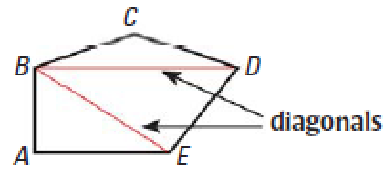
8.1 Find Angle Measures in Polygons

Obj.: To find angle measures in polygons.

Key Vocabulary

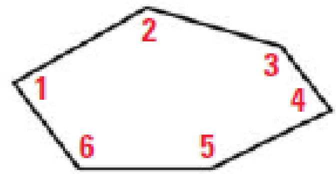


• **Diagonal -** A **diagonal** of a polygon is a segment that joins two nonconsecutive vertices. Polygon ABCDE has two diagonals from vertex B, BD and BE .

• **Interior angles -** The original angles are the **interior angles.**

• **Exterior angles -** When the sides of a polygon are extended, other angles are formed. The angles that form linear pairs with the interior angles are the **exterior angles**.

Polygon Interior Angles Theorem Poly. Int. ∠ Thm.



The sum of the measures of the interior angles of a convex *n*-gon is $(n-2)\bullet 180^0$.

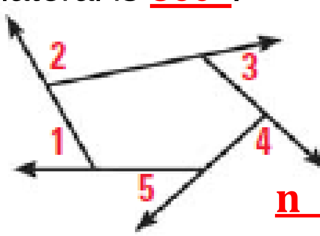
$m\angle 1 + m\angle 2 \ ... + m\angle n = (n-2)\bullet 180^0$.

n = 6

Interior Angles of a Quadrilateral

The sum of the measures of the interior angles of a quadrilateral is 360^0.

Polygon Exterior Angles Theorem



The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is 360^0.

$m\angle 1 + m\angle 2 \ ... + m\angle n = 360^0$.

n = 5

EXAMPLE 1 Find the sum of angle measures in a polygon

Find the sum of the measures of the interior angles of a convex hexagon.

Solution A hexagon has 6 sides. Use the Polygon Interior Angles Theorem.

$(n - 2)\cdot 180^\circ = (6-2)\cdot 180^\circ$ **Substitute 6 for *n*.**

$= 4 \cdot 180^\circ$ **Subtract.**

$= 720^\circ$ **Multiply.**

The sum of the measures of the interior angles of a hexagon is 720°.

EXAMPLE 2 Find the number of sides of a polygon

The sum of the measures of the interior angles of a convex polygon is 1260^0. Classify the polygon by the number of sides.

Solution

Use the Polygon Interior Angles Theorem to write an equation involving the number of sides *n*. Then solve the equation to find the number of sides.

$(n - 2)\cdot 180^\circ = 1260^\circ$ **Polygon Interior Angles Theorem**

$n - 2 = 7$ **Divide each side by 180°.**

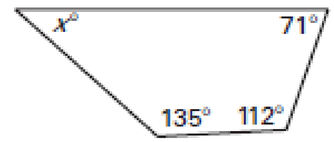
$n = 9$ **Add 2 to each side.**

The polygon has 9 sides. It is a nonagon.

EXAMPLE 3 Find an unknown interior angle measure

(8.1 cont.)

Find the value of x in the diagram shown.



Solution The polygon is a quadrilateral. Use the Corollary to the Polygon Interior Angles Theorem to write an equation involving *x*. Then solve the equation.

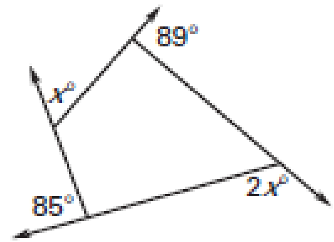
$x° +$ 135° $+$ 112° $+$ 71° $=$ 360° — Corollary to Theorem 8.1

$x +$ 318 $=$ 360 — Combine like terms.

$x =$ 42 — Subtract 318 from each side.

EXAMPLE 4 Find unknown exterior angle measures

Find the value of x in the diagram shown.



Solution Use the Polygon Exterior Angles Theorem to write and solve an equation.

$x° +$ 2x° $+$ 85° $+$ 89° $=$ 360° — Polygon Exterior Angles Theorem.

3 $x +$ 174 $=$ 360 — Combine like terms.

$x =$ 62 — Solve for *x*.

EXAMPLE 5 Find angle measures in regular polygons

Lamps The base of a lamp is in the shape of a regular 15-gon. Find (a) the measure of each interior angle and (b) the measure of each exterior angle.

Solution

a. a. Use the Polygon Interior Angles Theorem to find the sum of the measures of the interior angles.

$(n -$ 2 $) \cdot$ 180° $= ($ 15 $-$ 2 $) \cdot$ 180°

$=$ 2340°

Then find the measure of one interior angle. A regular 15-gon has 15 congruent interior angles. Divide 2340° by 15 : 2340° $\div$ 15 $=$ 156° .

The measure of each interior angle in the 15-gon is 156° .

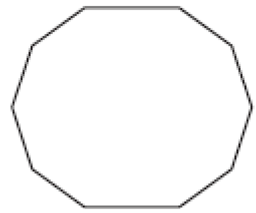
b. By the Polygon Exterior Angles Theorem, the sum of the measures of the exterior angles, one angle at each vertex, is 360° . Divide 360° by 15 : 360° $\div$ 15 $=$ 24° .

The measure of each exterior angle in the 15-gon is 24° .

8.1 Cont. (Write these on your paper)

Checkpoint **Complete the following exercise.**

1. Find the sum of the measures of the interior angles of the convex decagon.

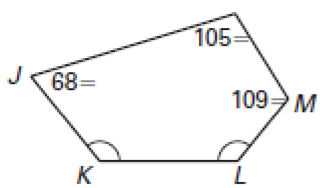


1440°

2. The sum of the measures of the interior angles of a convex polygon is 1620°. Classify the polygon by the number of sides.

11-gon

3. Use the diagram at the right. Find $m\angle K$ and $m\angle L$.



$m\angle K$ $m\angle L$ 129°

4. A convex pentagon has exterior angles with measures 66°, 77°, 82°, and 62°. What is the measure of an exterior angle at the fifth vertex?

73°

5. Find the measure of (a) each interior angle and (b) each exterior angle of a regular nonagon.

a. 140°

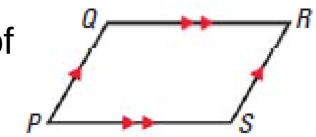
b. 40°

8.2 Use Properties of Parallelograms

Obj.: To find angle and side measures in parallelograms.

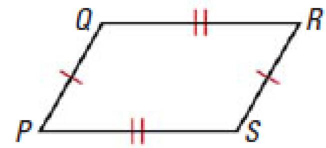
Key Vocabulary

• **Parallelogram -** A **parallelogram** is a quadrilateral with **both** pairs of opposite **sides parallel.** The term "parallelogram *PQRS*" can be written as ▱***PQRS*.**



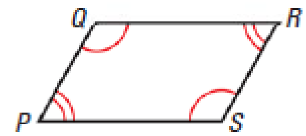
▱ →opp. sides ≅

If a quadrilateral is a **parallelogram**, then its opposite **sides** are **congruent**.
If *PQRS* is a parallelogram, then **PQ // RS** and **QR // PS** .



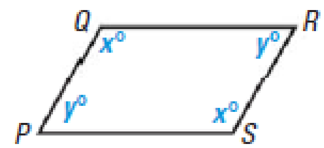
▱→opp. ∠'s ≅

If a **quadrilateral** is a parallelogram, then its opposite **angles** are **congruent**.
If *PQRS* is a parallelogram, then ***∠P ≅∠R*** and ***∠Q ≅ ∠S***.



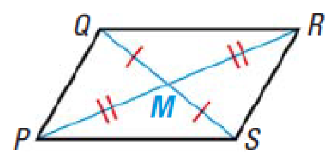
▱ →cons. ∠'s supp

If a quadrilateral is a **parallelogram**, then its **consecutive** angles are **supplementary**.
If *PQRS* is a parallelogram, then $x^0 + y^0 = 180^0$.



▱ →diag bis each other

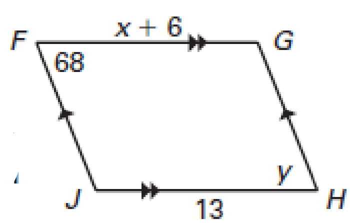
If a **quadrilateral** is a parallelogram, then its **diagonals bisect** each other.



$\overline{QM} \cong \overline{SM}$ and $\overline{PM} \cong \overline{QM}$

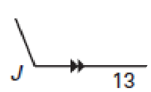
EXAMPLE 1 Use properties of parallelograms

Find the values of x and y.



Solution

FGHJ is a parallelogram by the definition of a parallelogram. Use Theorem 8.3 to find the value of *x*.



$FG =$ <u>*HJ*</u> **Opposite sides of a ▱ are ≅.**

$x + 6 =$ <u>13</u> **Substitute *x* + 6 for *FG* and <u>13</u> for <u>*HJ*</u> .**

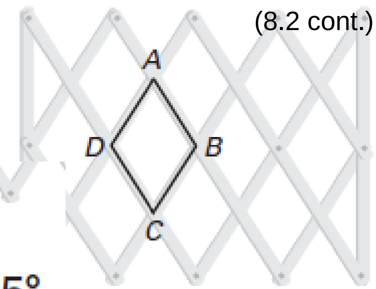
$x =$ <u>7</u> **Subtract 6 from each side.**

By Theorem 8.4, ∠*F* ≅ <u>∠*H*</u>, or *m*∠*F* = <u>*m*∠*H*</u>. So, $y° =$ <u>68°</u>.

In ▱*FGHJ*, *x* = <u>7</u> and *y* = <u>68</u>.

EXAMPLE 2 Use properties of a parallelogram

Gates As shown, a gate contains several parallelograms. Find the m∠ADC when m∠DAB = 65⁰.

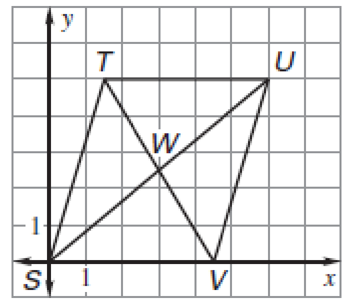


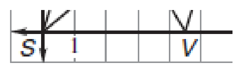
Solution

By Theorem 8.5, the consecutive angle pairs in ▱*ABCD* are supplementary. So, $m\angle ADC + m\angle DAB = 180^\circ$. Because $m\angle DAB = 65^\circ$, $m\angle ADC = 180^\circ - 65^\circ = 115^\circ$.

EXAMPLE 3

The diagonals of ▱ STUV intersect at point W. Find the coordinates of W.





Solution

By Theorem 8.6, the diagonals of a parallelogram bisect each other. So, *W* is the midpoint of the diagonals *TV* and *SU*. Use the Midpoint Formula.

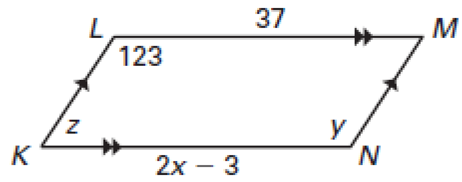
Coordinates of midpoint *W* of

$$SU = \left(\frac{6+0}{2}, \frac{5+0}{2}\right) = \left(3, \frac{5}{2}\right)$$

In Example 3, you can use either diagonal to find the coordinates of *W*. Using *SU* simplifies calculations because one endpoint is (0, 0).

8.2 Cont.

Checkpoint **Find the indicated measure in ▱*KLMN* shown at the right.**



1. x

$x = 20$

2. y

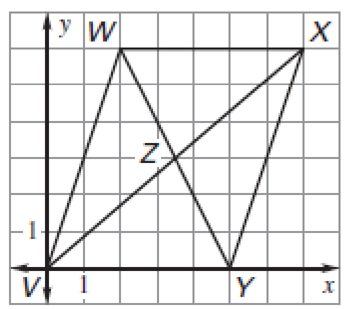
$y = 123$

3. z

$z = 57$

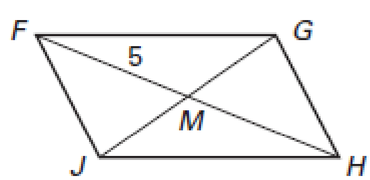
4. The diagonals of ▱*VWXY* intersect at point *Z*. Find the coordinates of *Z*.

$Z\left(\frac{7}{2}, 3\right)$



5. Given that ▱*FGHJ* is a parallelogram, find *MH* and *FH*.

$MH = 5, FH = 10$



8.3 Show that a Quadrilateral is a Parallelogram

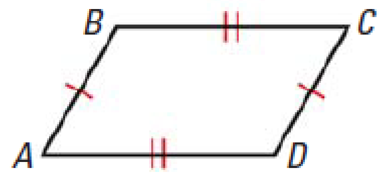
Obj.: Use properties to identify parallelograms.

Key Vocabulary

• **Parallelogram -** A **parallelogram** is a **quadrilateral** with both pairs of **opposite** sides **parallel**.

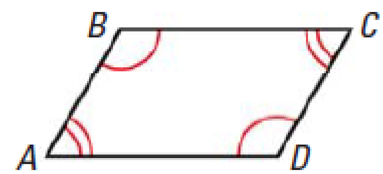
both pair opp. sides ≅ → ▱

If **both** pairs of opposite **sides** of a quadrilateral are **congruent**, then the quadrilateral is a **parallelogram**.
If $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$, then *ABCD* is a parallelogram.



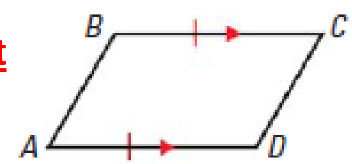
both pair opp ∠'s ≅ → ▱

If both **pairs** of **opposite** **angles** of a quadrilateral are **congruent**, then the **quadrilateral** is a parallelogram.
If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then *ABCD* is a parallelogram.



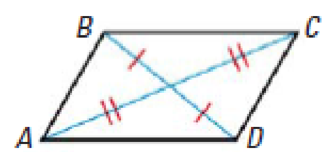
one pair opp sides ≅ & ∥→ ▱

If **one** pair of **opposite** sides of a quadrilateral are **congruent** and **parallel**, then the quadrilateral is a **parallelogram**.
If $\overline{BC} \cong \overline{AD}$ and $\overline{BC} \parallel \overline{AD}$, then **ABCD** is a parallelogram.



diag bis each other→ ▱

If the **diagonals** of a quadrilateral **bisect** each other, then the **quadrilateral** is a parallelogram. If $\overline{BD}$ and $\overline{AC}$ **bisect** each other, then *ABCD* **is** a parallelogram.

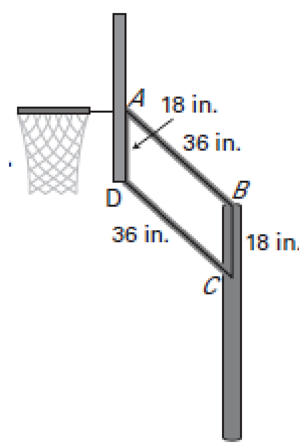


EXAMPLE 1 Solve a real-world problem

Basketball In the diagram at the right, $\overline{AB}$ and $\overline{DC}$ represent adjustable supports of a basketball hoop. Explain why $\overline{AD}$ is always parallel to $\overline{BC}$.

Solution The shape of quadrilateral *ABCD* changes as the adjustable supports move, but its side lengths do not change. Both pairs of opposite sides are congruent, so *ABCD* is a parallelogram by Theorem 8.7.

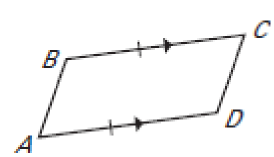
By the definition of a parallelogram, $\overline{AD} \parallel \overline{BC}$.



EXAMPLE 2 Identify a parallelogram

Lights The headlights of a car have the shape shown at the right. Explain how you know that ∠B ≅ ∠D.

Solution In the diagram, $\overline{BC} \parallel \overline{AD}$ and $\overline{BC} \cong \overline{AD}$. By Theorem 8.9, quadrilateral *ABCD* is a parallelogram. By Theorem 8.4, you know that opposite angles of a parallelogram are congruent. So, $\angle B \cong \angle D$.

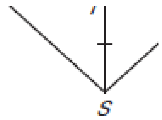


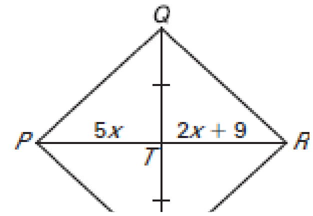
EXAMPLE 3 Use algebra with parallelograms

(8.3 cont.)

ALGEBRA For what value of x is quadrilateral PQRS a parallelogram?

Solution





By Theorem 8.10, if the diagonals of *PQRS* __bisect__ each other, then it is a parallelogram. You are given that $\overline{QT} \cong$ __$\overline{ST}$__. Find x so that $\overline{PT} \cong$ __$\overline{RT}$__.

$PT =$ __RT__ Set the segment lengths equal.

$5x =$ __$2x + 9$__ Substitute $5x$ for PT and __$2x + 9$__ for __RT__.

__3__ $x =$ __9__ Subtract __$2x$__ from each side.

$x =$ __3__ Divide each side by __3__.

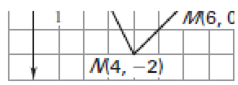
When $x =$ __3__, $PT = 5($__3__$) =$ __15__ and $RT = 2($__3__$) + 9 =$ __15__.

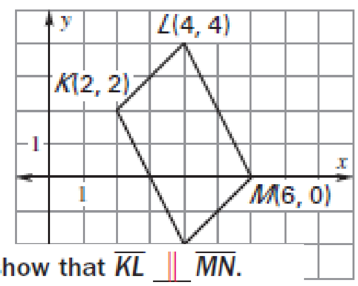
Quadrilateral *PQRS* is a parallelogram when $x =$ __3__.

EXAMPLE 4 Use coordinate geometry

Show the quadrilateral KLMN is a parallelogram.

Solution





One way is to show that a pair of sides are congruent and parallel. Then apply __Theorem 8.9__.

First use the Distance Formula to show that $\overline{KL}$ and $\overline{MN}$ are __congruent__.

$KL = \sqrt{(4-2)^2 + (4-2)^2} = \sqrt{8}$

$MN = \sqrt{(6-4)^2 + [0-(-2)]^2} = \sqrt{8}$

Because $KL = MN = \sqrt{8}$, $\overline{KL}$ __$\cong$__ $\overline{MN}$.

ı use the slope formula to show that $\overline{KL}$ __$\parallel$__ $\overline{MN}$.

e of $\overline{KL} = \dfrac{4-2}{4-2} =$ __1__

Slope of $\overline{MN} = \dfrac{0-(-2)}{6-4} =$ __1__

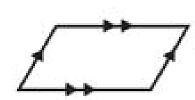
$\overline{KL}$ and $\overline{MN}$ have the same slope, so they are __parallel__.

$\overline{KL}$ and $\overline{MN}$ are congruent and parallel. So, *KLMN* is a parallelogram by __Theorem 8.9__.

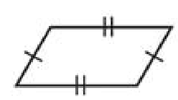
CONCEPT SUMMARY

Ways to Prove a Quadrilateral is a Parallelogram

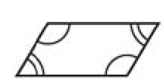
1. Show **both** pairs of opposite sides are **parallel**.
(Def. of ▱)



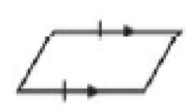
2. Show **both** pairs of opposite sides are **congruent**.
(both pair opp. sides ≅ → ▱)



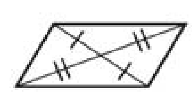
3. Show **both** pairs of **opposite** angles are congruent.
(both pair opp ∠'s ≅ → ▱)



4. Show **one pair** of opposite sides are congruent and parallel.
(one pair opp sides ≅ & ∥→ ▱)



5. Show the **diagonals** bisect each other.
(diag bis each other→ ▱)



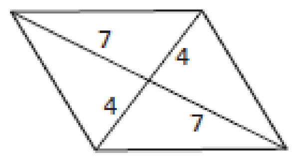
8.3 Cont. (Write these on your paper)

Checkpoint **Complete the following exercises.**

1. In quadrilateral *GHJK*, $m\angle G = 55°$, $m\angle H = 125°$, and $m\angle J = 55°$. Find $m\angle K$. What theorem can you use to show that *GHJK* is a parallelogram?

$m\angle K = 125°$; Theorem 8.8

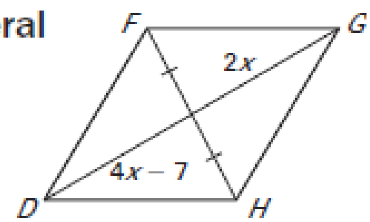
2. What theorem can you use to show that the quadrilateral is a parallelogram?



Theorem 8.10

Checkpoint **Complete the following exercises.**

3. For what value of *x* is quadrilateral *DFGH* a parallelogram?



$x = 3.5$

4. *Explain* another method that can be used to show that quadrilateral *KLMN* in Example 4 is a parallelogram.

Sample Answer: Draw the diagonals and find the point of intersection. Show the diagonals bisect each other and apply Theorem 8.10.

8.4 Properties of Rhombuses, Rectangles, and Squares

Obj.: Use properties of rhombuses, rectangles, and squares.

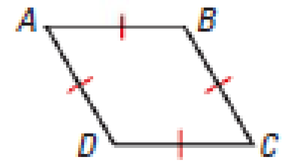
Key Vocabulary

- **Rhombus –** A **rhombus** is a parallelogram with four congruent sides.
- **Rectangle –** A **rectangle** is a parallelogram with four right angles.
- **Square –** A **square** is a parallelogram with four congruent sides and four right angles.

RHOMBUS COROLLARY rhom coro

A quadrilateral is a rhombus if and only if it has four congruent sides.

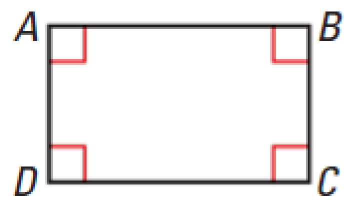
ABCD is a rhombus if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$



RECTANGLE COROLLARY rect coro

A quadrilateral is a rectangle if and only if it has four right angles.

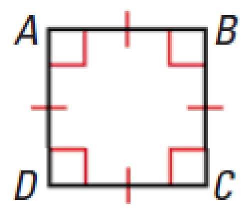
ABCD is a rectangle if and only if $\angle A, \angle B, \angle C,$ and $\angle D$ are right angles.

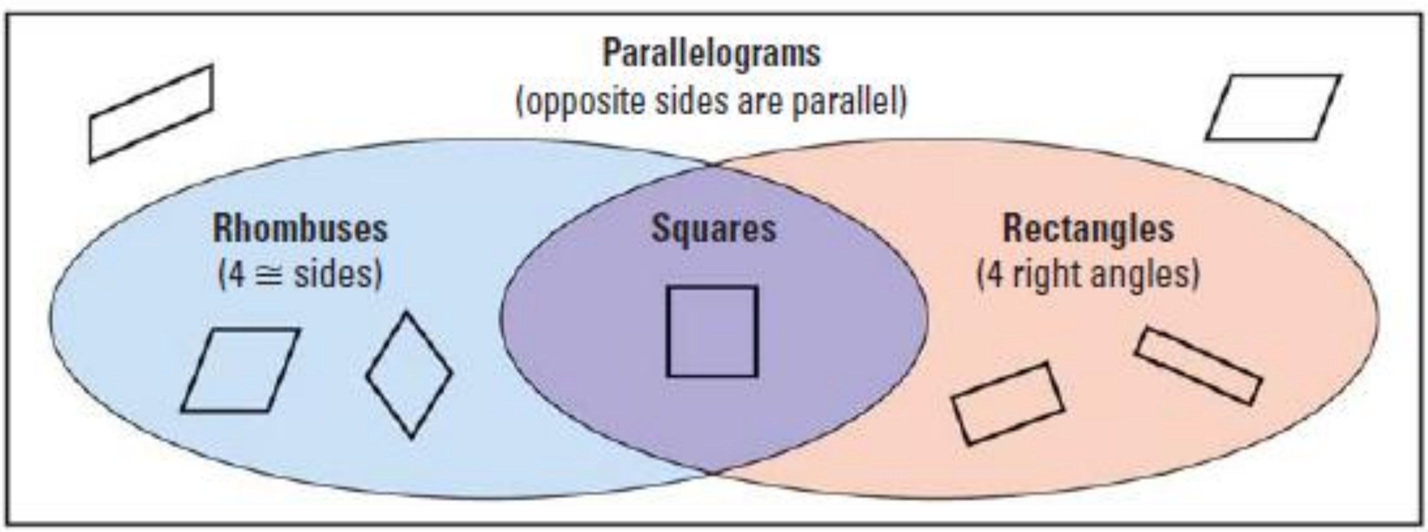


SQUARE COROLLARY rhomb. coro

A quadrilateral is a square if and only if it is a rhombus and a rectangle.

ABCD is a square if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ and $\angle A, \angle B, \angle C,$ and $\angle D$ are right angles.



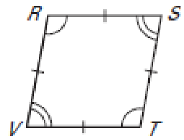


EXAMPLE 1 Use properties of special quadrilaterals

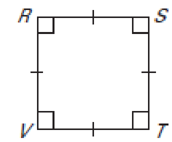
For any rhombus RSTV, decide whether the statement is *always* or *sometimes* true. Draw a sketch and explain your reasoning.

a. ∠S ≅ ∠V **b.** ∠T ≅ ∠V

a. By definition, a rhombus is a parallelogram with four congruent sides. By Theorem 8.4, opposite angles of a parallelogram are congruent. So, ∠*S* ≅ ∠*V*. The statement is always true.

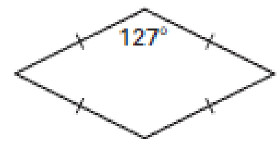


b. If rhombus *RSTV* is a square, then all four angles are congruent right angles. So ∠*T* ≅ ∠*V* if *RSTV* is a square. Because not all rhombuses are also squares, the statement is sometimes true.



EXAMPLE 2 Classify special quadrilaterals

Classify the special quadrilateral. Explain your reasoning.



Solution

The quadrilateral has four congruent sides. One of the angles is not a right angle, so the rhombus is not also a square. By the Rhombus Corollary, the quadrilateral is a rhombus.

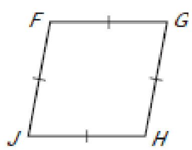
EXAMPLE 3 List properties of special parallelograms

Sketch rhombus FGHJ. List everything that you know about it.

Solution

By definition, you need to draw a figure with the following properties:

- The figure is a parallelogram.
- The figure has four congruent sides.

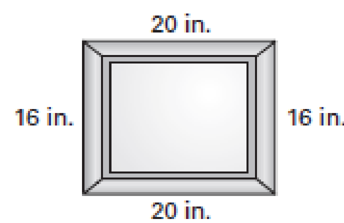


Because *FGHJ* is a parallelogram, it has these properties:

- Opposite sides are parallel and congruent.
- Opposite angles are congruent. Consecutive angles are supplementary.
- Diagonals bisect each other.

By Theorem 8.11, the diagonals of *FGHJ* are perpendicular. By Theorem 8.12, each diagonal bisects a pair of opposite angles.

EXAMPLE 4 Solve a real-world problem



Framing You are building a frame for a painting. The measurements of the frame are shown at the right.

a. The frame must be a rectangle. Given the measurements in the diagram, can you assume that it is? Explain.

b. You measure the diagonals of the frame. The diagonals are 25.6 inches. What can you conclude about the shape of the opening?

Solution

a. No, you cannot. The boards on opposite sides are the same length, so they form a parallelogram. But you do not know whether the angles are right angles.

b. By Theorem 8.13, the diagonals of a rectangle are congruent. The diagonals of the frame are congruent, so the frame forms a rectangle.

8.4 Cont

1. For any square *CDEF*, is it *always* or *sometimes* true that $\overline{CD} \cong \overline{DE}$? *Explain* your reasoning.

Always; a square has four congruent sides.

2. A quadrilateral has four congruent sides and four congruent angles. Classify the quadrilateral.

square

3. Sketch rectangle *WXYZ*. List everything that you know about it.

WXYZ is a parallelogram with four right angles. Opposite sides are parallel and congruent. Opposite angles are congruent and consecutive angles are supplementary. The diagonals are congruent and bisect each other.

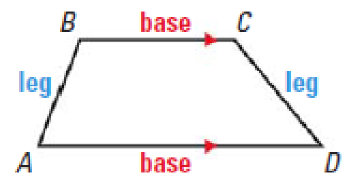
4. Suppose the diagonals of the frame in Example 4 are not congruent.

Could the frame still be a rectangle? *Explain*.

No; by Theorem 8.13, a rectangle must have congruent diagonals.

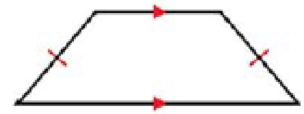
8.5 Use Properties of Trapezoids and Kites

Obj.: Use properties of trapezoids and kites.



Key Vocabulary

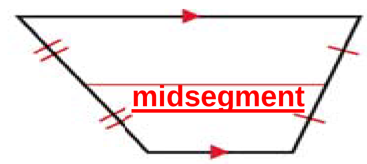
Trapezoid, bases - A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are the bases.


isosceles trapezoid

• **Base angles -** A trapezoid has two pairs of **base angles**. For example, in trapezoid *ABCD*, ∠*A* and ∠*D* are one pair of base angles, and ∠*B* and ∠*C* are the second pair.

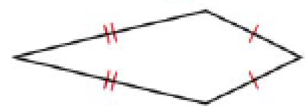
• **Legs -** The nonparallel sides are the legs of the trapezoid.

• **Isosceles trapezoid -** If the legs of a trapezoid are congruent, then the trapezoid is an isosceles **trapezoid**.



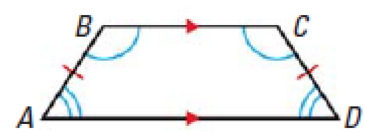
• **Midsegment of a trapezoid -** The **midsegment of a trapezoid** is the segment that connects the midpoints of its legs.

• **Kite -** A **kite** is a quadrilateral that has two pairs of consecutive congruent sides, but opposite sides are not congruent.



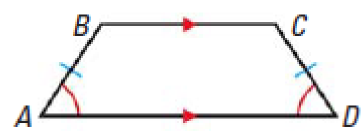
Trap isosc → base ∠s ≅

If a trapezoid is isosceles, then each pair of base angles is congruent. If trapezoid *ABCD* is isosceles, then ∠A ≅ ∠D and ∠B ≅ ∠C.



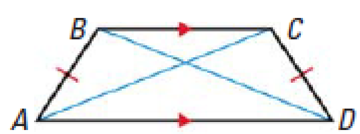
base ∠s ≅ → Trap isosc

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid. If ∠A ≅ ∠D (or if ∠B ≅ ∠C), then trapezoid *ABCD* is isosceles.



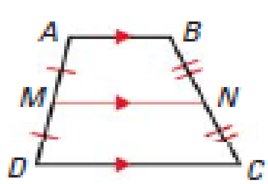
Trap isosc → diags ≅

A trapezoid is isosceles if and only if its diagonals are congruent. Trapezoid *ABCD* is isosceles if and only if $\overline{AC} \cong \overline{BD}$.



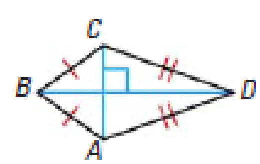
Midsegment Theorem for Trapezoids Trap midseg Thm.

The midsegment of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases. If $\overline{MN}$ is the midsegment of trapezoid *ABCD*, then $\overline{AB} \parallel \overline{MN} \parallel \overline{DC}$, and *MN* = ½(*AB* + *CD*).



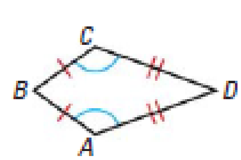
kite→⊥diags

If a quadrilateral is a kite, then its diagonals are perpendicular. If quadrilateral *ABCD* is a kite, then $\overline{AC} \perp \overline{BD}$.



kite→ one pair opp ∠s ≅

If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent. If quadrilateral *ABCD* is a kite and $\overline{BC} \cong \overline{BA}$, then ∠*A* ≅ ∠*C* and ∠*B* ≇ ∠*D*

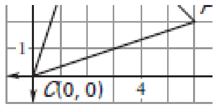


EXAMPLE 1 Use a coordinate plane

Show that CDEF is a trapezoid.

Solution

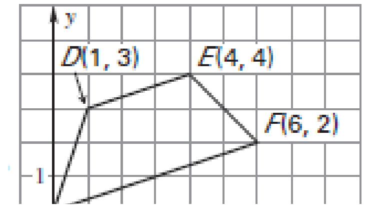
Compare the slopes of opposite sides.



Slope of $\overline{DE} = \dfrac{4-3}{4-1} = \dfrac{1}{3}$

Slope of $\overline{CF} = \dfrac{2-0}{6-0} = \dfrac{2}{6} = \dfrac{1}{3}$

The slopes of $\overline{DE}$ and $\overline{CF}$ are the same, so $\overline{DE}$ ∥ $\overline{CF}$.



Slope of $\overline{EF} = \dfrac{2-4}{6-4} = \dfrac{-2}{2} = -1$

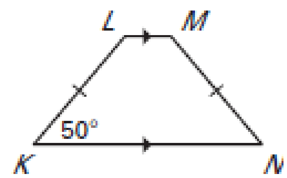
Slope of $\overline{CD} = \dfrac{3-0}{1-0} = \dfrac{3}{1} = 3$

The slopes of $\overline{EF}$ and $\overline{CD}$ are not the same, so $\overline{EF}$ is not parallel to $\overline{CD}$.

Because quadrilateral *CDEF* has exactly one pair of parallel sides, it is a trapezoid.

EXAMPLE 2 Use properties of isosceles trapezoids

Kitchen A shelf fitting into a cupboard in the corner of a kitchen is an isosceles trapezoid. Find m∠ N, m∠ L, and m∠ M.



Solution

Step 1 **Find $m\angle N$.** *KLMN* is an isosceles trapezoid, so ∠*N* and ∠*K* are congruent base angles, and $m\angle N = m\angle K = 50°$.

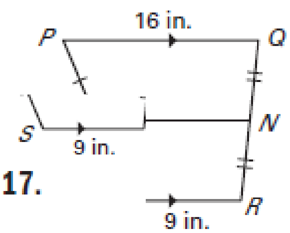
Step 2 **Find $m\angle L$.** Because ∠*K* and ∠*L* are consecutive interior angles formed by $\overleftrightarrow{KL}$ intersecting two parallel lines, they are supplementary. So, $m\angle L = 180° - 50° = 130°$.

Step 3 **Find $m\angle M$.** Because ∠*M* and ∠*L* are a pair of base angles, they are congruent, and $m\angle M = m\angle L = 130°$.

So, $m\angle N = 50°$, $m\angle L = 130°$, and $m\angle M = 130°$.

EXAMPLE 3 Use the midsegment of a trapezoid

In the diagram, $\overline{MN}$ is the midsegment of trapezoid PQRS. Find MN.



Solution

Use Theorem 8.17 to find *MN*.

$MN = \dfrac{1}{2}(PQ + SR)$ **Apply Theorem 8.17.**

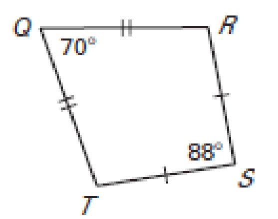
$= \dfrac{1}{2}(16 + 9)$ **Substitute 16 for *PQ* and 9 for *SR*.**

$= 12.5$ **Simplify.**

The length *MN* is 12.5 inches.

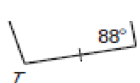
EXAMPLE 4 Apply kite→ one pair opp ∠s ≅

Find m∠ T in the kite shown at the right.



Solution

By Theorem 8.19, *QRST* has exactly one pair of congruent opposite angles. Because ∠*Q* ≇ ∠*S*, ∠*R* and ∠*T* must be congruent. So, $m\angle R = m\angle T$. Write and solve an equation to find $m\angle T$.



$m\angle T + m\angle R + 70° + 88° = 360°$ **Corollar Theorem 8.1**

$m\angle T + m\angle T + 70° + 88° = 360°$ **Substitute $m\angle T$ for $m\angle R$.**

$2(m\angle T) + 158° = 360°$ **Combine like terms.**

$m\angle T = 101°$ **Solve for $m\angle T$.**

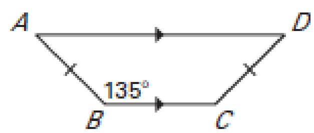
8.5 Cont.

Checkpoint **Complete the following exercises.**

1. In Example 1, suppose the coordinates of point E are (7, 5). What type of quadrilateral is $CDEF$? *Explain.*

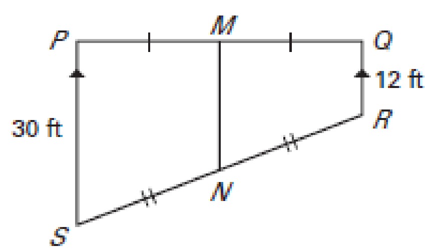
Parallelogram; opposite pairs of sides are parallel.

2. Find $m\angle C$, $m\angle A$, and $m\angle D$ in the trapezoid shown.



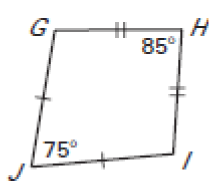
$m\angle C = 135°$, $m\angle A = 45°$, $m\angle D = 45°$

3. Find MN in the trapezoid at the right.



$MN = 21$ ft

4. Find $m\angle G$ in the kite shown at the right.



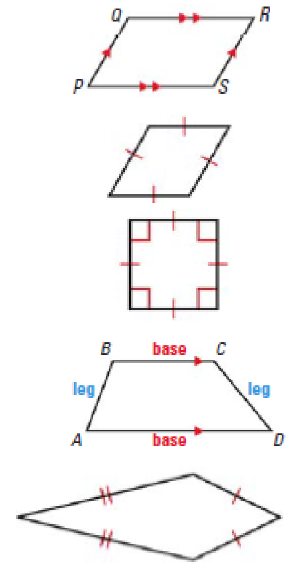
$m\angle G = 100°$

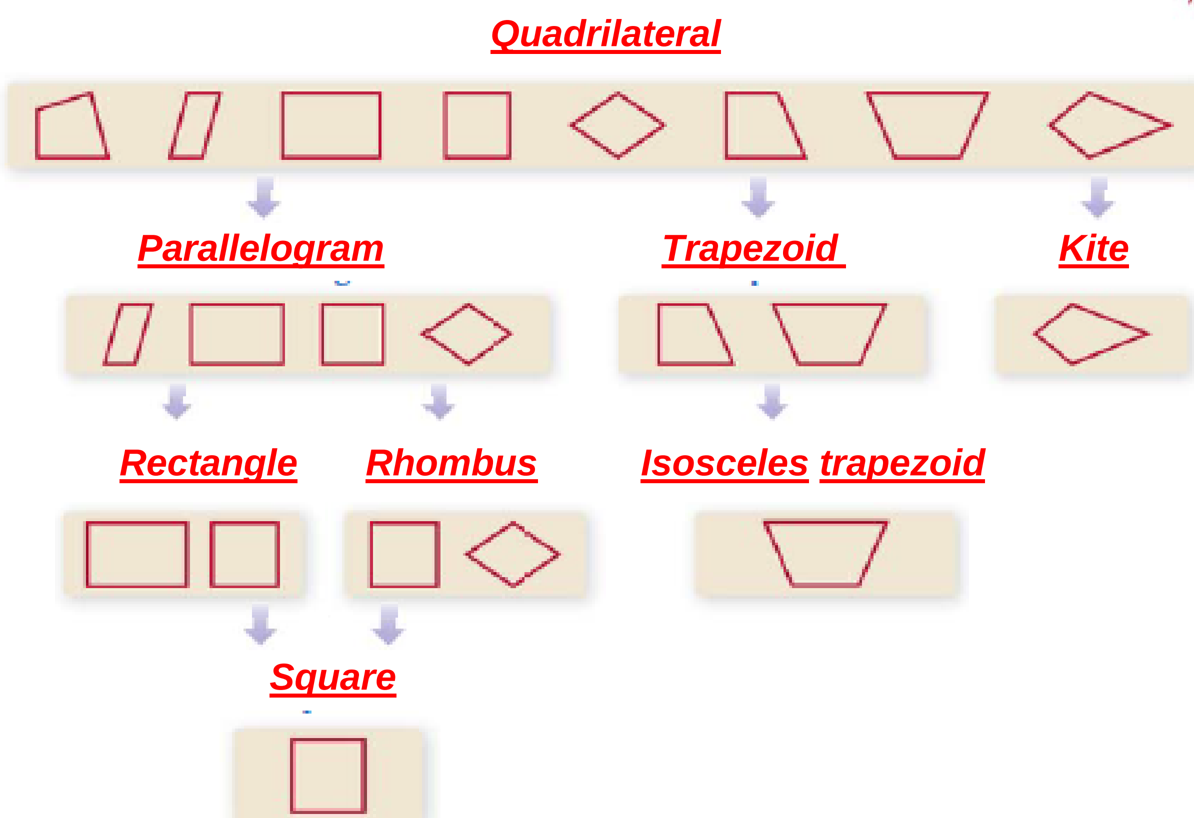
8.6 Identify Special Quadrilaterals

Obj.: Identify special quadrilaterals.

Key Vocabulary

- **Parallelogram -** A **parallelogram** is a quadrilateral with **both** pairs of **opposite** sides **parallel**.
- **Rhombus -** A rhombus is a parallelogram with **four congruent sides**.
- **Rectangle -** A rectangle is a **parallelogram** with four **right angles**.
- **Square -** A square is a parallelogram with four **congruent sides** and four **right angles**.
- **Trapezoid -** A **trapezoid** is a quadrilateral with **exactly one** pair of **parallel sides**.
- **Kite -** A **kite** is a **quadrilateral** that has two pairs of **consecutive congruent** sides, but opposite **sides** are **not** congruent.

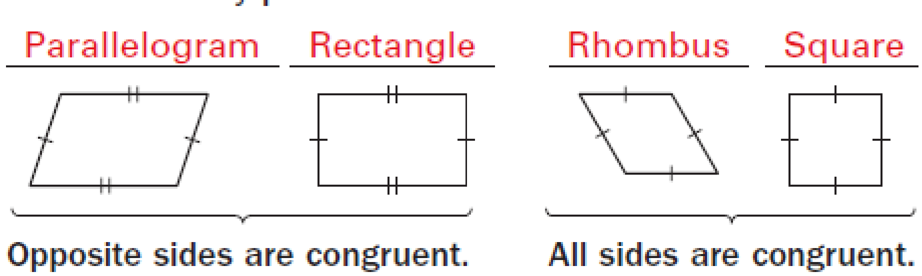




EXAMPLE 1 Identify quadrilaterals

Quadrilateral ABCD has both pairs of opposite sides congruent. What types of quadrilaterals meet this condition?

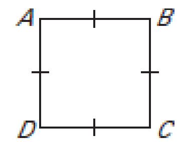
Solution There are many possibilities.



EXAMPLE 2 Identify a quadrilateral

What is the most specific name for quadrilateral ABCD?

Solution



In Example 2, *ABCD* is shaped like a square. But you must rely only on marked information when you interpret a diagram.

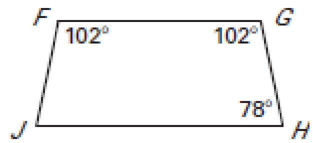
The diagram shows that both pairs of opposite sides are congruent. By Theorem 8.7, *ABCD* is a parallelogram. All sides are congruent, so *ABCD* is a rhombus by definition.

Squares are also rhombuses. However, there is no information given about the angle measures of *ABCD*. So, you cannot determine whether it is a square.

EXAMPLE 3 Identify a quadrilateral

Is enough information given in the diagram to show that quadrilateral FGHJ is an isosceles trapezoid? Explain.

Solution



Step 1 **Show that *FGHJ* is a trapezoid.** $\angle G$ and $\angle H$ are supplementary but $\angle F$ and $\angle G$ are not. So, $\overline{FG} \parallel \overline{HJ}$, but $\overline{FJ}$ is not parallel to $\overline{GH}$. By definition, *FGHJ* is a trapezoid.

Step 2 **Show that trapezoid *FGHJ* is isosceles.** $\angle F$ and $\angle G$ are a pair of congruent base angles. So, *FGHJ* is an isosceles trapezoid by Theorem 8.15.

Yes, the diagram is sufficient to show that *FGHJ* is an isosceles trapezoid.

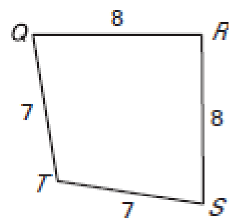
8.6 Cont.

Checkpoint **Complete the following exercise.**

1. Quadrilateral *JKLM* has both pairs of opposite angles congruent. What types of quadrilaterals meet this condition?

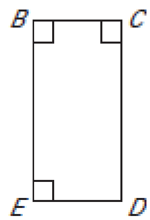
parallelogram, rectangle, square, rhombus

2. What is the most specific name for quadrilateral *QRST*? *Explain* your reasoning.



Kite; there are two pairs of consecutive congruent sides.

3. Is enough information given in the diagram to show that quadrilateral *BCDE* is a rectangle? *Explain.*



Yes; you know that $m\angle D = 90°$ by the Triangle Sum Theorem. Both pairs of opposite angles are congruent, so *BCDE* is a parallelogram by Theorem 8.8. By definition, *BCDE* is a rectangle.