8. Measurement

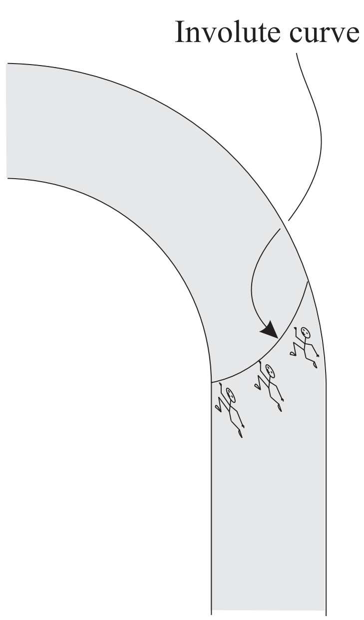
Track and Field

In 1905, encouraged by President Theodore Roosevelt, the Intercollegiate Athletic Association of the United States was founded in New York City. Their original purpose was to reform college football.

In 1910, they changed their name to the National College Athletic Association, and in 1921 the first National Collegiate Track and Field Championships were held.

The NCAA Track and Field Rule Book defines the rules, events, equipment, personnel and even the construction of the track facilities. It states that there must be a visible starting line, which should measure 2 inches (5.08 cm) wide. For races not run in lanes, the starting line should be curved.

This means that if there are more runners than there are lanes, the start line must be curved. The start is then called a "waterfall start," and the curved start line is the involute of a circle.

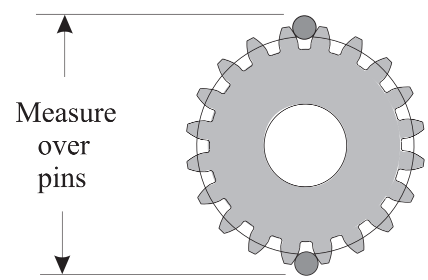


Measuring Gear Teeth

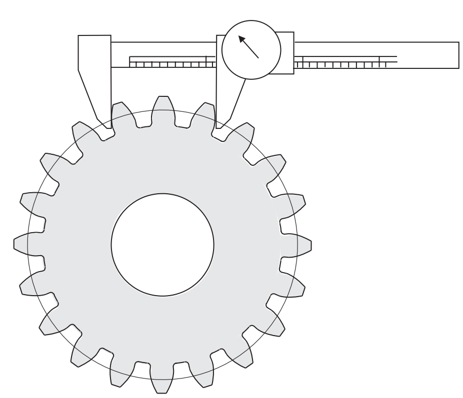
Gear teeth can be measured using special tooth calipers, or by meshing with special precision master gears, or with profile testers.

There are two ways of measuring gear tooth thickness, however, that require no special equipment.

You can hold round pins or balls in the space between gear teeth, and measure over the pins to determine tooth thickness.

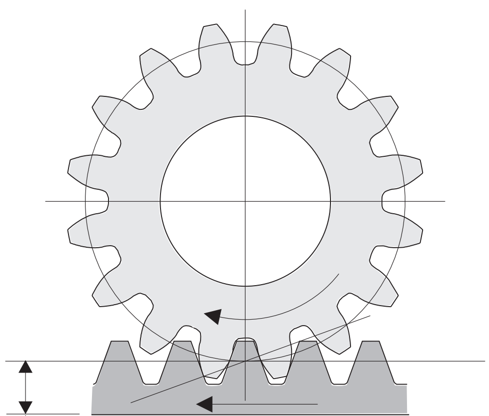


You can measure across two or more teeth, from the involute curve of one tooth, to the involute curve of another.

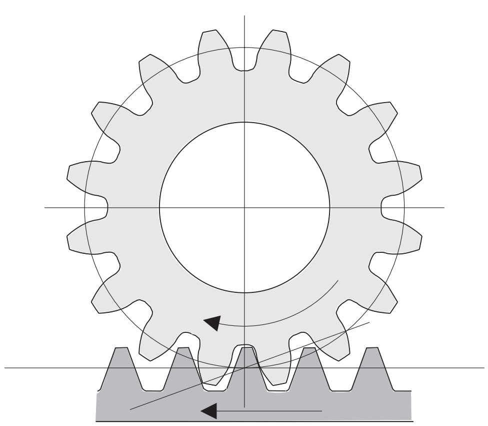


Measuring Thickness of Rack Teeth

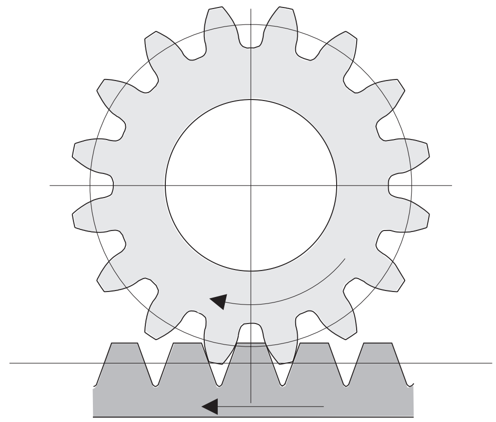
Imagine a rack mounted on a base plate. It is important that the distance from the pitch line to the back of the rack is correct. It is important that the teeth are the correct thickness. If too thick, the pinion will mesh too tightly, or be too tight to mesh at all. If too loose, there will be too much backlash between rack and pinion.



Measure pitch line to back

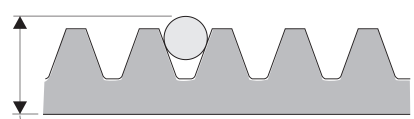


Rack teeth too thin.



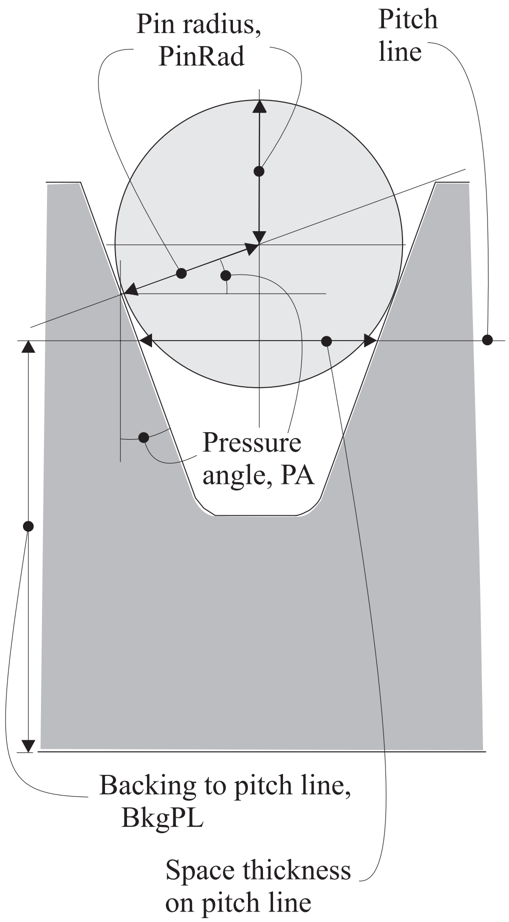
Rack teeth too thick

To measure the thickness of rack teeth, we can put a round pin or ball in the tooth space, and measure from the top of the pin to the back of the rack.



Measure over pin

Measurement over a pin works out like this.



Space thickness on pitch line =
Circular pitch – Tooth thickness on pitch line
[abbreviated CircP – TThkPL]

Measure over pin =

$$PinRad\left(1 + \sin PA + \frac{\cos PA}{\tan PA}\right) - \frac{CircP - TThkPL}{2(\tan PA)} + BkgPL$$

From trigonometry we know that:

$$(\sin PA)^2 + (\cos PA)^2 = 1$$

$$\tan PA = \frac{\sin PA}{\cos PA}$$

Measure over pin =

$$PinDia\left(\frac{\sin PA + 1}{2(\sin PA)}\right) - \frac{CircP - TThkPL}{2(\tan PA)} + BkgPL$$

Maximum pin diameter =

$$\frac{(2)\tan PA\,(Add) + CircP - TThkPL}{\cos PA}$$

Ideal pin diameter is about $\sim \frac{CircP - TThkPL}{\cos PA}$

For **14½° involute teeth**, equations work out like this:

Ideal pin diameter $= \frac{CircP - TThkPL}{0.968148}$

Measure over pin = 2.49696 (Pin dia.) –
1.93336 (CircP – TThkPL) + BackingPL

For **20° involute teeth**, equations work out like this:

Ideal pin diameter $= \frac{CircP - TThkPL}{0.939693}$

Measure over pin = 1.96190 (Pin dia.) –
1.37374 (CircP – TThkPL) + BackingPL

For example, picture 2.5 mod, 20° PA rack cut with no backlash allowance. Dimension from pitch line of rack to the back of the rack is 12.0 mm.

Assuming we have on hand a 4.900 mm diameter pin, here are equations for pin measurement.

Ideal pin diameter is about ~ $\dfrac{CircP - TThkPL}{0.939693}$

$CircP = 2.5 \times \pi = 7.85398$ mm

Since we are allowing nothing for backlash:

$TThkPL = 7.85398 \div 2 = 3.92699$ mm

Ideal pin diameter $= \dfrac{3.92699}{0.939693} = 4.17901$ mm

Measure over pin $= 1.96190\ (4.900\ ) - 1.37374\ (\ 3.92699) + 12.0 =$
16.219 mm measure over 4.900 mm diameter pin to back of rack.

Using general equations:
Tooth thickness on pitch line $= 2.5 \times \pi \div 2 = 3.92699$ mm

sin 20° = 0.342020
cos 20° = 0.939693
tan 20° = 0.363970

Backing to pitch line of rack = 12.0 mm

Measure over pin =

$$PinDia\left(\frac{\sin PA + 1}{2(\sin PA)}\right) - \frac{CircP - TThkPL}{2(\tan PA)} + BkgPL$$

Measure over pin =

$$4.900\left(\frac{1.34202}{2 \times 0.342020}\right) - \frac{3.92699}{(2)0.363970} + 12.0 =$$

= 16.219 mm measure over pin to back of rack

Study Questions Measuring Thickness of Rack Teeth

1. Imagine a rack, 4 mod, 20º pressure angle, cut into a 25 mm square steel bar with no backlash.

   What would a measurement over a 7.00 mm diameter pin be?

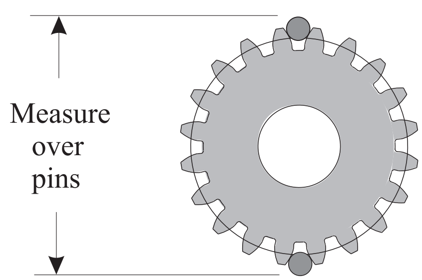
2. a. Imagine a rack, 6 DP, 14½º pressure angle, cut into a 1.000" square steel bar with no backlash.

   What would a measurement over a 0.2500" diameter pin be?

b. How far above the top of the rack tooth does the pin project?

Measuring Gear Teeth Over Pins

Gear tooth thickness can be measured with pins or balls placed in tooth spaces.

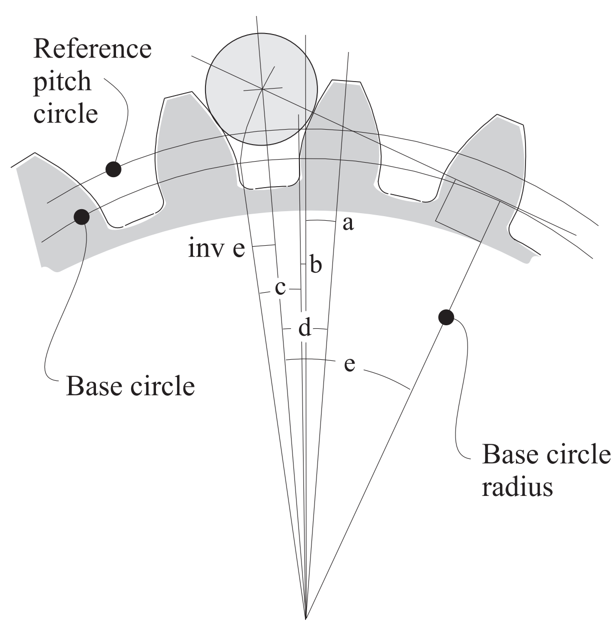


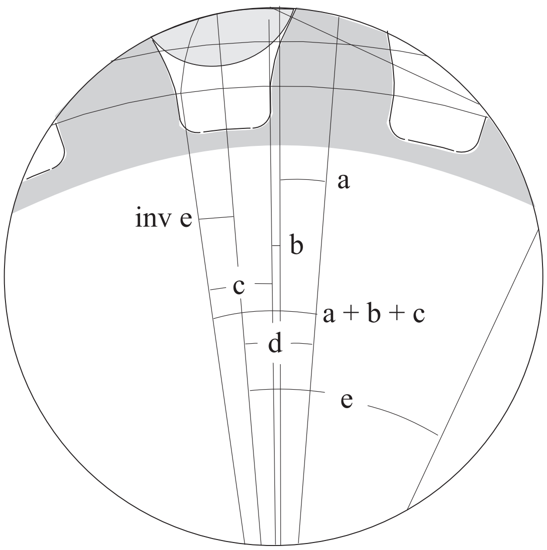
A computer program, Gear Measurements, for calculating measurement over pins or balls for spur gears and helical gears is given on www.salemcompany.com, and is on a CD available from Salem Co.

The program allows you to calculate the pin measurement for any inch size or metric size gear and any pin diameter. Use it. It will save much tedious hand calculator work.

Here's how to make these calculations by hand.

Picture a round pin or steel ball resting in a space between two teeth.





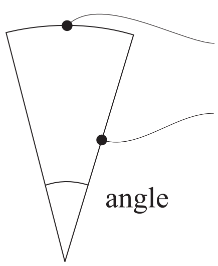
We can see that angle a + angle b + angle c – angle d = inv e.

$$a + b + c - d = inv\ e$$

Knowing inv e, we can calculate angle e.

Knowing angle e and the base circle radius, we can calculate the length from the center of the gear to the center of the ball.

The pitch circle, tooth thickness and pressure angle below refer to reference dimensions. Angles are measured in radians.



Remember - when measuring angles in radians, the size of the angle, in radians, equals this length divided by this length.

From the diagram we can see that:

$$\textbf{Angle a} = \frac{\textit{Half the tooth thickness on the pitch circle}}{\textit{Pitch radius}} = \frac{\textit{Tooth thickness}}{\textit{PD}}$$

angle b = *involute (PA at Pitch Radius)*

$$\textbf{angle c} = \frac{\textit{Ball or pin radius}}{\textit{Base radius}} = \frac{\textit{Ball dia.}}{\textit{PD (cos PA)}}$$

$$\textbf{angle d} = \frac{\textit{Half of circular pitch}}{\textit{Pitch radius}} = \frac{\pi}{\textit{Number of teeth in gear}}$$

Then we can calculate involute e:

inv e = angle a + angle b + angle c – angle d

$$\textbf{\textit{inv e}} = \frac{Thk}{PD} + inv\ PA + \frac{\textit{Pin diameter}}{\textit{Base diameter}} - \frac{\pi}{T}$$

Calculate angle e from inv e.

Then calculate radius to ball center:

$$\textit{Radius to Ball or Pin Center} = \frac{\textit{Base radius}}{\cos e}$$

If there are an even number of teeth in the gear,

Measurement over pins =

(Radius to pin center) (2)+ Pin diameter =

$$\left(\frac{\textit{Base radius}}{\cos e}\right)(2) + \textit{Pin Diameter}$$

Pitch diameter (PD) is the reference pitch diameter. Tooth thickness, circular pitch, and pressure angle are measured at the reference pitch diameter (PD). T is the number of teeth in the gear.

We can use the form below to make these calculations.

Example:

Calculate measurement over pins:

Spur gear
24 Teeth
4 DP
20° PA
0.008" backlash allowance

0.4200" Pin diameter

Example:

Calculate inv e:

T = Number of Teeth in Gear = <u>*24*</u>
DP = Diametral Pitch = <u>*4*</u>
PA = Pressure Angle, degrees = <u>*20°*</u>

p = PA in radians = $\frac{PA \times \pi}{180}$ = <u>*0.349 066*</u>
cosPA = cosine PA = <u>*0.939 693*</u>
tanPA = tangent PA = <u>*0.363 970*</u>
Pin Dia = <u>*0.4200*</u>
X Dimension = <u>*0*</u>
Backlash Allowance = <u>*0.0080*</u>

PD = Pitch Dia = $\frac{T}{DP}$ = <u>*6.0000*</u>
BD = Base Dia = *PD (cosPA)* = <u>*5.638 16*</u>

Thk = Tooth Thickness = $\frac{\pi}{2(DP)} - \frac{Backlash}{Allowance} \pm$ *2(tanPA)(X Dimension)*= <u>*0.384 699*</u>

$h = \frac{Thk}{PD}$ = <u>*+ 0.064 117*</u>

j = inv PA = $\tan p - p$ = <u>*+ 0.014 904*</u>

$k = \frac{Pin\ Dia}{BD}$ = <u>*+ 0.074 492*</u>

$m = \frac{\pi}{T}$ = <u>*– 0.130 900*</u>
inv e = $h + j + k - m$ = <u>*0.022 613*</u>

Calculation of angle e, given inv e:

* i = inv e = <u>*0.022 613*</u>

$A = (3\ i)^{0.\overline{3}}$ = <u>*+ 0.407 843*</u>

$B = 0.4\ i$ = <u>*– 0.009 045*</u>

$C = (0.2616\ i)^{1.\overline{6}}$ = <u>*+ 0.000 193*</u>

$D = (0.1721\ i)^{2.\overline{3}}$ = <u>*– 0.000 002*</u>

** $e_1 = A - B + C - D$ = <u>*0.398 989*</u>

i_1 = inv e_1 = tan $e_1 - e_1$ = <u>*0.022 613*</u>

E = error = $i_1 - i$ = <u>*0*</u>

If E is small, $e = e_1$, otherwise continue below.

$F = (3\ i)^{-0.\overline{6}} - 0.4 =$ ____________________

$F \times E =$ ________________________________

** $e = e_2 = e_1 - (F \times E) =$ __________________

i_2 = inv e_2 = tan $e_2 - e_2$ = ________________

$E = i_2 - i_1 =$ ____________________________

Notes:

p = Angle, in radians

$0.\overline{3} = 0.33333333$ . . . *threes repeat forever*

$1.\overline{6} = 1.66666666$ . . . *sixes repeat forever*

$2.\overline{3} = 2.33333333$ . . . *threes repeat forever*

$$(3\ i)^{-0.\overline{6}} = \frac{1}{(3\ i)^{0.\overline{6}}}$$

Calculate Measure over Pins:

cos e = <u>*0.921 454*</u>

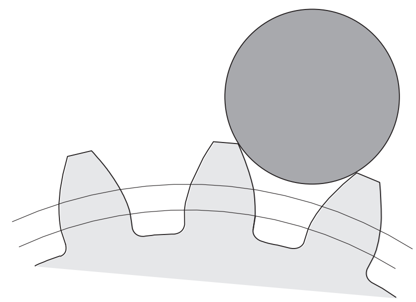
Measure over Pins, **Even** Number of Teeth =

$$\frac{BD}{\cos e} + \text{Pin Dia} = \underline{6.5388}$$

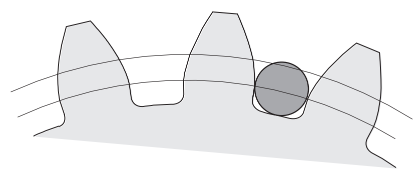
Measure over 0.4200" dia. pins = 6.5388"

If we use a pin or ball that's too small, it will fall into the tooth space and rest on the root diameter of the teeth, or it will not project above the gear outside diameter, and we won't be able to fit a micrometer or caliper tip between the teeth to get a measurement.

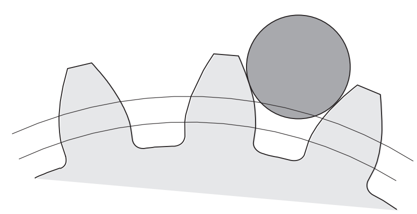
If it's too big, it will rest on the tips of the teeth, not on the involute flanks.



Ball or pin is too big.



Ball or pin is too small.



Ball or pin is okay.

There are two standards for pins to use for any size spur or helical tooth.

$$\text{Pin diameter} = \frac{1.680}{DP} \text{ or } 1.680 \times \text{mod}$$

$$\text{Pin diameter} = \frac{1.728}{DP} \text{ or } 1.728 \times \text{mod}$$

If you use these pin or ball sizes, you can find tables that will give exact measurements for spur gears of any number of teeth, for 20º or 14½º teeth.

If you don't have a pin or ball of this exact size, you can use something close (approximately $\frac{1.7}{DP}$ or 1.7 × module), and calculate the measurement from the formulas in this book, or use the computer program, Gear Measurements, on our web site, www.salemcompany.com, or available on CD from Salem Company.

Study Questions Measuring Gear Teeth over Pins

1. What should measurement over pins be for a 56 tooth spur gear, 10 DP, 20º pressure angle, zero backlash allowance, 0.1728" pin diameter. You can use the Gear Measurements computer program to check your answer.

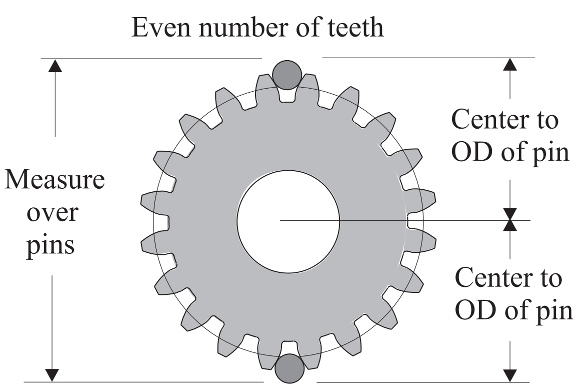
2. What should measurement over pins be for this gear:

   60 teeth, 4 mod, 20º pressure angle spur gear
   pin diameter = 6.912 mm
   Zero backlash allowance.

When measuring over pins or balls, there is a difference between a gear with an even number of teeth (evenly divisible by 2), and one with an odd number of teeth.

We have learned how to calculate a measurement from the center of a gear to the outside diameter of a pin or ball.

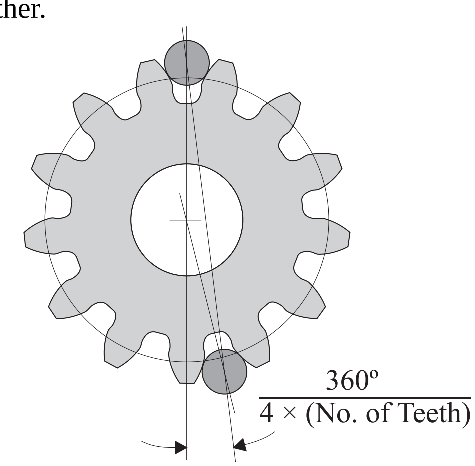
With an even number of teeth, measurement over pins equals twice the dimension from center of gear to OD of pin.



For **even** numbers of **teeth**:

Measure over pins =
(Measure to center of pin × 2) + Pin diameter

With **odd** numbers of **teeth**, we have to correct for the fact that the pins are not exactly opposite each other.



For **odd** numbers of **teeth**:

Measure over pins =

$$(\textit{Measure to pin center})\left(\cos\left(\frac{90°}{T}\right)\right)(2) + \textit{Pin diameter}$$

T = Number of teeth in the gear.

Notice that $\cos\left(\frac{90°}{T}\right)$ is cosine of an angle in degrees, not radians.

For example:

35 teeth, 8 DP, 14½° PA, no backlash
0.2160" pin diameter.

Measure over pins is 4.6773". See calculations below.

Calculate inv e:

T = Number of Teeth in Gear = $\underline{35}$
DP = Diametral Pitch = $\underline{8}$
PA = Pressure Angle, degrees = $\underline{14½°}$

p = PA in radians = $\frac{PA \times \pi}{180}$ = $\underline{0.253\ 072\ 7}$
$cosPA$ = cosine PA = $\underline{0.968\ 147\ 6}$
$tanPA$ = tangent PA = $\underline{0.258\ 617\ 6}$
Pin Dia = $\underline{0.2160}$
X Dimension = $\underline{0}$
Backlash Allowance = $\underline{0}$

PD = Pitch Dia = $\frac{T}{DP}$ = $\underline{4.3750}$

BD = Base Dia = $PD\ (cosPA)$ = $\underline{4.235\ 65}$

Thk = Tooth Thickness = $\frac{\pi}{2(DP)} - \frac{Backlash}{Allowance} \pm$ $2(tanPA)(X\ Dimension)$ = $\underline{0.196\ 350}$

$h = \frac{Thk}{PD}$ = $\underline{+\ 0.044\ 88}$

j = inv PA = $tan\ p - p$ = $\underline{+\ 0.005\ 544\ 8}$

$k = \frac{Pin\ Dia}{BD}$ = $\underline{+\ 0.050\ 995\ 7}$

$m = \frac{\pi}{T}$ = $\underline{-\ 0.089\ 759\ 8}$

$inv\ e = h + j + k - m$ = $\underline{0.011\ 660\ 7}$

Calculation of angle e, given inv e:

* i = inv e = $\underline{0.011\ 660\ 7}$

A = $(3\ i)^{0.\overline{3}}$ = $\underline{+\ 0.327\ 050\ 9}$

B = 0.4 i = $\underline{-\ 0.004\ 664\ 3}$

C = $(0.2616\ i)^{1.\overline{6}}$ = $\underline{+\ 0.000\ 064\ 2}$

D = $(0.1721\ i)^{2.\overline{3}}$ = $\underline{-\ 0.000\ 000\ 5}$

** e_1 = A – B + C – D = $\underline{0.322\ 450\ 3}$

i_1 = inv e_1 = tan $e_1 - e_1$ = $\underline{0.011\ 660\ 7}$

E = error = $i_1 - i$ = $\underline{0}$

If E is small, e = e_1, otherwise continue below.

F = $(3\ i)^{-0.\overline{6}} - 0.4$ = ____________________

F × E = ____________________

** e = e_2 = e_1 – (F × E) = ____________________

i_2 = inv e_2 = tan $e_2 - e_2$ = ____________________

E = $i_2 - i_1$ = ____________________

Notes:
p = Angle, in radians
$0.\overline{3}$ = 0.33333333 . . . *threes repeat forever*
$1.\overline{6}$ = 1.66666666 . . . *sixes repeat forever*
$2.\overline{3}$ = 2.33333333 . . . *threes repeat forever*
$(3\ i)^{-0.\overline{6}} = \frac{1}{(3\ i)^{0.\overline{6}}}$

Calculate Measure over Pins:

$cos\ e$ = $\underline{0.948\ 461\ 8}$

Measure over Pins, **Even** Number of Teeth =
$\frac{BD}{cos\ e} + Pin\ Dia$ = ____________________

Measure over Pins, **Odd** Number of Teeth =
$\left(\frac{BD}{cos\ e}\right)\left(cos\left(\frac{90°}{T}\right)\right) + Pin\ Dia$ = $\underline{4.6773}$

Measure over pins is 4.6773".

Study Questions

Odd Number of Teeth

1. What should measurement over pins be for this spur gear?

   75 Teeth, 2.5 mod, 20º PA, 0.100 mm backlash allowance, pin diameter 4.500 mm.

2. What is measurement over pins for a spur gear with 21 teeth, 4 DP, 20º pressure angle, 0.4380" diameter pins. Allowance for backlash is 0.008".

3. Imagine a spur gear, 40 teeth, 5 mod, 14½º PA. Using 9.000 mm diameter pins, you measure 212.800 mm over the pins. About how much backlash allowance is cut into the gear teeth?

4. A spur gear has 24 teeth, 8 DP, 20º PA.
   What should it measure over 0.2160" diameter pins, with no backlash allowance?

If we know the pin measurement of a gear, we can calculate tooth thickness by rearranging the tooth thickness equation. Use the form below:

Calculate Tooth Thickness:

T = Number of Teeth in Gear = ____________

DP = Diametral Pitch = __________________ or
mod = Module __________________________

PA = Pressure Angle, degrees = ____________

p = PA in radians = $\frac{PA \times \pi}{180}$ = ______________
cosPA = cosine PA = ______________________
tanPA = tangent PA = _____________________

Pin Dia = ______________________________

Measure over Pins = ____________________

PD = Pitch Dia = $\frac{T}{DP}$ = ___________________or
PD = Pitch Dia = *T* × *mod* = _______________

BD = Base Dia = *PD (cosPA)* = _____________

Even Number of Teeth:

$\cos e = \frac{BD}{(\text{Measure over pin} - \text{Pin dia})} =$

____

Odd Number of Teeth:

$\cos e = \frac{BD}{(\text{Measure over pin} - \text{Pin dia})} \times \cos(\frac{90°}{T}) =$

____

e = *ArcCos(cos e)* = _______________________

inv e = *tan e* – *e* = +_______________________

j = inv PA = *tan p* – *p* = –_________________

$k = \frac{\text{Pin Dia}}{BD}$ = –__________________________

$m = \frac{\pi}{T}$ = +_______________________________

Thk = ***Tooth Thickness at PD*** =
PD (inv e – j – k + m) = _____________

For example, consider a spur gear, 30 Teeth, 10 mod, 20° PA, Measure over 17.00 mm diameter pins = 322.00 mm. What is tooth thickness?

Calculate Tooth Thickness:

T = Number of Teeth in Gear = *30*
mod = Module = *10*
PA = Pressure Angle, degrees = *20°*

p = PA in radians = $\frac{PA \times \pi}{180}$ = *0.349 066*
cosPA = cosine PA = *0.939 693*
tanPA = tangent PA = *0.363 970*

Pin Dia = *17.00 mm*

Measure over Pins = *322.00 mm*

PD = Pitch Dia = T × mod = *300.00 mm*

BD = Base Dia = *PD (cosPA)* = *281.908*

Even Number of Teeth:

$\cos e = \frac{BD}{(\text{Measure over pin} - \text{Pin dia})} =$ *0.924 289*

Odd Number of Teeth:

$\cos e = \frac{BD}{(\text{Measure over pin} - \text{Pin dia})} \times \cos(\frac{90°}{T}) =$

____

e = *ArcCos(cos e)* = *0.391 628*

inv e = *tan e* – *e* = *+0.021 331 3*

j = inv PA = *tan p* – *p* = *– 0.014 904 4*

$k = \frac{\text{Pin Dia}}{BD}$ = *– 0.060 303 4*

$m = \frac{\pi}{T}$ = *+ 0.104 720*

Thk = ***Tooth Thickness at PD*** =
PD (inv e – j – k + m) = ***15.253 mm***

Consider a spur gear, 25 Teeth, 4 DP, 20° PA, Measure 6.7765 over 0.4200 dia pins. What is tooth thickness?

Calculate Tooth Thickness:

T = Number of Teeth in Gear = _25_
DP = Diametral Pitch = _4_
PA = Pressure Angle, degrees = _20°_

p = PA in radians = $\frac{PA \times \pi}{180}$ = _0.349 066_
$cosPA$ = cosine PA = _0.939 693_
$tanPA$ = tangent PA = _0.363 970_

$Pin\ Dia$ = _0.4200_

$Measure\ over\ Pins$ = _6.7765_

PD = Pitch Dia = $\frac{T}{DP}$ = _6.2500_
BD = Base Dia = $PD\ (cosPA)$ = _5.8731_

Even Number of Teeth:

$$\cos e = \frac{BD}{(Measure\ over\ pin - Pin\ dia)} = \underline{\hspace{4cm}}$$

Odd Number of Teeth:

$$\cos e = \frac{BD}{(Measure\ over\ pin - Pin\ dia)} \times \cos\left(\frac{90°}{T}\right) = \underline{0.922\ 13}$$

$e = ArcCos(\cos e)$ = _0.397 25_

$inv\ e = \tan e - e$ = _+0.022 305_

j = inv PA = $\tan p - p$ = _– 0.014 904_
$k = \frac{Pin\ Dia}{BD}$ = _– 0.071 512_

$m = \frac{\pi}{T}$ = _+ 0.125 66_

Thk = ***Tooth Thickness at PD*** =
$PD\ (inv\ e - j - k + m)$ = **_0.3847_**

Study Questions

Calculate Tooth Thickness

1. Consider a spur gear with 46 teeth, 10 DP, 14½° PA.
   Measure over 0.1728" diameter pins is 4.8344".

   What is tooth thickness?

2. Consider this spur gear, 45 teeth, 4 mod, 20° PA.
   Measure over 6.912 mm diameter pins is 191.003 mm.

   What is tooth thickness?

Blank Form, Measure over Pins

Calculate inv e:

T = Number of Teeth in Gear = ____________
DP or mod =
Diametral Pitch or module = ____________

PA = Pressure Angle, degrees = ____________

p = PA in radians = $\frac{PA \times \pi}{180}$ = ______________
cosPA = cosine PA = ______________________
tanPA = tangent PA = _____________________
Pin Dia = ______________________________
X Dimension = ___________________________
Backlash Allowance = ____________________

PD = Pitch Dia = $\frac{T}{DP}$ *or T (mod)* = ___________
BD = Base Dia = *PD (cosPA)* = _____________

Thk = Tooth Thickness = $\frac{\pi}{2(DP)}$ or $\frac{\pi\,(mod)}{2}$ – $\frac{Backlash}{Allowance}$ ± *2(tanPA)(X Dimension)*=
____

$h = \frac{Thk}{PD}$ = +__________________________
j = inv PA = *tan p – p* = +_________________
$k = \frac{Pin\ Dia}{BD}$ = +__________________________

$m = \frac{\pi}{T}$ = –_______________________________
inv e = *h + j + k – m* = __________________

Calculation of angle e, given inv e:

* i = inv e = __________________________

A = $(3\ i)^{0.\overline{3}}$ = +________________________

B = 0.4 i = –__________________________

C = $(0.2616\ i)^{1.\overline{6}}$ = +_________________

D = $(0.1721\ i)^{2.\overline{3}}$ = –____________________

** e_1 = A – B + C – D = __________________

i_1 = inv e_1 = tan e_1 – e_1 = ________________

E = error = i_1 – i = _______________________

If E is small, e = e_1, otherwise continue below.

F = $(3\ i)^{-0.\overline{6}}$ – 0.4 = ___________________

F × E = _________________________________

** e = e_2 = e_1 – (F × E) = _________________

i_2 = inv e_2 = tan e_2 – e_2 = ________________

E = i_2 – i_1 = ____________________________

Notes:
p = Angle, in radians
$0.\overline{3}$ = 0.33333333 . . . *threes repeat forever*
$1.\overline{6}$ = 1.66666666 . . . *sixes repeat forever*
$2.\overline{3}$ = 2.33333333 . . . *threes repeat forever*
$(3\ i)^{-0.\overline{6}} = \frac{1}{(3\ i)^{0.\overline{6}}}$

Calculate Measure over Pins:

cos e = _________________________________

Measure over Pins, **Even** Number of Teeth =
$\frac{BD}{cos\ e}$ +*Pin Dia* = ________________________

Measure over Pins, **Odd** Number of Teeth =
$\left(\frac{BD}{cos\ e}\right)\left(cos\left(\frac{90°}{T}\right)\right)$ + *Pin Dia* = ____________

Notes: Standard Pin Sizes are $\frac{1.680}{DP}$ or $\frac{1.728}{DP}$
or 1.680 × mod, or 1.728 × mod

Summary

In this section we learned two methods to measure the thickness of gear teeth. One is by **measuring over pins** or balls placed in the tooth spaces. The other is by **measuring from tooth flank to tooth flank** across two or more teeth.

Rack tooth thickness is determined by measuring over pins or balls. The exact calculation is straightforward for both 20° and 14½° teeth. Be careful that the micrometer spindle used to measure over the pins does not touch the rack itself, only the pin.

Pin measurements for spur gears have been printed in tables for standard pin sizes. A computer program, **Gear Measurements**, for both spur and helical gears for any pin size can be run from Salem Company's web site, www.salemcompany.com, or run from a CD disk available from Salem Company. Use rack tooth approximations to estimate tooth thickness change for a given change in pin measurement.

Be sure the pin touches the involute flanks of the gear teeth, not the bottom or tip of the tooth.

Measurements over pins for spur and helical gears can be calculated using **involute functions**.

Forms are shown to calculate pin measurements for spur gears with even or odd numbers of teeth and any possible pin size.

Forms are shown to calculate tooth thickness from pin measurements.

In the next section we will learn how to use **span measurements** to find tooth thickness. A span measurement is a measure over two or more teeth, from the involute curved flank of one tooth to the involute curved flank of another. Span measurements can be made with micrometers or vernier calipers.

A span measurement is actually a measurement along a line tangent to the base circle. It is sometimes called a **base tangent** measurement.

Tables of span measurements for spur gears are available, for 14½° and 20° teeth.

A computer program for span measurements of both spur and helical gear teeth can be run from Salem Company's web site, salemcompany.com.

A change in span measurement of 0.0010" or 0.01 mm equals a change in **backlash allowance** of 0.0010" or 0.01 mm, where the backlash is measured at the base circle diameter.

Circular pitch, diametral pitch and pressure angle of helical gears can be measured in two directions:

1. ***Normal*** (at right angles) to the tooth
2. ***Circumferential*** (around the circumference of the gear). Sometimes called **transverse**.

Span measure calculations can be made using involute functions, and are fairly simple if you use a scientific type calculator. **Forms** for calculating span measurements for both spur and helical gears are shown. A **form for calculating tooth thickness** from span measure for spur gears is shown.

Derivation of the equations for span measurements for spur and helical gears is shown.