

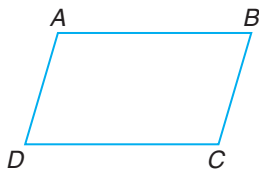
What You'll Learn

You'll learn to identify and use tests to show that a quadrilateral is a parallelogram.

Why It's Important

Crafts Quilters often use parallelograms when designing their quilts. See Exercise 17.



Theorem 8-3 states that the opposite sides of a parallelogram are congruent. Is the converse of this theorem true? In the figure below, \overline{AB} is congruent to \overline{DC} and \overline{AD} is congruent to \overline{BC} .



You know that a parallelogram is a quadrilateral in which both pairs of opposite sides are parallel. If the opposite sides of a quadrilateral are congruent, then is it a parallelogram?

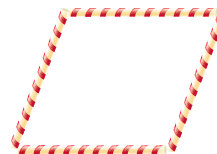
In the following activity, you will discover other ways to show that a quadrilateral is a parallelogram.

Hands-On Geometry

Materials:  straws  scissors  pipe cleaners
 ruler

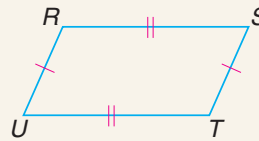
Step 1 Cut two straws to one length and two straws to a different length.

Step 2 Insert a pipe cleaner in one end of each straw. Connect the pipe cleaners at the ends to form a quadrilateral.

**Try These**

- How do the measures of opposite sides compare?
- Measure the distance between the top and bottom straws in at least three places. Then measure the distance between the left and right straws in at least three places. What seems to be true about the opposite sides?
- Shift the position of the sides to form another quadrilateral. Repeat Exercises 1 and 2.
- What type of quadrilateral have you formed? Explain your reasoning.

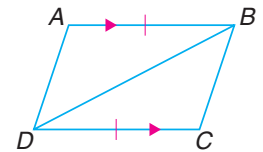
This activity leads to Theorem 8-7, which is related to Theorem 8-3.

Theorem 8-7**Words:** If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.**Model:****Symbols:** $\overline{RS} \cong \overline{UT}$,
 $\overline{RU} \cong \overline{ST}$

You can use the properties of congruent triangles and Theorem 8-7 to find other ways to show that a quadrilateral is a parallelogram.

Example**1**

In quadrilateral $ABCD$, with diagonal BD , $\overline{AB} \parallel \overline{CD}$, $\overline{AB} \cong \overline{CD}$. Show that $ABCD$ is a parallelogram.



Explore You know $\overline{AB} \parallel \overline{CD}$ and $\overline{AB} \cong \overline{CD}$. You want to show that $ABCD$ is a parallelogram.

Plan One way to show $ABCD$ is a parallelogram is to show $\overline{AD} \cong \overline{CB}$. You can do this by showing $\triangle ABD \cong \triangle CDB$. Make a list of statements and their reasons.

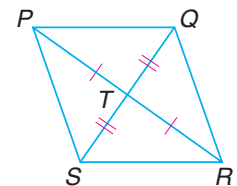
- Solve**
- $\angle ABD \cong \angle CDB$ *If two \parallel lines are cut by a transversal, then each pair of alternate interior angles is \cong .*
 - $\overline{BD} \cong \overline{BD}$ *Reflexive Property*
 - $\overline{AB} \cong \overline{CD}$ *Given*
 - $\triangle ABD \cong \triangle CDB$ *SAS*
 - $\overline{AD} \cong \overline{CB}$ *CPCTC*
 - $ABCD$ is a parallelogram. *Theorem 8-7*

Look Back

Alternate Interior Angles: Lesson 4-2

Your Turn

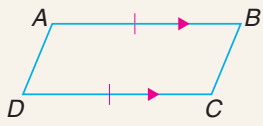
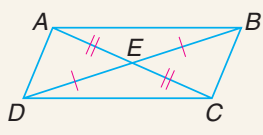
In quadrilateral $PQRS$, \overline{PR} and \overline{QS} bisect each other at T . Show that $PQRS$ is a parallelogram by providing a reason for each step.



- $\overline{PT} \cong \overline{TR}$ and $\overline{QT} \cong \overline{TS}$
- $\angle PTQ \cong \angle RTS$ and $\angle STP \cong \angle QTR$
- $\triangle PQT \cong \triangle RST$ and $\triangle PTS \cong \triangle RTQ$
- $\overline{PQ} \cong \overline{RS}$ and $\overline{PS} \cong \overline{RQ}$
- $PQRS$ is a parallelogram.

These examples lead to Theorems 8-8 and 8-9.

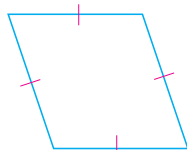


Theorem	Words	Models and Symbols
8-8	If one pair of opposite sides of a quadrilateral is parallel and congruent, then the quadrilateral is a parallelogram.	 $\overline{AB} \cong \overline{DC}, \overline{AB} \parallel \overline{DC}$
8-9	If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.	 $\overline{AE} \cong \overline{EC}, \overline{BE} \cong \overline{ED}$

Examples

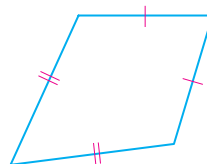
Determine whether each quadrilateral is a parallelogram. If the figure is a parallelogram, give a reason for your answer.

2



The figure has two pairs of opposite sides that are congruent. The figure is a parallelogram by Theorem 8-7.

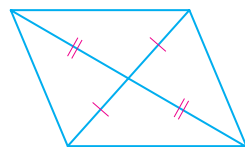
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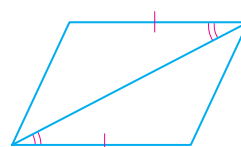
The figure has two pairs of congruent sides, but they are *not* opposite sides. The figure is *not* a parallelogram.

Your Turn

f.



g.



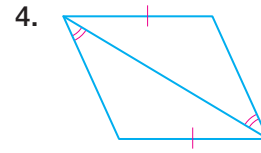
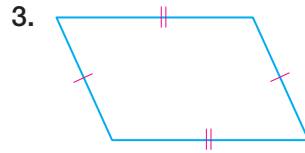
Check for Understanding

Communicating Mathematics

- Draw a quadrilateral that meets each set of conditions and is *not* a parallelogram.
 - one pair of parallel sides
 - one pair of congruent sides
 - one pair of congruent sides and one pair of parallel sides
- Writing Math** List four methods you can use to determine whether a quadrilateral is a parallelogram.

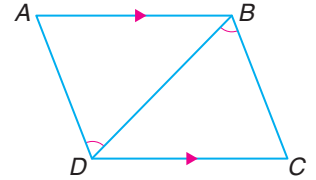
Guided Practice

Examples 2 & 3



Example 1

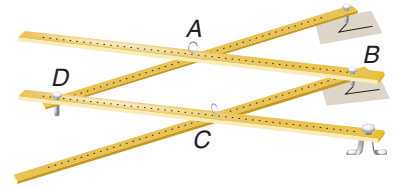
5. In quadrilateral $ABCD$, $\overline{BA} \parallel \overline{CD}$ and $\angle DBC \cong \angle BDA$. Show that quadrilateral $ABCD$ is a parallelogram by providing a reason for each step.



- $\overline{BC} \parallel \overline{AD}$
- $ABCD$ is a parallelogram.

Examples 2 & 3

6. In the figure, $\overline{AD} \cong \overline{BC}$ and $\overline{AB} \cong \overline{DC}$. Which theorem shows that quadrilateral $ABCD$ is a parallelogram?

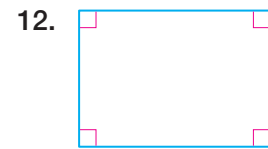
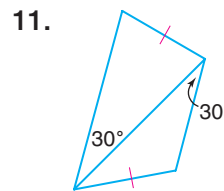
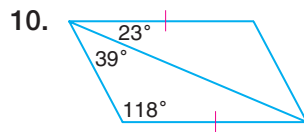
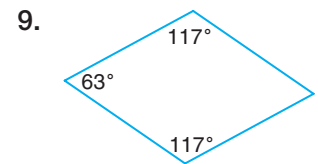
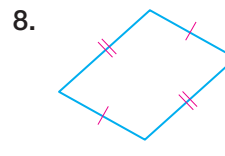
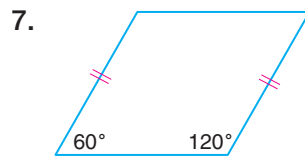


Exercises

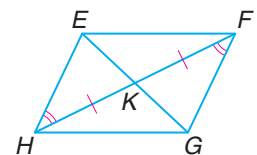
Practice

Determine whether each quadrilateral is a parallelogram. Write *yes* or *no*. If *yes*, give a reason for your answer.

Homework Help	
For Exercises	See Examples
7–12, 14–16	2, 3
13	1
17	2
Extra Practice	
See page 740.	

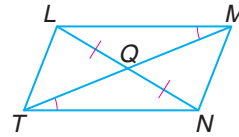


13. In quadrilateral $EFGH$, $\overline{HK} \cong \overline{KF}$ and $\angle KHE \cong \angle KFG$. Show that quadrilateral $EFGH$ is a parallelogram by providing a reason for each step.

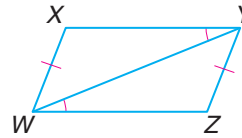


- $\angle EKH \cong \angle FKG$
- $\triangle EKH \cong \triangle FKG$
- $\overline{EH} \cong \overline{FG}$
- $\overline{EH} \parallel \overline{FG}$
- $EFGH$ is a parallelogram.

14. Explain why quadrilateral $LMNT$ is a parallelogram. Support your explanation with reasons as shown in Exercise 13.

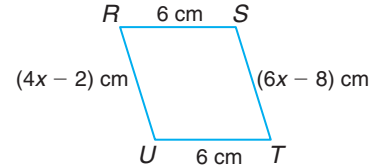


15. Determine whether quadrilateral $XYZW$ is a parallelogram. Give reasons for your answer.

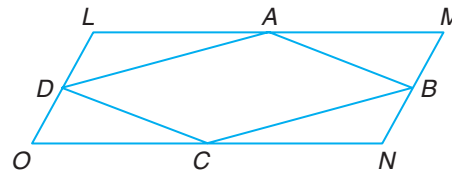


Applications and Problem Solving

16. **Algebra** Find the value for x that will make quadrilateral $RSTU$ a parallelogram.



17. **Quilting** Faith Ringgold is an African-American fabric artist. She used parallelograms in the design of the quilt at the left. What characteristics of parallelograms make it easy to use them in quilts?
18. **Critical Thinking** Quadrilateral $LMNO$ is a parallelogram. Points A , B , C , and D are midpoints of the sides. Is $ABCD$ a parallelogram? Explain your reasoning.



Faith Ringgold, #4 *The Sunflowers Quilting Bee at Arles*

Mixed Review

In $\square ABCD$, $m\angle D = 62$ and $CD = 45$. Find each measure. (*Lesson 8-2*)

19. $m\angle B$ 20. $m\angle C$ 21. AB

22. **Drawing** Use a straightedge and protractor to draw a quadrilateral with exactly two obtuse angles. (*Lesson 8-1*)

23. Find the length of the hypotenuse of a right triangle whose legs are 7 inches and 24 inches. (*Lesson 6-6*)

Standardized Test Practice

A B C D

24. **Grid In** In order to “curve” a set of test scores, a teacher uses the equation $g = 2.5p + 10$, where g is the curved test score and p is the number of problems answered correctly. How many points is each problem worth? (*Lesson 4-6*)

25. **Short Response** Name two different pairs of angles that, if congruent, can be used to prove $a \parallel b$. Explain your reasoning. (*Lesson 4-4*)

