Big Idea(s)/ Topic(s)

- Understand and apply the Pythagorean Theorem.

Standard(s) Alignment

- **MCC.8.G.7** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Diagnostic Assessment

The diagnostic assessment (see Appendix A) can be used to assess students' understanding of the Pythagorean Theorem and proportional relationships.

Instructional Design

Desmos Activity: [Grade 8 Taco Truck: Pythagorean Theorem]

Overview: The purpose of this activity is for students to use the Pythagorean Theorem to reason about distances and speeds. The lesson will allow students to calculate diagonal distances in context. Students will solve real-world problems involving the Pythagorean Theorem, missing measures, and proportions.

Materials:

- Access to the diagnostic assessment
- Access to the Desmos activity (or a printout)
- Graph paper
- Sticky notes
- Create a discussion board for this learning activity using platforms such as Kialo Edu or Yo Teach for asynchronous learners

Teacher Notes: Use the diagnostic assessment to measure students prior knowledge. The lesson uses proportional reasoning which should be assessed prior (via #3) in the diagnostic assessment. Provide scaffolded instruction on proportional reasoning and calculating Pythagorean Theorem, before the lesson. The Desmos activity may take up to 45 minutes. Follow up the lesson with highlights of student thinking and working.

Description of Learning: In this activity, students will find their intuition of finding the shortest path will be challenged! Coupled with the use of proportions, students will solve problems involving straight and diagonal distances. Students will determine who will get to the taco truck first!

Engage

Alma is going to walk through the park from Point A to Point B.

What distance will she walk?

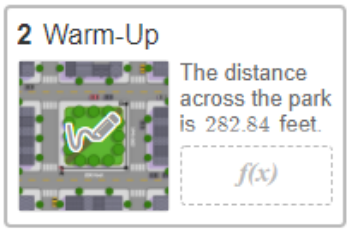
Teacher Moves

Invite students to look at the image when you put it up on the projector. Before giving students 2–3 minutes of quiet work time, consider showcasing that it's possible to enter either numbers *or* expressions in the math input. Show that when entering an expression, the math input works like a calculator.

Facilitation
Consider using pacing to restrict students to Screens 1–2.

Sample Responses

282.8 ft. or the equivalent, such as $\sqrt{200^2 + 200^2}$.



The distance across the park is 282.84 feet.

Alma walks 5 feet per second.

How long will it take for her to walk across the park?

Teacher Moves

After students have some time to think about the questions on Screens 1–2, pause the class for a brief discussion. Consider using the snapshots tool to show the different expressions and numbers that students entered for both screens.

In addition to understanding the Pythagorean theorem, understanding the relationship between distance, rate, and time is important for this lesson. Students may not immediately recall how to calculate the time given a distance and a rate. Consider using a quick example with simple numbers to remind students that distance equals rate multiplied by time. If distance and rate are known, as is the case here, the time can be found by dividing the distance by the rate.

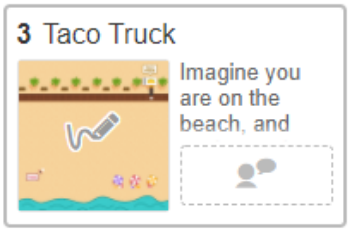
Once students have completed this screen, consider pausing to ask, "Is it faster to take the hypotenuse? Is that always the case?" This lesson may challenge students' assumptions on those questions.

Sample Responses

56.6 seconds or the equivalent, such as $\frac{282.84}{5}$.

- **Synchronous:** Restrict students to screen 1 and highlight student responses. Then add the following screen.
- **Asynchronous**: Introduce the lesson to students in a virtual platform; this can be done via e-document or video. Students who struggled with #1 and #2 on the diagnostic should be provided supports prior to this screen. Support could include addressing misconceptions with solving Pythagorean theorem in the introductory video.
- **Unplugged/ Offline:** Provide the image and question from screens 1 and 2 for students to engage in the task. Have students share ideas through email/text/phone. Provide feedback to students and share other students' ideas before engaging in the remaining screens.

Explore



Imagine you are on the beach, and you're getting hungry.

Use the sketch tool to show the route you would take to the taco truck.

Explain the reasoning behind your sketch.

Teacher Moves

Use the snapshots tool to showcase interesting and unique sketches as well as students' reasonings.

In particular, consider showcasing one or more sketches that "take the hypotenuse" (similar to the warm-up). Ask students if they think that route will be the fastest. It's okay—even desirable—to lack consensus at this stage. Encourage participation from students who think that the difficulty of walking on sand is a factor worth taking into account.

Facilitation
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

• The quickest way to get from one point to another is to follow a straight line between them.
• I hate walking on sand, so I would walk to the boardwalk as quickly as possible and walk the rest of the way on the boardwalk.

Student Supports

For students who benefit from extra processing time, provide them the images to review prior to implementation of this activity.

After watching Meyer's video, Daniel thought Meyer would arrive at the taco truck first and Ben thought Brandon would.

Who do you think will reach the taco truck first?

Teacher Moves

Tell students that the first part of this activity is about exploring two different routes to the taco truck. After giving 1–2 minutes of silent think-time, arrange students into pairs to discuss, focusing especially on what information would be helpful to determine the first question.

Highlight a few unique responses for the class. Ask students to explain why they requested certain information and to attend to precision. For example, if a student asks, "How slow does sand make you go?" or "What could Meyer's and Ben speeds be?" ask them for the units they are interested in.

Facilitation
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

• Both Meyer and Ben's speeds.
• The distance to the taco truck.
• The distance to the boardwalk from the starting point and the distance along the boardwalk to the taco truck.

- **Synchronous:** Unrestrict screen 3 and follow teacher moves guidance. Give students time to complete the screens and provide feedback. Ensure that enough time is provided for students to participate. Unrestrict screen 4 and allow students to adjust prior thinking. Students should recognize sand will slow your pace and the measurements are needed to solve this problem.
- **Asynchronous:** Encourage students to record their thinking in preparation for a group share. After a determined amount of allotted time, instruct students to work within small collaborative groups. Consider using Google Slides, or Padlet to organize the groups, ensure each group has a space to record their thinking. You may want to encourage students to record their name next to their thinking for accountability purposes.
- **Unplugged/ Offline**: Provide paper/electronic versions of the images and questions presented on screens 3 and 4. Allow students time to complete the work and submit through email/text or other means. Provided feedback and share with others.

Apply

5 Calculate

To settle the debate, the pair

To settle the debate, the pair decided to test it out for themselves!

Their speed on the BOARDWALK is 5 feet per second.

Their speed on the SAND is 3 feet per second.

The dimensions are shown in the image.

Determine the amount of time it will take for Daniel and for Brandon to get to the taco truck.

Use paper and/or a calculator as necessary.

Teacher Moves

Tell students that this screen provides them with some of the information they requested from the previous screen to help them know whether Brandon or Dan will arrive first. Give students 2–4 minutes of quiet think-time. Then arrange them into pairs or small groups to share their initial thinking and solve the problem together.

This is a great place to use the teacher dashboard to monitor student progress. Offer support where needed. If enough students are struggling, prepare to lead a whole-class discussion about strategies.

Facilitation
Consider using pacing to restrict students to this screen.

Sample Responses

Brandon will reach the taco truck first.

- Daniel's time can be found by using the expression $\frac{327.6}{3} + \frac{489}{5}$, which is 207 seconds.
- Brandon's time can be found by using the expression $\frac{\sqrt{327.6^2 + 489^2}}{3}$, which is about 196.2 seconds.

Student Supports

Read all statements aloud. Students who both listen to and read the information will benefit from extra processing time.

6 Zoe's Route

Other routes are possible, too. Some may be faster than Brandon's and Daniel's.

Other routes are possible, too. Some may be faster than Brandon's and Daniel's.

Use the movable point to choose a route for Zoe that you think will be faster than the routes that Brandon and Daniel used.

When you're done, go to the next screen.

Teacher Moves

Refer back to students' sketches from the beginning of this activity to remind students that Brandon's and Daniel's routes were just two of many different possible routes. Encourage students to think about whether there might be a route that is faster than those two. Call attention to how the movable point on this screen can help students explore different routes.

Use the overlay feature on the dashboard to show the range of responses on this screen. Tell students that later in the activity, all of these routes will race against one another.

Facilitation
Consider using pacing to restrict students to Screens 6–8.

Sample Responses

Responses vary.

Any route that aims Zoe *in between* where Brandon and Daniel reach the boardwalk will be faster than those two routes.

7 Taco Time

The route you chose for Zoe on the previous

The route you chose for Zoe on the previous screen is shown. Determine the amount of time this route will take.

Recall:
- The speed on the BOARDWALK is 5 feet per second.
- The speed on the SAND is 3 feet per second.

Use paper and/or a calculator as necessary.

Teacher Moves

Give students 2–4 minutes of quiet think-time. Then arrange them back into pairs or small groups to share their initial thinking and decide how they'll solve the problem together.

Use the snapshots tool to show students' chosen routes and the accompanying expressions or calculations. Ask students to explain their responses and to critique each other's reasoning (MP3).

Use the dashboard to monitor progress. Students who successfully calculate the timing for their route may go back one screen to seek a faster route. Another worthwhile challenge you could suggest is to find different routes for Zoe that will land her in 1st, 2nd, and 3rd place.

Sample Responses

Responses vary.

Student correctness for this screen is available in the teacher view of the dashboard.

- If the movable point is moved 100 ft. to the right, Zoe's time is about 192 seconds.
- If the movable point is moved 300 ft. to the right, Zoe's time is about 186 seconds.

8 The Race

The route you chose for Zoe is shown in blue. The routes chosen by your

The route you chose for Zoe is shown in blue. The routes chosen by your classmates are also shown.

Press "Race" to see how your classmates' routes compare to yours.

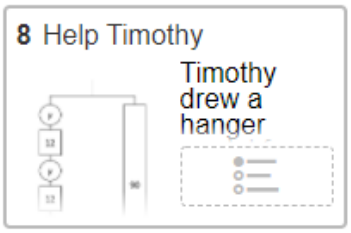
Teacher Moves

This screen will animate each student's route compared to those of their classmates. Note that students will not be able to identify who they are in the race unless they correctly predicted the time of their route.

- **Synchronous:** Using the teacher dashboard, unrestrict screens 5 through 8. Give students time to complete the screens and provide feedback. Ensure that enough time is provided for students to participate and respond to your feedback and edit responses as needed. Follow teacher move guidance.
- **Asynchronous:** Consider using a collaborative document or discussion post for students to work

through the assignment together. Plan to review students' work in a later video.

- **Unplugged/ Offline:** Provide students with the images and questions on screens 4 through 9. Allow students to apply their knowledge in skills throughout the tasks. Ask students to complete the questions and have them submit responses via email/text/phone. Provide feedback, share these responses with other students, and share other students' responses with them.



Timothy drew a hanger model for the story problem below.

Lin had 90 flyers to hang up around the school. She gave 12 flyers to each of three volunteers. Then she took the remaining flyers and divided them up equally between the three volunteers.

Do you agree with Timothy? Explain your thinking.

Teacher Moves

Have students articulate what the variable and number represents in this equation.
Have students read their classmate responses and possibly revise their own thinking.
Encourage struggling students to write the equation from the hanger model and explain how it could or could not fit in the story when explaining their answer.

Challenge students by asking them to describe another way to draw the hanger model.

Sample Responses

Agree

Each 12 represent the amount of flyers each volunteer received, "Y" represents the remaining flyers passed out to each volunteers. Ninety represents the total amount of flyers.

Student Supports

Hanger Diagram Applet:
https://www.geogebra.org/m/B89AaQqx#material/snnMT5Ga
Tape Diagram Applet:
https://www.geogebra.org/m/B89AaQqx#material/reYPNn3V

Reflect

9 Reflection

A student used the following expression to

A student used the following expression to calculate the time for Zoe's route:

$$\frac{\sqrt{327.6^2+100^2}}{3}+\frac{389}{5}$$

First, move the draggable point to show the path that matches the expression.

Then, explain what each part of the expression means.

Teacher Moves

Launch
With the activity paused, show the class some expressions from Screen 7, either from students or from you. Here are some examples:

- $\frac{\sqrt{327.6^2+200^2}}{3}+\frac{289}{5}$
- $\frac{\sqrt{50^2+327.6^2}}{3}+\frac{439}{5}$

Ask students to look back at their work on this problem and consider what similarities and differences they see between all of the work shown (their own included). Note that leaving problems like this written as expressions makes it hard to see the answer but makes it much easier to analyze the different parts of a problem. Then unpause and direct students to complete the synthesis question.

Teacher Moves
Use the teacher view of the dashboard to highlight unique answers for the class. Support students in seeing structure, calling attention to the fact that expressions like $\sqrt{327.6^2+100^2}$ often signify a calculation of distance using the Pythagorean theorem.

If time permits, lead the class in resolving the question of the fastest possible route. One approach to this is to write a generalized expression for Zoe's time, where the movable point has been moved x feet to the right. In this scenario, her time can be represented with a function: $\frac{\sqrt{327.6^2+x^2}}{3}+\frac{489-x}{5}$. The optimal point can be spotted when that function is graphed in an appropriate window. Here is a link to such a graph.

Facilitation
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

This student aims Zoe by dragging the movable point 100 feet to the right. $\sqrt{327.6^2 + 100^2}$ represents the diagonal distance traveled on sand, which gets divided by the speed on sand, 3.389 is the remaining distance to walk on the boardwalk, which gets divided by the speed on the boardwalk, 5.

- **Synchronous: Think-pair-share.** First, students work independently to answer the questions on the screen. Next, students pair up and share their work with each other. Finally, students engage in a large group discussion to discuss their answers.
- **Asynchronous: Virtual Think-Pair-Share.** First, students work independently to think about what they know and if you're able to group your students, you might consider having them share their work .
- **Unplugged/ Offline:** Provide students with the question on screen 9 and instruct them to complete it. Allow students time to complete the work and submit through email/text or other means

Extension

When we saw Daniel versus Brandon, Brandon won.

Determine the speed on the boardwalk that would make Daniel and Brandon arrive at the same time.

Use paper and/or a calculator as necessary.

Teacher Moves

This screen is designed to help differentiate the lesson by giving students who finish Screens 2–9 ahead of time another challenge. Because only a subset of your students will complete this screen, we recommend you don't discuss it with the entire class.

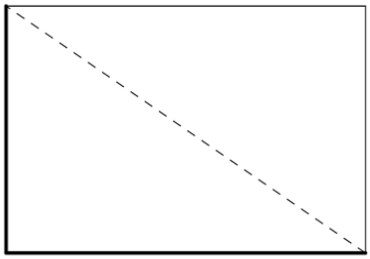
Sample Responses

About 5.6 feet per second

Evidence of Student Success

There are many opportunities to assess student success throughout the Desmos activity. Pay particular attention to screen 9 and use the Sample Response as guidance. Students could be presented with the question below. Teachers could gauge understanding based on the strengths and weaknesses in the student's justification.

In a huge parking lot Lynn helped Bethany learn how to skate. The parking lot is 120 meters long and 70 meters wide. Once Bethany got the hang of it, the friends, decided to race from one corner to the opposite corner of a parking lot. Since Bethany had on skates and Lynn did not, Bethany skated along the sidewalk that hugs the parking lot while Lynn ran the shorter distance directly across the lot. Lynn can run at around 6.5 meters per second, and Bethany can skate around 9 meters per second.



A. Bethany would win the race.	B. Lynn would win the race.	C. The friends would tie.	D. Not enough information

Select a response above and justify your thinking below.

Student Learning Supports

Establish mathematics goals to focus learning.

- Make instructions and expectations clear for the activities.
- Make explicit connections between current and prior lessons or units.

Facilitate meaningful mathematical discourse.

- Explicitly model and teach good "discussion board" etiquette.

Pose purposeful questions.

- Predetermine when you will call on the student or use the pause feature within the activities.
- Break class into small discussion groups to work collaboratively and then have groups report back to the whole group.

Anticipate Misconceptions.

- If the class is struggling to get started, scaffold the concept through a video.
- Be prepared to discuss speed misconceptions.
- Provide working mats to organize student work.

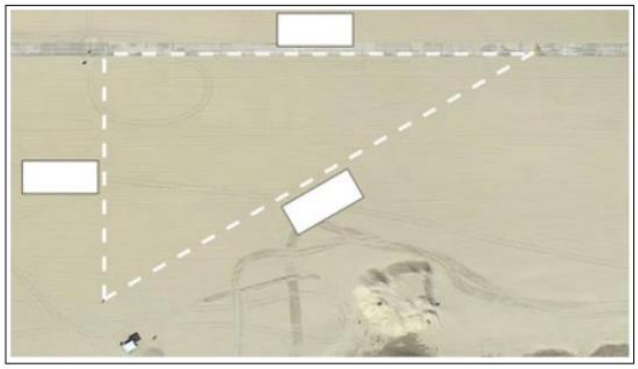


Fig. 7: *Working Place Mat (Nyguen, 2013).*

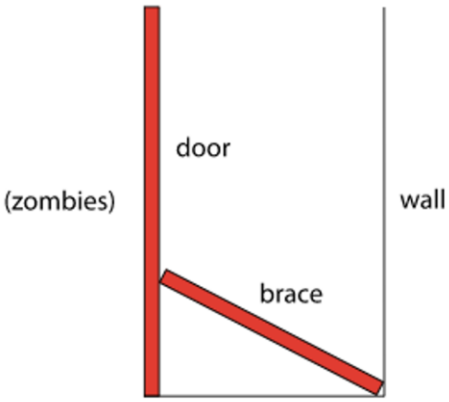
Source: http://fawnnguyen.com/mr-meyers-taco-cart/

Engaging Families

There are many ways to get the family involved with learning about applications of the Pythagorean Theorem. Watch this broadcasting with family members to see how math is found everywhere! Deepen the concept of Pythagorean Theorem by practicing this link. Be the teacher, the Taco Truck is a famous math problem, educate a family member by walking them through this link. These resources to engage families and many more are found on the Georgia Home Classroom website.

Student Name: _____ **Grade:** _____ **Teacher:** _____

Diagnostic Assessment
Grade 8 – Pythagorean Theorem

Problem	Explain your thinking (and show your work).
1. Jonah wants to keep the zombies from being able to get inside his house! He wants to cut a length of wood that will brace the door against a wall so that the door can't be opened. If the brace he cut is approximately 4.5 feet long, how far away is the door from the brace and how high does the brace rest against the wall? *If you do not have enough information, make an estimate and explain what information you need to give a more precise answer.*  	
2. Below are the side lengths of four different triangles. Circle the ones that are right triangles and explain how you know. A. 8, 7, 15 B. 4, 10, $\sqrt{84}$ C. $\sqrt{8}$, 11, $\sqrt{129}$ D. $\sqrt{1}$, 2, $\sqrt{3}$	
3. As the Earth revolves around the sun, it travels at a rate of approximately 30 km per second. At this rate, how many miles will the Earth travel in 30 minutes? Show how you know.	