8th Grade Mathematics

Linear Equations and Systems
Unit 4 Curriculum Map: May 2nd – June 23rd

ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

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Unit 4 Overview

In this unit, students will ….

- Model situations with symbolic statements
- Recognize when two or more symbolic statements represent the same context
- Use the properties of real numbers, such as the Distributive Property, to write equivalent expressions
- Determine if different symbolic expressions are mathematically equivalent
- Interpret the information that equivalent expressions represent in a given context
- Determine the equivalent expression or equation that is most helpful in answering a particular question about a relationship
- Use algebraic equations to describe the relationship among the volumes of cylinders, cones and spheres that have the same height and radius
- Solve linear equations involving parentheses
- Determine if a linear equation has a finite number of solutions, an infinite number of solutions, or no solution
- Develop understanding and some fluency with factoring quadratic expressions
- Solve quadratic equations by factoring
- Recognize how and when to use symbols, rather than tables or graphs, to display relationships, generalizations, and proofs
- Recognize linear equations in two variables in standard form $Ax+By=C$
- Recognize that a linear equation in the form $Ax+By=C$ has infinitely many solutions (x,y) and the graph of those solutions is always a straight line
- Recognize that the form $Ax+By=C$ of linear equations is equivalent to the form $y=mx+b$ for linear equations
- Recognize that systems of linear equations in the form
  $\{Ax+By=C$
  $\{Dx+Ey=F$
  may have exactly one solution, which is the intersection point of the lines represented by the equations; infinitely many solutions, which is represented by a single line for both equations; or no solution, which is represented by two parallel lines
- Continue to develop skills in solving a linear equation in two variables by graphing and with algebraic methods
- Recognize that solving a system of linear equations is equivalent to finding values of the variables that will simultaneously satisfy all equations in the system
- Develop skills in solving systems of linear equations by graphing solutions of separate equations; writing the system of equations in equivalent y=mx+b form; or using combinations of the system to eliminate one variable
- Choose between graphing and symbolic methods to efficiently find the solution to a particular system of linear equations
- Gain fluency with symbol manipulation in solving systems of linear equations
- Solve problems that involve systems of linear equations

Enduring Understandings

- Patterns and relationships can be represented graphically, numerically, and symbolically.
- Several ways of reasoning, all grounded in sense making, can be generalized into algorithms for solving proportion problems.
- Linear equations in one variable can have one solution, infinitely many solutions, or no solutions.
- Linear functions are characterized by a constant rate of change. Reasoning about the similarity of "slope triangles" allows deducing that linear functions have a constant rate of change and a formula of the type   for constants m and b.
- Two or more expressions may be equivalent, even when their symbolic forms differ. A relatively small number of symbolic transformations can be applied to expressions to yield equivalent expressions.
- Variables have many different meanings, depending on context and purpose.
- Using variables permits writing expressions whose values are not known or vary under different circumstances.
- Using variables permits representing varying quantities. This use of variables is particularly important in studying relationships between varying quantities.
- The equals sign can indicate that two expressions are equivalent. It is often important to find the value(s) of a variable for which two expressions represent the same quantity. Finding the value(s) of a variable for which two expressions represent the same quantity is known as solving an equation.
- There are situations that require two or more equations to be satisfied simultaneously.
- There are several methods for solving systems of equations.
- Solutions to systems can be interpreted algebraically, geometrically, and in terms of problem contexts.
- The number of solutions to a system of equations or inequalities can vary from no solution to an infinite number of solutions.

Unit 4 Pacing Guide

Activity	Common Core Standards	Estimated Time
Unit 4 Diagnostic	7.EE.A.1, 7.EE.A.2, 7.EE.B.3, 7.EE.B.4a	½ Block
Say It With Symbols (CMP3) Investigation 1	8.EE.C.7, 8.EE.C.7b, 8.F.A.3	4½ Blocks
Unit 4 Performance Task 1	8.EE.B.5	½ Block
Say It With Symbols (CMP3) Investigation 2	8.EE.C.7, 8.EE.C.7b, 8.F.A.1, 8.F.A.2, 8.F.A.3, 8.G.C.9	4½ Blocks
Unit 4 Performance Task 2	8.EE.B.6	½ Block
Say It With Symbols (CMP3) Investigation 3.1 & 3.2 only	8.EE.A.2, 8.EE.C.7, 8.EE.C.7a, 8.EE.C.7b, 8.EE.C.8, 8.EE.C.8a, 8.EE.C.8b, 8.EE.C.8c, 8.F.A.1	2½ Blocks
Say It With Symbols (CMP3) Investigation 4.1 & 4.2 only	8.EE.C.7, 8.EE.C.7b, 8.F.A.1, 8.F.A.2, 8.F.A.3, 8.F.B.4, 8.F.B.5	2½ Blocks
Unit 4 Performance Task 3	8.EE.C.7	½ Block
Say It With Symbols (CMP3) Investigation 5	8.EE.C.7b, 8.F.A.1, 8.F.A.2, 8.F.B.4	3½ Blocks
Unit 4 Assessment 1	8.EE.B.5, 8.F.A.2, 8.F.B.4, 8.F.B.5	1 Block
It's In The System (CMP3) Investigation 1	8.EE.C.8, 8.EE.C.8a, 8.EE.C.8b, 8.EE.C.8c, 8.F.A.3	4½ Blocks
Assessment: Unit 4 Check Up 1 (CMP3)	8.EE.C.8, 8.EE.C.8a, 8.EE.C.8b, 8.EE.C.8c, 8.F.A.3	½ Block
It's In The System (CMP3) Investigation 2	8.EE.C.8, 8.EE.C.8a, 8.EE.C.8b, 8.EE.C.8c	4½ Blocks
Unit 4 Performance Task 4	8.EE.C.8	½ Block
Unit 4 Assessment 2	8.EE.C.7, 8.EE.C.8	1 Block
Total Time		**31½ Blocks**

Major Work Supporting Content Additional Content

Pacing Calendar

MAY						
Sunday	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**	**Saturday**
1	*2* Unit 4: Linear Eqs & Systems *Unit 4 Diagnostic*	*3*	*4*	*5*	*6*	7
8	*9* *Unit 4 Performance Task 1 Due*	*10*	*11*	*12*	*13*	14
15	*16* *Unit 4 Performance Task 2 Due*	*17*	*18*	*19*	*20*	21
22	*23* *Unit 4 Performance Task 3 Due*	24	25	26	27	28
29	30 Memorial Day District Closed	31 Snow Day Make up District Closed				

June

			Wednesday	Thursday	Friday	Saturday
			1 *Assessment: Unit 4 Assessment 1*	2	3	4
5	6 **Snow Day Make up** **District Closed**	7	8	9 *Assessment: Unit 4 Check Up 1*	10	11
12	13	14	15	16 *Unit 4 Performance Task 4 Due*	17 *Assessment: Unit 4 Assessment 2*	18
19	20 **Solidify Unit 4 Concepts**	21 12:30 pm **Student Dismissal**	22 12:30 pm **Student Dismissal**	23 **Last day for students**	24	25
26	27	28				

CMP3 Unit 4 Math Background

In this Unit, *Say It With Symbols*, the emphasis shifts to using the properties of numbers to look at equivalent expressions and the information each expression represents in a given context and to interpreting the underlying patterns that a symbolic equation or statement represents. Students look critically at each part of an expression and how each part relates to the original expression. They examine the graph and table of an expression as well as the context the expression is modeling. The properties of equality and numbers are used extensively in this Unit as students write and interpret equivalent expressions, combine expressions to form new ones, predict patterns of change represented by an equation or expression, and solve equations. Students continue to develop their algebraic skills in the remaining Grade 8 Units.

This Unit develops students' facility in reasoning with purely symbolic expressions. They observe how symbolic expressions are used in real situations in the Problems. This Unit provides meaningful settings to motivate conventional algebraic notation and techniques. Thinking with symbolic expressions, in situations where mathematics is applied, plays a significant role in developing a student's "symbolic sense" or fluency with symbolic statements.

The Unit is organized around five aspects of symbolic expressions: creating and interpreting equivalent expressions, combining expressions, solving equations, observing patterns of change, and reasoning with symbols. Throughout all of the Investigations, the Problems require students to write symbolic statements to model a situation, interpret symbolic statements, write equivalent symbolic expressions, and make predictions using symbolic statements.

In *Variables and Patterns* and *Moving Straight Ahead*, students explored ways in which relationships can be expressed in tables, graphs, and equations. The contextual clues or the patterns in tables or graphs strongly influenced the construction of a single equation or expression, so students did not gain experience with equivalent equations. In this Unit, students are deliberately presented with situations in which contextual clues can be interpreted in several ways to produce different equations or expressions that are equivalent.

In Investigation 2, students combine expressions to write new expressions either by adding or subtracting expressions, or by substituting an equivalent expression for a given quantity in another expression that contains the quantity.

Students are quite comfortable using tables or graphs to solve equations, and they can solve simple linear equations of the form $y=mx+b$, $mx+b=nx+c$, or simple equations with parentheses, such as $y=a(x+b)$. In Investigation 3, students solve more complicated equations which involve more work with the use of parentheses.

In Investigation 5, the central idea is to use symbolic statements and appropriate mathematical properties to confirm conjectures.

The second half of the unit involves solving systems of equations. One of the standard topics in elementary algebra is the variety of techniques available for solving systems of two linear equations in two unknowns. In *Moving Straight Ahead,* students used the simplest form of a linear equation and were asked to find the intersection point of two lines. The standard solution strategies for systems of equations are important for students later in mathematics, so a conceptual understanding is a useful foundation that students can build on.

Unit 4 Math Background (Continued)

The graphic method involves graphing all the equations in a system on a single coordinate grid in order to read the coordinates of the intersection point(s) as the solution(s). Since this method relies on a visual representation of the equations, coordinates of an intersection point can only be estimated and may not even appear in the graphing window you select to display. If a display does not show any intersection point(s), you may still have to resort to hand calculations to determine a graphing window that shows the solution(s). Since the coordinates of an intersection point are estimates, it is important to check the solution(s) in the original equations. When you suspect that lines are parallel, you will want to find the slopes of each line to see if their slopes are equal.

The equivalent form can be ideal for students when the equations in a system are given in standard form, $Ax+By=C$, they can be rewritten in $y=ax+b$ form. When the arithmetic is easy, this is a good strategy.

Linear Combination method of solving linear systems relies on two basic principles: 1)The solutions to any linear equation $ax+by=c$ are identical to the solutions of the equation $kax+kby=kc$, where $k\neq0$. That is, multiplying each side of a linear equation by the same nonzero number does not change the set of solutions, or $kax+kby=kc$, is equivalent to $ax+by=c$. 2)The solution to any system of linear equations is unchanged if one of the equations is replaced by a new equation formed by adding the two original equations. This property is often explained by invoking a general "equals added to equals are equal" rule, though careful justification requires more substantial reasoning. Using the two properties of the linear combination method, one of the two variables in a system can be eliminated. That is why this method is commonly referred to as the elimination method.

When solving a system of two linear equations symbolically using any method, students will encounter cases where there is no solution or where there are infinitely many solutions.

PARCC Assessment Evidence Statements

CCSS	Evidence Statement	Clarification	Math Practices	Calculator?
8.EE.5-1	Graph proportional relationships, interpreting the unit rate as the slope of the graph.	i) Pool should contain tasks with and without context. ii) The testing interface can provide students with a calculation aid of the specified kind for these tasks.	1, 5	Yes
8.F.1-2	[Understand that] the graph of a function is the set of ordered pairs consisting of an input and the corresponding output.	i) Functions are limited to those with inputs and outputs in the real numbers. ii) Tasks do not require knowledge of the concepts or terms domain and range. iii) 80% of tasks require students to graph functions in the coordinate plane or read inputs and outputs from the graph of a function in the coordinate plane. iv) 20% of tasks require students to tell whether a set of points in the plane represents a function.	2, 5	No
8.F.5-1	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear).	i) Tasks may or may not have a context.	2, 5	No
8.EE.7.b	Solve linear equations in one variable. b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms	i) Tasks do not have a context.	6, 7	No

8.EE.8a	Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.	i) Tasks do not have a context.	2, 5, 6, 7	No
8.C.1.1	Base reasoning on the principle that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. Content Scope: Knowledge and skills articulated in 8.EE.6	i) Note especially the portion of 8.EE.6 after the semicolon	2, 3, 7, 8	Yes
8.C.1.2	Base reasoning on the principle that the graph of an equation is two variables is the set of all its solutions plotted in the coordinate plane. Content Scope: Knowledge and skills articulated in 8.EE.8a	None	2, 3, 5, 6, 7	Yes
8.C.2	Given an equation or system of equations, present the solution steps as a logical argument that concludes with the set of solutions (if any). Content Scope: Knowledge and skills articulated in 8.EE.7a, 8.EE.7b, 8.EE.8b	None	3, 6	Yes

8.C.4.1	Present solutions to multi-step problems in the form of valid chains of reasoning, using symbols such as equals signs appropriately (for example, rubrics award less than full credit for the presence of nonsense statements such as 1+4=5+7=12, even if the final answer is correct), or identify or describe errors in solutions to multi-step problems and present corrected solutions. Content Scope: Knowledge and skills articulated in 8.EE.8c	i) See ITN Appendix F, section A, "Illustrations of innovative task characteristics," sub-section 6, "Expressing mathematics," sub-section "Illustrative tasks that require students to express mathematical reasoning," the problem of the two shepherds.	1, 2, 3, 6, 7	Yes
8.C.5.1	Apply geometric reasoning in a coordinate setting, and/or use coordinates to draw geometric conclusions. Content Scope: Knowledge and skills articulated in 8.EE.6	i) Note especially the portion of 8.EE.6 before the semicolon.	2, 3, 5	Yes

Connections to the Mathematical Practices

1	**Make sense of problems and persevere in solving them**
	- Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, —What is the most efficient way to solve the problem? —Does this make sense, and —Can I solve the problem in a different way?
2	**Reason abstractly and quantitatively**
	- Students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. - Students examine patterns in data and assess the degree of linearity of functions. - Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
3	**Construct viable arguments and critique the reasoning of others**
	- Students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, graphs, and tables. - They pose questions like —How did you get that? —Why is that true? —Does that always work? - They explain their thinking to others and respond to others' thinking.
4	**Model with mathematics**
	- Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. - Students solve systems of linear equations and compare properties of functions provided in different forms - Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context
5	**Use appropriate tools strategically**
	- Students use graphs, tables, and equations to represent relationships
6	**Attend to precision**
	- Students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. - Students use appropriate terminology when referring to functions, variables, equations, and graphs.
7	**Look for and make use of structure**
	- Students apply properties to generate equivalent expressions and solve equations. - Students examine patterns in tables and graphs to generate equations and describe relationships.
8	**Look for and express regularity in repeated reasoning**
	- During multiple opportunities to solve and model problems, they notice that the slope of a line and rate of change are the same value. - Students flexibly make connections between covariance, rates, and representations showing the relationships between quantities.

Vocabulary

Term	Definition
Commutative Property of Addition	A mathematical property that states that the order in which quantities are added does not matter. It states that $a + b = b + a$ for any two real numbers a and b. For example, $5 + 7 = 7 + 5$ and $2x + 4 = 4 + 2x$.
Commutative Property of Multiplication	A mathematical property that states that the order in which quantities are multiplied does not matter. It states that $ab = ba$ for any two real numbers a and b. For example, $5 \times 7 = 7 \times 5$ and $2x(4) = (4)2x$.
Distributive Property	A mathematical property used to rewrite expressions involving addition and multiplication. The Distributive Property states that for any three numbers a, b, and c, $a(b+c)=ab+ac$. If an expression is written as a factor multiplied by a sum, you can use the Distributive Property to multiply the factor by each term in the sum. $4(5+x)=4(5)+4(x)=20+4x$ If an expression is written as a sum of terms and the terms have a common factor, you can use the Distributive Property to rewrite the expression as the common factor multiplied by a sum. This process is called factoring. $20+4x=4(5)+4(x)=4(5+x)$
Properties of Equality	For all real numbers a, b, and c: Addition: If $a = b$, then $a + c = b + c$. Subtraction: If $a = b$, then $a - c = b - c$. Multiplication: If $a = b$, then $a \cdot c = b \cdot c$. Division: If $a = b$ and $c \neq 0$, then $a/c = b/c$.
Additive Inverses	Two numbers whose sum is 0 are additive inverses of one another. Example: ¾ and -3/4 are additive inverses of one another because ¾ + (-3/4) = (-3/4)+3/4 = 0
Algebraic Expression	A mathematical phrase involving at least one variable. Expressions can contain numbers and operation symbols.
Addition Property of Equality	For real numbers a, b, and c, if a = b, then a + c = b + c. In other words, adding the same number to each side of an equation produces an equivalent equation.
Equation	A mathematical sentence that contains an equals sign.
Evaluate an Algebraic Expression	To perform operations to obtain a single number or value.
Inverse Operation	Pairs of operations that undo each other, for example, addition and subtraction are inverse operations and multiplication and division are inverse operations.
Like Terms	Monomials that have the same variable raised to the same power. Only the coefficients of like terms can be different.

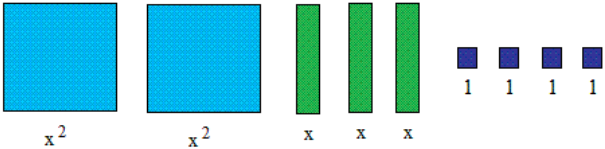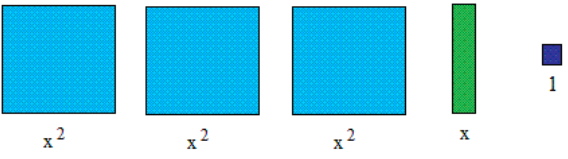| Linear Equation in One Variable	An equation that can be written in the form ax + b = c where a, b and c are real numbers and a ≠ 0.
Multiplication Property of Equality	For real numbers a, b, and c (c≠0), if a = b, then ac = bc. In other words, multiplying both sides of an equation by the same number produces an equivalent expression.
Multiplicative Inverses	Two numbers whose product is 1 are multiplicative inverses of one another. Example: ¾ and 4/3 are multiplicative inverses of one another because ¾ x 4/3 = 4/3 x ¾ = 1
Solution	The value or values that make an equation a true statement.
Solve	Identify the value that when substituted for the variable makes the equation a true statement.
Variable	A letter or symbol used to represent a number.
Linear Equation in Slope-Intercept Form	The slope-intercept form of a linear equation is $y = mx + b$, where m is the slope and b is the y-intercept.
Linear Equation in Standard Form	The standard form of a linear equation is $Ax + By = C$, where A, B, and C are integers and A and B are not both zero. The equation $6x + 3y = 12$ is in standard form. Although the slope-intercept form, $y = mx + b$, is common and useful, it is not considered the "standard form."
Solution of the System	A set of values for the variables that makes all the equations or inequalities true.
System of Linear Equations	Two or more linear equations that represent constraints on the variables used. A solution of a system of equations is a pair of values that satisfies all the equations in the system. For example, the ordered pair (1, 2) is the solution of the system because it satisfies both equations. {6x+3y=12 {−2x+y=0

Potential Student Misconceptions

- Students may think that only the letters x and y can be used for variables.

- Students may think that you always need a variable = a constant as a solution.

- The variable is always on the left side of the equation.

- Equations are not always in the slope intercept form, $y=mx+b$

- Students confuse one-variable and two-variable equations

- Students often struggle with the difference between combining like terms on one side of an equation and eliminating a variable on one side of an equation.

- Students may think any term can be combined no matter what the variables are.

- When you multiply or divide to solve equations students get confused if they're not taught to either put brackets around all the terms before they divide or multiply or if you do not emphasize the fact that you must multiply or divide every single term in the equation.

- Combination method for solving systems students may add up the variables in the equations as they are, instead of making them cancel out.

- Substitution method for solving systems students may re-arrange the equation and then substituting it back into itself. This will make everything cancel out.

- Students may only check a solution for only one of the equations in the system

Teaching to Multiple Representations

CONCRETE REPRESENTATIONS	
Bar Model	
Algebra Tiles	
PICTORIAL REPRESENTATIONS	
Tabular Representation	
Graphing	
ABSTRACT REPRESENTATIONS	

Simplifying Expressions/Combining Like Terms
Generating an Equation
a+.05a=1.05a

Properties of Operations:
Commutative
Associative
Distributive

Teaching Multiple Representations (Continued)

CONCRETE REPRESENTATIONS	
Algebra Tiles	$2x^2 + 3x + 4 \rightarrow$ $x^2 \quad x^2 \quad x \quad x \quad x \quad 1 \quad 1 \quad 1 \quad 1$
	$3x^2 + x + 1 \rightarrow$ $x^2 \quad x^2 \quad x^2 \quad x \quad 1$
Counters	
Grid/Graph Paper	
ABSTRACT REPRESENTATIONS	
• Equations, y = mx + b • Graphing Calculator – Used to demonstrate the connection between an equation and the respective graph	

Unit 4 Assessment Framework

Unit 4 Assessment Framework				
Assessment	**CCSS**	**Estimated Time**	**Format**	**Graded ?**
Unit 4 Diagnostic (Beginning of Unit)	7.EE.A.1, 7.EE.A.2, 7.EE.B.3, 7.EE.B.4a	½ Block	Individual	No
Unit 4 Assessment 1 (After Investigation 5) *Say It With Symbols*	8.EE.B.5, 8.F.A.2, 8.F.B.4, 8.F.B.5	1 Block	Individual	Yes
Unit 4 Check Up 1 (After Investigation 1) *It's In The System*	8.EE.C.8, 8.EE.C.8a, 8.EE.C.8b, 8.EE.C.8c, 8.F.A.3	½ Block	Individual	Yes
Unit 4 Assessment 2 (After Investigation 2) *It's In The System*	8.EE.C.7, 8.EE.C.8	1 Block	Individual	Yes
Unit 4 Check Up 1 (optional) *Say It With Symbols*	8.EE.C.7, 8.EE.C.7b, 8.F.A.3	½ Block	Teacher Discretion	Yes, if administered
Unit 4 Partner Quiz (optional) *Say It With Symbols*	8.EE.C.7, 8.EE.C.7b, 8.F.A.1, 8.F.A.2, 8.F.A.3, 8.G.C.9	½ Block	Teacher Discretion	Yes, if administered
Unit 4 Check Up 2 (optional) *Say It With Symbols*	8.EE.A.2, 8.EE.C.7, 8.EE.C.7a, 8.EE.C.7b, 8.EE.C.8, 8.EE.C.8a, 8.EE.C.8b, 8.EE.C.8c, 8.F.A.1	½ Block	Teacher Discretion	Yes, if administered
Unit 4 Check Up 2 (optional) *It's In The System*	8.EE.C.8, 8.EE.C.8a, 8.EE.C.8b, 8.EE.C.8c	½ Block	Teacher Discretion	Yes, if administered

Unit 4 Assessment Framework (continued)

Unit 4 Performance Assessment Framework				
Assessment	**CCSS**	**Estimated Time**	**Format**	**Graded ?**
Unit 4 Performance Task 1 (Early May) *Who Has the Best Job?*	8.EE.B.5	½ Block	Individual	Yes; Rubric
Unit 4 Performance Task 2 (Mid May) *Slopes*	8.EE.B.6	½ Block	Individual	Yes: rubric
Unit 4 Performance Task 3 (Late May) *Solving Equations*	8.EE.C.7	½ Block	Group	Yes; Rubric
Unit 4 Performance Task 4 (Late June) *Swimming*	8.EE.C.8	½ Block	Individual	Yes; Rubric
Unit 4 Performance Task Option 1 (optional)	8.EE.B.5	Teacher Discretion	Teacher Discretion	Yes, if administered
Unit 4 Performance Task Option 2 (optional)	8.EE.C.8	Teacher Discretion	Teacher Discretion	Yes, if administered

Performance Tasks

Unit 4 Performance Task 1

Who Has The Best Job? (8.EE.B.5)

Kell works at an after-school program at an elementary school. The table below shows how much money he earned every day last week.

	Monday	Wednesday	Friday
Time worked	1.5 hours	2.5 hours	4 hours
Money earned	$12.60	$21.00	$33.60

a. Mariko has a job mowing lawns that pays $7 per hour. Who would make more money for working 10 hours? Explain or show work.

b. Draw a graph that represents y, the amount of money Kell would make for working x hours, assuming he made the same hourly rate he was making last week.

c. Using the same coordinate axes, draw a graph that represents y, the amount of money Mariko would make for working x hours.

d. How can you see who makes more per hour just by looking at the graphs? Explain.

Solution:

a. Mariko would make $7 \times 10 = 70$ dollars for working 10 hours.

Kell's hourly rate can be found by dividing the money earned by the hours worked each day.

	Monday	Wednesday	Friday
Time worked	1.5 hours	2.5 hours	4 hours
Money earned	$12.60	$21.00	$33.60
Pay rate	$8.40 per hour	$8.40 per hour	$8.40 per hour

If Kell works for 10 hours at this same rate, he will earn $8.4 \times 10 = 84$ dollars. So Kell will earn more money for working 10 hours.

Alternatively, we could reason proportionally without computing the unit rate. Since Mariko earned $21.00 for 2.5 hours, she will earn four times as much for working four times as long ($10 = 4 \times 2.5$), for a total of $4 \times \$21 = \$84$.

b.
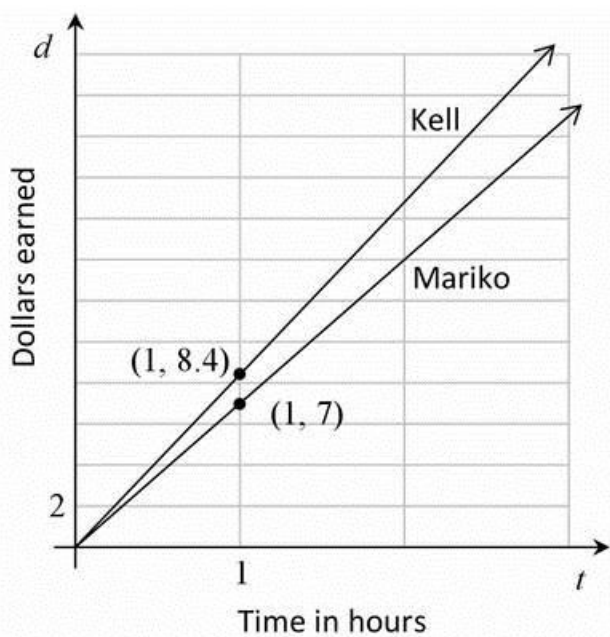
c. See the figure above

d. You can see that Kell will make more per hour if you look at the points on the graph where $x = 1$. Since this will tell you how much money each person will make for working 1 hour, you can see that Kell's line is above Mariko's line. This makes her line steeper than Mariko's line. You can also compare the slopes of the two graphs which are equal to the hourly rate.

Unit 4 Performance Task 1 PLD Rubric

SOLUTION
- **The student indicates Mariko will make $70.00, computes Kell's hourly rate and indicates that she will make 84.00, so she earns more money if she works 10 hours.**
- **Student creates a table with all the points for both Mariko and Kell. Students graph both situations by indicating x axis as time and y axis as dollars earned.**
- **The student indicates that at t = 1 Kell is making $8.40 vs. Mariko who is making $7.00. Or that Kell's line is above Mariko's line, which makes her line steeper. Or the value of the slope of Kell's line is bigger than the slope of Mariko's line.**

| Level 5: Distinguished Command	Level 4: Strong Command	Level 3: Moderate Command	Level 2: Partial Command	Level 1: No Command
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# Unit 4 Performance Task 2

Slopes (8.EE.B.6)

The slope between two points is calculated by finding the change in y-values and dividing by the change in x-values. For example, the slope between the points (7, -15) and (-8, 22) can be computed as follows:

- The difference in the y-values is $-15-22=-37$.
- The difference in the x-values is $7-(-8)=15$.
- Dividing these two differences, we find that the slope $\dfrac{-37}{15}$

Eva, Carl, and Maria are computing the slope between pairs of points on the line shown below.
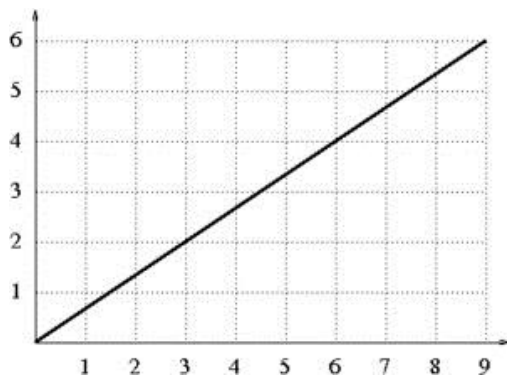
Eva finds the slope between the points (0,0) and (3,2). Carl finds the slope between the points (3,2) and (6,4). Maria finds the slope between the points (3,2) and (9,6). They have each drawn a triangle to help with their calculations (shown below).
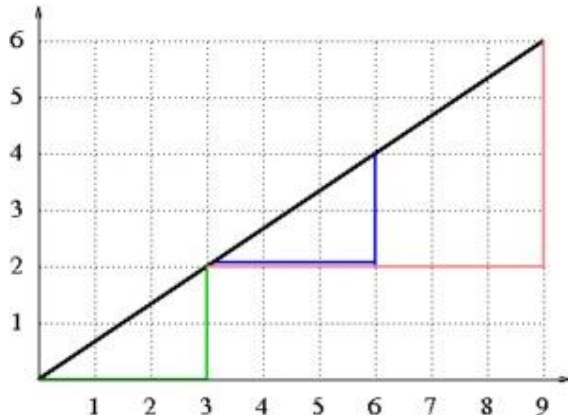
a. Which student has drawn which triangle? Finish the slope calculation for each student. How can the differences in the x- and y-values be interpreted geometrically in the pictures they have drawn?

b. Consider any two points (x1,y1) and (x2,y2) on the line shown above. Draw a triangle like the triangles drawn by Eva, Carl, and Maria. What is the slope between these two points? Why should this slope be the same as the slopes calculated by the three students?

Solution:

a. Eva is using the green triangle, since two of the vertices of the triangle are at $(0,0)$ and $(3,2)$. She will next find the length of each leg in the right triangle. The horizontal leg length is the difference between the x-coordinates, which is 3. The vertical leg length is the difference between the y-coordinates, which is 2. So the line rises by 2 units for every horizontal increase of 3 units. Therefore the slope is $\frac{2}{3}$.
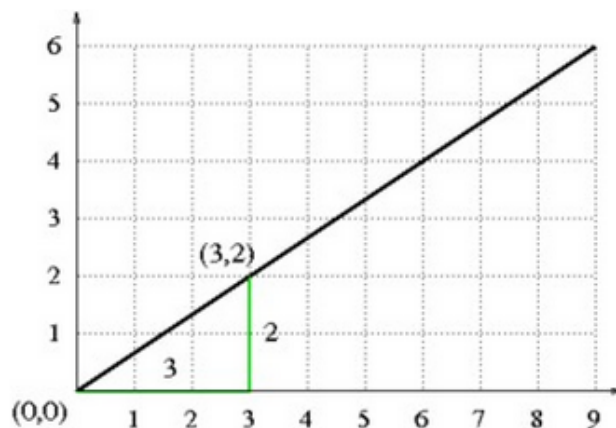

Carl is using the blue triangle, since two of the vertices of the triangle are at $(3,2)$ and $(6,4)$. He will next find the length of each leg in the right triangle. The horizontal leg length is the difference between the x-coordinates, which is 3. The vertical leg length is the difference between the y-coordinates, which is 2. So the line rises by 2 units for every horizontal increase of 3 units. Therefore the slope is $\frac{2}{3}$.

Maria is using the red triangle, since two of the vertices of the triangle are at $(3,2)$ and $(9,6)$. She will next find the length of each leg in the right triangle. The horizontal leg length is the difference between the x-coordinates, which is 6. The vertical leg length is the difference between the y-coordinates, which is 4. So the line rises by 4 units for every horizontal increase of 6 units. Therefore, the slope is $\frac{4}{6}$.

b. To compute the slope between two points, we are computing the quotient of the lengths of the legs in a right triangle. We can see that the blue and the green triangles are congruent since we can translate the green triangle along the line until it lines up with the blue triangle. Therefore, the quotient of the lengths of the legs must be the same.

The red triangle is not congruent to the blue triangle but it is similar to it. We can dilate the blue triangle by a factor of 2 to line it up with the red triangle. The sides in similar triangles also have the same proportion. Therefore, the quotient of the lengths of the legs of the two triangles must be the same.
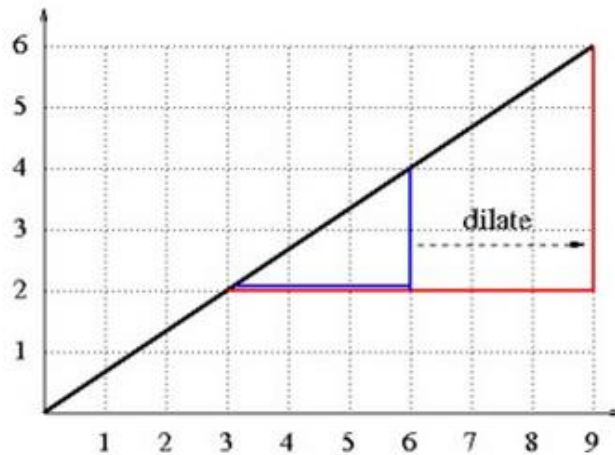
In order to draw a new triangle we can dilate the green triangle by a factor of 3 and get a similar triangle. In similar triangle ratio of the corresponding sides are equivalent, so the new triangle that I drew should have the same slope as the one Eva drew.
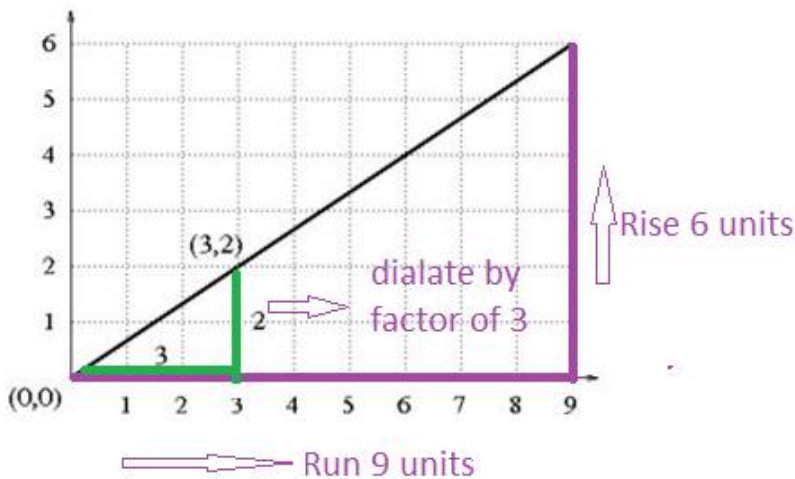
Slope of the Green Triangle

$$\frac{Rise}{Run} = \frac{2}{3}$$

Slope of the Dilated/Similar Triangle

$$\frac{Rise}{Run} = \frac{2\times3}{3\times3} = \frac{6}{9}$$

Unit 4 Performance Task 2 PLD Rubric

SOLUTION
- **The student indicates that the green one was drawn by Eva by refereeing to the coordinate and vertical and horizontal length. Carl drew the blue triangle and refers to the coordinate and the vertical and horizontal length.**
- **The student indicates the slopes of the triangle that Eva and Carl drew are equal because Carl's triangle was translated, which makes the triangles congruent. Student indicates that Maria's triangle is the dilation of Carl's Triangle by factor of 2, which makes her triangle similar and similar triangles are proportional.**
- **The student draws a new triangle by translating or by dilating and mentions the reason why it's the same slope as the 3 triangles that were drawn.**

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Unit 4 Performance Task 3

Solving Equations (8. EE.C.7)

In elementary school, students often draw pictures of the arithmetic they do. For instance, they might draw the following picture for the problem 2+3:


In this picture, each square represents a tile.
We can do the same thing for algebraic expressions, but we need to be careful about how we represent the unknown. If we assume that an unknown number of tiles x are contained in a bag, we could draw the following picture for 2x+3:

When we have an equation to solve, we assume that the two sides of the equation are equal. We can represent this by showing them level on a balance. For example, the equation 2x+3=7 could be shown as:
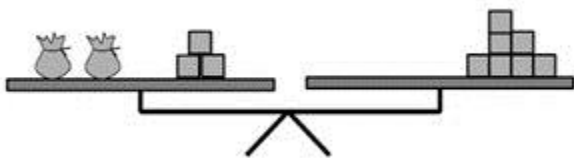
When we solve equations, we can add, subtract, multiply or divide both sides of the equation by the same thing in order to maintain the equality. This can be shown in pictures by keeping the balance level. For example, we could solve the equation 2x+3=7 using pictures by first removing (subtracting) 3 from each side, and then splitting (dividing) the remaining blocks into two equal groups:
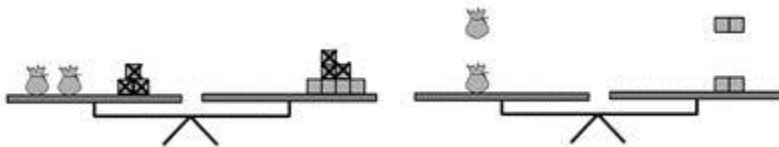
From this picture, we can see that, in order to keep the balance level, each bag must contain 2 tiles, which means that x=2.

a. Solve 5x+1=2x+7 in two ways: symbolically, the way you usually do with equations, and also with pictures of a balance. Show how each step you take symbolically is shown in the pictures.

b. Solve the equation 4x=x+1 using pictures and symbols. Discuss any issues that arise.

c. What issues arise when you try to solve the equation $2=2x-4$ using pictures? Do the same issues arise when you solve this equation symbolically?

d. Make up a linear equation that has no solutions. What would happen if you solved this equation with pictures? How is this different than an equation that has infinitely many solutions?

Use pictures to show why the following solution to the equation $2x+4=10$ is incorrect:

$$\frac{2x}{2} + 4 = \frac{10}{2}$$
$$x + 4 = 5$$
$$\frac{-4 \quad -4}{x \qquad = 1}$$

Solution:

a.

Symbolically:

$$5x + 1 = 2x + 7$$

$$\underline{\quad -1 \qquad -1 \quad}$$

$$5x \qquad = 2x + 6$$

$$\underline{-2x \qquad -2x \quad}$$

$$\frac{3x}{3} \quad = \quad \frac{6}{3}$$

$$x \quad = \quad 2$$
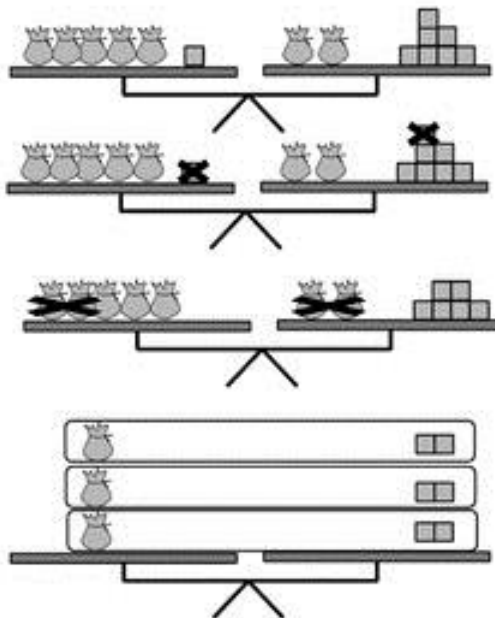
With pictures:

b. When you solve this equation with pictures, you end up with 3 bags balancing with 1 tile. In order to do the division, you have to cut the tile, leading to the fraction 1/3, which is the solution you get symbolically.

c. In order to solve this equation with pictures, you have to have some way of representing the subtraction in $2x-4$. If students have experience with integer chips, they can transfer that knowledge to this situation to show $2x + -4$, but otherwise they may struggle with the idea. The pictures give us a nice model for understanding the operations we do to solve equations, but it is only smooth for problems with "nice" numbers. This is one reason why we want to move to the symbolic approach.
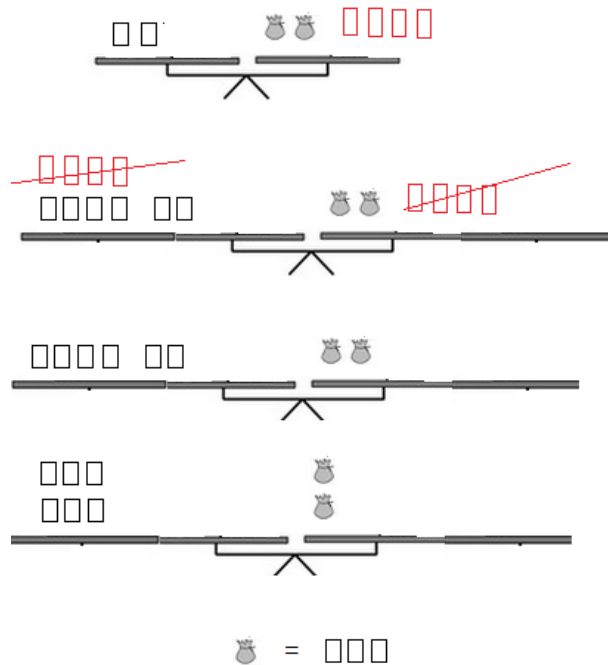
d. A linear equation will have no solution if there are the same number of x's and different constants on each side. For example: $2x + 4 = 2x + 1$. If you solve this with pictures, when you take away the $2x$ from both sides you will end up with $4 = 1$, which clearly cannot be balanced. If the equation had infinitely many solutions, you would find that you had exactly the same picture on the two sides of the balance.

e. The mistake is in the first step - the student divided only part of the left-hand-side of the equation by 2. You can see in the picture that splitting the equation this way will not keep the balance level (assuming the two bags are equal):
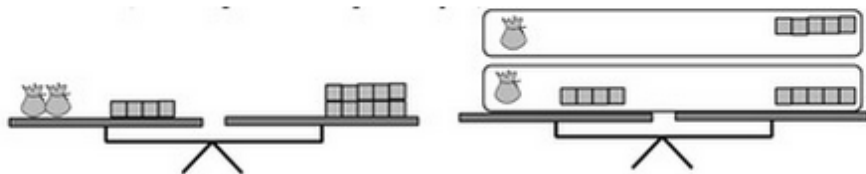
Unit 4 Performance Task 3 PLD Rubric

SOLUTION
- **The student solves the problem both symbolically and with pictures by showing each steps with correct procedures and pictures with the answer x = 2**
- **The student indicates that the problem starts with 3 bags and one tile. You would have to break the tile into one third and one will get the same solution if they solve this equation symbolically.**
- **Student indicates that it is hard to solve this equation symbolically due to negative tiles on one side of the equation. In order to solve this equation you would have to add 4 negatives and 4 positives to the left hand side. The student can either explain it verbally or show it symbolically.**
- **The student indicates that any equations that have the same variables on both sides of the equal sign with different constant terms have no solution. The student explains how it doesn't make sense to set different number of tiles equal to each other.**
- **The student indicates the mistake took place in step one with division and draws the picture with division.**

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Unit 4 Performance Task 4

Summer Swimming (8.EE.C.8)

The local swim center is making a special offer. They usually charge $7 per day to swim at the pool. This month swimmers can pay an enrollment fee of $30 and then the daily pass will only be $4 per day.

a. Suppose you do not take the special offer. Write an equation that represents the amount of money you would spend based on how many days you go to the pool if the passes were bought at full price. Explain what your variables represent

b. Write a second equation that represents the amount of money you would spend if you decided to take the special offer. Explain what your variables represent.

c. Graph your two equations from part (a) and (b).

d. After how many days of visiting the pool will the special offer be a better deal? How can you tell algebraically? How can you see this graphically? Show your work.

e. You only have $60 to spend for the summer on visiting this pool. Which offer would you take? Explain.

Solution:

a. Let m represent the amount of money you spend, and let d represent the number of days you go to the pool. At full price, you would spend $7 per day, so $m = 7d$ represents the amount of money you would spend without the special offer.

b. Again, let m represent the amount of money you spend, and let d represent the number of days you go to the pool. With the special offer, you would spend $30 once, and then $4 per day, so $m = 4d + 30$ represents the amount of money you would spend with the special offer.

c.
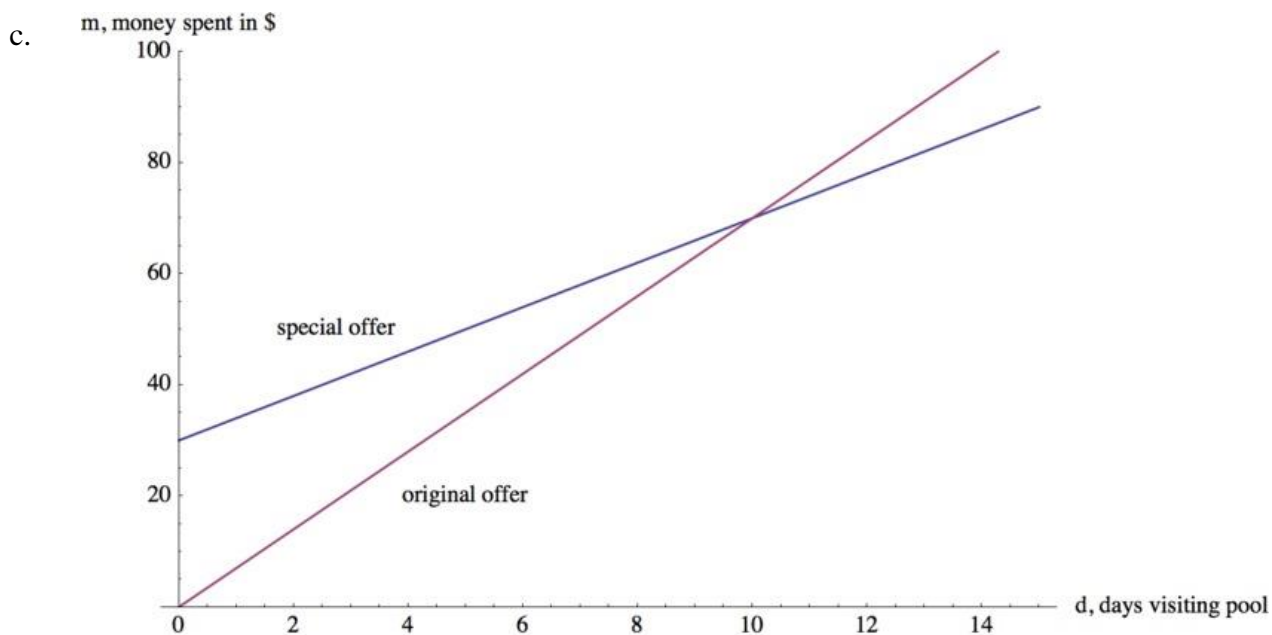
d. When considering when the special offer will become a better deal, we want to find out after how many days we will be spending less money than with the full price passes. For example, after 3 days, the full price offer will cost us $21 dollars and the special offer will cost us $39, so the special will cost more. We see that this is because of the initial fee of $30.

Algebraically, we can determine exactly when the special offer costs the same amount as the standard price by setting our two expressions for m equal to each other and solving for d.

$$4d + 30 = 7d$$

$$30 = 3d$$

$$d = 10.$$

So, when $d = 10$, or on the 10th day, the special offer and the original will have cost us the same amount of money:

$$m = 7(10) = 70 \quad \text{and} \quad m = 4(10) + 30 = 70.$$

Because on the 11th day, and every day thereafter, the special offer will cost less ($4 a day versus $7 a day), we can conclude that after the 10th day is when the special offer becomes a better deal.

Graphically, we can see that the graph of the special offer drops below the graph of the original offer after $d = 10$, and as lower translates into less money spent, we can see our conclusion in the graph, as well.

e. $60 = 4d + 30$

    $60 - 30 = 4d + 30 - 30$

    $30 = 4d$

    $30/4 = d$

    7.5 days $= d$

    You can't go to swimming for half day, so $d = 7$ days. Given $60.00, with the special offer you can go to the pool for 7 days.

    $60 = 7d$

    $60/7 = 8.571$

    You can't go to swimming for half day, so $d = 8$ days. Given $60.00, without the special offer you can go to the pool for 8 days.

Unit 4 Performance Task 4 PLD Rubric

SOLUTION
- **Student indicates the equation for full price, *m* = 7*d* or *y* =7*x***
- **Student indicates the equation for special offer m = 4*d* + 30 or *y* = 4*x* + 30**
- **Student graphs both of the equations by labeling the axis correctly and plotting the points accurately**
- **Student indicates that after 10 days, the price with the special offer is better by referring to the points on the graph or the point of intersection(break-even point)**
- **Student sets up both equations equal to each other and explains algebraically by showing all the steps and indicates that after *x* = 10 days or *d* = 10 days the special offer price is better OR student substitutes the values of *d* = 10 and *d*= 11 to show why after 10 days the special offer price is better**
- **Student indicates that it is better to go without the special offer because you can go to pool for 8 days rather than 7 days with the special offer.**
- **Student also explains how he/she arrived to this conclusion.**

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Unit 4 Performance Task Option 1

Stuffing Envelopes (8.EE.B.5)

Anna and Jason have summer jobs stuffing envelopes for two different companies. Anna earns $14 for every 400 envelops she finishes. Jason earns $9 for every 300 envelopes he finishes.

 a. Draw graphs and write equations that show the earnings, y as functions of the number of envelopes stuffed, n for Anna and Jason.

 b. Who makes more from stuffing the same number of envelopes? How can you tell this from the graph?

 c. Suppose Anna has savings of $100 at the beginning of the summer and she saves all her earnings from her job. Graph her savings as a function of the number of envelopes she stuffed, n. How does this graph compare to her previous earnings graph? What is the meaning of the slope in each case?

Unit 4 Performance Task Option 2

Cell Phone Plans (8.EE.C.8)

You are a representative for a cell phone company and it is your job to promote different cell phone plans.

a. Your boss asks you to visually display three plans and compare them so you can point out the advantages of each plan to your customers.

   - Plan A costs a basic fee of $29.95 per month and 10 cents per text message
   - Plan B costs a basic fee of $90.20 per month and has unlimited text messages
   - Plan C costs a basic fee of $49.95 per month and 5 cents per text message
   - All plans offer unlimited calling
   - Calling on nights and weekends are free
   - Long distance calls are included

b. A customer wants to know how to decide which plan will save her the most money. Determine which plan has the lowest cost, given the number of text messages a customer is likely to send.

Extensions and Sources

Assessment Resources

http://dashweb.pearsoncmg.com
- Online Connected Math 3 Resources

http://www.illustrativemathematics.org/standards/k8
- Performance tasks, scoring guides

Online Resources

http://www.ixl.com/math/grade-8
- Interactive, visually appealing fluency practice site that is objective descriptive

https://www.khanacademy.org/
- Tracks student progress, video learning, interactive practice

http://www.doe.k12.de.us/assessment/files/Math_Grade_8.pdf
- Common Core aligned assessment questions, including Next Generation Assessment
  Prototypes

https://www.georgiastandards.org/Common-Core/Pages/Math-6-8.aspx
- Special Needs designed tasks, assessment resources

http://www.parcconline.org/sites/parcc/files/PARCCMCFMathematicsGRADE8_Nov2012V3_FINAL.pdf
- PARCC Model Content Frameworks Grade 8

http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_ee_2011_04_25.pdf
- Document Progressions