8. Short Columns

INTRODUCTION : AXIAL COMPRESSION

LATERAL TIES AND SPIRALS

COMPRESSION + BENDING OF REC. COLUMNS

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BALANCED FAILURE

DISTRIBUTED REINFORCEMENT

CIRCULAR COLUMNS

KCI CODE PROVISION FOR COLUMN DESIGN

DESIGN AIDS

447.328

Theory of Reinforced Concrete and Lab. II

Fall 2007

8. Short Columns

AXIAL COMPRESSION

- Columns – defined as members that carry loads CHIEFLY in compression ; compression member
- Compression members includes

  arch ribs, rigid frame members inclined, compression members in truss structure, shells that carry axial compression

- Three types of RC compression members are

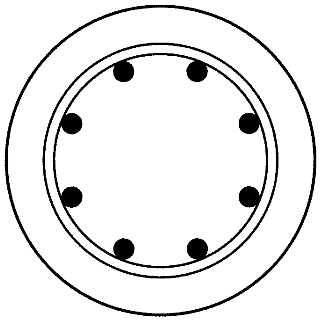
  i) Members reinforced with longitudinal bars and ties.

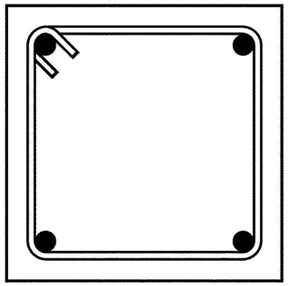
  ii) Members reinforced with longitudinal bars and spirals.

  iii) Composite compression members.

8. Short Columns

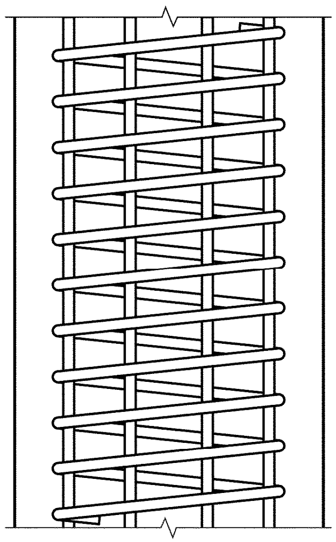
AXIAL COMPRESSION

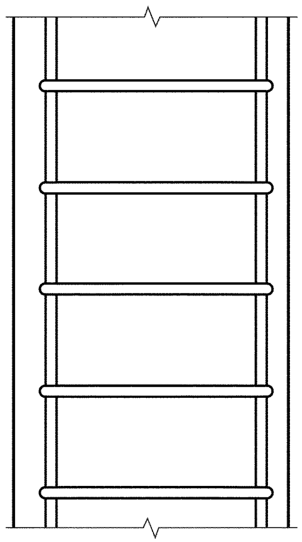




Longitudinal bars and spiral reinforcement

Longitudinal bars and lateral ties





8. Short Columns

AXIAL COMPRESSION

- The ratio of longitudinal steel area A_{st} to gross cross section A_g is 0.01~0.08 (=1%~8%)

  - lower limit 0.01

    ; to ensure resistance to bending moment not considered in the analysis

    ; to reduce the effects of creep and shrinkage of concrete

  - upper limit 0.08

    ; for economy

    ; to avoid the congestion of the reinforcement particularly where the steel must be spliced.

8. Short Columns

AXIAL COMPRESSION

- Short Column : the strength is governed by the strength of material and the geometry of the cross section
- Long Column (slender column) : the strength may be significantly reduced by lateral deflections

<u>Note</u>

90% of columns braced against sidesway and 40% of unbraced columns could be designed as SHORT Columns. In spite of high-strength material and improved methods of dimensioning members, most column including slender columns are considered (designed) as short column.

8. Short Columns

AXIAL COMPRESSION

General Principle and Requirement

- for both concrete and steel remain elastic

$$f_s = nf_c \qquad (1)$$

where modular ratio $n = E_s / E_c$, and the axial force P is

$$P = f_c \underbrace{\left[A_g + (n-1)A_{st} \right]}_{\text{transformed section area}} \qquad (2)$$

- The nominal strength of an axially loaded column

$$P_n = 0.85 f_{ck} A_c + A_{st} f_y \quad \text{or}$$
$$= 0.85 f_{ck} (A_g - A_{st}) + A_{st} f_y \qquad (3)$$

8. Short Columns

AXIAL COMPRESSION

General Principle and Requirement

- KCI Code 3.3.3 provides ϕ factor of 0.70 for spirally reinforced columns and 0.65 for tied columns

Note

KCI factors are lower for columns than for beams (Φ=0.85), because of their greater importance in a structure.

8. Short Columns

AXIAL COMPRESSION

General Principle and Requirement

- Upper limit of the axial load allowing for accidental eccentricities of loading not considered in the analysis (KCI Code 6.2.2)

  for spirally reinforced columns

$$\phi P_{n(\max)} = 0.85\phi[0.85 f_{ck}(A_g - A_{st}) + f_y A_{st}] \qquad (4a)$$

with ϕ=0.70

  for tied columns

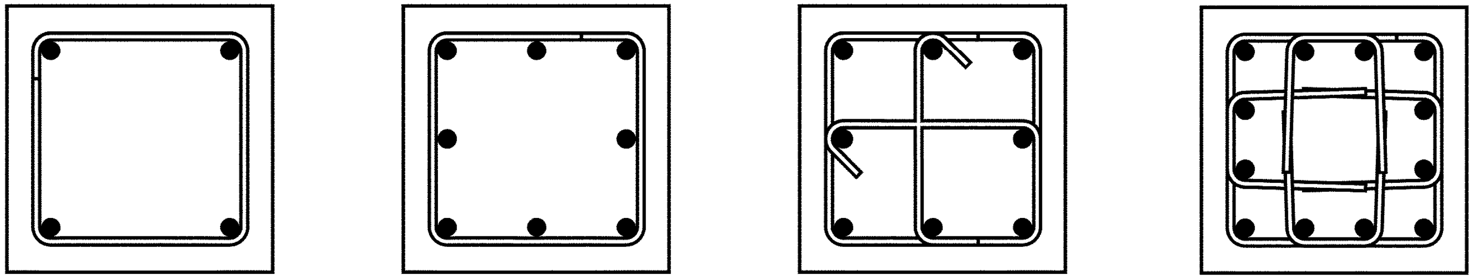
$$\phi P_{n(\max)} = 0.80\phi[0.85 f_{ck}(A_g - A_{st}) + f_y A_{st}] \qquad (4b)$$

with ϕ =0.65

8. Short Columns

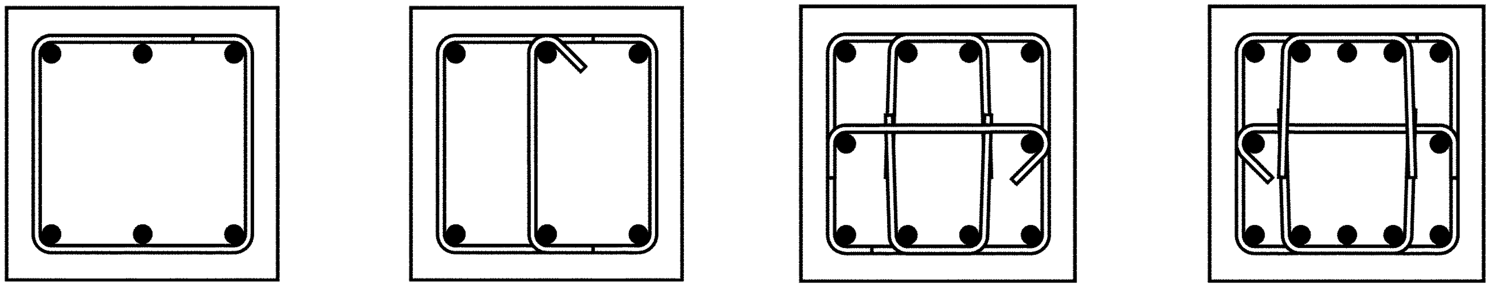
LATERAL TIES AND SPIRALS

Members with large axial forces and small moments



Spacing < 6"   Spacing > 6"

Members with large bending moments



Spacing < 6"   Spacing > 6"

8. Short Columns

LATERAL TIES AND SPIRALS

Functions of Ties and Spirals

i) To hold the longitudinal bars in position in the forms while the concrete is being placed.

ii) To prevent the highly stressed, slender longitudinal bars from buckling outward by bursting the thin cover.

Note

closely spaced spirals serve above two functions. Tie must be designed that those two requirements are met.

⇨ small spacing to prevent buckling and a sufficient number of ties to position and hold all bar.

8. Short Columns

LATERAL TIES AND SPIRALS

KCI Code Requirement for Lateral Ties

KCI Code 5.5.2 (3) writes

(1) Tie bar size ≥ D10 for longitudinal bars ≤ D32

≥ D13 for (longitudinal bars / bundled longitudinal bars) ≥ D35

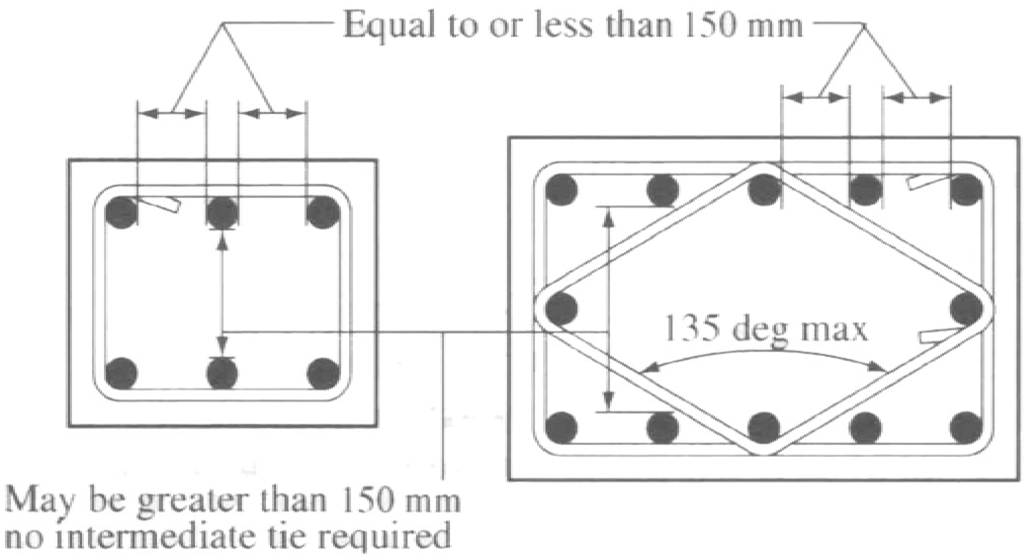
(2) Vertical spacing of ties, *s* ,

s ≤ 16 longitudinal bar diameters,
48 tie bar or wire diameters,
least lateral dimension of compression member

8. Short Columns

(3) At least every other longitudinal bar shall have lateral support from the corner of a tie with an included angle ≤ 135°

No longitudinal bar shall be more than 150mm clear on either side from support bar.



8. Short Columns

KCI Code Requirement for Spirals

KCI Code 5.5.2 (2) writes

(1) For cast in place construction, size of spirals ≥ 10mm

(2) 25mm ≤ clear spacing between spirals ≤ 75mm

(3) Anchorage of spirals shall be provided by 1.5 extra turns of spiral bars or wire at each end of spiral unit.

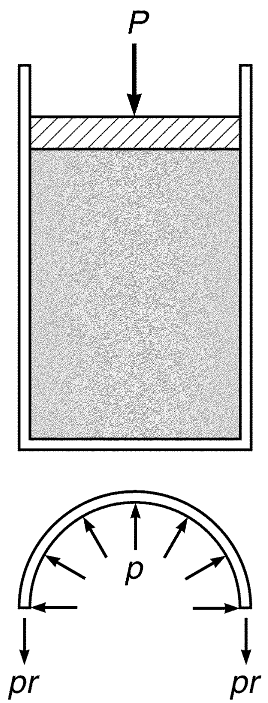
(4) Lap splices not less than the larger of 300mm and the length indicated in one of the followings.

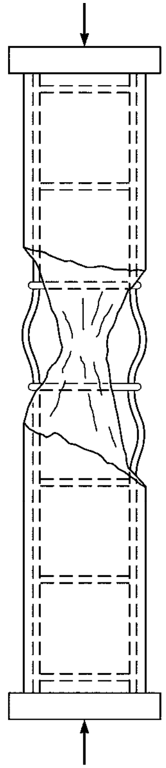
i) $48d_b$ for deformed uncoated bar or wire

ii) $72d_b$ for plain uncoated bar or wire

8. Short Columns

Structural Effect of Spiral and Tie





Spiral restrains lateral expansion

(Poisson's effect)

Tie supports long bars (reduce buckling)

Negligible restraint to lateral exposure of core

8. Short Columns

Behavior of Tied / Spiraled Column

- A TIED column fails at the load given by Eq.(3)

  ; at this load the concrete fails by crushing and shearing outward along the inclined planes

  ; the longitudinal bars by buckling outward between ties.

- A SPIRALLY reinforced column at the same load

  ; the longitudinal steel and the concrete within the core are prevented from moving outward by the spiral. (confining effect)

  ; but outer shell (concrete cover) spalls off.

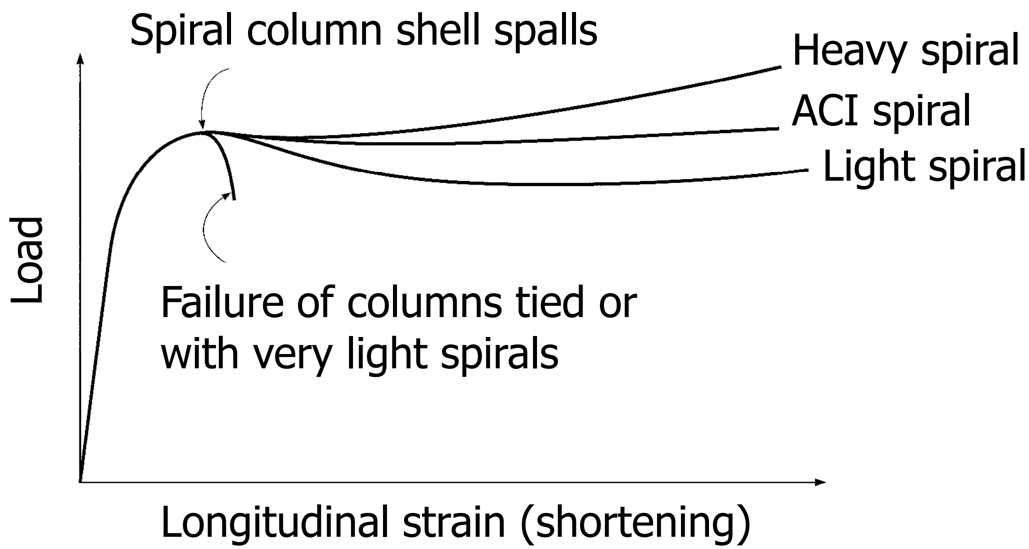
- A spirally reinforced column is more ductile than tied column

8. Short Columns

Behavior of Tied / Spiraled Column

- Excessive capacity beyond the spalling load is wasted in terms of serviceability.

  ⇨ KCI Code provides a minimum spiral reinforcement of such an amount that its contribution to the carrying capacity is just slightly larger than that of the concrete in shell.



8. Short Columns

Spiral Reinforcement Ratio

- The increase in compressive strength of the core concrete provided the confining effect of spiral steel.
(Empirical equation)

$$f_c^* - 0.85 f_{ck} = 4.0 f_2' \qquad (5)$$

where f_c^*=compressive strength of spirally confined core concrete

$0.85 f_{ck}$=compressive strength of unconfined concrete

f_2'=lateral confinement stress in core concrete produced by spiral

8. Short Columns

Spiral Reinforcement Ratio

- The confinement stress f_2' can be calculated assuming that the spiral steel reaches f_y when the column fails.

From a hoop tension analysis

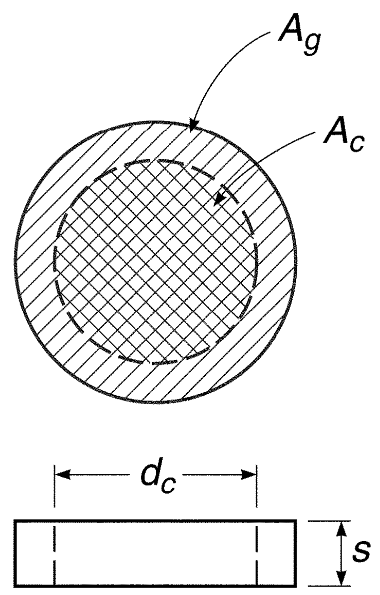
$$f_2' = \frac{2A_{sp} f_y}{d_c s} \qquad (6)$$

A_{sp}=cross-sectional area of spiral wire

f_y=yield strength of spiral steel

d_c=outside diameter of spiral (core concrete diameter)

s=spacing or pitch of spiral wire



Confinement of core concrete due to hoop tension

8. Short Columns

Spiral Reinforcement Ratio

- Spiral reinforcement ratio is defined as

$$\rho_s = \frac{\text{Vol of spiral}}{\text{Vol of core}} = \frac{\pi d_c A_{sp}}{\frac{\pi d_c^2 s}{4}} = \frac{4A_{sp}}{d_c s} \qquad (7)$$

from which

$$A_{sp} = \frac{\rho_s d_c s}{4} \qquad (8)$$

Substitute Eq.(8) into Eq.(6)

$$f_2^{'} = \frac{\rho_s f_y}{2} \qquad (9)$$

8. Short Columns

Spiral Reinforcement Ratio

- Strength contribution of the shell=$0.85f_{ck}(A_g\text{-}A_c)$ (10)

  Substitute Eq.(9) into Eq.(5) and multiplying by A_c, then

  compressive strength provided by the spiral=$4.0f_2'A_c$

  $=2\rho_s f_y A_c$ (11)

Note

The basic design concept of spiral is that the strength gain by the spiral should be at least equal to that lost when the shell spalls.

8. Short Columns

Spiral Reinforcement Ratio

- Therefore, Eq.(10) should be equal to Eq.(11)

$$0.85 f_{ck} (A_g - A_c) = 2\rho_s f_y A_c$$

$$\Rightarrow \quad \rho_s = 0.425\left(\frac{A_g}{A_c} - 1\right)\frac{f_{ck}}{f_y} \qquad (12)$$

- KCI Code 6.4.2 (3) provides

$$\rho_s \geq 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f_{ck}}{f_y} \qquad (13)$$

where $f_y \leq 700$MPa. For $f_y > 400$MPa, lap splicing shall not be used.

8. Short Columns

COMPRESSION + BENDING OF REC. COLUMNS

- Compression members should be almost always designed to resist against bending.

  ⇦ building continuity : the girders are resisted by the abutting column

  wind forces

  loads carried eccentrically on column bracket

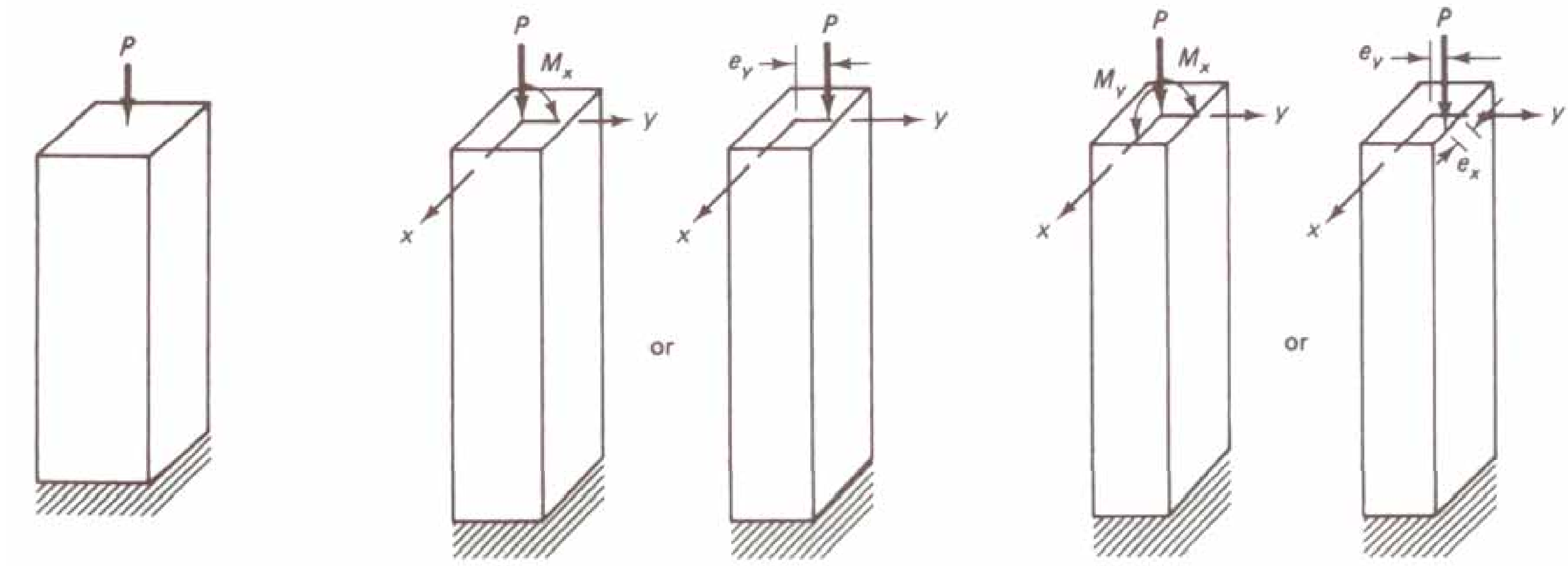
  arch axis does not coincide with the pressure line

  imperfection of construction (eccentricity)

8. Short Columns

COMPRESSION + BENDING OF REC. COLUMNS

usually bending moment is represented by axial load times eccentricity.



Concentrically loaded column

axial load + uniaxial moment

axial load + biaxial moment

8. Short Columns

COMPRESSION + BENDING OF REC. COLUMNS

- Columns with small eccentricity, *e* ,

  - compression over the entire concrete section at lower loads

  - fail by crushing of the concrete and yielding of the steel in compression

- Columns with large eccentricity, *e* ,

  - tension over at least a part of the section

  - fail due to tension yielding of the steel on the side farthest from the loading point.

8. Short Columns

COMPRESSION + BENDING OF REC. COLUMNS

- For columns

  - load stages below the ultimate are NOT important.
  - cracking of concrete due to large eccentricity is not a serious problem.
  - lateral deflections at service load levels are seldom important.
  - Just satisfy

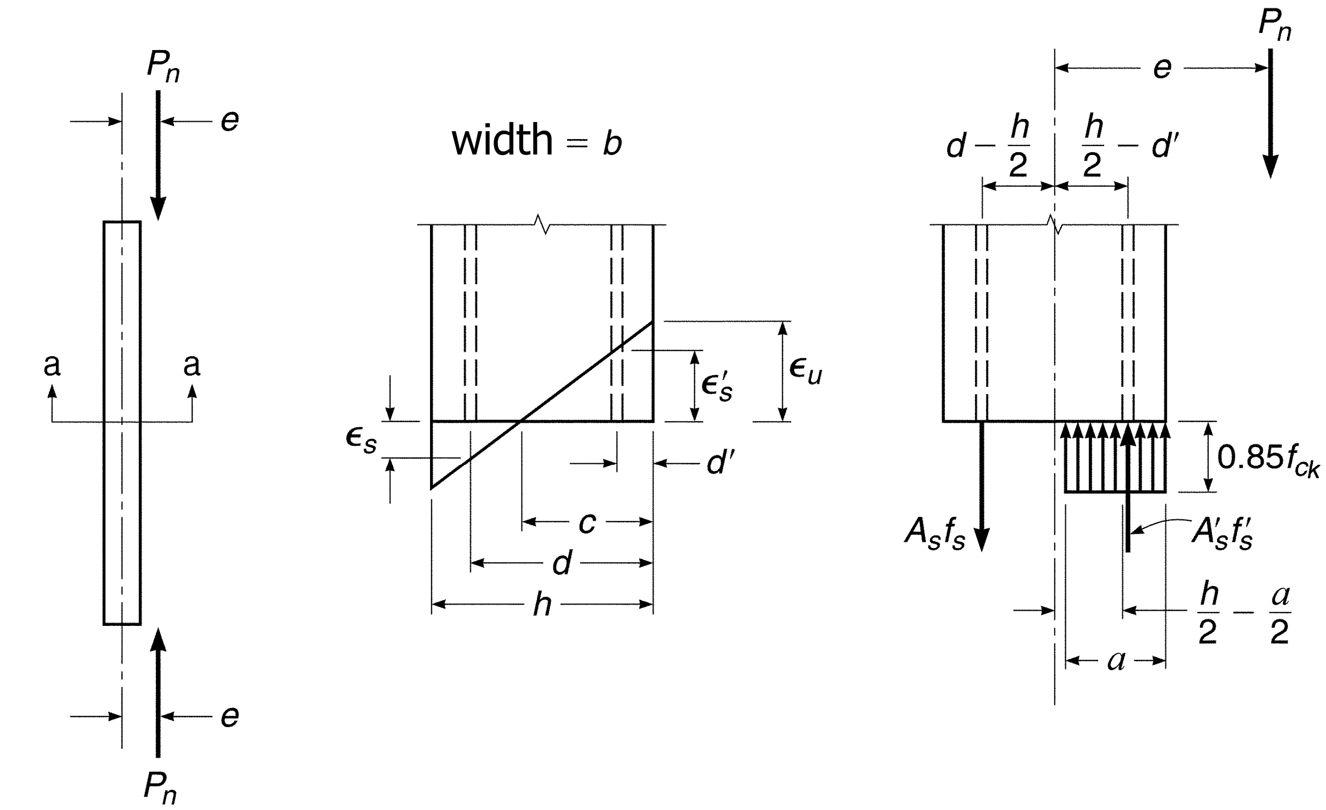
$$\phi M_n \geq M_u$$

$$\phi P_n \geq P_u$$

Q) What does this mean?

8. Short Columns

INTERACTION DIAGRAM



loaded column

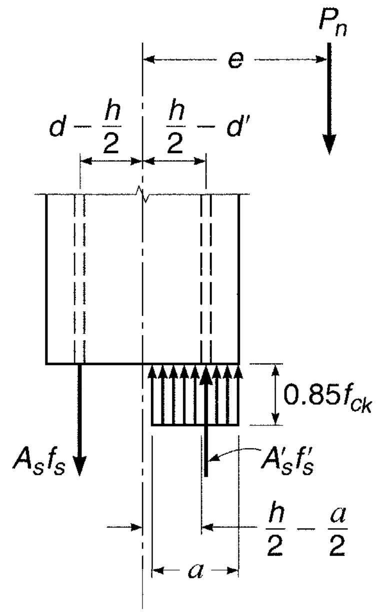
strain distribution at section a-a

stresses and forces at nominal strength

8. Short Columns

INTERACTION DIAGRAM

Basic Relations for Rectangular Eccentrically Compressed Member



- Equilibrium between external and internal axial forces requires that

$$P_n = 0.85 f_{ck} ab + A_s' f_s' - A_s f_s \qquad (14)$$

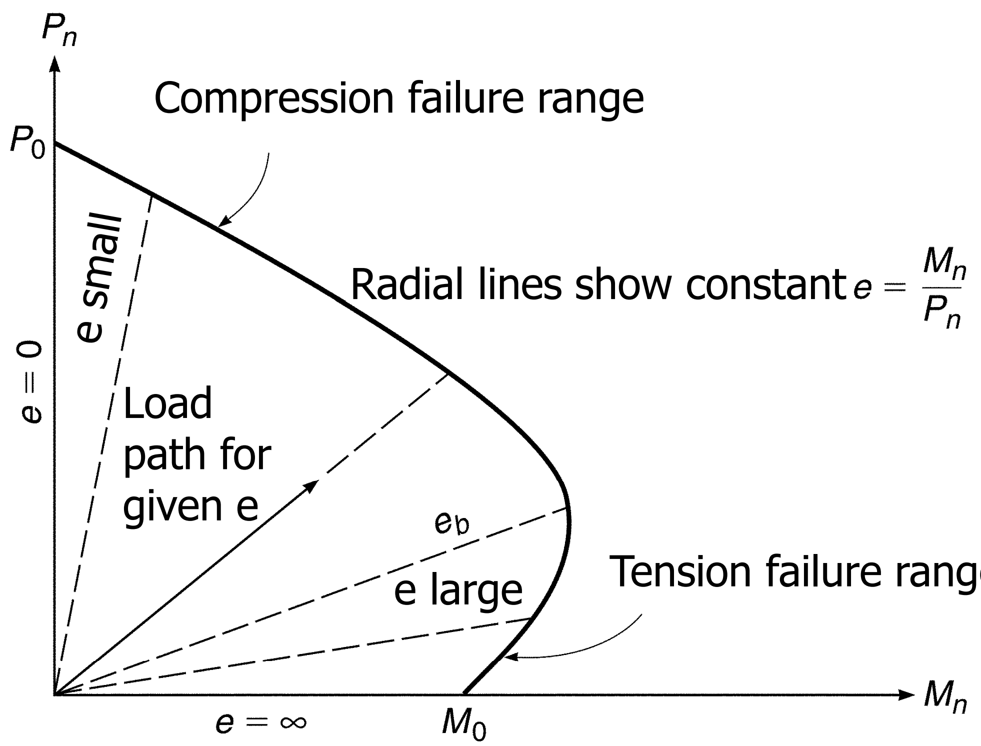
- Moment about the centerline of the section

$$\underbrace{M_n = P_n e}_{\text{External}} = \underbrace{0.85 f_{ck} ab\left(\frac{h}{2} - \frac{a}{2}\right) + A_s' f_s'\left(\frac{h}{2} - d'\right) + A_s f_s\left(d - \frac{h}{2}\right)}_{\text{Internal}} \qquad (15)$$

8. Short Columns

Strength Interaction Diagram

Defines the failure load and failure moment for a given column for the full range of eccentricities 0 to ∞



8. Short Columns

Strength Interaction Diagram

How to obtain *P-M* interaction diagram

- Start with a selection of neutral axis distance, c , then, for the tension steel

$$\varepsilon_s = \varepsilon_u \frac{d-c}{c} \tag{16}$$

$$f_s = \varepsilon_u E_s \frac{d-c}{c} \le f_y \tag{17}$$

while for the compression steel

$$\varepsilon_s' = \varepsilon_u \frac{c-d'}{c} \tag{18}$$

$$f_s' = \varepsilon_u E_s \frac{c-d'}{c} \le f_y \tag{19}$$

8. Short Columns

- The concrete stress block has depth

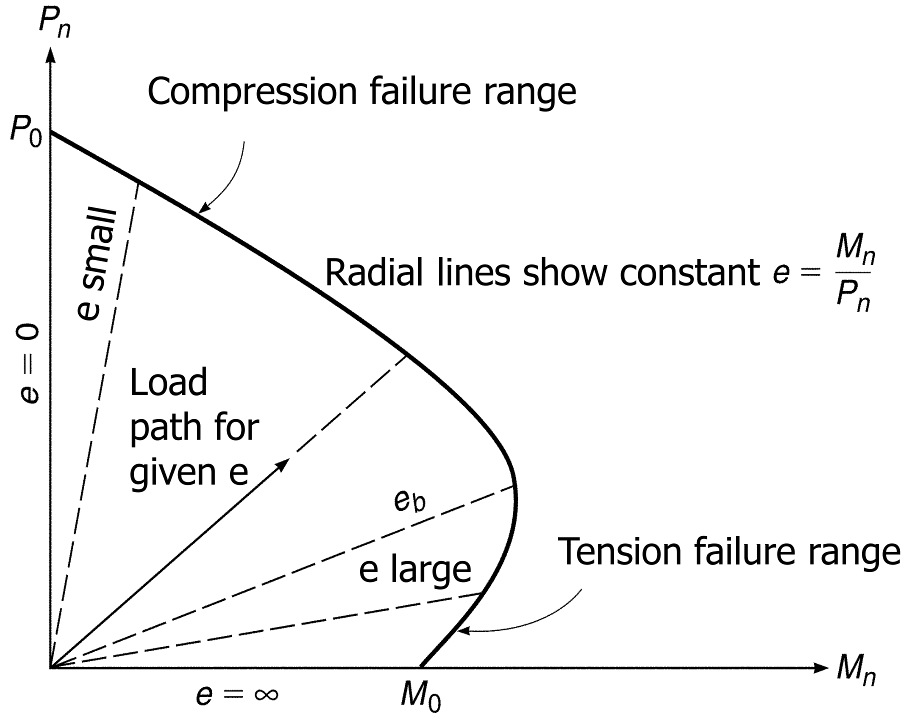
$$a = \beta_1 c \le h \qquad (20)$$

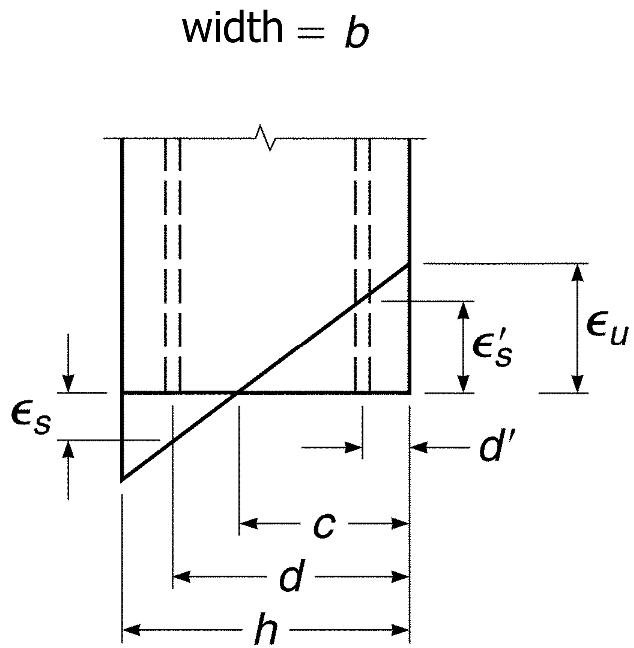
then, the concrete compressive resultant is

$$C = 0.85 f_{ck} ab \qquad (21)$$

- Calculate the nominal axial force P_n and nominal moment M_n corresponding to the selected c from Eq.(14) & (15)
- Repeat the above procedure for successive choices of neutral axis.

BALANCED FAILURE





$$c = c_b = d\frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} \tag{22}$$

$$a = a_b = \beta_1 c_b \tag{23}$$

8. Short Columns

Example 8.1

A 300∗500mm column is reinforced with four D22 bars of area 387mm^2 each. The concrete cylinder strength f_{ck}=24MPa and the steel yield strength f_y=350MPa

(1) Determine the P_b, M_b, and corresponding e_b for balanced failure.

(2) Determine the load and moment for a representative point in the tension failure region of the interaction curve.

(3) Repeat (2) for compression.

(4) Determine the axial load strength for zero eccentricity.

8. Short Columns

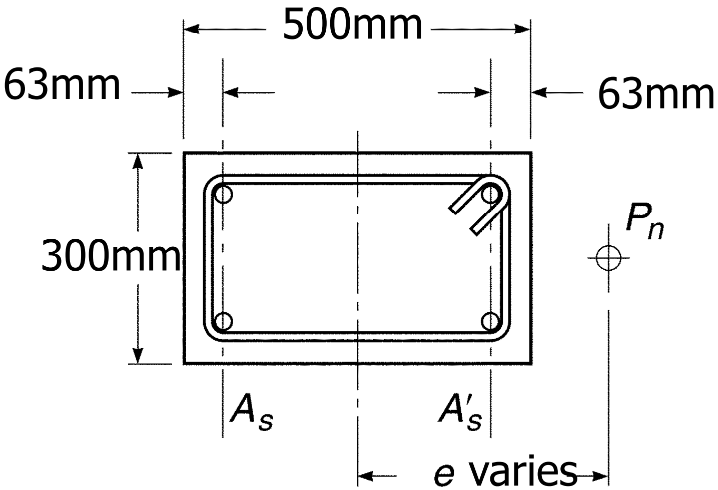
Example 8.1 (cont.)

(5) Sketch the interaction diagram

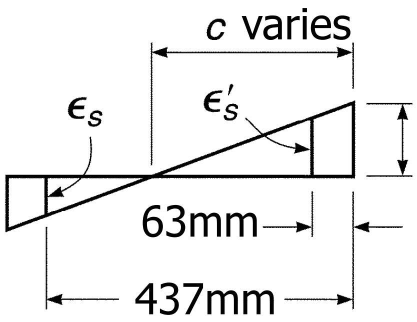
(6) Design the transverse reinforcement based on KCI Code.

8. Short Columns

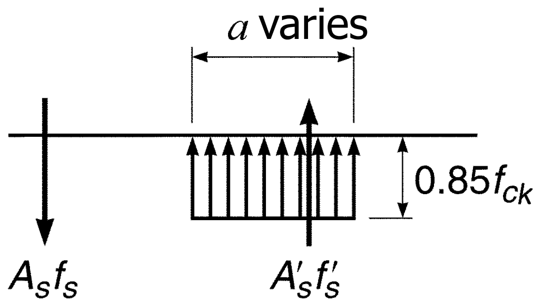
Solutions



<Cross section>



<Strain distribution>



<Stresses and forces>

8. Short Columns

Solution (a)

- The neutral axis for balanced failure

$$c_b = d\frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} = (437)\frac{0.003}{0.003 + \dfrac{350}{2.0\times10^5}} = 276mm \qquad (22)$$

$$\Rightarrow \quad a_b = \beta_1 c_b = (0.85)(276) = 234mm$$

- The compressive steel stress is

$$f_s^{'} = \varepsilon_u E_s \frac{c_b - d^{'}}{c_b} = (0.003)(2.0\times10^5)\left(\frac{276-63}{276}\right) \qquad (19)$$

$$= 463N/mm^2 \geq 350N/mm^2$$

$\therefore$ compressive steel also yields.

8. Short Columns

Solution (a)

- The concrete compressive resultant is

$$C = 0.85 f_{ck} a_b b = (0.85)(24)(234)(300) \tag{21}$$

$$= 1,423,080N = 1,423kN$$

- The balanced load P_b is

$$P_b = 0.85 f_{ck} ab + A_s' f_y - A_s f_y \tag{14}$$

$$= 1,432 + (774 \times 350 - 774 \times 350) \times 10^{-3}$$

$$= 1,432kN$$

8. Short Columns

Solution (a)

- The balanced moment M_b is

$$M_b = 0.85 f_{ck} a_b b\left(\frac{h}{2}-\frac{a_b}{2}\right)+A_s' f_y\left(\frac{h}{2}-d'\right)+A_s f_y\left(d-\frac{h}{2}\right) \qquad (15)$$

$$=(1,432)\left(\frac{500}{2}-\frac{234}{2}\right)+(774)(350)\left(\frac{500}{2}-63\right)(10^{-6})$$

$$+(774)(350)\left(437-\frac{500}{2}\right)(10^{-6})$$

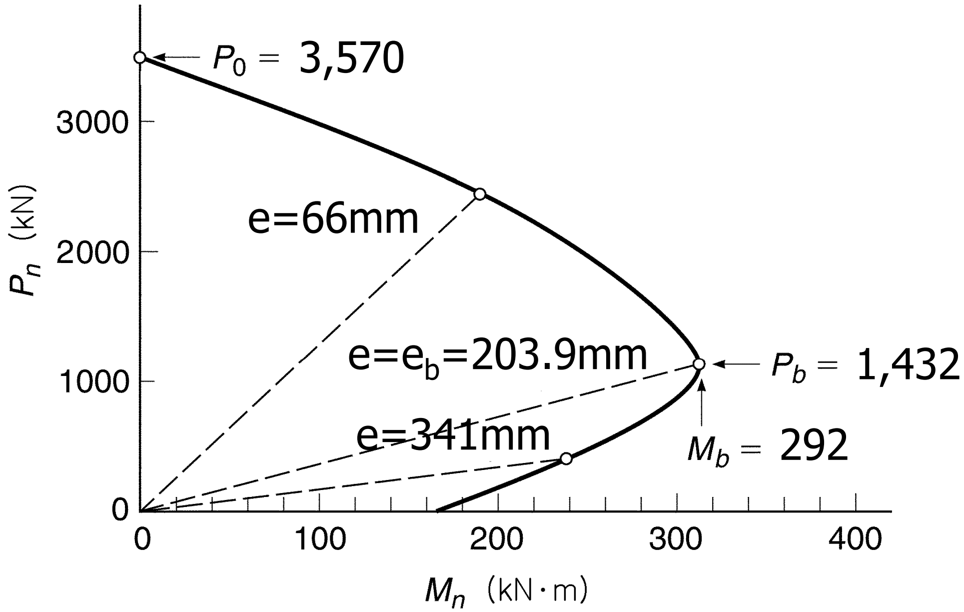
$$=292 kN \bullet m$$

8. Short Columns

Solution (a)

- The corresponding eccentricity e_b is

$$e_b = \frac{M_b}{P_b} = \frac{292}{1,432} = 0.2039m = \underline{204mm}$$



8. Short Columns

Solution (b)

- Choose any c smaller than c_b=276mm.

  For example, c=130mm. By definition, $f_s=f_y$

$$f_s' = \varepsilon_u E_s \frac{c-d'}{c} = (0.003)(2.0\times 10^5)\left(\frac{130-63}{130}\right)$$

$$= 309.2 N/mm^2$$

and $a = \beta_1 c = (0.85)(130) = 111mm$

- The corresponding resultant is

$$C = (0.85)(24)(111)(300)(10^{-3}) = 680kN$$

8. Short Columns

Solution (b)

- Force equilibrium

$$P_n = 0.85 f_{ck} ab + A_s' f_s' - A_s f_y$$

$$= 650 + (774)(309)(10^{-3}) - (774)(350)(10^{-3})$$

$$= 648 kN$$

- The moment capacity

$$M_n = (680)\left(\frac{500}{2} - \frac{111}{2}\right)(10^{-3}) + (774)(309.2)(10^{-3})\left(\frac{500}{2} - 63\right)(10^{-3})$$

$$+ (774)(350)(10^{-3})\left(437 - \frac{500}{2}\right)(10^{-3})$$

$$= 228 kN \bullet m$$

8. Short Columns

Solution (b)

- The corresponding eccentricity

$$e = \frac{228}{648} = 0.352m = \underline{352mm}$$

8. Short Columns

Solution (c)

- Choose any c larger than c_b=276mm

  For example, c=460mm and then a=(0.85)(460)=391mm

- Compression resultant is

$$C = (0.85)(24)(391)(300)(10^{-3}) = 2,393kN$$

- From Eq.(17) the stress in the steel at the bottom of the column cross section is

$$f_s = \varepsilon_u E_s \frac{d-c}{c} = (0.003)(2.0\times10^5)\left(\frac{437-460}{460}\right) = -30N/mm^2$$

Note that the negative f_s indicates that A_s is in compression if c is greater than d.

8. Short Columns

Solution (c)

- The compressive steel stress is

$$f_s^{'} = (0.003)(2.0\times10^5)\left(\frac{460-63}{460}\right) = 518N/mm^2$$

but f_s' should be less than f_y=350N/mm^2

$$\therefore\ f_s^{'} = 350N/mm^2$$

- Column capacity is

$$P_n = 2,393 + (774)(350)(10^{-3}) - (774)(-30)(10^{-3}) = 2,687kN$$

$$M_n = (2,393)\left(\frac{500}{2}-\frac{391}{2}\right)(10^{-3}) + (774)(350)(10^{-3})\left(\frac{500}{2}-63\right)(10^{-3})$$

$$+(774)(-30)(10^{-3})\left(437-\frac{500}{2}\right)(10^{-3}) = 177kN\bullet m$$

8. Short Columns

Solution (c)

- The corresponding eccentricity

$$e = \frac{177}{2,687} = 0.066m = \underline{66mm}$$

8. Short Columns

Solution (d)

- If concentrically loaded, the axial strength of the column corresponds to e=0 and c=∞

  For this case

$$P_n = \left[(0.85)(24)(300)(500) + (4 \times 387)(350)\right](10^{-3}) = 3,602kN$$

Note

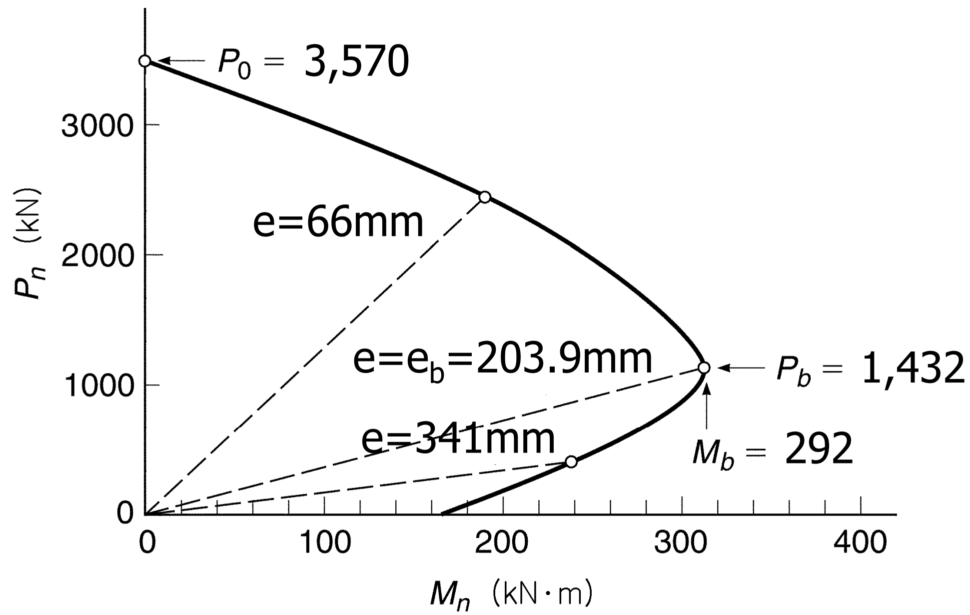
For this as well as preceding calculations, subtraction of the concrete displaced by the steel has been neglected.

The difference can be neglected, except for columns with the reinforcement ratio close to 8 percents.

8. Short Columns

Solution (e)

Results from (a) ~ (e) plus similar repetitive. Calculations will give a *P-M* interaction diagram as below



8. Short Columns

Solution (f)

- The design of the column will be carried out of following the KCI Code.

  For the minimum permitted tie diameter of D10, the tie spacing should not to exceed.

$$48d_t = (48)(10) = 480mm$$

$$16d_l = (16)(22) = 352mm$$

$$b = \underline{300mm}$$

Note

To save the steel, the 1st and 2nd restrictions can be reduced by using smaller bar diameter.

Is it O.K? No, this would not meet KCI code restriction.

DISTRIBUTED REINFORCEMENT

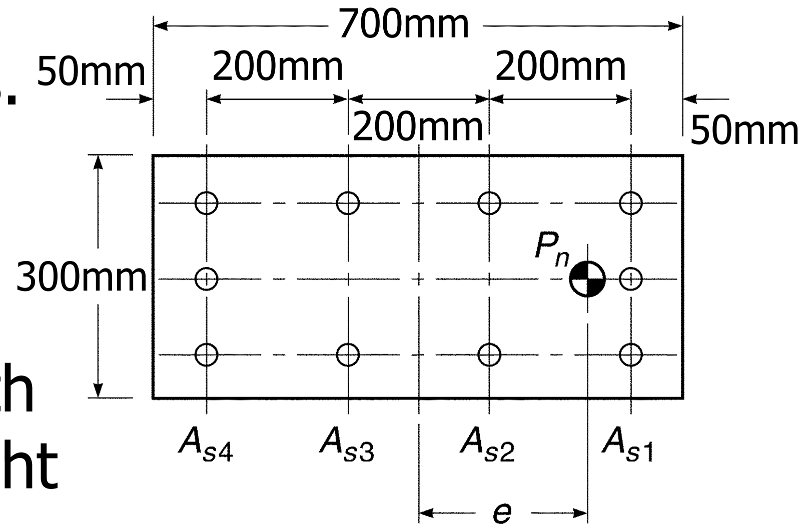
Example 8.2 Analysis of Eccentrically Loaded Column with Distributed Reinforcement.

Consider a column which is reinforced with ten D35 rebars distributed around the perimeter as shown.

Load P_n will be applied with eccentricity e about the strong axis.

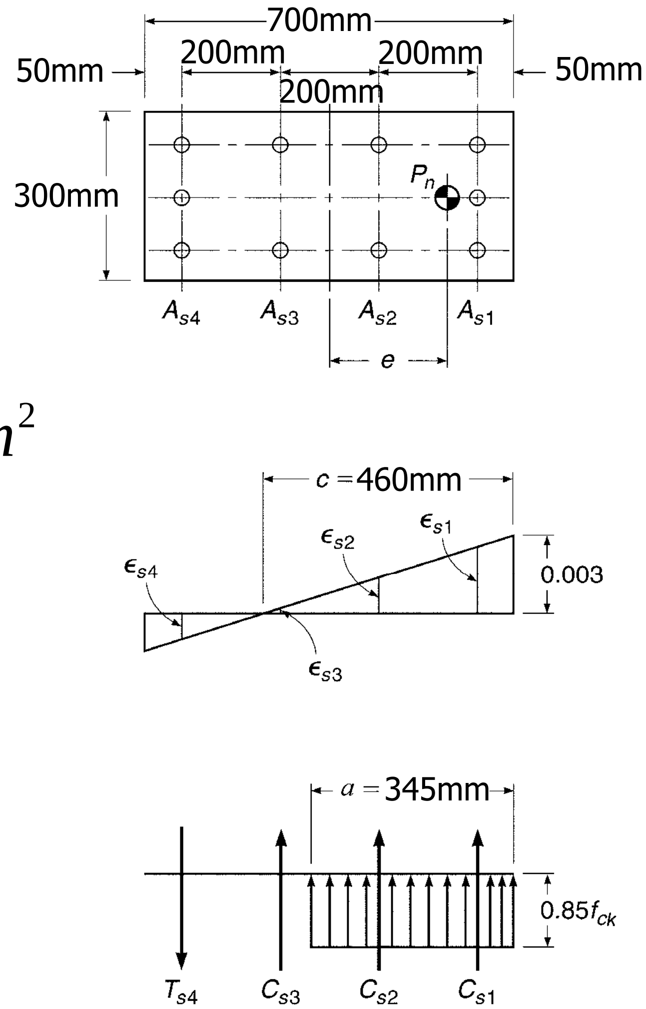
f_{ck}=42MPa and f_y=550MPa.

Find the load and moment corresponding to a failure point with neutral axis c=460mm from the right face.



Solution

When the concrete reaches 0.003, the strains at the each locations of the four bar groups are found from similar triangles (strain compatibility)



$\varepsilon_{s1} = 0.00267$ ⇨ $f_{s1} = \varepsilon_{s1} \cdot E_s = 534 N/mm^2$

$\varepsilon_{s2} = 0.00140$ ⇨ $f_{s2} = 280 N/mm^2$

$\varepsilon_{s3} = 0.0000652$ ⇨ $f_{s3} = 13 N/mm^2$

$\varepsilon_{s4} = .000124$ ⇨ $f_{s4} = 248 N/mm^2$

It should be noted that ε_{si} should not exceed f_y=550MPa

8. Short Columns

- For f_{ck}=42MPa, β_1=0.75 and the depth of the equivalent rectangular stress block is a=0.75*460=345mm. The concrete compression resultant is

$$C = 0.85 f_{ck} ab$$

$$= (0.85)(42)(345)(300)(10^{-3}) = 3,695kN$$

- The respective rebar force resultants are

$$C_{s1} = A_{s1} \cdot f_{s1} = (956.6 \times 3)(534)(10^{-3}) = 1,533kN$$

$$C_{s2} = A_{s2} \cdot f_{s2} = (956.6 \times 2)(280)(10^{-3}) = 535kN$$

$$C_{s3} = A_{s3} \cdot f_{s3} = (956.6 \times 2)(13)(10^{-3}) = 24.8kN$$

$$T_{s4} = A_{s4} \cdot f_{s4} = (956.6 \times 3)(248)(10^{-3}) = 712kN$$

8. Short Columns

- The axial load and moment that would produce failure for a neutral axis 460mm are

$$P_n = 3,695 + 1,533 + 535 + 24.8 - 712$$

$$= \underline{5,076 kN}$$

$$M_n = [3,695 \times \left(\frac{700}{2} - \frac{345}{2}\right) + 1,533 \times \left(\frac{700}{2} - 50\right) + 535 \times \left(\frac{700}{2} - 250\right)$$

$$-24.8 \times \left(\frac{700}{2} - 250\right) + 712 \times \left(\frac{700}{2} - 50\right)] \times 10^{-3}$$

$$= \underline{1,380 kN \cdot m}$$

- The corresponding eccentricity e=1,380/5,076=0.272m

8. Short Columns

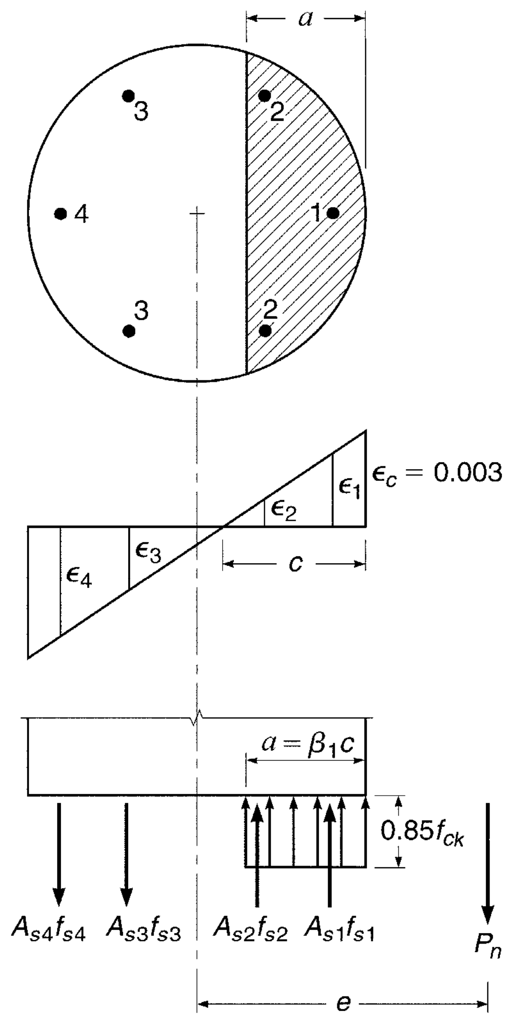
Homework 2

Construct a *P-M* interaction diagram for the column given in Example 8.2

8. Short Columns

CIRCULAR COLUMNS

- When load eccentricities are small, spirally reinforced column show greater toughness.
- Spirally reinforced columns permit a somewhat more economical utilization of the materials.
- Automated fabrication is available.



8. Short Columns

KCI CODE PROVISION FOR COLUMN DESIGN

Strength Reduction Factor (KCI 3.3.3)

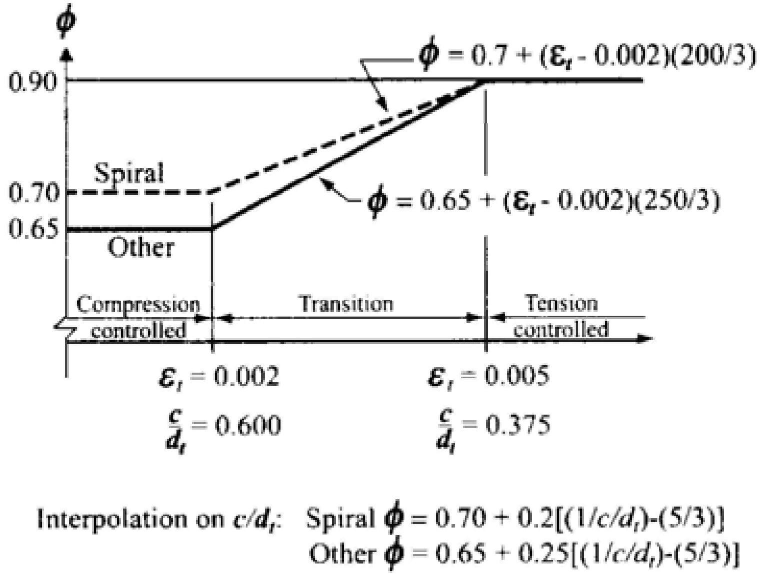
(1) Axial tension, and axial tension with flexure, Φ=0.85

(2) Axial compression and axial compression with flexure

for spiralled members Φ=0.70

for the others Φ=0.65

(3) For intermediate net strain ε_t Φ varies.



8. Short Columns

KCI CODE PROVISION FOR COLUMN DESIGN

Strength Reduction Factor (KCI 3.3.3)

Why less than Φs for flexure and shear?

- Underreinforced flexural members are dominated by the "guaranteed" yield strength of the steel.
- Columns depends on relatively poor concrete.

  - cast-in place

  - material segregation

  - loss of cross section : electrical and the other conduits

- Column fail is more catastrophic than that of a single beam.

8. Short Columns

KCI CODE PROVISION FOR COLUMN DESIGN

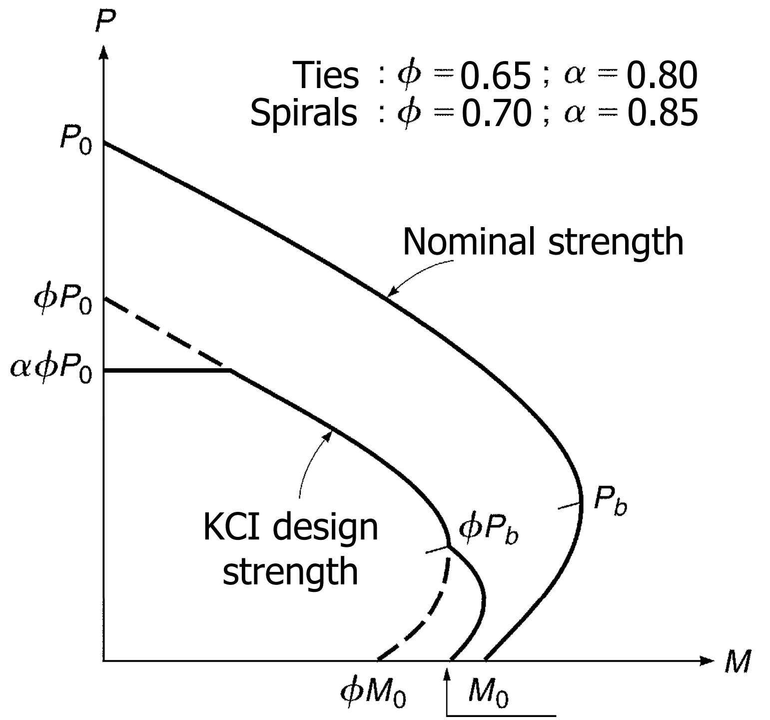
Strength Reduction Factor (KCI 3.3.3)

α factor

- For columns with very small or zero eccentricity

  α=0.80 for Ties

  α=0.85 for spirals



⇦ Accidental construction misallignment and the other unforeseen factors may produce actual eccenricity.

8. Short Columns

DESIGN AIDS

- *P-M* diagram permits the direct design of eccentrically loaded columns throughout the common range of strength and geometric variables.
- One of the following ways can be used for a given factored load P_u and equivalent eccentricity $e=M_u/P_u$

8. Short Columns

DESIGN AIDS

Method 1

- Select trial dimension b and h
- Calculate the ratio γ and select the corresponding column design chart (P-M diagram), where γ is the ratio of distance between centroids of outer rows of bars and thickness of cross section in the direction of bending.
- Calculate

$$K_n = \frac{P_u}{\phi f_{ck} A_g} \qquad R_n = \frac{P_u e}{\phi f_{ck} A_g h}$$

- From P-M diagram, read the required reinforcement ratio ρ_g
- Calculate the total steel area $A_{st} = \rho_g bh$

8. Short Columns

DESIGN AIDS

Method 2

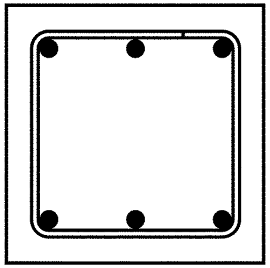
- Select the reinforcement ratio ρ_g.
- Choose a trial value of h and calculate e/h and γ.
- From the corresponding chart, read K_n and calculate the required A_g.
- Calculate $b=A_g/h$
- Revise the trial value of h if necessary to obtain well-proportioned section
- Calculate the total steel area $A_{st}=\rho_g bh$

8. Short Columns

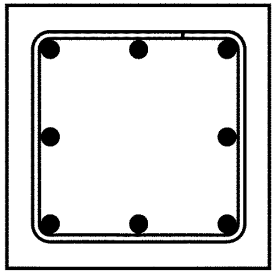
DESIGN AIDS

Choice of Column Type

- For *e/h* < 0.1 : spirally reinforced columns are more efficient but are more expensive (form work + cost of spirals)
- For *e/h* > 0.2 : tied column with bars on two faces



- For *e/h* < 0.2 : tied column with bars on four faces. Also when biaxial bending is expected.



8. Short Columns

Example 8.3 Selection of reinforcement for column of given size.

In a two-story building, a exterior column is to be designed for a service dead load of 1,000kN, maximum live load of 1,500kN, dead load moment of 60kN·m, and live load moment of 90kN·m.

The minimum live load compatible with the full live load moment is 800kN, obtained when no live load is placed on the roof but a full live loading placed on the second floor.

Architectural considerations require that a rectangular column be used with b=400mm, and h=500mm

8. Short Columns

Example 8.3

Material strengths are f_{ck}=27MPa and f_y=400MPa.

(1) Find the required column reinforcement for the condition that the full live load acts.

(2) Check to ensure that the column is adequate for the condition of no live load on the roof.

8. Short Columns

Solution (1)

- According to the KCI Code provision (2007),

  a factored load P_u is

$$P_u = 1.2 \times 1,000 + 1.6 \times 1,500 = 3,600 kN$$

  a factored moment M_u is

$$M_u = 1.2 \times 60 + 1.6 \times 90 = 216 kN \cdot m$$

- Reinforcement will be placed on opposite end faces and bar cover is estimated to be 75mm.

8. Short Columns

- The column parameters assuming bending about the strong axis.

$$\frac{P_u}{A_g} = \frac{3,600}{(400)(500)}(10^3) = 18MPa$$

$$\frac{M_u}{A_g h} = \frac{216}{(400)(500)(500)}(10^6) = 2.16MPa$$

<u>Note</u>

$$K_n = \frac{P_u}{\phi f_{ck} A_g} \qquad R_n = \frac{M_u}{\phi f_{ck} A_g h}$$

8. Short Columns

- With 75mm cover, the parameter γ is

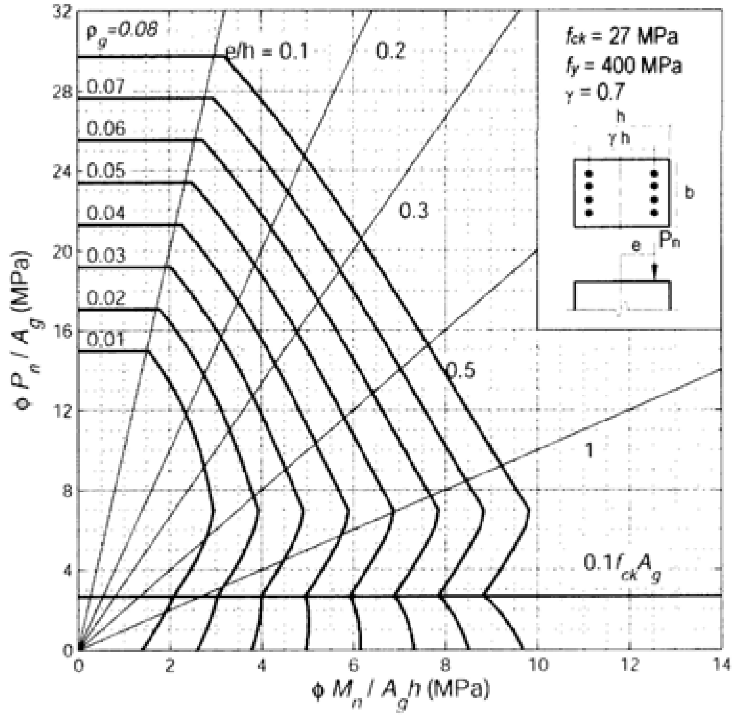
$$\gamma = \frac{(500-150)}{500} = 0.70$$

- From *P-M* diagram

$\rho_g \approx 0.025$

Thus, the required reinforcement is

$A_{st} = (0.025)(400)(500) = 5{,}000mm^2$



- Five D25(=2,534mm^2) bars will be used on each end side.

8. Short Columns

Solution (2)

- With the roof live load absent

$$P_u = 1.2 \times 1,000 + 1.6 \times 800 = 2,480 kN$$

$M_u = 216 kN \cdot m$ as before

- The column parameters

$$\frac{P_u}{A_g} = \frac{2,480}{(400)(500)}(10^3) = 12.4 MPa$$

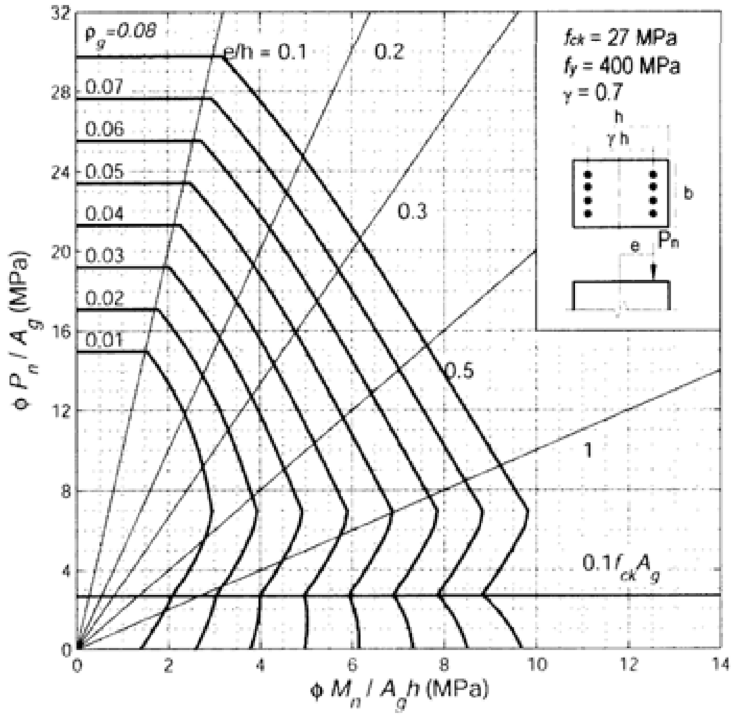
$$\frac{M_u}{A_g h} = \frac{216}{(400)(500)(500)}(10^6) = 2.16 MPa$$

8. Short Columns

- With the same condition, from *P-M* diagram

  $\rho_g \approx 0.01$ is sufficient

  for this condition, less than that required in (1), so no modification is required.



8. Short Columns

Solution (*) Tie design

- Selecting D10 ties, the maximum tie spacing is

$$48d_t = 480mm$$

$$16d_l = \underline{400mm}$$

min. dimension = <u>400mm</u>

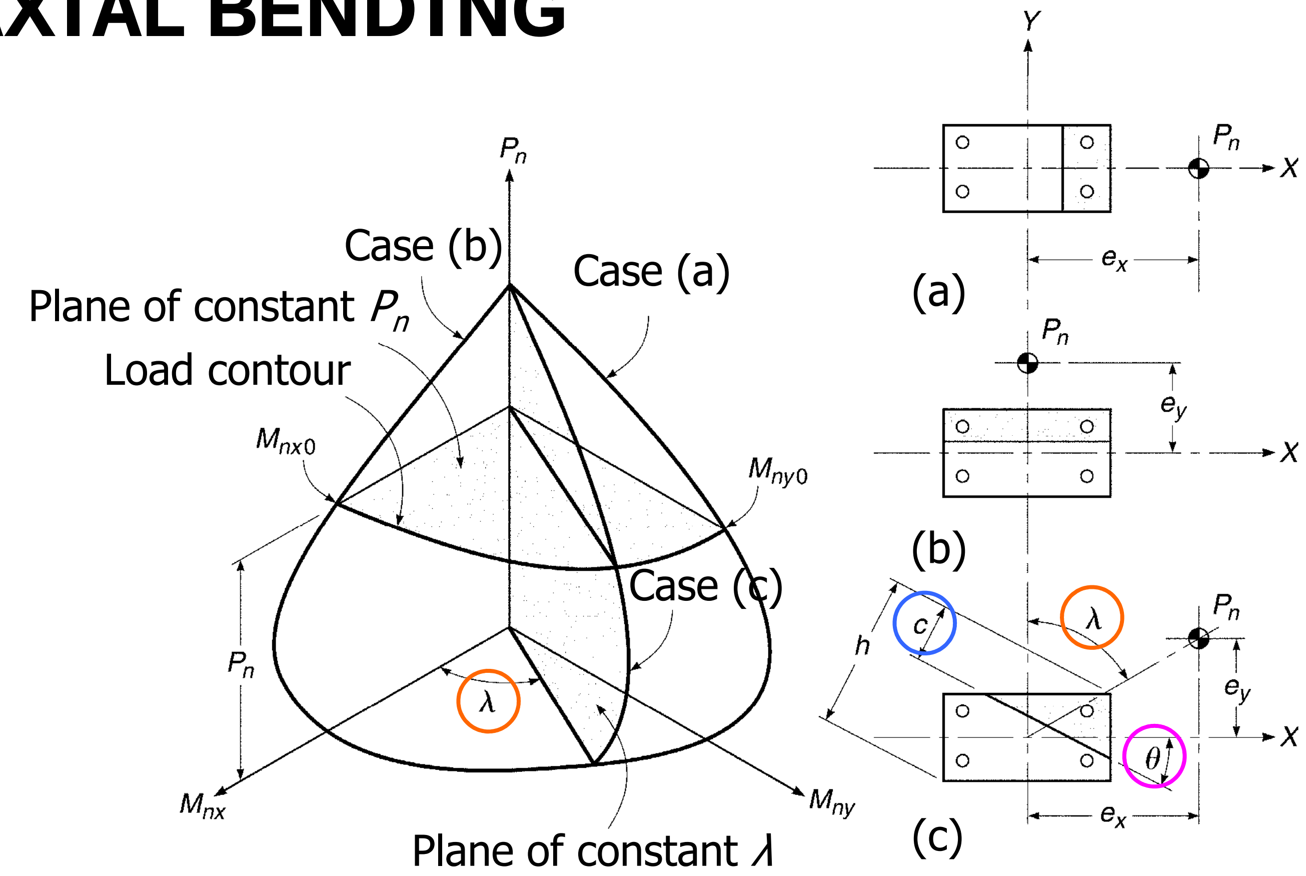
Note

$$e = \frac{M_u}{P_u} = \frac{216}{3,600} = 0.06m = 60mm$$

$$\therefore \frac{e}{h} = \frac{60}{500} = 0.12$$

Therefore, tied column with bars on four sides are preferable!

BIAXIAL BENDING



$$\lambda = \arctan \frac{e_x}{e_y} = \arctan \frac{M_{ny}}{M_{nx}}$$

8. Short Columns

BIAXIAL BENDING

Note

- The neutral axis will not be perpendicular to the resultant eccentricity. ($\lambda \neq \theta$)

  ; For each successive choice of neutral axis, there are unique values of P_n, M_{nx}, and M_{ny}, and only for special cases will the ratio M_{ny}/M_{nx} be such that the eccentricity resultant is perpendicular to the neutral axis chosen.

- The result is that, for successive choices of c for any given θ, the value of λ will vary.

BIAXIAL BENDING

Analysis Methods

- Iterative method using computer
- Approximate method (Handout #2)

  - Load Contour method

  - Reciprocal Load method

LOAD CONTOUR METHOD

This method is based on representing the failure surface by a family of curves for constant P_n.

The general form of non dimensional interaction equation is

$$\left(\frac{M_{nx}}{M_{nx0}}\right)^{\alpha 1} + \left(\frac{M_{ny}}{M_{ny0}}\right)^{\alpha 2} = 1$$

where $M_{nx} = P_n e_y$

$M_{nx0} = M_{nx}$ when $M_{ny} = 0$

$M_{ny} = P_n e_x$

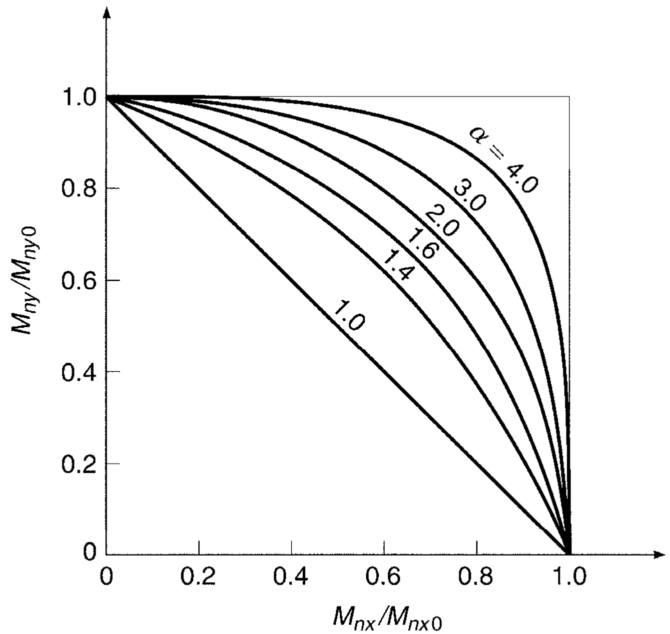
$M_{ny0} = M_{ny}$ when $M_{nx} = 0$

8. Short Columns

LOAD CONTOUR METHOD

a_1 and a_2 are dependent on

column dimension
amount / distribution of steel reinforcement
steel / concrete stress-strain relation
concrete cover
size of ties / spirals, etc.



8. Short Columns

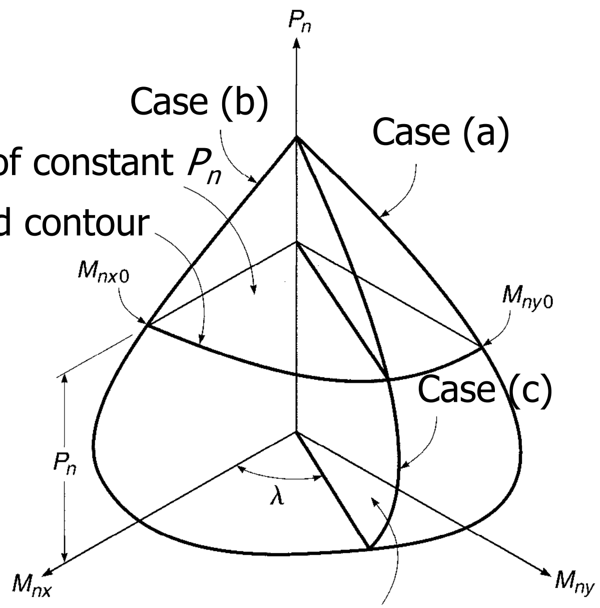
LOAD CONTOUR METHOD

How to use interaction contour

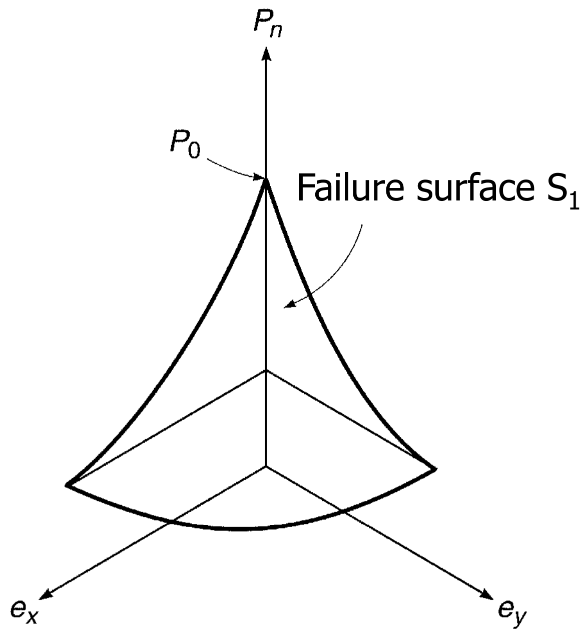
1. P_u, M_{ux}, M_{uy} are known from the structural analysis.
2. For a trial column section, M_{nx0} and M_{ny0} corresponding P_u/ϕ can be obtained by the usual methods for uniaxial bending.
3. By plotting $(M_{nx}/\phi)/M_{nx0}$ and $(M_{ny}/\phi)/M_{ny0}$ in previous figure, check the validity of design.

8. Short Columns

RECIPROCAL LOAD METHOD



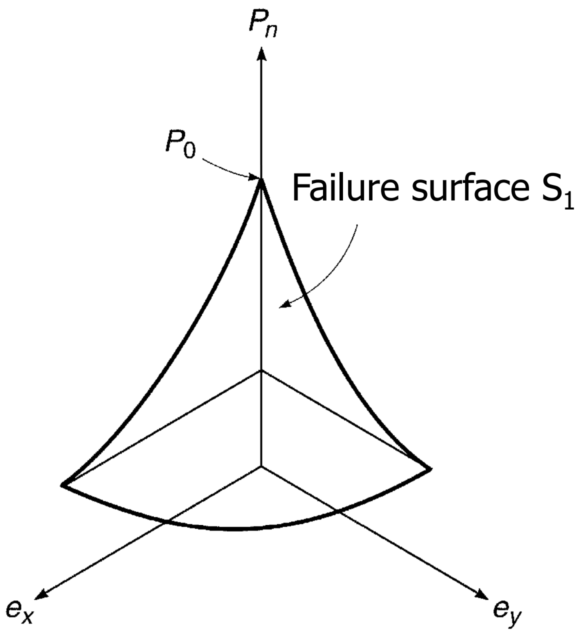
⇨



also see Fig 1 in HO#2

$$e_x = \frac{M_{ny}}{P_n} \quad \text{and} \quad e_y = \frac{M_{nx}}{P_n}$$

RECIPROCAL LOAD METHOD



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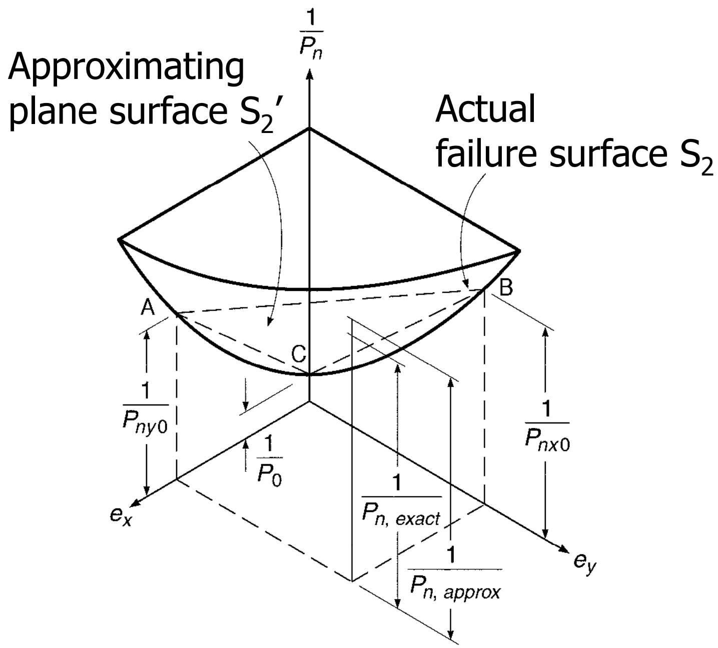


also see Fig 1 in HO#2

also see Fig 2 and Fig 4 in HO#2

$$P_n \quad ⇨ \quad \frac{1}{P_n}$$

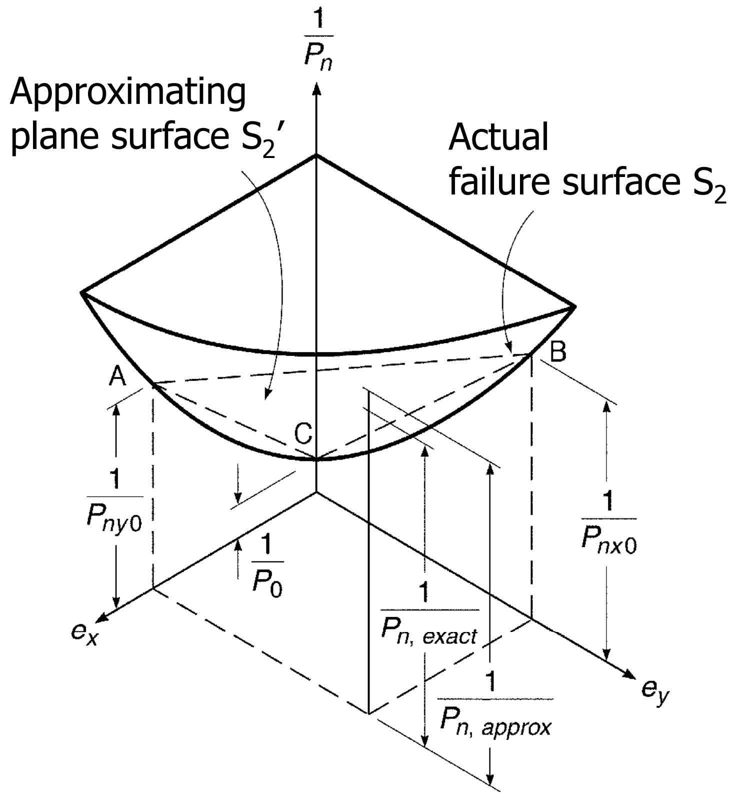
8. Short Columns

RECIPROCAL LOAD METHOD

- The vertical ordinate $1/P_{n,exact}$ to the true failure surface will always be CONSERVATIVELY estimated by the distance $1/P_{n,approx}$

$$\frac{1}{P_{n,approx}} > \frac{1}{P_{n,exact}}$$

$$\Rightarrow \quad P_{n,approx} < P_{n,exact}$$



8. Short Columns

RECIPROCAL LOAD METHOD

Bresler's Reciprocal Load Equation

$$\frac{1}{P_n} \approx \frac{1}{P_{nx0}} + \frac{1}{P_{ny0}} - \frac{1}{P_0}$$

P_n = approx. value of nominal load in biaxial bending with eccentricities e_x and e_y

P_{ny0} = nominal load when only e_x exists (e_y=0)

P_{nx0} = nominal load when olny e_y exists (e_x=0)

P_0 = nominal load for concentrically loaded column

Note Bresler's equation is valid for $P_n \geq 0.1 P_0$

Homework #3 Design of column for biaxial tension

The 400*500mm column is reinforced with eight D29 bars. A factored load P_u of 1,700kN is applied with eccentricities e_y=75mm, e_x=150mm.

f_{ck}=27MPa, f_y=400MPa

Check the adequacy of the trial design (1) using the reciprocal load method and (2) using the contour method.