

Exponential Functions

1. Plan

Objectives

- To evaluate exponential functions
- To graph exponential functions

Examples

- Evaluating an Exponential Function
- Real-World Problem Solving
- Graphs of Exponential Functions
- Real-World Problem Solving



Math Background

Exponential functions can model many naturally occurring phenomena, such as the growth of a colony of bacteria and the decay of radioactive polonium.

More Math Background: p. 428D

Lesson Planning and Resources

See p. 428E for a list of the resources that support this lesson.

Bell Ringer Practice



Check Skills You'll Need

For intervention, direct students to:

Slope-Intercept Form

Lesson 6-2: Example 4

Extra Skills and Word

Problem Practice, Ch. 6

Zero and Negative Exponents

Lesson 8-1: Example 1

Extra Skills and Word

Problem Practice, Ch. 8

What You'll Learn

- To evaluate exponential functions
- To graph exponential functions

... And Why

To use an exponential model for a population of rabbits, as in Example 2



Check Skills You'll Need

Graph each function. 1-3. See back of book.

1. $y = 3x$

2. $y = 4x$

3. $y = -2x$

Simplify each expression.

4. 3^2 **9**

5. 5^{-3} $\frac{1}{125}$

6. $2 \cdot 3^4$ **162**

7. $2 \cdot 3^{-2}$ $\frac{2}{9}$

8. $3 \cdot 2^{-1}$ $\frac{3}{2}$

9. $10 \cdot 3^2$ **90**



New Vocabulary • exponential function



GO for Help Lessons 6-2 and 8-1

1

Evaluating Exponential Functions

The rules you wrote in Lesson 8-6 to describe geometric sequences, such as $A(n) = 3 \cdot 4^n - 1$, are examples of exponential functions.



Key Concepts

Definition

Exponential Function

An **exponential function** is a function in the form $y = a \cdot b^x$, where a is a nonzero constant, b is greater than 0 and not equal to 1, and x is a real number.

Examples $y = 0.5 \cdot 2^x$ $f(x) = -2 \cdot 0.5^x$

You can evaluate an exponential function for given values of the domain to find the corresponding values of the range.

1

EXAMPLE Evaluating an Exponential Function

Evaluate each exponential function.

a. $y = 5^x$ for $x = 2, 3, 4$

b. $t(n) = 4 \cdot 3^n$ for the domain $\{-3, 6\}$

| x | 5^x | y |
|-----|-------------|-----|
| 2 | $5^2 = 25$ | 25 |
| 3 | $5^3 = 125$ | 125 |
| 4 | $5^4 = 625$ | 625 |

| n | $4 \cdot 3^n$ | $t(n)$ |
|-----|--|----------------|
| -3 | $4 \cdot 3^{-3} = 4 \cdot \frac{1}{27} = \frac{4}{27}$ | $\frac{4}{27}$ |
| 6 | $4 \cdot 3^6 = 4 \cdot 729 = 2916$ | 2916 |



For: Exponential Functions Activity
Use: Interactive Textbook, 8-7



Quick Check

1

Evaluate each exponential function for the domain $\{-2, 0, 3\}$.

a. $y = 4^x$
 $\frac{1}{16}, 1, 64$

b. $f(x) = 10 \cdot 5^x$
 $\frac{2}{5}, 10, 1250$

c. $g(x) = -2 \cdot 3^x$
 $-\frac{2}{9}, -2, -54$

468 Chapter 8 Exponents and Exponential Functions

Differentiated Instruction Solutions for All Learners

Special Needs **L1**

Have students search for examples of exponential functions in, for example, newspapers and magazines and science texts. Ask volunteers to present and describe their findings, including graphs and functions.

learning style: verbal

Below Level **L2**

Ask students what $f(x)$ and x represent in the function $f(x) = 1.5^x$ in Example 4. Help them to see what effect the value of the base, 1.5, has on the graph of the function.

learning style: visual



Real-World Connection

Rabbits were brought to Australia in 1860. Their numbers increased exponentially.

You can evaluate exponential functions to solve real-world problems.

2 EXAMPLE Real-World Problem Solving

Gridded Response Suppose 20 rabbits are taken to an island. The rabbit population then triples every half year. The function $f(x) = 20 \cdot 3^x$, where x is the number of half-year periods, models this situation. How many rabbits would there be after 2 years?

$$\begin{aligned} f(x) &= 20 \cdot 3^x \\ &= 20 \cdot 3^4 && \text{In 2 years, there are 4 half years. Evaluate the function for } x = 4. \\ &= 20 \cdot 81 && \text{Simplify powers.} \\ &= 1620 && \text{Simplify.} \end{aligned}$$

| | | | |
|---|---|---|---|
| 1 | 6 | 2 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 |
| 2 | 2 | 2 | 2 |
| 3 | 3 | 3 | 3 |
| 4 | 4 | 4 | 4 |
| 5 | 5 | 5 | 5 |
| 6 | 6 | 6 | 6 |
| 7 | 7 | 7 | 7 |
| 8 | 8 | 8 | 8 |
| 9 | 9 | 9 | 9 |

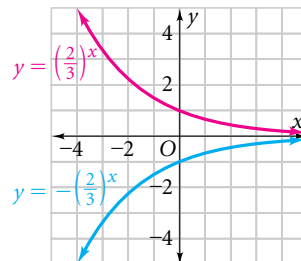
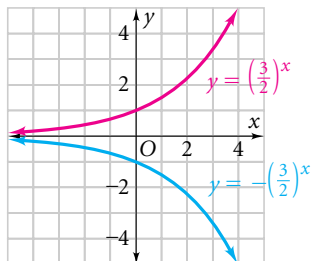
- After two years, there would be 1620 rabbits.

Quick Check

- 2 Suppose 10 animals are taken to an island, and then the population of these animals quadruples every year. Use the function $f(x) = 10 \cdot 4^x$. How many animals would there be after 6 years? **40,960 animals**

2 Graphing Exponential Functions

Here are two graphs that show what exponential functions generally look like.

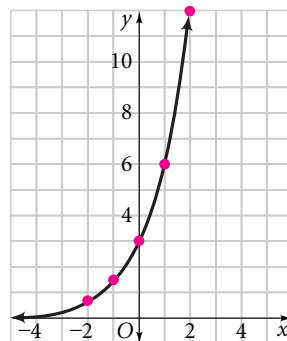


To graph an exponential function, make a table of values. Plot the points. Then join the points to form a smooth curve.

3 EXAMPLE Graphs of Exponential Functions

Graph $y = 3 \cdot 2^x$.

| x | $3 \cdot 2^x$ | (x, y) |
|-----|---|----------------------|
| -2 | $3 \cdot 2^{-2} = \frac{3}{2^2} = \frac{3}{4}$ | $(-2, \frac{3}{4})$ |
| -1 | $3 \cdot 2^{-1} = \frac{3}{2^1} = 1\frac{1}{2}$ | $(-1, 1\frac{1}{2})$ |
| 0 | $3 \cdot 2^0 = 3 \cdot 1 = 3$ | $(0, 3)$ |
| 1 | $3 \cdot 2^1 = 3 \cdot 2 = 6$ | $(1, 6)$ |
| 2 | $3 \cdot 2^2 = 3 \cdot 4 = 12$ | $(2, 12)$ |



Test-Taking Tip

Organize your work. Use a column to show your work when evaluating a function.

Quick Check

- 3 Graph each exponential function. a. $y = 0.5 \cdot 2^x$ b. $y = -0.5 \cdot 2^x$
a-b. See margin p. 470.

Lesson 8-7 Exponential Functions 469

2. Teach

Guided Instruction



Additional Examples

- Evaluate each exponential function.
 - $y = 3^x$ for $x = 2, 3, 4$ **9, 27, 81**
 - $p(q) = 3 \cdot 4^q$ for the domain $\{-2, 3\}$ **$\frac{3}{16}, 192$**
- Suppose two mice live in a barn. If the number of mice quadruples every 3 months, how many mice will be in the barn after 2 years? **131,072**



Additional Examples

- Graph $y = 2 \cdot 3^x$. See back of book.
- The function $f(x) = 1.25^x$ models the increase in size of an image being copied over and over at 125% on a photocopier. Graph the function. See back of book.

Resources

- Daily Notetaking Guide 8-7 **L3**
- Daily Notetaking Guide 8-7—Adapted Instruction **L1**

Closure

Ask students to explain why $y = x^4$ is not an exponential function. **The exponent, not the base, must be a variable.** Have students explain how the graphs of $y = x^4$ and $y = 4^x$ differ. **The graph of $y = x^4$ is a u-shape in which the y-values increase on both sides of the y-axis as you move away from the y-axis. The graph of $y = 4^x$ is a smooth curve in which the y-values increase quickly as you move to the right side of the y-axis and decrease slowly as you move to the left of the y-axis.**

Advanced Learners **L4**

Lead students in a discussion of the graph of $y = a^x$, when a is greater than or equal to 1 and when a is between 0 and 1.

learning style: visual

English Language Learners **ELL**

Be sure students understand the term *exponential function*. Ask them to write a function that describes y as a function of x , and then a function for y as an exponential function of x . Discuss how the functions differ.

learning style: verbal

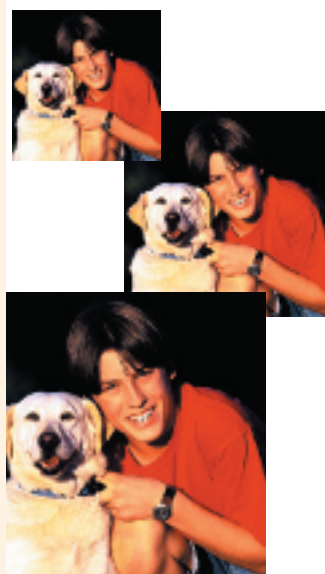
3. Practice

Assignment Guide

- 1 A B** 1-11, 25-34, 36-42
2 A B 12-24, 35, 43
C Challenge 44-50
 Test Prep 51-55
 Mixed Review 56-65

Homework Quick Check
 To check students' understanding of key skills and concepts, go over Exercises 6, 24, 34, 38, 42.

Careers
Exercises 9–11 A financial planner helps people plan how to manage and invest their money to pay for present and future needs. Investments may be used to earn money for a business venture, to buy property, to save for retirement, or to pay for any large purchase.

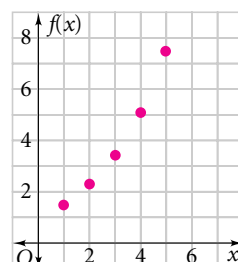


You can graph exponential functions to model real-world situations.

4 EXAMPLE Real-World Problem Solving

Photocopying Many copiers allow you to choose how large you want an image to be. The function $f(x) = 1.5^x$ models the new size of an image being copied over and over at 150%, where x is the number of enlargements. Graph the function.

| x | 1.5^x | $(x, f(x))$ |
|-----|-------------------------------|-------------|
| 1 | $1.5^1 = 1.5$ | (1, 1.5) |
| 2 | $1.5^2 = 2.25 \approx 2.3$ | (2, 2.3) |
| 3 | $1.5^3 = 3.375 \approx 3.4$ | (3, 3.4) |
| 4 | $1.5^4 = 5.0625 \approx 5.1$ | (4, 5.1) |
| 5 | $1.5^5 = 7.59375 \approx 7.6$ | (5, 7.6) |



Quick Check

- 4 a.** You can also make images that are smaller than the original on a copier. The function $f(x) = 0.9^x$ models the new size of an image being copied over and over at 90%. Graph the function. **See margin.**
b. Critical Thinking Explain why the function in Example 4 models discrete data. **It is not possible to have a fractional number of enlargements.**

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice*.

Practice and Problem Solving

A Practice by Example

Example 1
(page 468)



Example 2
(page 469)

Example 3
(page 469)

Evaluate each function rule for the given value.

- $f(x) = 6^x$ for $x = 3$ **216**
- $g(t) = 2 \cdot 3^t$ for $t = -2$ **$\frac{2}{9}$**
- $y = 20 \cdot (0.5)^x$ for $x = 3$ **2.5**
- $h(w) = 0.5 \cdot 4^w$ for $w = 3$ **32**
- $y = 50 \cdot (0.3)^x$ for $x = 2$ **4.5**
- $f(x) = 1.8 \cdot 2^x$ for $x = 6$ **115.2**
- $y = 100 \cdot \left(\frac{1}{2}\right)^x$ for $x = -4$ **1600**
- $y = 9 \cdot \left(\frac{5}{2}\right)^x$ for $x = -3$ **0.576**
- Finance** Suppose an investment of \$10,000 doubles in value every 13 years. How much is the investment worth after 52 years? After 65 years? **See left.**
- Finance** Suppose an investment of \$500 doubles in value every 15 years. How much is the investment worth after 30 years? After 45 years? **See left.**
- Finance** Suppose an investment of \$2000 doubles in value every 8 years. How much is the investment worth after 24 years? After 32 years? **\$16,000, \$32,000**

Match each table with the function that models the data.

12. $y = 3x$ **A**

A.

| x | y |
|-----|-----|
| 1 | 3 |
| 2 | 6 |
| 3 | 9 |
| 4 | 12 |

13. $y = x^3$ **C**

B.

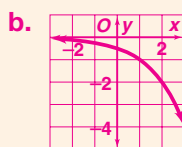
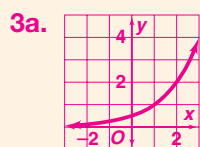
| x | y |
|-----|-----|
| 1 | 3 |
| 2 | 9 |
| 3 | 27 |
| 4 | 81 |

14. $y = 3^x$ **B**

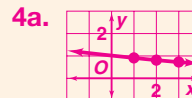
C.

| x | y |
|-----|-----|
| 1 | 1 |
| 2 | 8 |
| 3 | 27 |
| 4 | 64 |

page 469 Quick Check



Quick Check



Differentiated Instruction Resources

GPS Guided Problem Solving **L3**

Enrichment **L4**

Reteaching **L2**

Adapted Practice **L1**

Practice **L3**

Practice 8-7 Exponential Functions

Complete the table for each exercise.

| Time | Value of Investment | Time | Number of Animals | Time | Amount of Matter |
|---------|---------------------|---------|-------------------|---------|------------------|
| Initial | \$800 | Initial | 18 | Initial | 1200 g |
| 5 yr | \$1200 | 3 mo | 36 | 1 yr | 1600 g |
| 10 yr | \$1600 | 6 mo | 72 | 2 yr | 2400 g |
| 15 yr | \$2700 | 9 mo | 108 | 3 yr | 3600 g |
| 20 yr | — | 12 mo | — | 4 yr | — |
| 25 yr | — | — | — | — | — |
| — | — | — | — | — | — |
| — | — | — | — | — | — |

Evaluate each function for the domain $\{-2, 0, 1, 2, 4\}$.

| | | |
|--------------------------------------|---|---------------------------------|
| 4. $y = 2^x$ | 5. $y = 31^x$ | 6. $y = 0.8^x$ |
| 7. $y = 2^{-4^x}$ | 8. $y = 10^{-2^x}$ | 9. $y = 25^{-5^x}$ |
| 10. $y = \left(\frac{1}{9}\right)^x$ | 11. $y = 100 \cdot \left(\frac{1}{10}\right)^x$ | 12. $y = \frac{1}{4} \cdot 4^x$ |

Graph each function.

| | | |
|---------------------------------|---------------------------------|--|
| 13. $y = 3^x$ | 14. $y = 4^x$ | 15. $y = 1.5^x$ |
| 16. $y = 2^x$ | 17. $y = 10^{-5^x}$ | 18. $y = 16 \cdot 0.5^x$ |
| 19. $y = \frac{1}{2} \cdot 2^x$ | 20. $y = \frac{1}{4} \cdot 4^x$ | 21. $y = 8 \cdot \left(\frac{1}{8}\right)^x$ |

Evaluate each function rule for the given values.

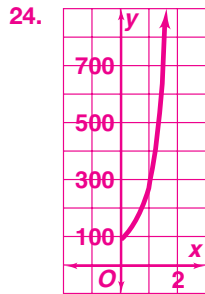
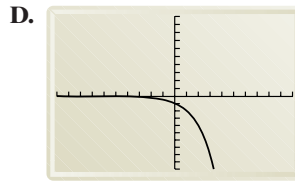
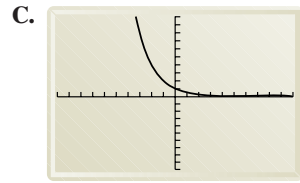
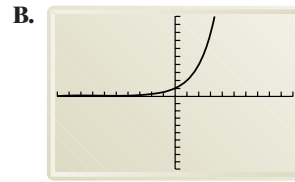
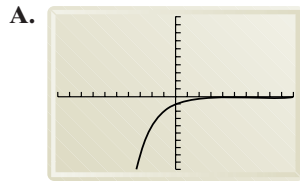
| | |
|---|---|
| 22. $y = 5.5^x$ for $x = 1, 3,$ and 4 | 23. $y = 4 \cdot 1.5^x$ for $x = 2, 4,$ and 5 |
| 24. $y = 3 \cdot 4^x$ for $x = 1, 3,$ and 5 | 25. $y = 6^x$ for $x = 2, 3,$ and 4 |
| 26. $y = 0.7^x$ for $x = 1, 3,$ and 4 | 27. $y = 3.5^x$ for $x = 1, 2,$ and 3 |
| 28. $y = 180 \cdot 0.5^x$ for $x = 0, -2,$ and $-\frac{1}{2}$ | 29. $y = 4.5^x$ for $x = -2, -1,$ and 0 |
| 30. $y = 100 \cdot 0.1^x$ for $x = -4, -1,$ and 2 | 31. $y = 5^x$ for $x = -2, -3,$ and 4 |

Solve each equation.

| | |
|--------------------------|----------------------------------|
| 32. $5^x = 625$ | 33. $2 \cdot 4^x = 128$ |
| 34. $4^x = \frac{1}{16}$ | 35. $4 \cdot 9^x = \frac{1}{81}$ |

Match each function rule with the graph of the function.

15. $y = 2^x$ **B** 16. $y = -(2^x)$ **D** 17. $y = (\frac{1}{2})^x$ **C** 18. $y = -(\frac{1}{2})^x$ **A**



Example 4
(page 470)

B Apply Your Skills

25. 0.04, 0.2, 1, 5, 25, 125; increase
 26. 0.16, 0.4, 1, 2.5, 6.25, 15.625; increase
 27. 100, 10, 1, 0.1, 0.01, 0.001; decrease
 28. 0.3125, 1.25, 5, 20, 80, 320; increase
 29. 4, 2, 1, 0.5, 0.25, 0.125; decrease
 30. $\frac{9}{4}, \frac{3}{2}, 1, \frac{2}{3}, \frac{4}{9}, \frac{8}{27}$; decrease
 31. 0.04, 0.4, 4, 40, 400, 4000; increase
 32. $1111.\bar{1}, 333.\bar{3}, 100, 30, 9, 2.7$; decrease

Graph each function. 19–22. See margin.

19. $y = 10 \cdot 2^x$ 20. $y = 0.1 \cdot 2^x$ 21. $y = \frac{1}{4} \cdot 2^x$ 22. $y = 4^x$

23. **Photocopying** Suppose you are photocopying an image, reducing it to 85% its original size. The function $y = 0.85^x$ models the size of an image after x number of times it is reduced. Graph the function. See margin.

24. **Science** A population of 100 insects triples in size every month. The function $y = 100 \cdot 3^x$ models the population after x months. Graph the function. See left above.

Evaluate each function for the domain $\{-2, -1, 0, 1, 2, 3\}$. As the values of the domain increase, do the values of the range increase or decrease? 25–32. See left.

25. $f(x) = 5^x$ 26. $y = 2.5^x$ 27. $h(x) = 0.1^x$ 28. $f(x) = 5 \cdot 4^x$
 29. $y = 0.5^x$ 30. $y = (\frac{2}{3})^x$ 31. $g(x) = 4 \cdot 10^x$ 32. $y = 100 \cdot 0.3^x$

33. **Multiple Choice** The population of Texas in 2000 was about 20.852 million people. The function $p(n) = 20.852(1.02071)^n$ estimates the population where $n = 0$ corresponds to the year 2000. Which is a reasonable estimate in millions of the population of Texas in 2020? **B**

- (A) 21.284 (B) 31.420 (C) 41.740 (D) 49.842

34. Biology A certain species of bacteria in a laboratory culture begins with 75 cells and doubles in number every 20 min.

- GPS** 75 cells and doubles in number every 20 min.
 a. Copy, complete, and extend the table to find when there will be more than 5,000 bacteria cells. See back of book.

| Time (min) | Number of 20-min Time Periods | Pattern | Number of Bacteria Cells |
|------------|-------------------------------|----------------------|--------------------------|
| Initial | 0 | 75 | 75 |
| 20 | 1 | $75 \cdot 2$ | $75 \cdot 2^1 = \square$ |
| 40 | \square | $75 \cdot 2 \cdot 2$ | $75 \cdot 2^2 = \square$ |
| 60 | \square | \square | $75 \cdot 2^3 = \square$ |
| \square | \square | \square | \square |

- b. Write a function rule to model the situation.
 $y = 75 \cdot 2^x$, where x is the number of 20-min time periods

Exercises 15–18 Suggest students review graphs from the lesson and make generalizations about their shapes before doing these exercises.

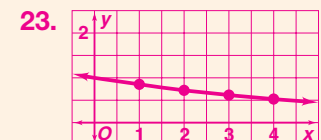
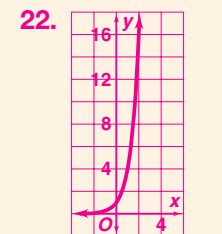
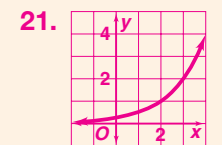
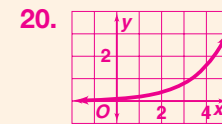
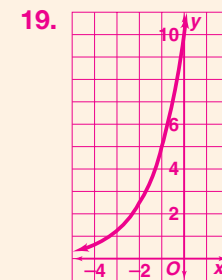
Technology Tip

Exercise 34 Suggest that students write an equation for the problem. Let x equal the number of 20-min time periods. Let y equal the number of bacteria cells. Students can check their answer to the exercise by using the **TABLE** function on a graphing calculator.

Error Prevention!

Exercise 41 Some students may begin by multiplying 100 by 10 and then squaring the product. Remind them that the order of operations is: parentheses, exponents, multiplication and division, addition and subtraction.

pages 470–473 Exercises

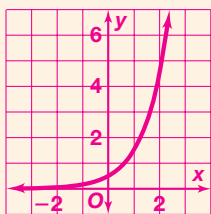


4. Assess & Reteach

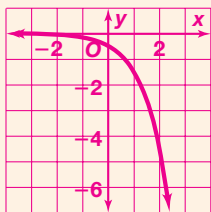
PowerPoint

Lesson Quiz

- Evaluate each function rule for the given value.
 - $y = 0.5^x$ for $x = 3$ **0.125**
 - $f(x) = 4 \cdot 3^x$ for $x = -2$ **$\frac{4}{9}$**
- Suppose an investment of \$5000 doubles every 12 years.
 - How much is the investment worth after 24 years? **\$20,000**
 - After 48 years? **\$80,000**
- Graph $y = 0.5 \cdot 3^x$.



- Graph $y = -0.5 \cdot 3^x$.



Alternative Assessment

Write $y = 3x$ on a transparency and project it with an overhead projector. Give students three seconds to look at the equation and write on their own paper whether the equation is *exponential* or *not exponential*. Repeat with various functions. Cover each function as you proceed. Include exponential, linear, quadratic, and absolute value functions. At the end of the activity, uncover the whole list of functions. Have students compare their answers with those of classmates and determine which are correct.

Test Prep

Resources

For additional practice with a variety of test item formats:

- Standardized Test Prep, p. xxx
- Test-Taking Strategies, p. xxx
- Test-Taking Strategies with Transparencies

GO Online
Homework Help
 Visit: PHSchool.com
 Web Code: ate-0807

35c. No; there is no value of x for which $y = 0$.

d. If the base is >1 , the graph gets steeper as the base increases. If the base is <1 , the graph gets steeper as the base decreases.

37a.

| x | y |
|-----|-----|
| 1 | -2 |
| 2 | 4 |
| 3 | -8 |
| 4 | 16 |
| 5 | -32 |

b. Every other value is negative. The absolute value of one term is double the previous term.

Challenge

- Graph $y = 2^x$, $y = 4^x$, and $y = (0.25)^x$. **See margin.**
 - What point is on each graph? **(0, 1)**
 - Does the graph of an exponential function intersect the x -axis? Explain.
 - Critical Thinking** How does the graph of an exponential function change as the base increases or decreases? **c-d. See left.**

36. Ecology In 50 days, a water hyacinth can generate 1000 offspring (the number of plants is multiplied by 1000). **a. 1,000,000,000 plants**

- How many hyacinth plants could there be after 150 days?
- How many hyacinth plants could there be after 200 days?

37. a. Make a table of values for the domain $\{1, 2, 3, 4, 5\}$ of the function $y = (-2)^x$. **See left.**

- What pattern do you see in the outputs? **See left.**

c. Critical Thinking Is $y = (-2)^x$ an exponential function? Justify your answer. **No; in $y = a \cdot b^x$, $b > 0$. $-2 < 0$, so it is not exponential.**

Which function is greater at the given value? 38. $y = x^5$ 39. $f(t) = 200 \cdot t^2$

38. $y = 5^x$ or $y = x^5$ at $x = 3$ 39. $f(t) = 10 \cdot 2^t$ or $f(t) = 200 \cdot t^2$ at $t = 7$

40. $y = 3^x$ or $y = x^3$ at $x = 4$ 41. $f(x) = 2^x$ or $f(x) = 100x^2$ at $x = 10$
 $y = 3^x$ $f(x) = 100x^2$

42. Writing Analyze the range of the function $f(x) = 500 \cdot 1^x$ using the domain $\{1, 2, 3, 4, 5\}$. Explain why the definition of *exponential function* includes the restriction that $b \neq 1$. **{500}; $b = 1$ produces a linear graph.**

43. a. Graphing Calculator Graph the functions $y = x^2$ and $y = 2^x$.

- What happens to the graphs between $x = 1$ and $x = 3$? **a-b. See margin.**

c. Critical Thinking How do you think the graph of $y = 6^x$ would compare to the graphs of $y = x^2$ and $y = 2^x$?

The graph of $y = 6^x$ is steeper than $y = x^2$ and $y = 2^x$.

Solve each equation.

44. $3^x = 9$ 2

45. $3^x = \frac{1}{27}$ -3

46. $2^x = 64$ 6

47. $3 \cdot 2^x = 24$ 3

48. $2 \cdot 3^x = 162$ 4

49. $5 \cdot 2^x - 152 = 8$ 5

50. Suppose $(0, 4)$ and $(2, 36)$ are on the graph of an exponential function.

- Use $(0, 4)$ in the general form of an exponential function $y = a \cdot b^x$ to find the value of the constant a . **4**

b. Use your answer from part (a) along with $(2, 36)$ to find the value of the constant b . **3**

c. Write a rule for the function. **$y = 4 \cdot 3^x$**

d. Evaluate the function for $x = -2$ and $x = 4$. **$\frac{4}{9}$, 324**



Test Prep

Multiple Choice

51. For the function $y = -3^x$, what is the value of y when $x = -2$? **B**

A. -9

B. $-\frac{1}{9}$

C. $\frac{1}{9}$

D. 9

52. Which function contains the points $(1, 3)$ and $(3, 6.75)$? **G**

F. $y = 1.675x + 1.325$

G. $y = 2 \cdot 1.5^x$

H. $y = 1.5 \cdot 2^x$

J. $y = 1.325x + 1.675$

53. Which function has the same y -intercept as $y = 2^x$? **A**

A. $y = x + 1$

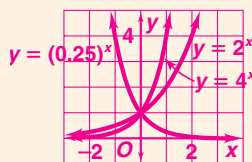
B. $y = 2x$

C. $y = x$

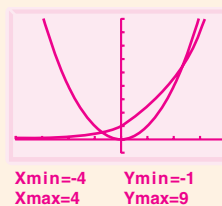
D. $y = 2(x + 1)$

pages 470–473 Exercises

35a.



43a.



b. Between $x = 1$ and $x = 3$, the graph of $y = x^2$ rises faster than the graph of $y = 2^x$. The graphs intersect at $x = 2$.



Checkpoint Quiz

Use this Checkpoint Quiz to check students' understanding of the skills and concepts of Lessons 8-5 through 8-7.

Resources

Grab & Go

- Checkpoint Quiz 2

54. A population of 6000 doubles in size every 10 years. Which equation relates the size of the population y to the number of 10-year periods x ? **H**

F. $y = 6000 \cdot 10^x$

G. $y = 10 \cdot 2^x$

H. $y = 6000 \cdot 2^x$

J. $y = 2 \cdot 100^x$

Short Response

55. Between what two integer values of x do the graphs of $y = 20(0.5)^x$ and $y = 0.5 \cdot 4^x$ intersect? Show your work. **See margin.**

Mixed Review



Lesson 8-6

Find each common ratio. Then find the next three terms in each sequence. **56–59. See left.**

56. 2, 10, 50, 250, ...

57. 7, -21, 63, -189, ...

58. -0.2, -0.4, -0.8, -1.6, ...

59. 27, -9, 3, -1, ...

60. 450, 45, 4.5, 0.45, ...

61. 7168, 1792, 448, 112, ...

0.1; 0.045, 0.0045, 0.00045

0.25; 28, 7, 1.75

Lesson 6-6

Write an equation for the line that passes through the given point and is parallel to the given line.

56. **5; 1250, 6250, 31,250**

62. $y = 5x + 1$; (0, 0) **$y = 5x$**

63. $y = 3x - 2$; (0, 1) **$y = 3x + 1$**

57. **-3; 567, -1701, 5103**

58. **2; -3.2, -6.4, -12.8**

64. $y = -2x + 5$; (4, 0) **$y = -2x + 8$**

65. $y = 0.4x + 5$; (2, -3) **$y = 0.4x - 3.8$**

59. **$-\frac{1}{3}; \frac{1}{3}, -\frac{1}{9}, \frac{1}{27}$**



Checkpoint Quiz 2

Lessons 8-5 through 8-7

Simplify each expression.

1. $\left(\frac{3^2}{3^{-1}}\right)^4$ **3^{12}**

2. $\left(\frac{x^2}{y^3}\right)^{-5} \frac{y^{15}}{x^{10}}$

3. $\left(\frac{10m^{-3}}{25n^{-6}}\right)^2 \frac{4n^{12}}{25m^6}$

4. $\left(\frac{6^2t^{-3}}{6^2t^0t^2}\right)^2 \frac{1}{t^{10}}$

Determine whether each sequence is *arithmetic* or *geometric*.

5. 22, 11, 5.5, 2.75, ...

6. 5, 10, 20, 40, 80, ...

7. 5, 10, 15, 20, 25, ...

geometric

geometric

arithmetic

8. Use the sequence -100, 20, -4, ...

a. What is the first term? **-100**

b. What is the common ratio? **$-\frac{1}{5}$ or -0.2**

c. Write a rule for the sequence. **$A(n) = -100 \cdot (-0.2)^{n-1}$**

d. Use your rule to find the fifth and seventh terms in the sequence. **-0.16; -0.0064**

9. **Physics** On the first swing, a pendulum swings through an arc of length 40 cm.

On each successive swing, the length of the arc is 85% of the length of the previous swing.

a. Write a rule to model this situation. **$A(n) = 40 \cdot (0.85)^{n-1}$**

b. Find the length of the arc on the fifth swing. Round your answer to the nearest millimeter. **209 mm**

10. **Commuting** Refer to the information at the left.

a. Write the number of vehicles that crossed the George Washington Bridge in scientific notation. **1.08×10^8**

b. The Port Authority collected about \$249 million in tolls from this bridge. Write this number in scientific notation. **2.49×10^8**

c. What was the average toll per vehicle? **about \$2.31**



Real-World Connection

About 108 million vehicles cross the George Washington Bridge between New York and New Jersey in a year.

55. [2]

| x | $y = 20 \cdot 0.5^x$ | $y = 0.5 \cdot 4^x$ |
|-----|----------------------|---------------------|
| 0 | 20 | 0.5 |
| 1 | 10 | 2 |
| 2 | 5 | 8 |

The graphs intersect between $x = 1$ and $x = 2$ (OR equivalent explanation).

[1] answer with no work shown