8th Grade Mathematics

Exponential Equations: Expressions & equations work with radicals and integer exponents

Unit 2 Curriculum Map: November 28th – January 18th

ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

A STORY OF UNITS

	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN
1										
2										
3										
4										
5										
6										
7										
8	Geometry		Expressions & Equations			Thinking with Mathematical Models		Linear Equations & Systems		

Geometry: Understand congruence and similarity using physical models, transparencies, or geometry software

Expressions and Equations: Expressions and Equations Work with radicals and interger exponents

Thinking with Mathematical Models: Define, evaluate, and compare functions, use similar triangles to explain slope, investigate patterns of association in bivariate data

Linear Equations and Systems: Use functions to model relationships between quantities, analyze and solve linear equations and pairs of simulataneous linear equations

Table of Contents

Unit 2 Overview

<u>In this unit students will ….</u>

- Identify situations that can be modeled with an exponential function
- Identify the pattern of change (growth/decay factor) between two variables that represent an exponential function in a situation, table, graph, or equation
- Represent an exponential function with a table, graph, or equation
- Compare the growth/decay rate and growth/decay factor for an exponential function and recognize the role each plays in an exponential situation
- Identify the growth/decay factor and initial value in problem situations, tables, graphs, and equations that represent exponential functions
- Determine whether an exponential function represents a growth (increasing) or decay (decreasing) pattern, from an equation, table, or graph that represents an exponential function
- Determine the values of the independent and dependent variables from a table, graph, or equation of an exponential function
- Use an exponential equation to describe the graph and table of an exponential function
- Predict the *y*-intercept from an equation, graph, or table that represents an exponential function
- Solve problems about exponential growth and decay from a variety of different subject areas, including science and business, using an equation, table, or graph
- Observe that one exponential equation can model different contexts
- Write and interpret exponential expressions that represent the dependent variable in an exponential function
- Develop the rules for operating with rational exponents and explain why they work
- Write, interpret, and operate with numerical expressions in scientific notation
- Write and interpret equivalent expressions using the rules for exponents and operations
- Solve problems that involve exponents, including scientific notation

<u>Enduring Understandings</u>

- Make connections among the patterns of change in a table, graph, and equation of an exponential function
- Real world situations involving exponential relationships can be solved using multiple representations.
- There is a specific order of operations in the real number system that must be followed for all computations.
- Scientific notation will help demonstrate very large and very small numbers when solving real world application problems.
- Numbers can be represented in scientific notation and still be manipulated using operations such as addition, subtraction, multiplication, and division.

Pacing Guide

Activity	New Jersey Student Learning Standards (NJSLS)	Estimated Time
Unit 2 Diagnostic	6.EE.A.2.C, 7.EE.B.3; 7.EE.B.4.A, 7.NS.A.2.A, 7.NS.A.2.B	½ Block
Growing, Growing, Growing (CMP3) Investigation 1	8.EE.3, 8.EE.4, 8.F.2, 8.F.4	3 Blocks
Growing, Growing, Growing (CMP3) Investigation 2	8.EE.3, 8.F.5	3 Blocks
Unit 2 Performance Task 1	8.EE.A.1, 8.F.A.1, 8.F.B.5	½ Block
Exponents and Exponential Models (Agile Mind) Lesson 19.1	8.EE.A.1, 8.EE.A.2	1½ Blocks
Exponents and Exponential Models (Agile Mind) Topic 19.2	8.EE.A.1	2½ Block
Exponents and Exponential Models (Agile Mind) Topic 19.3	8.EE.A.1	2½ Blocks
Unit 2 Performance Task 2	8.EE.1	½ Block
Unit 2 Assessment 1	8.EE.A.1	1 Block
Magnitude & Scientific Notation (EngageNY) Module 1 Topic B Lesson 8	8.EE.3	1 Block
Magnitude & Scientific Notation (EngageNY) Module 1 Topic B Lesson 9	8.EE.4	1 Block
Magnitude & Scientific Notation (EngageNY) Module 1 Topic B Lesson 10	8.EE.4	1 Block
Unit 2 Assessment 2	8.EE.3	1 Block
Growing, Growing, Growing (CMP3) Investigation 5.4	8.EE.3, 8.EE.4	2½ blocks
Unit 2 Assessment 3	8.EE.4	1 Block
Unit 2 Performance Task 3	8.EE.3, 8.EE.4	½ Block
Total Time		**23 Blocks**

Major Work Supporting Content Additional Content

Pacing Calendar

NOVEMBER						
Sunday	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**	**Saturday**
		1	**2**	**3**	**4**	**5**
6	**7**	**8**	**9**	**10** **NJEA Convention** **District Closed**	**11** **NJEA Convention** **District Closed**	**12**
13	**14**	**15**	**16** **12:30pm Student Dismissal**	**17**	**18**	**19**
20	**21**	**22**	**23** **12:30 pm Dismissal**	**24** **Thanksgiving Recess** **District Closed**	**25** **Thanksgiving Recess** **District Closed**	**26**
27	**28** **Unit 2: Exponential Equations** ***Unit 2 Diagnostic***	**29**	**30**			

DECEMBER

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
				1	**2**	**3**
4	**5**	**6** ***Performance Task 1 Due***	**7**	**8**	**9**	**10**
11	**12**	**13**	**14**	**15** ***Performance Task 2 Due***	**16** ***Assessment: Unit 2 Assessment 1***	**17**
18	**19**	**20**	**21**	**22**	**23 12:30 pm Dismissal**	**24**
25	**26 Holiday Recess** **District Closed**	**27 Holiday Recess** **District Closed**	**28 Holiday Recess** **District Closed**	**29 Holiday Recess** **District Closed**	**30 Holiday Recess** **District Closed**	**31**

JANUARY

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1	**2** **Holiday Recess** **District Closed**	**3**	**4** ***Assessment: Unit 2 Assessment 2***	**5**	**6**	**7**
8	**9** ***Performance Task 3 Due***	**10** ***Assessment: Unit 2 Assessment 3***	**11** **Solidify Unit 1 Concepts** **Project Based Learning**	**12** **Solidify Unit 1 Concepts** **Project Based Learning**	**13** **Solidify Unit 1 Concepts** **Project Based Learning**	**14**
15	**16** **MLK Birthday** **District Closed**	**17** **Solidify Unit 1 Concepts** **Project Based Learning**	**18** **Solidify Unit 1 Concepts** **Project Based Learning**	**19**	**20**	**21**
22	**23**	**24**	**25** **12:30 pm Student Dismissal**	**26** **12:30 pm Student Dismissal**	**27**	**28**
29	**30**	**31**				

Math Background

This algebra Unit, *Growing, Growing, Growing*, focuses students' attention on exponential functions. Students look at studies of biological populations, from bacteria and amoebas to mammals (including humans), which often reveal exponential patterns of growth. Such populations may increase over time and at increasing rates of growth. This same pattern of growth at increasing rates is seen when money is invested in accounts paying compound interest, or when growth of amoeba is tracked.

In Growing, Growing, Growing, equivalence occurs naturally as students generate two or more symbolic expressions for the dependent variable in an exponential function. Students use patterns from the table, graph, or verbal descriptions to justify the equivalence.

In Investigation 5, after the rules for operation with exponents are developed, students use properties of exponents and operations to demonstrate that two expressions are equivalent.

Students begin to develop understanding of the rules of exponents by examining patterns in the table of powers for the first 10 whole numbers.

Since exponential growth patterns can grow quite fast, students may encounter scientific notation on their calculators. Therefore, scientific notation is introduced in Investigation 1 and then used throughout the Unit. The ACE exercises in every Investigation also continue to reinforce the skills need to work with scientific notation. Sometimes if the numbers are very large or very small, you might get an approximation. Students use exponential notation and the rules of exponents to solve problems with scientific notation.

PARCC Assessment Evidence Statements

CCSS	Evidence Statement	Clarification	Math Practices	Calculator?
8.EE.1	Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 1/3^3 = 1/27$	i) Tasks do not have a context. ii) Tasks center on the properties and equivalence, not on simplification. For example, a task might ask a student to classify expressions according to whether or not they are equivalent to a given expression. iii) 50% of expressions should involve one property iv) 30% of expressions should involve two properties v) 20% of expressions should involve three properties vi) Tasks should involve a single common base	7	No
8.EE.3	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.	None	4	No
8.EE.4-1	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used.	i) Tasks have "thin context" or no context. ii) Rules or conventions for significant figures are not assessed. iii) 20% of tasks involve both decimal and scientific notation, e.g., write $120 + 3 \times 10^4$ in scientific notation.	6, 7, 8	No

<u>8.EE.4-2</u>	Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	i) Task have “thin context”. ii) The testing interface can provide students with a calculation aid of the specified kind for these tasks. iii) Tasks may require students to recognize 3.7E-2 (or 3.7e-2) from technology as 3.7×10^{-2}	6, 7, 8	Yes

Connections to the Mathematical Practices

1	Make sense of problems and persevere in solving them
	- Students use tools, conversions, and properties to solve problems
2	Reason abstractly and quantitatively
	- Students use concrete numbers to explore the properties of numbers in exponential form and then prove that the properties are true for all positive bases and all integer exponents using symbolic representations for bases and exponents. - Use symbols to represent integer exponents and make sense of those quantities in problem situations. - Students refer to symbolic notation in order to contextualize the requirements and limitations of given statements (e.g., letting m, n represent positive integers, letting a, b represent all integers, both with respect to the properties of exponents)
3	Construct viable arguments and critique the reasoning of others
	- Students reason through the acceptability of definitions and proofs (e.g., the definitions of x^0 and x^{-b} for all integers b and positive integers x). - New definitions, as well as proofs, require students to analyze situations and break them into cases. - Examine the implications of definitions and proofs on existing properties of integer exponents. Students keep the goal of a logical argument in mind while attending to details that develop during the reasoning process.
4	Model with mathematics
	- When converting between measurements in scientific notations, students understand the scale value of a number in scientific notation in one unit compared to another unit
5	Use appropriate tools strategically
	- Understand the development of exponent properties yet use the properties with fluency - Use unit conversions in solving real world problems
6	Attend to precision
	- In exponential notation, students are required to attend to the definitions provided throughout the lessons and the limitations of symbolic statements, making sure to express what they mean clearly. Students are provided a hypothesis, such as $x < y$, for positive integers x, y, and then asked to evaluate whether a statement, like $-2 < 5$, contradicts this hypothesis.
7	Look for and make use of structure
	- Students understand and make analogies to the distributive law as they develop properties of exponents. Students will know $x^m \cdot x^n = x^{m+n}$ as an analog of $m^x + n^x = (m + n)$ and $(x^m) = x^{m \times n}$ as an analog of $n \times (m \times x) = (n \times m) \times x$.
8	Look for and express regularity in repeated reasoning
	- While evaluating the cases developed for the proofs of laws of exponents, students identify when a statement must be proved or if it has already been proven. - Students see the use of the laws of exponents in application problems and notice the patterns that are developed in problems.

Vocabulary

Term	Definition
Algebraic Expression	A mathematical phrase involving at least one variable. Expressions can contain numbers and operation symbols.
Cube Root	If the cube root of a number b is a (i.e. $\sqrt[3]{b} = a$), then $a^3 = b$.
Decimal Expansion	The decimal expansion of a number is its representation in base 10 (i.e., the decimal system). For example, the decimal expansion of 25^2 is 625, or π is 3.14159…, and of 1/9 is 0.1111…
Equation	A mathematical sentence that contains an equals sign
Exponent	The number of times a base is used as a factor of repeated multiplication
Evaluate an Algebraic Expression	To perform operations to obtain a single number or value
Inverse Operation	Pairs of operations that undo each other, for example, addition and subtraction are inverse operations and multiplication and division are inverse operations.
Like Terms	Monomials that have the same variable raised to the same power. Only the coefficients of liker terms can be different
Multiplicative Inverses	Two numbers whose product is 1 are multiplicative inverses of one another. Example: ¾ and 4/3 are multiplicative inverses of one another because ¾ x 4/3 = 4/3 x ¾ = 1.
Multiplication Property of Equality	For real numbers a, b, and c (c ≠ 0), if a = b, then ac = bc. In other words, multiplying both sides of an equation by the same number produces an equivalent expression.
Perfect Square	A number that has a rational number as its square root.
Radical	A symbol √ that is used to indicate square roots
Rational	A number that can be written as the ratio of two integers with a nonzero denominator
Square Root	One of two equal factors of a nonnegative number. For example, 5 is a square root of 25 because 5x5 = 25. Another square root of 25 is -5 because (-5) x (-5) = 25. The +5 is called the principle square root of 25 and is always assumed when the radical symbol is used.

Scientific Notation	A representation of real numbers as the product of a number between 1 and 10 and a power of 10, used primarily for very large or very small numbers.
Solution	The value or values of a variable that make an equation a true statement.
Solve	Identify the value that when substituted for the variable makes the equation a true statement.
Variable	A letter or symbol used to represent at number
Qualitative Variables	A variable whose values are not numerical. Examples include gender (male, female), paint color (red, black, blue), etc.
Bivariate Data	Two different response variables that are from the same population.
Quantitative Variables	A variable whose values are numerical. Examples include height, temperature, weight, grades, and etc.
Model	A mathematical representation of a process, device, or concept by means of a number of variables.
Interpret	To establish or explain the meaning or significance of something.
Linear	A relationship or function that can be represented by a straight line.
Non-Linear	A relationship which does not create a straight line
Slope	The measure of steepness of a line
Base	The number that is raised to a power in an exponential expression. In the expression 3^5, read "3 to the fifth power", 3 is the base and 5 is the exponent.
Standard Form	The most common way we express quantities. For example, 27 is the standard form of 3^3.
Exponential Form	A quantity expressed as a number raised to a power. In exponential form, 32 can be written as 2^5. The exponential form of the prime factorization of 5,000 is $2^3 \times 5^4$.
Growth Factor	The constant factor that each value in an exponential growth pattern is multiplied by to get the next value. The growth factor is the base in an exponential growth equation, and is a number greater than 1.
Growth Rate	The percent increase in an exponential growth pattern.
Decay Factor	The constant factor that each value in an exponential decay pattern is multiplied by to get the next value. The decay factor is the base in an exponential decay equation, and is a number between 0 and 1.

Decay Rate	The percent decrease in an exponential decay pattern. In general, for an exponential pattern with decay factor *b*, the decay rate is $1-b$.
Nth Root	The *n*th root of a number *b* is a number *r* which, when raised to the power of *n*, is equal to *b*.

Potential Student Misconceptions

- Students often mistake the exponent as the number of zeros to put on the end of the coefficient instead of realizing it represents the number of times they should multiply by ten.

- Students often move the decimal in the wrong direction when dealing with positive and negative powers. Also, students forget to the move the decimal past the first non-zero digit (or count it) for very small numbers.

- Students may make the relationship that in scientific notation, when a number contains one nonzero digit and a positive exponent, that the number of zeros equals the exponent. This pattern may incorrectly be applied to scientific notation values with negative values or with more than one nonzero digit. Students may mix up the product of powers property and the power of a power property.

- When writing numbers in scientific notation, students may interpret the negative exponent as a negative number.

- When multiplying or dividing numbers that are given in scientific notation, in which the directions say to write the answer in scientific notation, sometimes students forget to double check that the answer is in correct scientific notation.

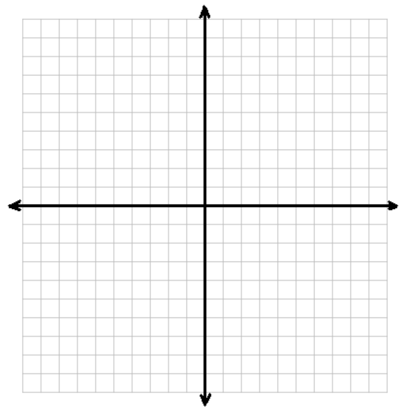
- When performing calculations on a calculator, in which the number transforms to scientific notation, students sometimes overlook the last part of the number showing scientific notation part and just notice the first part of the number, ignoring the number after E.

- Students will sometimes multiply the base and the exponent. For example, 2^6 is not equal to 12, it's 64.

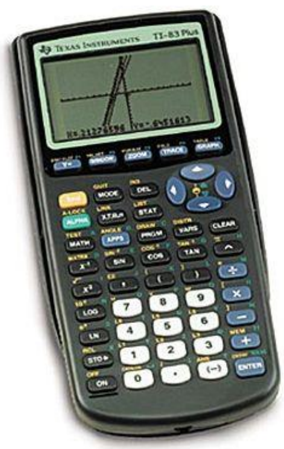
Teaching Multiple Representations

CONCRETE REPRESENTATIONS

- **Grid/Graph Paper**



- **Graphing Calculator**



- **Tables**

X	Y
2	7
3	5
4	2
4	3

X	Y
1	6
2	6
3	6
7	6

PICTORIAL REPRESENTATIONS	
• **Making sense of exponents**	$\frac{(2u^{2}v^{2})^{4}}{2u^{4}v^{4}\cdot 2u^{1}v^{1}} = \frac{16u^{8}v^{8}}{4u^{5}v^{5}} = 4u^{3}v^{3}$ $2\cdot 2\cdot 2\cdot 2 = 16$ $\frac{1m^{3}n^{4}\cdot 2m^{3}n^{4}}{(2m^{4}n^{3})^{4}} = \frac{2m^{6}n^{8}}{16m^{16}n^{12}} = \frac{1}{8m^{10}}$

ABSTRACT REPRESENTATIONS
• Equations • Scientific Notation • Integer powers of 10 • Properties of integer exponents

Assessment Framework

Unit 2 Assessment Framework				
Assessment	**NJSLS**	**Estimated Time**	**Format**	**Graded ?**
Unit 2 Diagnostic (Beginning of Unit)	6.EE.A.2.C, 7.EE.B.3; 7.EE.B.4.A,7.NS.A.2.A, 7.NS.A.2.B	½ Block	Individual	No
Unit 2 Assessment 1 (After Topic 19.3) *District Assessment*	8.EE.A1, 8.EE.A2	½ Block	Individual	Yes
Unit 2 Assessment 2 (After Lesson 10) *District Assessment*	8.EE.3	1 Block	Individual	Yes
Unit 2 Assessment 3 (After Investigation 5.4) *District Assessment*	8.EE.4	1 Block	Individual	Yes
Unit 2 Check Up (Optional) *Growing, Growing, Growing CMP3*	8.EE.3, 8.EE.4, 8.F.2, 8.F.4, 8.F.5	½ Block	Individual	Yes
Unit 2 Partner Quiz (Optional) *Growing, Growing, Growing CMP3*	8.F.A.1, 8.F.A.2, 8.F.A.3, 8.F.B.5	½ Block	Group	Yes

Unit 2 Performance Assessment Framework				
Assessment	**NJSLS**	**Estimated Time**	**Format**	**Graded ?**
Unit 2 Performance Task 1 (Early December) *Extending the Definition of Exponents*	8.EE.A.1, 8.F.A.1, 8.F.B.5	½ Block	Individual w/ Interview Opportunity	Yes; Rubric
Unit 2 Performance Task 2 (Mid-December) *Raising to zero and negative powers*	8.EE.A.1	½ Block	Group (Possible Reflection)	Yes: rubric
Unit 2 Performance Task 3 (Mid-January) *Giant burgers*	8.EE.A.3, 8.EE.A.4	½ Block	Individual w/ Interview Opportunity	Yes; Rubric
Unit 2 Performance Task Option 1 (optional)	8.EE.A.1, 8.EE.A.4	Teacher Discretion	Teacher Discretion	Yes, if administered
Unit 2 Performance Task Option 2 (optional)	8.EE.A.3	Teacher Discretion	Teacher Discretion	Yes, if administered

Performance Tasks

Unit 2 Performance Task 1

Extending the Definition of Exponents (8.EE.A.1)

Marco and Seth are lab partners studying bacterial growth. They were surprised to find that the population of the bacteria doubled every hour.

a. The table shows that there were 2,000 bacteria at the beginning of the experiment. What was the size of population of bacteria after 1 hour? After 2, 3 and 4 hours? Enter this information into the table:

Hours into study				0	1	2	3	4
Population (thousands)				2				

b. If you know the size of the population at a certain time, how do you find the population one hour later?

c. Marco said he thought that they could use the equation P=2t+2 to find the population at time t. Seth said he thought that they could use the equation P=2 · 2t. Decide whether either of these equations produces the correct populations for t=1,2,3,4.

d. Assuming the population doubled every hour before the study began, what was the population of the bacteria 1 hour *before* the students started their study? What about 3 hours before?

e. If you know the size of the population at a certain time, how do you find the population one hour *earlier*?

f. What number would you use to represent the time 1 hour before the study started? 2 hours before? 3 hours before? Finish filling in the table if you haven't already.

g. Now use Seth's equation to find the population of the bacteria 1 hour before the study started. Use the equation to find the population of the bacteria 3 hours before. Do these values produce results consistent with the arithmetic you did earlier?

h. Use the context to explain why it makes sense that 2−n=(12)n=12n. That is, describe why, based on the population growth, it makes sense to define 2 raised to a negative integer exponent as repeated multiplication by 12.

Solution:

a. What was the size of population of bacteria after 1 hour? After 2, 3 and 4 hours? Enter this information into the table:

Hours into study	0	1	2	3	4
Population (thousands)	2	4	8	16	32

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b. You multiply the previous value by 2 to get the next value, since it doubled.

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c. The values predicted by Seth's equation agree exactly with those in the table above; Seth's equation works because it predicts a doubling of the population every hour. Marco's doesn't because it doesn't double the new population you have – instead it is doubling the time. Marco's equation predicts a linear growth of only two thousand bacteria per hour.

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d. Since the population is multiplied by 2 every hour we would have to divide by 2 (which is the same as multiplying by 12) to work backwards. The population 1 hour before the study started would be $12 \cdot 2=1$ thousand, and the population 3 hours before the study started would be $12 \cdot 12 \cdot 12 \cdot 2=0.25$ thousand=250.

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e. Time before the study started would be negative time; for example one hour before the study began was $t=-1$.

Hours into study	-3	-2	-1	0	1	2	3	4
Population (thousands)	$12 \cdot 12=14=0.25$	$12 \cdot 1=0.5$	1	2	4	8	16	32

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f. Since one hour before the study started would be $t=-1$, we would simply plug this value into Seth's equation: $2 \cdot (2)^{-1} = 2 \cdot (1/2)^{1} = 1$ thousand. Three hours before would be $t=-3$. Using the equation: $2 \cdot (2)^{-3}=2/2^{3}=0.25$ thousand, giving us the same answers as we got through reasoning.

g. Since the bacteria double every hour, we multiply the population by two for every hour we go forward in time. So if we want to know what the population will be 8 hours after the experiment started, we need to multiply the population at the start (t=0) by 2 eight times. This explains why we raise 2 to the number of hours that have passed to find the new population; repeatedly doubling the population means we repeatedly multiply the population at t=0 by 2. In this context, negative time corresponds to time *before* the experiment started. To figure out what the population was before the experiment started we have to "undouble" (or multiply by 1/2) for every hour we have to go back in time. So if we want to know what the population was 8 hours before the experiment started, we need to multiply the population at the start (t=0) by 1/2 eight times. The equation indicates that we should raise 2 to a power that corresponds to the number of hours we need to go back in time. For every hour we go back in time, we multiply by 1/2. So it makes sense in this context that raising 2 to the -8 power (or any negative integer power) is the same thing as repeatedly multiplying 1/2 8 times (or the opposite of the power you raised 2 to). In other words, it makes sense in this context that $2^{-n} = 1/2^n = 1/2^n$.

Unit 2 Performance Task 1 PLD Rubric

SOLUTION

a. **Students indicate the thousandth of population for each hour or complete the table with the correct values for population**
b. **Student indicates you multiply the previous value by 2 to get the next value, since it doubled.**
c. **Student indicates that Seth's correct because his equation matches the table**
d. **Student indicates that the population 1 hour before the study started would be $1/2 \cdot 2=1$ thousand, and the population 3 hours before the study started would be $(1/2)^3 \cdot 2=0.25$ thousand = 250.**
e. **Time before the study started would be negative time; for example one hour before the study began was $t=-1$.**
f. **Student indicates In this context, negative time corresponds to time *before* the experiment started. To figure out what the population was before the experiment started we have to "undouble" (or multiply by 1/2) for every hour we have to go back in time. So if we want to know what the population was 8 hours before the experiment started, we need to multiply the population at the start (t=0) by 1/2 eight times.**

Level 5: Distinguished Command	**Level 4: Strong Command**	**Level 3: Moderate Command**	**Level 2: Partial Command**	**Level 1: No Command**
Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with minor computational error	Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with minor conceptual error	Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical, but incomplete, progression of steps • minor calculation errors • partial justification of a conclusion • a logical, but incomplete, progression of steps	Constructs and communicates an incomplete response based on concrete referents provided in the prompt such as: diagrams, number line diagrams or coordinate plane diagrams, which may include: • a faulty approach based on a conjecture and/or stated assumptions • An illogical and Incomplete progression of steps • majr calculation errors • partial justification of a conclusion	The student shows no work or justification.

Unit 2 Performance Task 2

Raising to zero and negative powers (8.EE.A.1)

In this problem c represents a positive number.

The quotient rule for exponents says that if m and n are positive integers with $m > n$, then

$$\frac{c^m}{c^n} = c^{m-n}.$$

After explaining to yourself why this is true, complete the following exploration of the quotient rule when $m \leq n$:

a. What expression does the quotient rule provide for $\frac{c^m}{c^n}$ when $m = n$?

b. If $m = n$, simplify $\frac{c^m}{c^n}$ without using the quotient rule.

c. What do parts (a) and (b) above suggest is a good definition for c^0?

d. What expression does the quotient rule provide for $\frac{c^0}{c^n}$?

e. What expression do we get for $\frac{c^0}{c^n}$ if we use the value for c^0 found in part (c)?

f. Using parts (d) and (e), propose a definition for the expression c^{-n}.

Unit 2 Performance Task 2 PLD Rubric

SOLUTION

a. **If we apply the quotient rule for exponents when m=n then the exponent is always 0**
b. **Without the quotient rule it would be the same value divided by itself which equals 1**
c. **Anything to the zero power has to equal 1**
d. **You will always have a negative exponent**
e. **1/value with a positive exponent**
f. **Negative exponents are the same as 1/positive exponent**

Level 5: Distinguished Command	**Level 4: Strong Command**	**Level 3: Moderate Command**	**Level 2: Partial Command**	**Level 1: No Command**
Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with minor computational error	Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with minor conceptual error	Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical, but incomplete, progression of steps • minor calculation errors • partial justification of a conclusion • a logical, but incomplete, progression of steps	Constructs and communicates an incomplete response based on concrete referents provided in the prompt such as:diagrams, number line diagrams or coordinate plane diagrams, which may include: • a faulty approach based on a conjecture and/or stated assumptions • An illogical and Incomplete progression of steps • major calculation errors • partial justification of a conclusion	The student shows no work or justification.

Unit 2 Performance Task 3

Giantburgers (8.EE.A.4)

This headline appeared in a newspaper.

Every day 7% of Americans eat at Giantburger restaurants

Decide whether this headline is true using the following information.

- There are about 8×103 Giantburger restaurants in America.
- Each restaurant serves on average 2.5×103 people every day.
- There are about 3×108 Americans.

Explain your reasons and show clearly how you figured it out.

Solution:
If there are about 8×10^3 Giant Burger restaurants in America and each restaurant serves about 2.5×10^3 people every day, then about $(8\times10^3)\times(2.5\times10^3) = (20\times10^6) = (2\times10^7)$ 2×10^7 People eat at a Giant Burger restaurant every day. Since there are about 3×10^8 Americans, the percent of Americans who eat at a Giant Burger restaurant every day can be computed by dividing the number of restaurant patrons by the total number of people: $\frac{\text{Number of Americans who eat at Giant Burger}}{\text{Total number of americans}}$ $\frac{2\times10^7}{3\times10^8}$ $\frac{2}{3}\times10^{-1}$ $\frac{2}{3}\times10^{-1}$ $\frac{2}{3}\times\frac{1}{10}$

$$\frac{2}{30} = \frac{1}{15} = 0.066 \cong 6.67\%$$

Our estimate is, about 6.67% of the Americans eat at Giant Burger Restaurant, which is close to 7%. Therefore the claim in the headline is reasonable.

Unit 2 Performance Task 3 PLD Rubric

SOLUTION

a. **The student indicates the number of Americans who eat at Giant Burgers and justifies the solution with Math**
b. **The student indicates that 6.67% percent of Americans eat at Giant Burgers by dividing the part/whole and justifies the answers**
c. **The student indicates that 6.67% is close to 7% so it is reasonable to say that 7% of the Americans eat at Giant Burgers.**

Level 5: Distinguished Command	**Level 4: Strong Command**	**Level 3: Moderate Command**	**Level 2: Partial Command**	**Level 1: No Command**
Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with minor computational error	Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with minor conceptual error	Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical, but incomplete, progression of steps • minor calculation errors • partial justification of a conclusion • a logical, but incomplete, progression of steps	Constructs and communicates an incomplete response based on concrete referents provided in the prompt such as: diagrams, number line diagrams or coordinate plane diagrams, which may include: • a faulty approach based on a conjecture and/or stated assumptions • An illogical and Incomplete progression of steps • majr calculation errors • partial justification of a conclusion	The student shows no work or justification.

Unit 2 Performance Task Option 1

Ants Versus Humans (8.EE.A.1, 8.EE.A.4)

The average mass of an adult human is about 65 kilograms while the average mass of an ant is approximately 4×10^{-3} grams. The total human population in the world is approximately 6.84 billion, and it is estimated there are currently about 10,000 trillion ants alive.[1]

Based on these values, how does the total the total mass of all living ants compare to the total mass of all living humans?

Unit 2 Performance Task Option 2

Orders of Magnitude (8.EE.A.3)

It is said that the average person blinks about 1000 times an hour. This is an *order-of-magnitude* estimate, that is, it is an estimate given as a power of ten. Consider:

- 100 blinks per hour, which is about two blinks per minute.
- 10,000 blinks per hour, which is about three blinks per second.

Neither of these are reasonable estimates for the number of blinks a person makes in an hour. Make order-of-magnitude estimates for each of the following:

a. Your age in hours.
b. The number of breaths you take in a year.
c. The number of heart beats in a lifetime.
d. The number of basketballs that would fill your classroom.

Can you think of others questions like these?

21st Century Career Ready Practices

CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP3. Attend to personal health and financial well-being.
CRP4. Communicate clearly and effectively and with reason.
CRP5. Consider the environmental, social and economic impacts of decisions.
CRP6. Demonstrate creativity and innovation.
CRP7. Employ valid and reliable research strategies.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP10. Plan education and career paths aligned to personal goals.
CRP11. Use technology to enhance productivity.
CRP12. Work productively in teams while using cultural global competence.

For additional details see **21st Century Career Ready Practices** .

Extensions and Sources

Assessment Resources

http://dashweb.pearsoncmg.com
- Online Connected Math 3 Resources

http://www.illustrativemathematics.org/standards/k8
- Performance tasks, scoring guides

Online Resources

http://www.agilemind.com/programs/mathematics/algebra-i/
-Lessons for Exponents and Exponential Models

http://www.ixl.com/math/grade-8
Interactive, visually appealing fluency practice site that is objective descriptive

https://www.khanacademy.org/
- Tracks student progress, video learning, interactive practice

http://www.doe.k12.de.us/assessment/files/Math_Grade_8.pdf
- Common Core aligned assessment questions, including Next Generation Assessment Prototypes

https://www.georgiastandards.org/Common-Core/Pages/Math-6-8.aspx
- Special Needs designed tasks, assessment resources

http://www.parcconline.org/sites/parcc/files/PARCCMCFMathematicsGRADE8_Nov2012V3_FINAL.pdf
- PARCC Model Content Frameworks Grade 8

http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_ee_2011_04_25.pdf
- Document Progressions