8-4 Sine and Cosine Ratios

What You'll Learn

- To use sine and cosine to determine side lengths in triangles

. . . And Why

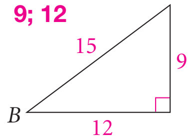
To use the sine ratio to estimate astronomical distances indirectly, as in Example 2

Check Skills You'll Need

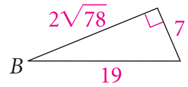
GO for Help Lesson 8-3

For each triangle, find (a) the length of the leg opposite ∠B and (b) the length of the leg adjacent to ∠B.

1. 9; 12

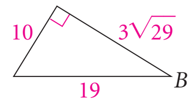


2.



7; $2\sqrt{78}$

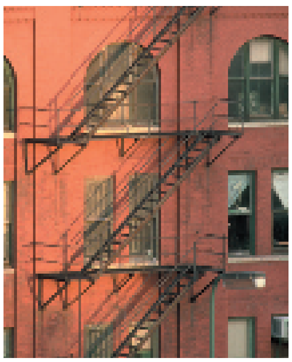
3.



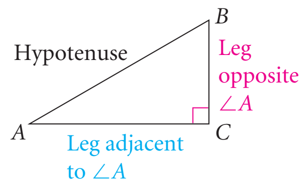
10; $3\sqrt{29}$

New Vocabulary • sine • cosine • identity

1 Using Sine and Cosine in Triangles



The tangent ratio, as you have seen, involves both legs of a right triangle. The sine and cosine ratios involve one leg and the hypotenuse.



$$\textbf{sine} \text{ of } \angle A = \frac{\text{leg opposite } \angle A}{\text{hypotenuse}}$$

$$\textbf{cosine} \text{ of } \angle A = \frac{\text{leg adjacent to } \angle A}{\text{hypotenuse}}$$

These equations can be abbreviated:

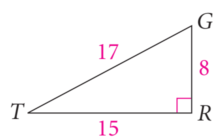
$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

Real-World Connection

For an angle of a given size, the sine and cosine ratios are constant, no matter where the angle is located.

1 EXAMPLE Writing Sine and Cosine Ratios

Use the triangle to write each ratio.



a. sin T $\quad \sin T = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{8}{17}$

b. cos T $\quad \cos T = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{15}{17}$

c. sin G $\quad \sin G = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{15}{17}$

d. cos G $\quad \cos G = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{8}{17}$

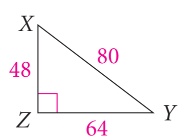
Quick Check

1 a. Write the sine and cosine ratios for ∠X and ∠Y. **See right.**

b. Critical Thinking When does sin X = cos Y? Explain.

sin X = cos Y when ∠X and ∠Y are complementary.

1a. $\sin X = \frac{64}{80}$; $\cos X = \frac{48}{80}$; $\sin Y = \frac{48}{80}$; $\cos Y = \frac{64}{80}$



8-4

1. Plan

Objectives

1 To use sine and cosine to determine side lengths in triangles

Examples

1 Writing Sine and Cosine Ratios
2 Real-World Connection
3 Using the Inverse of Cosine and Sine

Math Background

Professional Development

A *unit circle* has radius 1 and center (0,0) in the coordinate plane. For all real values of θ, the point that is reached by traveling θ radians from point (1,0) in a counterclockwise direction has coordinates ($\cos\theta$, $\sin\theta$).

More Math Background: p. 414D

Lesson Planning and Resources

See p. 414E for a list of the resources that support this lesson.

Bell Ringer Practice

PowerPoint

Check Skills You'll Need
For intervention, direct students to:

Writing Tangent Ratios
Lesson 8-3: Example 1
Extra Skills, Word Problems, Proof Practice, Ch. 8

Differentiated Instruction Solutions for All Learners

Special Needs L1
Present the mnemonic device SOHCAHTOA for the definition of the three trigonometric ratios: Sine is Opposite over Hypotenuse; Cosine is Adjacent over Hypotenuse; Tangent is Opposite over Adjacent.
learning style: verbal

Below Level L2
Have students draw and measure right triangles to make a table of sine and cosine values for the angles in the set {10°, 20°, ... , 80°}.
learning style: visual

2. Teach

Guided Instruction

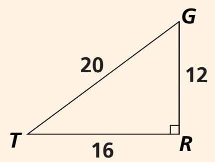
Visual Learners

Have students display a poster listing the trigonometric ratios.

PowerPoint

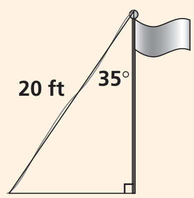
Additional Examples

1 Use the triangle to find sin T, cos T, sin G, and cos G.



$\sin T = \frac{12}{20}$, $\cos T = \frac{16}{20}$, $\sin G = \frac{16}{20}$, $\cos G = \frac{12}{20}$

2 A 20-ft wire supporting a flagpole forms a 35° angle with the flagpole. To the nearest foot, how high is the flagpole?



16 ft

3 A right triangle has a leg 1.5 units long and a hypotenuse 4.0 units long. Find the measures of its acute angles to the nearest degree. 22, 68

Resources

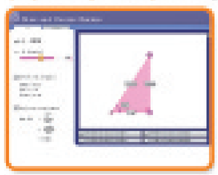
- Daily Notetaking Guide 8-4 L3
- Daily Notetaking Guide 8-4—Adapted Instruction L1

Closure

A right triangle whose hypotenuse is 18 cm long contains a 65° angle. Find the lengths of its legs to one decimal place. 16.3 cm, 7.6 cm

One way to describe the relationship of sine and cosine is to say that $\sin x° = \cos(90 - x)°$ for values of x between 0 and 90. This type of equation is called an **identity** because it is true for all the allowed values of the variable. You will discover other identities in the exercises.

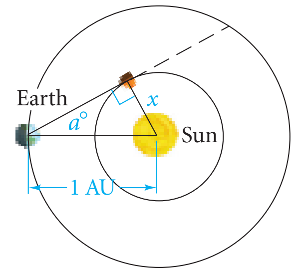
Online active math



For: Sine and Cosine Activity
Use: Interactive Textbook, 8-4

2 EXAMPLE Real-World Connection

Astronomy The trigonometric ratios have been known for centuries by peoples in many cultures. The Polish astronomer Nicolaus Copernicus (1473–1543) developed a method for determining the sizes of orbits of planets closer to the sun than Earth. The key to his method was determining when the planets were in the position shown in the diagram, and then measuring the angle to find a.



not to scale

If $a = 22.3$ for Mercury, how far is Mercury from the sun in astronomical units (AU)? One astronomical unit is defined as the average distance from Earth to the center of the sun, about 93 million miles.

$\sin 22.3° = \frac{x}{1}$ **Use the sine ratio.**

$x = \sin 22.3°$ **Solve for x.**

SIN 22.3 ENTER .37945616 **Use a calculator.**

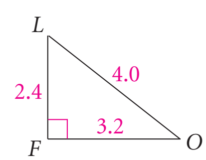
Mercury is about 0.38 AU from the sun.

Quick Check 2 **a.** If $a = 46$ for Venus, how far is Venus from the sun in AU? about 0.72 AU
b. About how many miles from the sun is Venus? Mercury? 66,960,000 mi; 35,340,000 mi

When you know the leg and hypotenuse lengths of a right triangle, you can use inverse of sine and inverse of cosine to find the measures of the acute angles.

3 EXAMPLE Using the Inverse of Cosine and Sine

Find $m\angle L$ to the nearest degree.



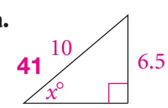
Problem Solving Hint

Think of $\cos^{-1}\left(\frac{2.4}{4.0}\right)$ as "the angle whose cosine is $\frac{2.4}{4.0}$," and $\sin^{-1}\left(\frac{3.2}{4.0}\right)$ as "the angle whose sine is the quotient $\frac{3.2}{4.0}$."

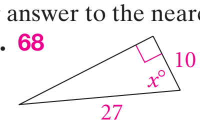
Method 1		Method 2
$\cos L = \frac{2.4}{4.0}$	← **Find the trigonometric ratio.** →	$\sin L = \frac{3.2}{4.0}$
$m\angle L = \cos^{-1}\left(\frac{2.4}{4.0}\right)$	← **Use the inverse.** →	$m\angle L = \sin^{-1}\left(\frac{3.2}{4.0}\right)$
$\cos^{-1}$ 2.4 ÷ 4.0 ENTER 53.130102	← **Use a calculator.** →	$\sin^{-1}$ 3.2 ÷ 4.0 ENTER 53.130102
$m\angle L \approx 53$		$m\angle L \approx 53$

Quick Check 3 Find the value of x. Round your answer to the nearest degree.

a. 41



b. 68



Differentiated Instruction Solutions for All Learners

Advanced Learners L4
Encourage students to make conjectures about the values of sin 0°, cos 0°, sin 90°, and cos 90°, defend their conjectures, and then check the values on a calculator.

learning style: verbal

English Language Learners ELL
Help students distinguish between *sine* and *inverse sine*. The sine of an angle is a ratio, or number. The inverse sine of a number, or ratio, is an angle measure. So, inverses are used to find angle measures.

learning style: verbal

EXERCISES

For more exercises, see *Extra Skill, Word Problem, and Proof Practice.*

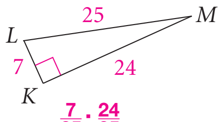
Practice and Problem Solving

A Practice by Example

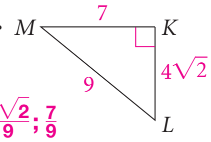
Example 1 (page 439)

GO for Help

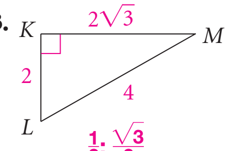
Write the ratios for sin M and cos M.

1. 

$\frac{7}{25}, \frac{24}{25}$

2. 

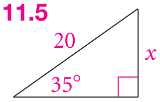
$\frac{4\sqrt{2}}{9}, \frac{7}{9}$

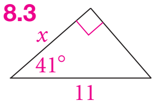
3. 

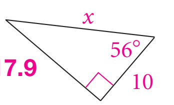
$\frac{1}{2}, \frac{\sqrt{3}}{2}$

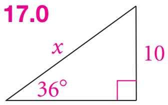
Example 2 (page 440)

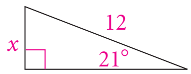
Find the value of x. Round answers to the nearest tenth.

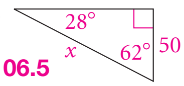
4. **11.5** 

5. **8.3** 

6. **17.9** 

7. **17.0** 

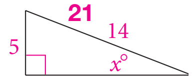
8. **4.3** 

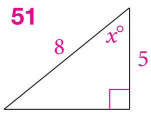
9. **106.5** 

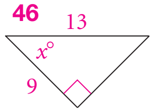
10. **Escalators** An escalator in the subway system of St. Petersburg, Russia, has a vertical rise of 195 ft 9.5 in., and rises at an angle of 10.4°. How long is the escalator? Round your answer to the nearest foot. **1085 ft**

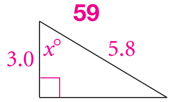
Example 3 (page 440)

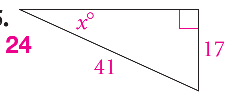
Find the value of x. Round answers to the nearest degree.

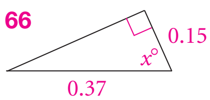
11. **21** 

12. **51** 

13. **46** 

14. **59** 

15. **24** 

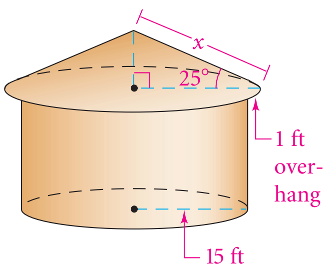
16. **66** 

B Apply Your Skills

Real-World Connection

Corn that fills the bin in Exercise 17 would make 28,500 gallons of ethanol.

17. **Construction** Carlos is planning to build a grain bin with a radius of 15 ft. He reads that the recommended slant of the roof is 25°. He wants the roof to overhang the edge of the bin by 1 ft. What should the length x be? Give your answer in feet and inches. **about 17 ft 8 in.**



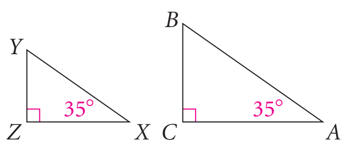
Use what you know about trigonometric ratios (and other identities) to show that each equation is an identity. **18–20. See margin.**

18. $\tan X = \frac{\sin X}{\cos X}$

19. $\sin X = \cos X \cdot \tan X$

20. $\cos X = \frac{\sin X}{\tan X}$

21. **Error Analysis** A student states that $\sin A > \sin X$ because the lengths of the sides of $\triangle ABC$ are greater than the lengths of the sides of $\triangle XYZ$. Is the student correct? Explain.
No; the △ are ~ and the sine ratio for 35° is constant.



18. $\sin X \div \cos X = \frac{\text{opp.}}{\text{hyp.}} \div \frac{\text{adj.}}{\text{hyp.}} = \frac{\text{opp.}}{\text{adj.}} = \tan X$

19. $\cos X \cdot \tan X = \frac{\text{adj.}}{\text{hyp.}} \cdot \frac{\text{opp.}}{\text{adj.}} = \frac{\text{opp.}}{\text{hyp.}} = \sin X$

20. $\sin X \div \tan X = \frac{\text{opp.}}{\text{hyp.}} \div \frac{\text{opp.}}{\text{adj.}} = \frac{\text{adj.}}{\text{hyp.}} = \cos X$

3. Practice

Assignment Guide

1 A B 1-30
C Challenge 31-36

Test Prep 37-40
Mixed Review 41-47

Homework Quick Check

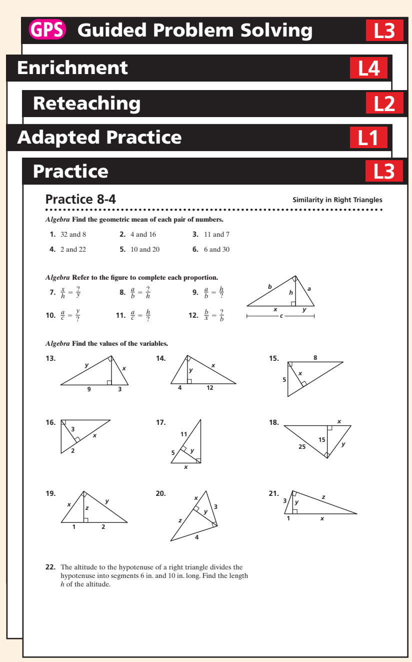
To check students' understanding of key skills and concepts, go over Exercises 2, 14, 17, 22, 27.

Error Prevention!

Exercises 6, 7 Some students may need help solving equations with the variable in the denominator. Review techniques such as cross-multiplication and taking the reciprocal of each side.

Exercise 25 Tell students that there is also a cotangent ratio. Ask: *What do you think is the cotangent ratio?* $\frac{\text{adjacent}}{\text{opposite}}$

Differentiated Instruction Resources

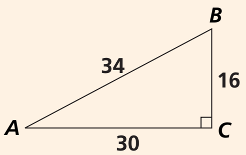


4. Assess & Reteach

PowerPoint

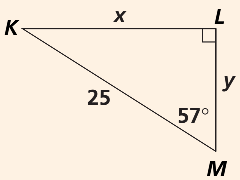
Lesson Quiz

Use this figure for Exercises 1 and 2.



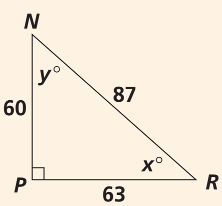
1. Write the ratios for sin A and sin B. **sin $A = \frac{16}{34}$, sin $B = \frac{30}{34}$**
2. Write the ratios for cos A and cos B. **cos $A = \frac{30}{34}$, cos $B = \frac{16}{34}$**

Use this figure for Exercises 3 and 4.



3. Find x to the nearest tenth. **21.0**
4. Find y to the nearest tenth. **13.6**

Use this figure for Exercises 5 and 6.



5. Find x to the nearest degree. **44**
6. Find y to the nearest degree. **46**

Alternative Assessment

Have students write two measurement problems involving distances in your school. Students also should show how to solve one problem using the sine ratio and the other problem using the cosine ratio.

GO Online
Homework Help
Visit: PHSchool.com
Web Code: aue-0804

25a. They are equal; yes; The sine and cosine of complementary ∠s are =.

25c. Sample: cosine of ∠A = sine of the compl. of ∠A.

27. Yes; use any trig. function and the known measures to find one other side. Use the Pythagorean Thm. to find the 3rd side. Subtract the acute ∠ measure from 90 to get the other ∠ measure.

28e. cos 30° = $\sqrt{3}$ sin 30°

28f. sin 60° = $\sqrt{3}$ cos 60°

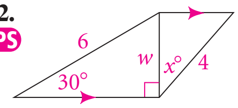
30b–d. Answers may vary. Samples are given.

30c. sin X = 1 for X = 89.9; no

Go Online
PHSchool.com
For: Graphing calculator procedures
Web Code: aue-2111

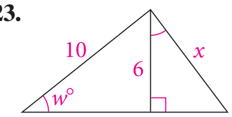
Find the values of w and then x. Round lengths to the nearest tenth and angle measures to the nearest degree.

22. GPS



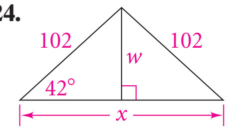
$w = 3$; $x = 41.4$

23.



$w = 37$; $x = 7.5$

24.

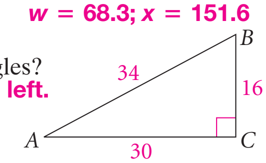


$w = 68.3$; $x = 151.6$

25. a. In △ABC, how does sin A compare to cos B? Is this true for the acute angles of other right triangles? **See left.**

b. Reading Math The word cosine is derived from the words *complement's sine* (see page 694). Which angle in △ABC is the complement of ∠A? Of ∠B? **∠B; ∠A**

c. Explain why the derivation of the word cosine makes sense. **See left.**

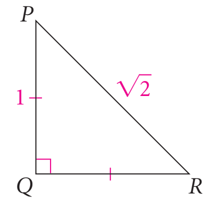


26. Find each ratio.

a. sin P **b.** cos P **c.** sin R **d.** cos R

a. $\frac{\sqrt{2}}{2}$ c. $\frac{\sqrt{2}}{2}$ b. $\frac{\sqrt{2}}{2}$ d. $\frac{\sqrt{2}}{2}$

e. Make a conjecture about how the sine and cosine of a 45° angle are related. **They are equal.**



27. Writing Leona said that if she had a diagram that showed the measure of one acute angle and the length of one side of a right triangle, she could find the measure of the other acute angle and the lengths of the other sides. Is she correct? Explain. **See left.**

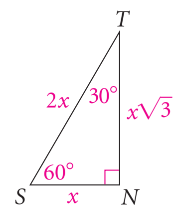
28. Find each ratio.

a. sin S **b.** cos S **c.** sin T **d.** cos T

a. $\frac{\sqrt{3}}{2}$ c. $\frac{1}{2}$ b. $\frac{1}{2}$ d. $\frac{\sqrt{3}}{2}$

e. Make a conjecture about how the sine and cosine of a 30° angle are related. **See left.**

f. Make a conjecture about how the sine and cosine of a 60° angle are related. **See left.**



Proof **29.** For right △ABC with right ∠C, prove each of the following.

a. sin $A < 1$, no matter how large ∠A is. **a–b. See margin.**

b. cos $A < 1$, no matter how small ∠A is.

30. Graphing Calculator Use the TABLE feature of your graphing calculator to study sin X as X gets close (but ≠) to 90. In the Y= screen, enter Y1 = sin X.

a. Use the TBLSET feature so that X starts at 80 and changes by 1. Access the TABLE. From the table, what is sin X for X = 89? **0.99985**

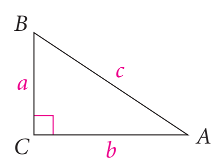
b. Perform a "numerical zoom in." Use the TBLSET feature, so that X starts with 89 and changes by 0.1. What is sin X for X = 89.9? **1**

c. Continue to numerically zoom in on values close to 90. What is the greatest value you can get for sin X on your calculator? How close is X to 90? Does your result contradict what you are asked to prove in Exercise 29a? **See left.**

d. Writing Use right triangles to explain the behavior of sin X found above. **See margin.**

Challenge

Show that each equation is an identity by showing that each expression on the left simplifies to 1. 31–34. See margin.



31. $(\sin A)^2 + (\cos A)^2 = 1$

32. $(\sin B)^2 + (\cos B)^2 = 1$

33. $\frac{1}{(\cos A)^2} - (\tan A)^2 = 1$

34. $\frac{1}{(\sin A)^2} - \frac{1}{(\tan A)^2} = 1$

35. Show that $(\tan A)^2 - (\sin A)^2 = (\tan A)^2 (\sin A)^2$ is an identity. **See margin.**

29. Answers may vary. Samples are given.

a. Since sin $A = \frac{\text{opp.}}{\text{hyp.}}$, if sin $A \geq 1$, then opp. ≥ hyp., which is impossible.

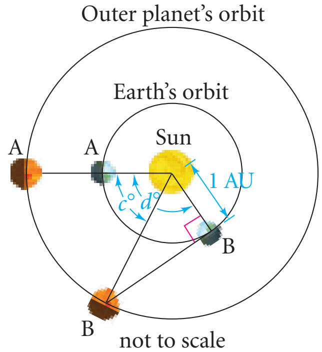
b. Since cos $A = \frac{\text{adj.}}{\text{hyp.}}$, if cos $A \geq 1$, then adj. ≥ hyp., which is impossible.

30. d. For ∠s that approach 90, the opp. side gets close to the hyp. in length, so $\frac{\text{opp.}}{\text{hyp.}}$ approaches 1.

Real-World Connection

Poland honored Copernicus with this 1000-zloty note, last used in 1995.

36. Astronomy Copernicus devised a method different from the one in Example 2 in order to find the sizes of the orbits of planets farther from the sun than Earth. His method involved noting the number of days between the times that a planet was in the positions labeled A and B in the diagram. Using this time and the number of days in each planet's year, he calculated c and d.



a. For Mars, $c = 55.2$ and $d = 103.8$. How far is Mars from the sun in astronomical units (AU)? **about 1.5 AU**

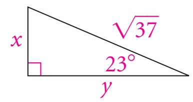
b. For Jupiter, $c = 21.9$ and $d = 100.8$. How far is Jupiter from the sun in astronomical units? **about 5.2 AU**

Test Prep

Multiple Choice

37. What is the value of x to the nearest whole number? **A**

A. 2 B. 3
C. 4 D. 6

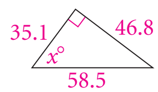


38. What is the value of y to the nearest tenth? **H**

F. 5.4 G. 5.5
H. 5.6 J. 5.7

39. What is the value of x to the nearest whole number? **A**

A. 53 B. 47
C. 43 D. 37

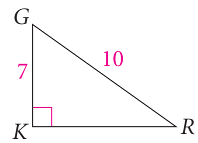


Short Response

40. Use the figure at the right.

a. Find $m\angle G$. Show your work. **a–b. See margin.**

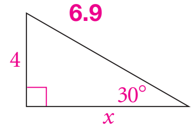
b. Find $m\angle R$ by two different methods. Show your work.



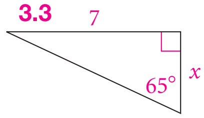
Mixed Review

Lesson 8-3 **Find the value of x. Round answers to the nearest tenth.**

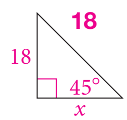
41. **6.9**



42. **3.3**



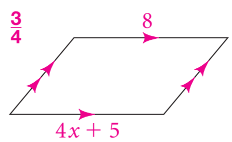
43. **18**



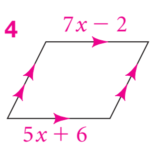
Lesson 7-2 **44.** The wall of a room is in the shape of a golden rectangle. If the height of the wall is 8 ft, what are the possible lengths of the wall to the nearest tenth? **12.9 ft or 4.9 ft**

Lesson 6-2 **Find the value of x for each parallelogram.**

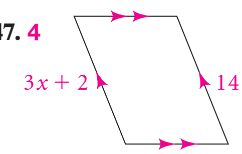
45. $\frac{3}{4}$



46. **4**



47. **4**



31. $(\sin A)^2 + (\cos A)^2 = \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = \frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{a^2 + b^2}{c^2} = \frac{c^2}{c^2} = 1$

Test Prep

Resources

For additional practice with a variety of test item formats:

- Standardized Test Prep, p. 465
- Test-Taking Strategies, p. 460
- Test-Taking Strategies with Transparencies

Exercise 36 Point out that Copernicus's method depends on the sun, Earth, and outer planets' being in a line at one point in time and forming a right angle at the other point in time.

32. $(\sin B)^2 + (\cos B)^2 = \left(\frac{b}{c}\right)^2 + \left(\frac{a}{c}\right)^2 = \frac{b^2}{c^2} + \frac{a^2}{c^2} = \frac{b^2 + a^2}{c^2} = \frac{c^2}{c^2} = 1$

33. $\frac{1}{(\cos A)^2} - (\tan A)^2$
$= \left(1 \div \frac{b^2}{c^2}\right) - \frac{a^2}{b^2}$
$= \frac{c^2}{b^2} - \frac{a^2}{b^2} =$
$\frac{c^2 - a^2}{b^2} = \frac{b^2}{b^2} = 1$

34. $\frac{1}{(\sin A)^2} - \frac{1}{(\tan A)^2} = \frac{1}{\left(\frac{a}{c}\right)^2} - \frac{1}{\left(\frac{a}{b}\right)^2} = \frac{c^2}{a^2} - \frac{b^2}{a^2} = \frac{c^2 - b^2}{a^2} = \frac{a^2}{a^2} = 1$

35. $(\tan A)^2 - (\sin A)^2 = \left(\frac{a}{b}\right)^2 - \left(\frac{a}{c}\right)^2 = \frac{a^2}{b^2} - \frac{a^2}{c^2} = \frac{a^2c^2}{b^2c^2} - \frac{a^2b^2}{b^2c^2} = \frac{a^2c^2 - a^2b^2}{b^2c^2} = \frac{a^2(c^2 - b^2)}{b^2c^2} = \frac{a^2 \cdot a^2}{b^2c^2} = \left(\frac{a}{b}\right)^2\left(\frac{a}{c}\right)^2 = (\tan A)^2(\sin A)^2$

40. [2] a. $\cos G = \frac{7}{10}$
$m\angle G = \cos^{-1}\left(\frac{7}{10}\right) \approx 46$

b. $m\angle R \approx 90 - 46 = 44$ OR $m\angle R = \sin^{-1}\left(\frac{7}{10}\right) \approx 44$

[1] one angle found correctly