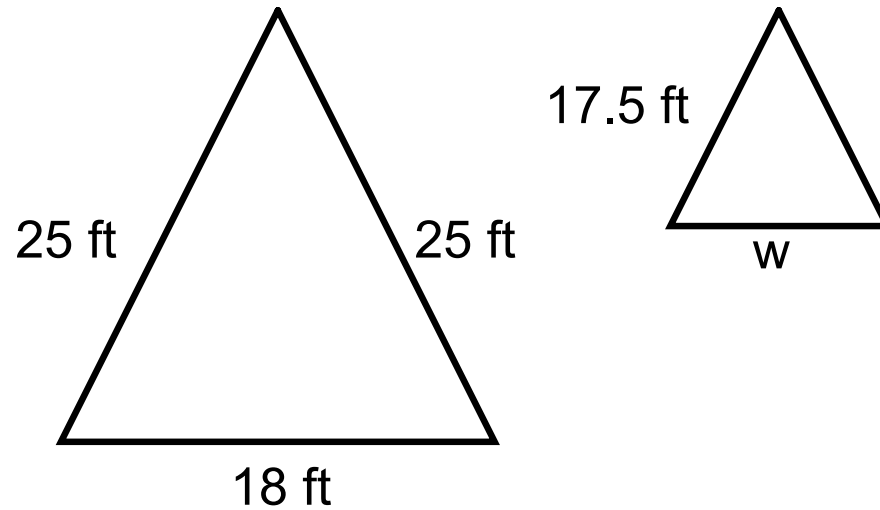
Answer

111 Find the measure of the missing value in the pair of similar polygons. (Shapes not drawn to scale.)



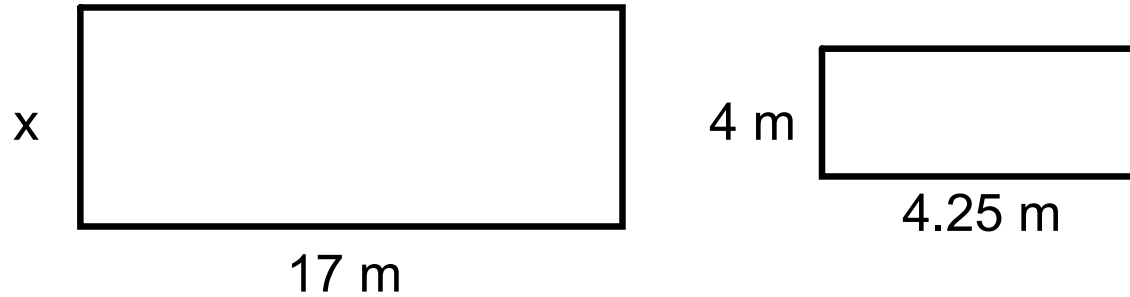
Answer

112 Find the measure of the missing value in the pair of similar polygons. (Shapes not drawn to scale.)



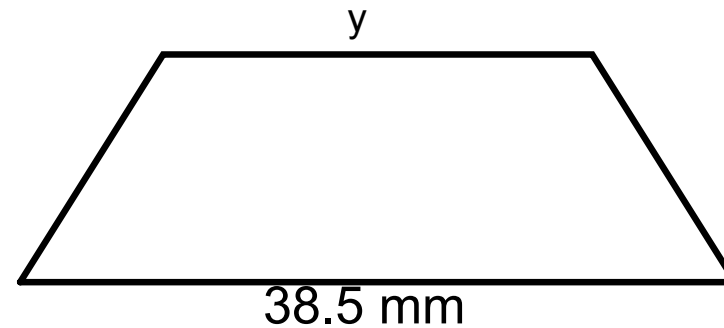
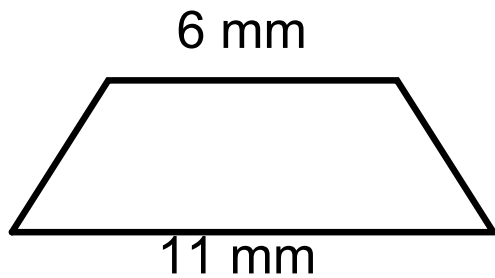
Answer

113 Find the measure of the missing value in the pair of similar polygons. (Shapes not drawn to scale.)



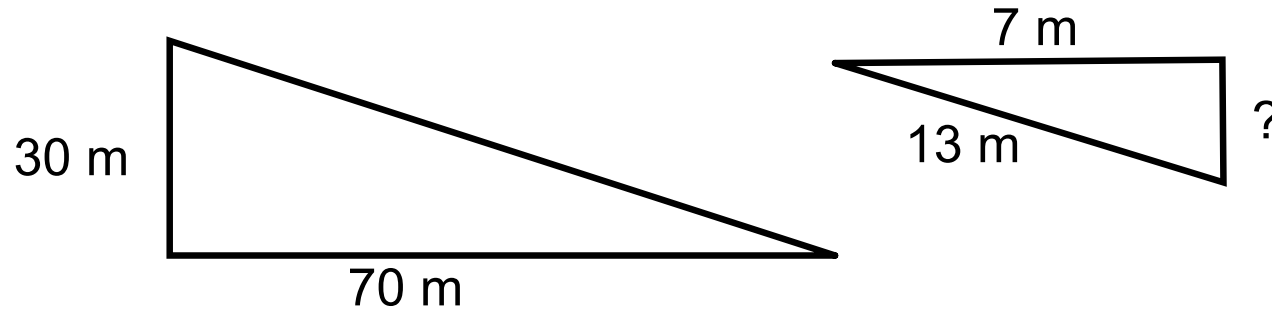
Answer

114 Find the measure of the missing value in the pair of similar polygons. (Shapes not drawn to scale.)



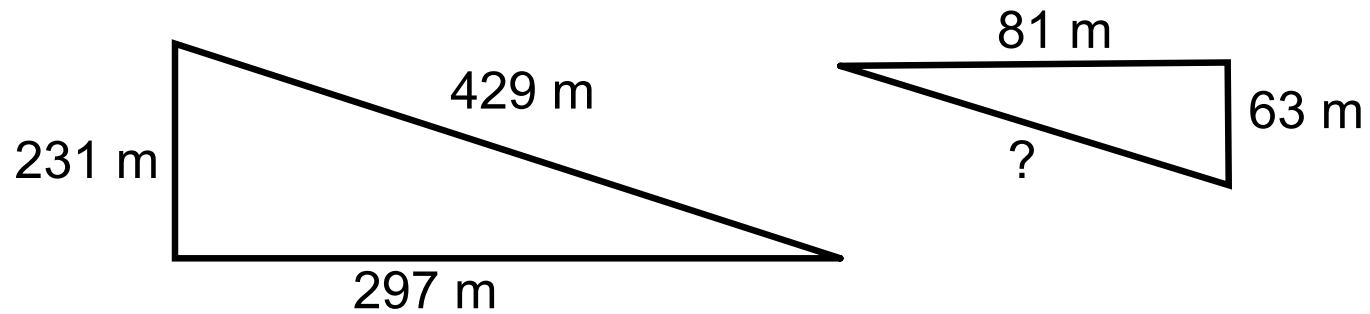
Answer

115 Find the measure of the missing value in the pair of similar polygons. (Shapes not drawn to scale.)



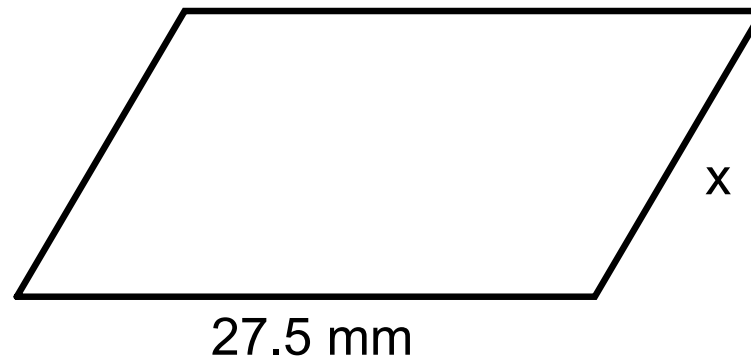
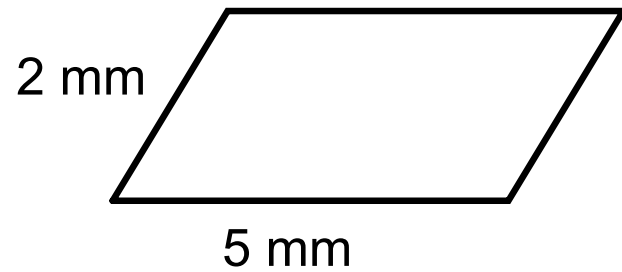
Answer

116 Find the measure of the missing value in the pair of similar polygons. (Shapes not drawn to scale.)



Answer

117 Find the measure of the missing value in the pair of similar polygons. (Shapes not drawn to scale.)



Answer

Glossary

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Constant of Proportionality

A constant ratio (unit rate) in any proportional relationship

<p><u>Equations:</u></p> $y = kx$ <p>or</p> $k = \frac{y}{x}$	$y = 5x$ <p>$k = 5$</p>	$\begin{matrix} (3, 45) \\ x \quad y \end{matrix}$ $y = kx$ $45 = k3$ $k = 15$
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Equivalent Ratios

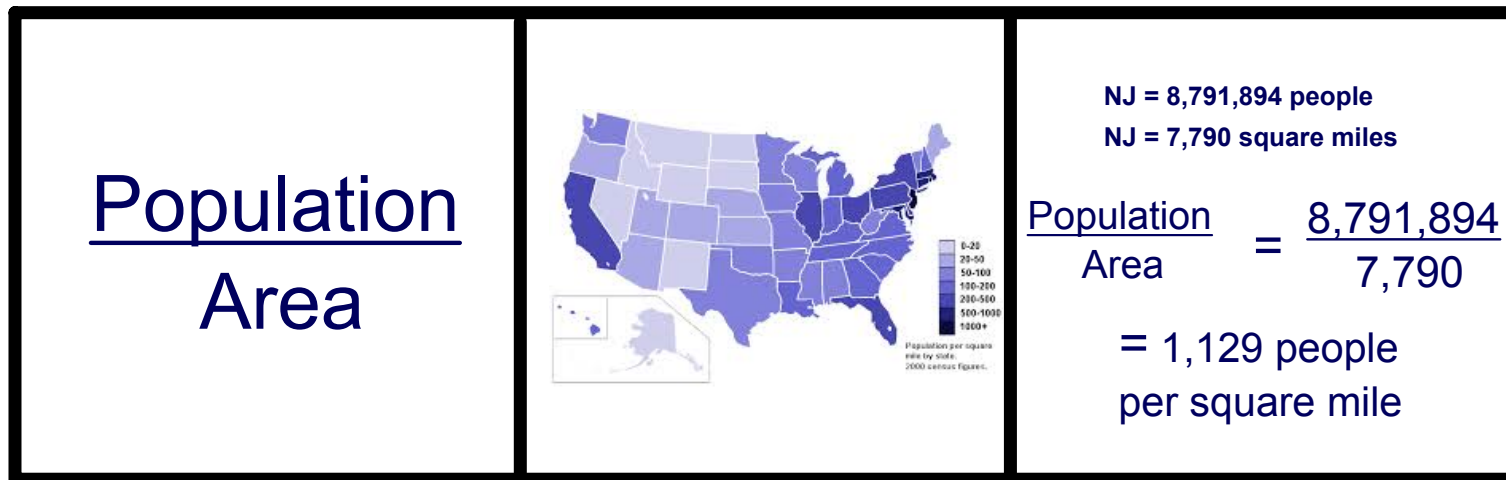
Ratios that have the same value.

$$\frac{3}{6} = \frac{1}{2} = \frac{4}{8}$$

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Population Density

A unit rate of people per square mile.



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Proportion

An equation that states that two ratios are equivalent.

$\frac{2}{3} = \frac{14}{21}$	$\frac{1}{2} = \frac{20}{40}$	$\frac{5}{8} \stackrel{\times 3}{=} \frac{15}{x}$ $x = 24$
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Rate


A ratio of two quantities measured in different units.

 <p>3 participants/2 teams</p> 	 <p>5 gallons/3 rooms</p> 	 <p>7 burgers/2 tomatoes</p> 
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Ratio


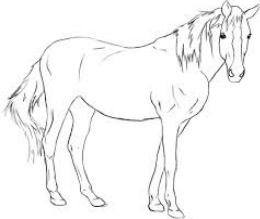
A comparison of two numbers by division.

<p>3 different ways: "the ratio of a to b"</p> <p>a to b</p> <p>a : b</p> <p>$\frac{a}{b}$</p>	<p>20 to 48</p> <p>20:48</p> <p>$\frac{20}{48}$</p> <p>There are 48 animals in the field. Twenty are cows and the rest are horses.</p> <p><i>What is the number of cows to the total number of animals?</i> </p>
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Scale

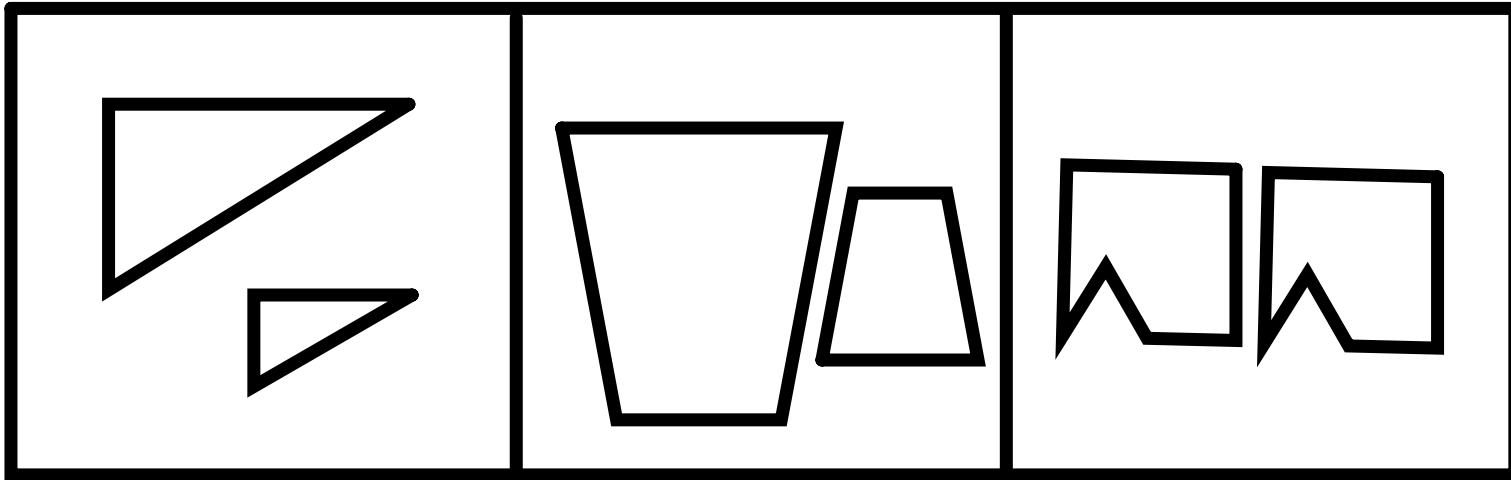
The ratio of a drawing to the real life measurement.

$\frac{\text{drawing}}{\text{real life (actual)}}$	 <p>Real Horse 1500mm high</p> <p>Scale- 1:10</p>  <p>Drawn Horse 150mm high</p>
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Similar

- Two figures that are the same shape.
- corresponding angles are congruent
 - corresponding sides are proportional



Unit Rate

Rate with a denominator of one.

 34 miles/gallon 	 3 cookies per person 	 62 words/minute 
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Standards for Mathematical Practices

MP1 Make sense of problems and persevere in solving them.

MP2 Reason abstractly and quantitatively.

MP3 Construct viable arguments and critique the reasoning of others.

MP4 Model with mathematics.

MP5 Use appropriate tools strategically.

MP6 Attend to precision.

MP7 Look for and make use of structure.

MP8 Look for and express regularity in repeated reasoning.

**Click on each standard to bring
you to an example of how to meet
this standard within the unit.**



