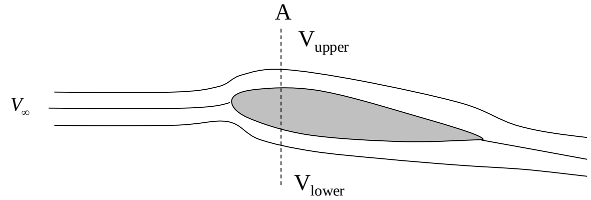
8. Aerodynamics: Two-Dimensional (the airfoil)

a) Simple Case: An airfoil as incompressible, steady, 1-D flow approaches from upstream:



Obstruction of airfoil causes constriction of stream tube above airfoil (with little affect on stream tube below). Continuity demands that flow above airfoil accelerate to match conditions of same streamtubes after the airfoil has been passed. Thus, from Bernoulli's equation:

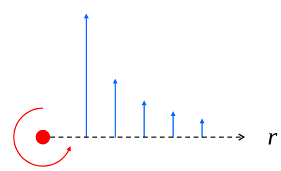
→

And when and lift force results.

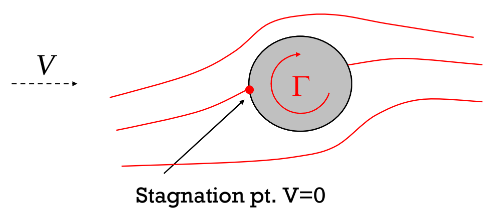
b) Aside: Mathematic Model for Lift Generation- Circulation and Potential Theory

The following are the essential results from potential theory in explaining the generation of lift. Please see any good aerodynamics text (e.g. *Fundamentals of Aerodynamics*, by J. Anderson, McGraw Hill publishers, 2001) for further details.

- Circulation- Flow rotating about a point- Vortex
- Related velocity (and thus lift) to vortex strength (Γ)
-



The superposition of a stationary cylinder in a flowfield with a rotating cylinder (with strength Γ) in no flow produces the analogy of flow around airfoil.



upper surface

$$V_U = V + \frac{\Gamma}{2\pi r}$$

lower surface

$$V_L = V - \frac{\Gamma}{2\pi r}$$

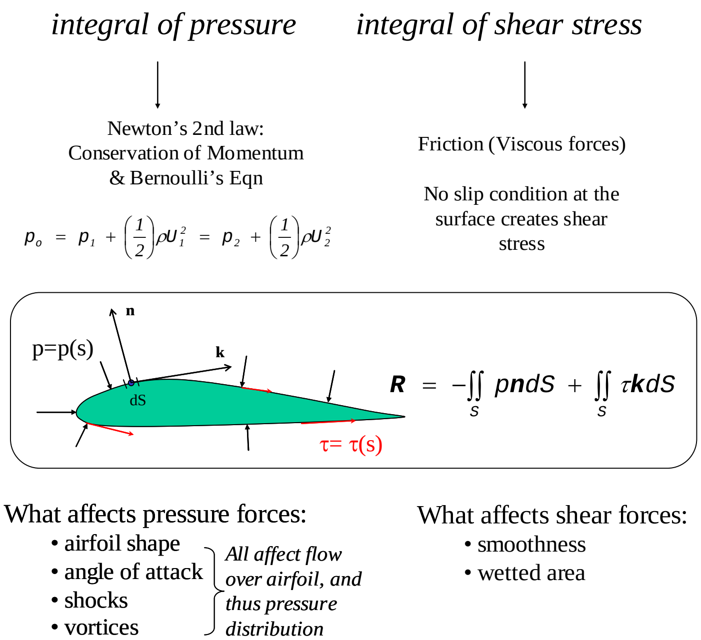
Finally, the Kutta-Jukowski Law relates lift and circulation (vortex strength):

- Implications:
    -
    -

c) The Total (Resultant) Aerodynamic Force

Only **<u>2 sources</u>** of the resultant aerodynamic force:

- **Pressure forces** (as we have just explained from Bernoulli (or K-J Law)
    - o
- **Shear forces** (the effects of viscosity)
    - o
-



These are the two "hands of nature" that affect the body in a fluid. Everything else (all types of lift and drag) is derived from these sources.

*d) Resolving the Resultant Aerodynamic Force (**R**)- Lift and Drag*

<u>Definitions:</u>

V_∞ = free stream velocity (relative wind)

α = angle of attack, angle made by chordline* and V_∞

Lift = L = component of **R** perpendicular to free stream

Drag = D = component of **R** parallel to free stream

Moment = M_x = Force*Distance; given a pivot point *x* on chordline, tendency to rotate due to **R** applied away from pivot; (positive moment >> increase in α)

e) Lift as summation of (normal) pressure distribution

From pgs. 76-77 in Brandt text, inspect chordwise velocity and normal pressure distributions:

- Difference in velocity at each chordwise location corresponds to net normal force (lift)

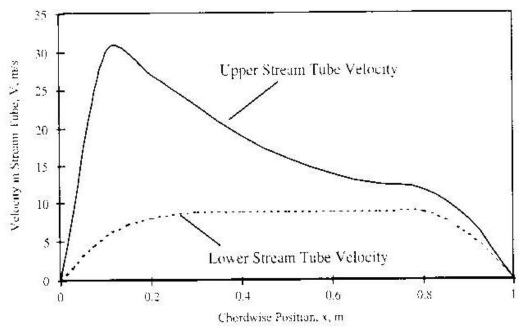


Fig. 3.11 Velocity distributions in stream tubes above and below airfoil.

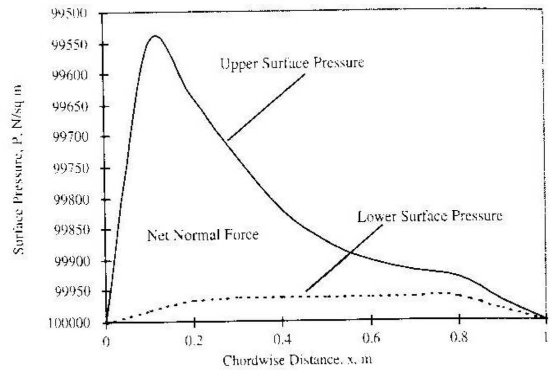
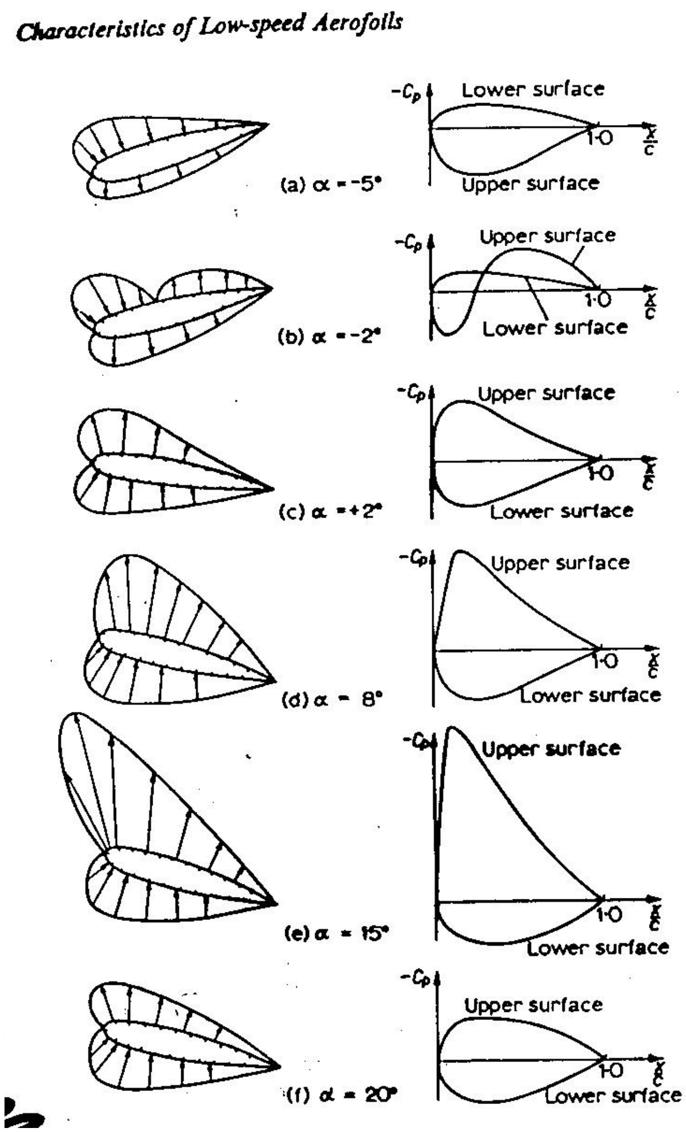


Fig. 3.12 Typical airfoil surface-pressure distribution.

If α changes, the flowfield changes, and thus the pressure dist. changes, and finally lift changes !

The series of pressure distributions shown at right (from *Aerodynamics*, by LJ Clancy, Longman Scientific & Technical, London, 1975) illustrate this cause-effect chain:



Some observations:

- "-Cp" is the negative of pressure coefficient;
-
- "Zero lift angle of attack", $\alpha_{L=0}$, is the α when no lift is generated.
- As α increases in positive regime, lift increases (noted by large "net normal force" area)
- Something bad happens between α=15 and α=20 degs.? What is it?

f) Aerodynamic Coefficients

With these definition, we could go right to the wind tunnel, but …..

$$\left.\begin{aligned} L &= L(\rho_\infty, V_\infty, S, \alpha, \mu_\infty, a_\infty) \\ D &= D(\rho_\infty, V_\infty, S, \alpha, \mu_\infty, a_\infty) \\ M &= M(\rho_\infty, V_\infty, S, \alpha, \mu_\infty, a_\infty) \end{aligned}\right\} \text{For a given shape}$$

. . . Too many variables.

$$C_L = \frac{L}{q_\infty S}, \quad C_D = \frac{D}{q_\infty S}, \quad C_M = \frac{M}{q_\infty S c}$$

$$\frac{N}{\left(\frac{kg}{m^3}\right)\left(\frac{m^2}{s^2}\right)m^2} = \frac{N}{(kg)\frac{m}{s^2}} = \frac{N}{N} = 1$$

Define dimensionless Coefficients !

where $q_\infty = \left(\frac{1}{2}\right)\rho V_\infty^2$ is called dynamic pressure

Now, define further the Reynolds Number (*Re*) and Mach number (M_∞)

$$\text{Re} = \frac{\rho_\infty V_\infty c}{\mu_\infty}, \quad M_\infty = \frac{V_\infty}{a_\infty}$$

. . . Apply the Buckingham-Pi Theorem (dimensional analysis) . .

$$\begin{aligned} C_L &= f_1(\alpha, \text{Re}, M_\infty) \\ C_D &= f_2(\alpha, \text{Re}, M_\infty) \\ C_M &= f_3(\alpha, \text{Re}, M_\infty) \end{aligned}$$

- On three dependent parameters !
- *Dynamic Similarity* . . . Enables sub-scale wind tunnel testing
- S is usually wing planform area

Conventions:

Lower case refers to airfoil (2-D) case, while upper case refer to whole wing/aircraft (3-D)

- c_l and c_d are airfoil lift and drag coefficients
- C_L and C_D are wing or aircraft lift and drag coefficients

g) Variations of lift with α, M

- The lift-curve slope relates lift to increments in α, and is linear in nature (theoretical result validated by experiment) up to stall limit:

(sketch diagram in class and see Fig 3.24 and 3.25 in Brandt text)

- Variation with mach number (sketch)
    - Compressibility takes effect when M>0.3
    - Use Prandtl-Glauert Correction
    - Only good until ~ M=0.8

$$c_l = \frac{(c_l)_{\text{incompressible}}}{\sqrt{1-M_0^2}}$$