EUNOIA JUNIOR COLLEGE

JC2 Preliminary Examination 2020

General Certificate of Education Advanced Level

Higher 1

CANDIDATE NAME	

CIVICS GROUP			-			INDEX NUMBER	

MATHEMATICS **8865/01**

Paper 1 [100 marks]

28 August 2020

3 hours

Candidates answer on the Question Paper

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name, civics group and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

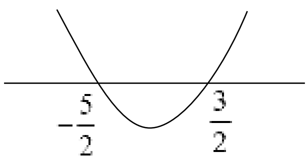
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **20** printed pages and **2** blank pages.

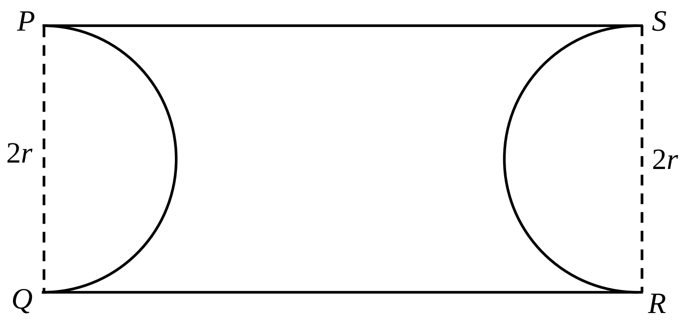
For markers' use:											
Q1	**Q2**	**Q3**	**Q4**	**Q5**	**Q6**	**Q7**	**Q8**	**Q9**	**Q10**	**Q11**	**Total**

Section A: Pure Mathematics [40 marks]

1 Find algebraically the set of values of k for which the equation $(k-2)+(1-2k)x-2x^2=0$ has real roots. [4]

Suggested Solution
For real roots, $b^2-4ac\geq 0$ $(1-2k)^2-4(-2)(k-2)\geq 0$ $1-4k+4k^2+8k-16\geq 0$ $4k^2+4k-15\geq 0$ $(2k+5)(2k-3)\geq 0$  $\left\{k\in\mathbb{R}:k\leq -\frac{5}{2} \quad \text{or} \quad k\geq \frac{3}{2}\right\}$

2



The diagram shows a ticket in the shape of a rectangle *PQRS* with two semicircle cut-outs. The diameters of the semicircles are *PQ* and *RS*. The length of $PQ = RS = 2r$ cm and the perimeter of the ticket is 20 cm.

(i) Show that the area of the ticket is $20r - 3\pi r^2$. [3]

(ii) Use a non-calculator method to find the maximum value of the area of the ticket, giving your answer in terms of π. Justify that this is the maximum value. [4]

Suggested Solution
Let A be the area of the ticket and let $PS = QR = L$ $A = 2rL - \pi r^2$ Perimeter of ticket $= 2L + 2\pi r = 20$ $\Rightarrow L = 10 - \pi r$ $\therefore\ A = 2r(10 - \pi r) - \pi r^2$ $= 20r - 2\pi r^2 - \pi r^2$ $= 20r - 3\pi r^2$

2(ii)

Suggested Solution
$A = 20r - 3\pi r^2$ $\frac{\mathrm{d}A}{\mathrm{d}r} = 20 - 6\pi r$ To find stationary points, $\frac{\mathrm{d}A}{\mathrm{d}r} = 0$ $20 - 6\pi r = 0$ $r = \frac{10}{3\pi} = 1.0610 \quad (5\text{ s.f.})$

r	$\left(\frac{10}{3\pi}\right)^-$	$\frac{10}{3\pi}$	$\left(\frac{10}{3\pi}\right)^+$
$\frac{\mathrm{d}A}{\mathrm{d}r}$	E.g. use $r = 1.06$	0	E.g. use $r = 1.07$

	$\frac{\mathrm{d}A}{\mathrm{d}r}=20-6\pi(1.06)$ $=0.019471>0$		$\frac{\mathrm{d}A}{\mathrm{d}r}=20-6\pi(1.07)$ $=-0.16902<0$
Slope	/	-	\

$\therefore$ At $r=\frac{10}{3\pi}$, the area of the ticket is a maximum.

Hence, the maximum area of the ticket is $=20\left(\frac{10}{3\pi}\right)-3\pi\left(\frac{10}{3\pi}\right)^2$

$$=\frac{200}{3\pi}-\frac{100}{3\pi}$$

$$=\frac{100}{3\pi}\text{ cm}^2$$

3 A curve C has equation $y = a\ln(x) + bx^2 + c$, where a, b and c are integers. The following table shows the coordinates of three points on C.

x	e^{-1}	1	e
y	3.86466	5	0.610944

(i) By writing down three equations, find the equation of curve C. [4]

(ii) Sketch the graph of C, stating the coordinates of any points of intersection with the axes, stationary points and the equation of the asymptote. [4]

Suggested Solution

$a\ln(e^{-1}) + b(e^{-1})^2 + c = 3.86466$

$-a + be^{-2} + c = 3.86466 ---------(1)$

$a\ln(1) + b(1)^2 + c = 5$

$b + c = 5 ---------(2)$

$a\ln(e) + b(e)^2 + c = 0.610944$

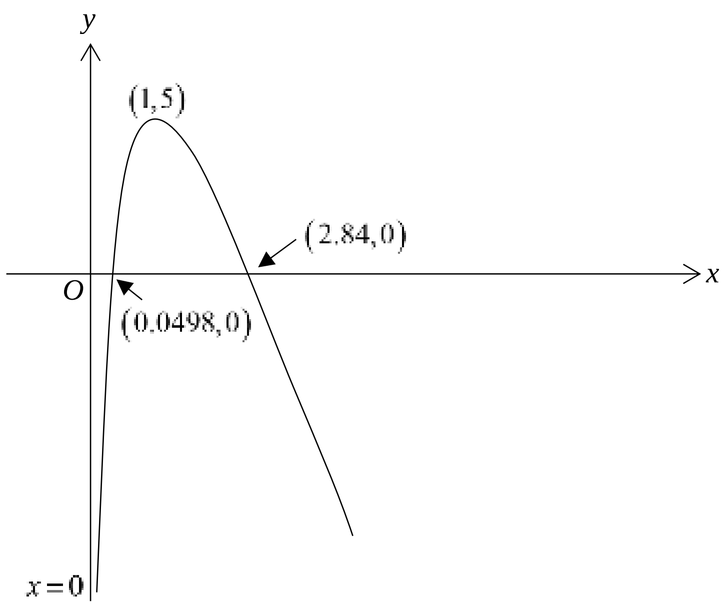
$a + be^2 + c = 0.610944 ---------(3)$

Using GC,

$a = 2, b = -1, c = 6$ (since a, b and c are real integers)

The equation of curve C is $y = 2\ln(x) - x^2 + 6$.

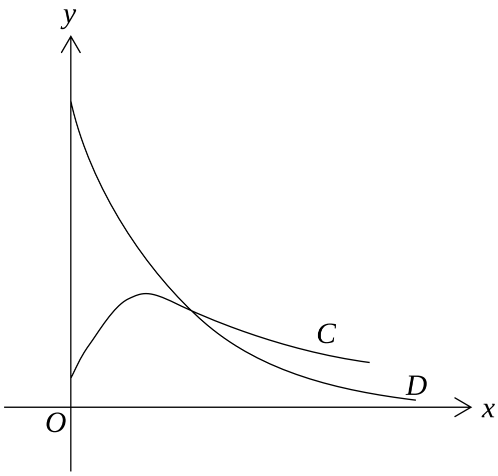
Suggested Solution



4 **(i)** Differentiate $y = \ln\sqrt{2x^2+1}$ with respect to x. [2]

(ii) Hence, find $\int_0^a \frac{2x^2+2x+1}{2x^2+1}\,\mathrm{d}x$, where $a > 0$. Give your answer in terms of a. [3]

The diagram shows the curve C with equation $y = \frac{2x^2+2x+1}{2x^2+1}$ and the curve D with equation $y = \frac{5}{\sqrt{1+8x}}$.



(iii) Write down as an integral an expression for the area of the region bounded by C, D and the y-axis. Hence, without the use of a graphing calculator, evaluate this integral. [4]

4(i)

Suggested Solution		
$y = \ln\sqrt{2x^2+1}$ $y = \frac{1}{2}\ln\left(2x^2+1\right)$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}\left(\frac{4x}{2x^2+1}\right)$ $= \frac{2x}{2x^2+1}$	OR	$y = \ln\sqrt{2x^2+1}$ $y = \ln\left(2x^2+1\right)^{\frac{1}{2}}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{1}{2}\left(2x^2+1\right)^{-\frac{1}{2}}(4x)}{\left(2x^2+1\right)^{\frac{1}{2}}}$ $= \frac{2x}{2x^2+1}$

4(ii)

Suggested Solution
From part (i), $\int \frac{2x}{2x^2+1}\,\mathrm{d}x = \ln\sqrt{2x^2+1} + C$ $\int_0^a \frac{2x^2+2x+1}{2x^2+1}\,\mathrm{d}x = \int_0^a \frac{2x}{2x^2+1} + \frac{2x^2+1}{2x^2+1}\,\mathrm{d}x$ $= \int_0^a \frac{2x}{2x^2+1} + 1\,\mathrm{d}x$ $= \left[\ln\sqrt{2x^2+1} + x\right]_0^a$ $= \ln\sqrt{2(a)^2+1} + a - \ln\sqrt{2(0)^2+1} - 0$ $= \ln\sqrt{2a^2+1} + a$

4(iii)

Suggested Solution
Using GC, point of intersection is at $x = 1$ Area of region $= \int_0^1 \frac{5}{\sqrt{1+8x}} - \frac{2x^2+2x+1}{2x^2+1}\,\mathrm{d}x$ $= 5\int_0^1 (1+8x)^{-\frac{1}{2}}\,\mathrm{d}x - \int_0^1 \frac{2x^2+2x+1}{2x^2+1}\,\mathrm{d}x$ $= 5\left[\frac{(1+8x)^{\frac{1}{2}}}{\frac{1}{2}(8)}\right]_0^1 - \left[\ln\sqrt{2(1)^2+1} + 1\right]$ (from part **(ii)**) $= \frac{5}{4}\left(\sqrt{1+8(1)} - \sqrt{1+8(0)}\right) - \left(\ln\sqrt{3} + 1\right)$ $= \frac{5}{2} - \ln\sqrt{3} - 1$ $= \frac{3}{2} - \ln\sqrt{3}$ units^2

5 An ecologist studying the population change of a particular prey and predator in a habitat collected data on how the populations of prey and predator interacted with each other. The ecologist started the collection of data when the population of the predator was one thousand.

The ecologist consulted a mathematician who modelled the population of the predator, P in thousands, at time t years after the start of data collection. The mathematician believes that $P = A - 5(1.6)^{-2t}$, where A is a positive constant.

(i) Explain why $A = 6$. [1]

(ii) Sketch the graph of P against t for $t \geq 0$, stating the coordinates of the point of intersection with the vertical axis and the equation of the asymptote. [2]

The population for the prey is modelled by the equation $P = 8(1.6)^{-t}$, where P is population in thousands, at time t years after the start of data collection.

(iii) On the same diagram as part **(ii)**, sketch the graph representing the population for the prey, stating the coordinates of the point of intersection with the vertical axis and the equation of the asymptote. [2]

(iv) The ecologist is interested to know when the populations of prey and predator became equal. By using the substitution $x = 1.6^{-t}$, form an equation in terms of x and hence determine the number of years it took for both populations of prey and predator to be equal. Leave your answer to 2 decimal places. [4]

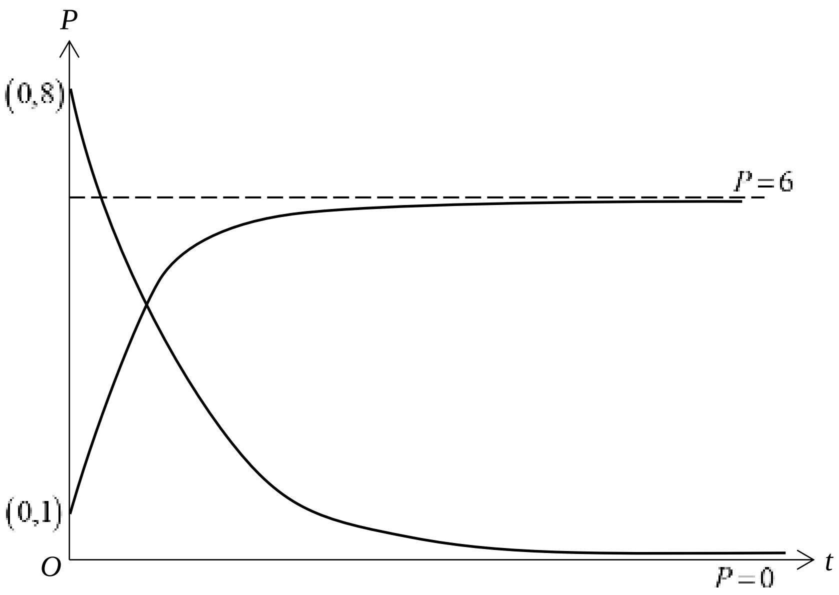
(v) Estimate the population of the prey after three years. How many months did it take for the population of the predator to reach the same value? Leave your answer to 1 decimal place. [2]

(vi) Comment on the feasibility of the mathematician's model for the population of the predator in the long run. [1]

5(i)

Suggested Solution
Since the population of the predator was one thousand at the start of data collection, at $t = 0, P = 1$. Therefore, $1 = A - 5(1.6)^{-2(0)}$ $A = 6$

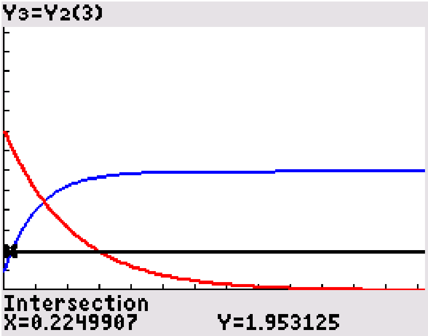
5(ii)&(iii)

Suggested Solution


5(iv)

Suggested Solution
Predator: $P = 6 - 5(1.6)^{-2t}$ Prey: $P = 8(1.6)^{-t}$ Require: $6 - 5(1.6)^{-2t} = 8(1.6)^{-t}$ Using $x = 1.6^{-t}$, $6 - 5x^2 = 8x$ $5x^2 + 8x - 6 = 0$ $x = 0.55647 \quad (5\text{ s.f.}) \quad \text{or} \quad -2.1565 \quad (\text{rej } \because x > 0)$ $\therefore 0.55647 = 1.6^{-t}$ $\Rightarrow t = -\dfrac{\ln 0.55647}{\ln 1.6} = 1.25 \text{ years} \quad (2\text{ d.p.})$

5(v)

Suggested Solution
When $t = 3$, using GC, Population of prey = 1953  Using GC, $t = 0.22499$ It took about 2.7 months to reach the same value. (1 d.p.)

5(vi)

Suggested Solution
The model for the population of the predator is not feasible in the long run because since the population of the prey approaches zero, the population of predator should decrease in the long run instead of approaching to 6000 as there would be fewer prey to feed on.

Section B: Probability & Statistics [60 marks]

6 The lengths of carrots sold by a grocer follow a normal distribution. It is known that there is an equal proportion of 6.68% of carrots that have a length less than 13.2 cm and carrots that have a length greater than 16.8 cm. Find the mean and variance of the distribution. [4]

Suggested Solution
Let μ and σ^2 be the mean and variance of the distribution. Let X be the length of a randomly chosen carrot. $X \sim \mathrm{N}\left(\mu, \sigma^2\right)$ Since $\mathrm{P}\left(X < 13.2\right) = \mathrm{P}\left(X > 16.8\right)$, $\mu = \frac{13.2 + 16.8}{2} = 15$ and since $\mathrm{P}\left(X < 13.2\right) = 0.0668$, $\mathrm{P}\left(Z < \frac{13.2 - 15}{\sigma}\right) = 0.0668$ Using GC, $\frac{13.2 - 15}{\sigma} = -1.5001 \quad (5 \text{ s.f.})$ $\sigma = 1.2000 \quad (5 \text{ s.f.})$ $\therefore \sigma^2 = 1.44 \quad (3 \text{ s.f.})$

7 A choir committee consisting of a President, a Vice-President and a Quartermaster are chosen from a group of 4 boys and 16 girls. Find the number of ways the committee can be formed if it comprises at least one girl. [2]

The committee, their conductor and two teachers are seated in a row to watch the choir rehearse. Find the number of seating arrangements such that

(i) the committee members must all sit next to each other, [2]

(ii) the teachers must be separated, [2]

(iii) the committee members alternate. [2]

7

Suggested Solution
No of ways to choose a committee of all boys $= {}^{4}C_3 \times 3! = 24$ No of ways to choose without restriction $= {}^{20}C_3 \times 3! = 6840$ No of ways to choose a committee with at least 1 girl $= 6840 - 24 = 6816$

7(i)

Suggested Solution
No. of ways to arrange the committee $= 3! = 6$ No. of ways to arrange everyone the committee grouped together $= 4! = 24$ No. of ways that the committee must sit next to each other $= 4! \times 3! = 144$

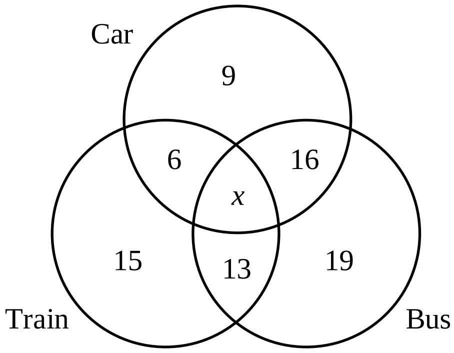
7(ii)

Suggested Solution
No. of ways to arrange the committee and conductor $= 4! = 24$ No. of ways to choose slots and arrange the teachers in-charge $= {}^{5}C_2 \times 2! = 20$ No. of ways teacher in-charge must be separated $= 4! \times {}^{5}C_2 \times 2! = 480$ OR by complement of group method: $6! - (2!)(5!) = 480$

7(iii)

Suggested Solution
No. of ways to arrange the conductor and teachers in-charge $= 3! = 6$ No. of ways to slot the committee members such that they alternate $= 2$ No. of ways to arrange the committee members $= 3! = 6$ No. of ways for the committee members to alternate $= 3! \times 3! \times 2 = 72$

8



A group of students were surveyed on the means of transport they use to reach school. The number of students who use a car, the train, the bus and different combinations of these means of transport to reach school are shown in the Venn diagram above. Every student uses at least one of these means of transport to reach school. The number of students who use all three means of transport is x. One of the students is chosen at random.

- C is the event that the student uses a car
- T is the event that the student uses the train
- B is the event that the student uses the bus

(i) Write down expressions for $\mathrm{P}(B)$ and $\mathrm{P}(C)$ in terms of x. [2]

(ii) Given that events B and C are independent, find the value of x. [3]

(iii) Find $\mathrm{P}(T \cap C')$ and explain, in the context of this question, what this probability represents. [2]

(iv) Given that the student does not use the bus to reach school, find the probability that the student uses a car to reach school. [3]

8(i)

Suggested Solution
$\mathrm{P}(B) = \dfrac{16+13+19+x}{9+6+16+x+15+13+19} = \dfrac{48+x}{78+x}$ $\mathrm{P}(C) = \dfrac{9+6+16+x}{78+x} = \dfrac{31+x}{78+x}$

8(ii)

Suggested Solution
Since events B and C are independent, $\mathrm{P}(B \cap C) = \mathrm{P}(B)\mathrm{P}(C)$ $\Rightarrow \quad \dfrac{16+x}{78+x} = \left(\dfrac{48+x}{78+x}\right)\left(\dfrac{31+x}{78+x}\right)$ $(16+x)(78+x) = (48+x)(31+x)$ $1248+94x+x^2 = 1488+79x+x^2$ $15x = 240$ $x = 16$

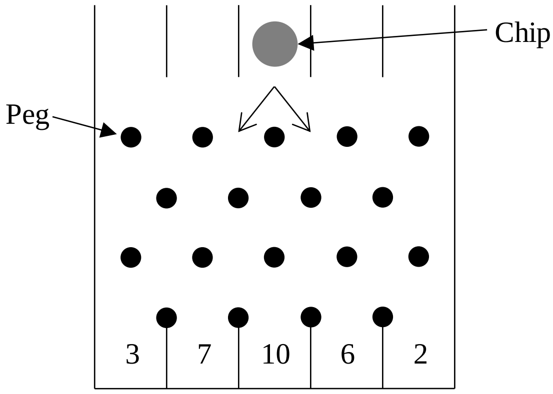
8(iii)

Suggested Solution
$\mathrm{P}\left(T \cap C'\right)=\dfrac{15+13}{78+16}=\dfrac{14}{47}$ This probability represents the probability that a randomly chosen student uses the train but not a car to reach school.

8(iv)

Suggested Solution		
$\mathrm{P}(C \mid B')$ $=\dfrac{\mathrm{P}(C \cap B')}{\mathrm{P}(B')}$ $=\dfrac{\frac{9+6}{78+16}}{\frac{9+6+15}{78+16}}$ $=\dfrac{1}{2}$	OR $\mathrm{P}(C \mid B')$ $=\dfrac{\mathrm{n}(C \cap B')}{\mathrm{n}(B')}$ $=\dfrac{9+6}{9+6+15}$ $=\dfrac{1}{2}$	OR $\mathrm{P}(C \mid B')$ $=\dfrac{\mathrm{P}(C \cap B')}{\mathrm{P}(B')}$ $=\dfrac{\frac{9+6}{78+16}}{1-\frac{16+16+13+19}{78+16}}$ $=\dfrac{1}{2}$

9 A Plinko board comprises a vertical board populated with offset rows of pegs. A player chooses one of five slots at the top of the board and drops a chip into it. As the chip falls through the board, it encounters a peg. Each time the chip encounters a peg, it will either fall to its left or right. Once the chip lands in one of five slots at the bottom of the board, the player scores the corresponding number of points. For example, if the chip starts from the middle slot and falls to the left four times, it will land in the slot that scores 3 points.



(i) Given that the chip has a probability of 0.4 to fall to the left of a peg and a probability of 0.6 to fall to right of a peg, show that the probability that a player scores 7 points when he drops the chip from the middle slot at the top is 0.1536. [2]

(ii) A player observes that the way the chip falls from the middle slot at the top can be modelled using a binomial distribution with random variable X. Given that X is defined as the number of times the chip falls to the left, out of four times, when it encounters a peg. State two assumptions for X to be well modelled by a binomial distribution. [2]

(iii) Find the probability that a player scores an odd number of points when he drops the chip from the middle slot at the top. [2]

(iv) A player drops a chip ten times from the middle slot at the top. Given that the outcomes are independent, find the probability that the player scores an even number of points fewer than eight times. [2]

(v) It is given instead that the probability that the chip will fall to the left of a peg is p. Given that $\mathrm{P}(X \le 2) = 0.366$, find the value of p. [2]

9(i)

Suggested Solution
L: Peg falls left R: Peg falls right Required probability = P(LLLR) + P(LLRL) + P(LRLL) + P(RLLL) $= 4(0.4)(0.4)(0.4)(0.6)$ $= 0.1536$

9(ii)

Suggested Solution
The probability that a chip falls to the left when it encounters a peg must be constant for every peg.

The event that a chip falls to the left when it encounters a peg must be independent of the direction the chip falls when it encounters any other peg.

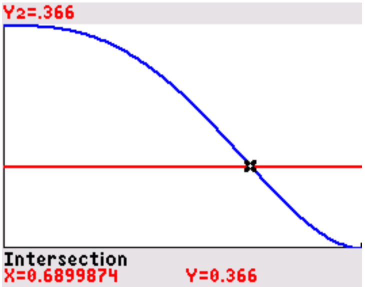
9(iii)

Suggested Solution
Let X be the number of times, out of 4, that the chip falls to the left of the peg. $X \sim \mathrm{B}(4, 0.4)$ To score an odd number of points, the chip needs to fall left 3 or 4 times.   Required probability = $\mathrm{P}(X \geq 3) = 1 - \mathrm{P}(X \leq 2)$ $= 0.1792$ OR: part (i)'s answer $+\mathrm{P}(X = 4) = 0.1792$

9(iv)

Suggested Solution
Let Y be the number of times, out of 10, that the players scores an even number of points. $Y \sim \mathrm{B}(10, 0.8208)$ Required probability = $\mathrm{P}(Y \leq 7) = 0.26048 \quad (5 \text{ s.f.})$ $= 0.260 \quad (3 \text{ s.f.})$

9(v)

Suggested Solution
$X \sim \mathrm{B}(4, p)$ Given $\mathrm{P}(X \leq 2) = 0.366$ Using GC,    $p = 0.690 \quad (3 \text{ s.f.})$

10 A researcher claims that youths spend an average of 8.5 hours watching WeTube weekly. The time spent by a random sample of 250 youths are summarised as follows, where t denotes the number of hours each youth spent watching WeTube weekly.

$$\sum t = 2049.4 \qquad \sum t^2 = 18806.2$$

(i) Find unbiased estimates of the population mean and variance. [3]

(ii) What do you understand by the term 'unbiased estimate'? [1]

(iii) Test the researcher's claim at the 5% significance level. [4]

(iv) Another researcher now claims that there is a decrease in the mean at the 5% significance level. Without conducting a test, determine if there is sufficient evidence to support the new claim. [2]

(v) A nationwide campaign is implemented in schools to reduce the amount of time youths spend watching WeTube and it is claimed that the average time spent has been reduced.

The researcher takes a new random sample of 250 youths and the mean amount of time this sample spent watching WeTube weekly is k hours. It is given that the population standard deviation is 3 hours. A test at the 5% significance level indicates that the average time spent has been reduced.

Find the range of values of k, giving your answer correct to the 2 decimal places. [4]

10(i)

Suggested Solution
Unbiased estimate of μ, $\bar{t} = \frac{\sum t}{n} = \frac{2049.4}{250} = 8.1976 \quad (\text{exact})$ Unbiased estimate of σ^2, $s^2 = \frac{1}{249}\left[18806.2 - \frac{2049.4^2}{250}\right]$ $= 8.0564 \quad (5 \text{ s.f.})$ $= 8.06 \quad (3 \text{ s.f.})$

10(ii)

Suggested Solution
Unbiased estimate is the numerical value of the unbiased estimator, calculated based on a sample, where the expectation of the unbiased estimator is equal to the actual population parameter being estimated.

10(iii)

Suggested Solution
Let T denote the number of hours each youth spent watching WeTube weekly. $H_0: \mu = 8.5$ $H_1: \mu \neq 8.5$ Level of significance: 5%

Under H_0, $\overline{T} \sim N\left(8.5, \frac{8.0564}{250}\right)$ approximately by Central Limit Theorem since $n = 250$ is large.

Test Statistic: $Z = \frac{\overline{T} - 8.5}{\sqrt{\frac{8.0564}{250}}} \sim N(0,1)$ approximately

Reject H_0 if p-value < 0.05

Given $\bar{t} = 8.1976$, from GC, p-value $= 0.092077 \quad (5 \text{ s.f.})$

Since p-value $= 0.0921 > 0.05$, we do not reject H_0. Hence, there is insufficient evidence at a 5% level of significance to conclude that the mean number of hours spent is not equal to 8.5 hours.

10(iv)

Suggested Solution

$H_0: \mu = 8.5$

$H_1: \mu < 8.5$

Level of significance: 5%

p-value
$= \frac{0.092077}{2}$
$= 0.046038 \quad (5 \text{ s.f.})$

Since p-value $= 0.0460 < 0.05$, we reject H_0. Hence, there is sufficient evidence at a 5% level of significance to conclude that the mean number of hours spent is less than 8.5 hours.

10(v)

Suggested Solution

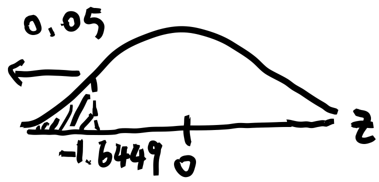
$H_0: \mu = 8.5$

$H_1: \mu < 8.5$

Level of significance: 5%

Under H_0, $\overline{T} \sim N\left(8.5, \frac{3^2}{250}\right)$ approximately by Central Limit Theorem since $n = 250$ is large.

Test Statistic: $Z = \frac{\overline{T} - 8.5}{\sqrt{\frac{3^2}{250}}} \sim N(0,1)$ approximately



Reject H_0 if z-value < -1.6449

Since H_0 is rejected,

$$\frac{k-8.5}{\frac{3}{\sqrt{250}}} < -1.6449$$

$k < 8.1879 \quad (4 \text{ d.p.})$

$k < 8.19 \quad (2 \text{ d.p.})$

11 The masses, in grams, of a randomly selected cookie and a randomly selected cupcake produced by a bakery are said to be normally distributed with mean and standard deviation given in the following table.

	Mean (grams)	Standard deviation (grams)
Cookies	45	0.5
Cupcakes	80	2

(i) Find the probability that the mass of a cookie chosen at random is within ± 0.8 g of the mean mass of cookies. [2]

3 cupcakes are chosen at random. Find the probability that

(ii) each of the three cupcakes has a mass of more than 78 g, and $[P(Y>78)]^3 = 0.596$ [2]

(iii) the total mass of the first two cupcakes is greater than twice the mass of the third cupcake by more than 5 g. $P(Y_1+Y_2-2Y_3>5)=$ $Y_1+Y_2-2Y_3 \sim N(\; , \;)$ [3]

Each 100 g of cookie contains 230 calories and each 100 g of cupcake contains 300 calories.

(iv) Find the probability that two randomly chosen cookies and three randomly chosen cupcakes contain less than 935 calories in total. [4]

$\mu=$

The bakery also bakes loaves of bread. The mass of a randomly chosen loaf of bread has a mean of 400 g and standard deviation $\sigma=$ of 3 g. Given that the probability that the average mass of 100 randomly chosen loaves of bread is at most α g is 0.8, find the value of α, correct to 2 decimal places. [3]

$\le \alpha$ $\quad$ 5

11(i)

Suggested Solution
Let X denote the mass of a randomly chosen cookie. $\mathrm{P}(-0.8 \le X - 45 \le 0.8)$ $= \mathrm{P}(44.2 \le X \le 45.8)$ $= 0.89040 \quad (5 \text{ s.f.})$ $= 0.890 \quad (3 \text{ s.f.})$

11(ii)

Suggested Solution
Let Y denote the mass of a randomly chosen cupcake. $Y \sim \mathrm{N}\left(80, 2^2\right)$ $\mathrm{P}(Y > 78)^3$ $= 0.59555 \quad (5 \text{ s.f.})$ $= 0.596 \quad (3 \text{ s.f.})$

70

11(iii)

Suggested Solution
Required probability $= \mathrm{P}(Y_1 + Y_2 - 2Y_3 > 5)$ $\mathrm{E}\left(Y_1 + Y_2 - 2Y_3\right) = 0$ $\mathrm{Var}\left(Y_1 + Y_2 - 2Y_3\right) = 2^2 + 2^2 + (-2)^2\left(2^2\right) = 24$ $\therefore Y_1 + Y_2 - 2Y_3 \sim \mathrm{N}\left(0, \sqrt{24}^2\right)$ $\mathrm{P}(Y_1 + Y_2 - 2Y_3 > 5)$ $= 0.15372 \quad (5 \text{ s.f.})$ $= 0.154 \quad (3 \text{ s.f.})$

11(iv)

Suggested Solution
Required probability $= \mathrm{P}\left(\text{the total calories of 3 cupcakes and 2 cookies} < 935\right)$ Let W denote the total calories of 3 cupcakes and 2 cookies. $W = \frac{X_1}{100}(230) + \frac{X_2}{100}(230) + \frac{Y_1}{100}(300) + \frac{Y_2}{100}(300) + \frac{Y_3}{100}(300)$ $= 2.3X_1 + 2.3X_2 + 3Y_1 + 3Y_2 + 3Y_3$ $= 2.3(X_1 + X_2) + 3(Y_1 + Y_2 + Y_3)$ $\mathrm{E}(W) = 2(2.3)(45) + 3(3)(80) = 927$ $\mathrm{Var}(W) = 2\left(2.3^2\right)\left(0.5^2\right) + 3\left(3^2\right)\left(2^2\right) = 110.645$ $W \sim \mathrm{N}\left(927, \sqrt{110.645}^2\right)$ $\mathrm{P}(W < 935)$ $= 0.77654 \quad (5 \text{ s.f.})$ $= 0.777 \quad (3 \text{ s.f.})$

Suggested Solution
Let S denote the mass of a randomly chosen loaf of bread. $\overline{S} = \frac{S_1 + S_2 + \ldots + S_{100}}{100}$ $\mathrm{E}\left(\overline{S}\right) = 400$ $\mathrm{Var}\left(\overline{S}\right) = \frac{3^2}{100} = \frac{9}{100} = 0.3^2$ Since $n = 100$ is large, by Central Limit Theorem, $\overline{S} \sim \mathrm{N}\left(400, 0.3^2\right)$ approximately. $\mathrm{P}\left(\overline{S} \le \alpha\right) = 0.8$ inv norm $\alpha = 400.2525 = 400.25 \quad (2 \text{ d.p.})$