
 NEW JERSEY CENTER  
FOR TEACHING & LEARNING

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NEW JERSEY CENTER  
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## 8th Grade

### The Number System and Mathematical Operations Part 2

2015-08-31

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## Squares of Numbers Greater than 20

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### Square Root of Large Numbers

Think about this...

What about larger numbers?

How do you find  $\sqrt{1156}$  ?

### Square Root of Large Numbers

It helps to know the squares of larger numbers such as the multiples of tens.

$10^2 = 100$   
 $20^2 = 400$   
 $30^2 = 900$   
 $40^2 = 1600$   
 $50^2 = 2500$   
 $60^2 = 3600$   
 $70^2 = 4900$   
 $80^2 = 6400$   
 $90^2 = 8100$   
 $100^2 = 10000$

What pattern do you notice?

## Square Root of Large Numbers

For larger numbers, determine between which two multiples of ten the number lies.

$$\begin{array}{ll} 10^2 = 100 & 1^2 = 1 \\ 20^2 = 400 & 2^2 = 4 \\ 30^2 = 900 & 3^2 = 9 \\ 40^2 = 1600 & 4^2 = 16 \\ 50^2 = 2500 & 5^2 = 25 \\ 60^2 = 3600 & 6^2 = 36 \\ 70^2 = 4900 & 7^2 = 49 \\ 80^2 = 6400 & 8^2 = 64 \\ 90^2 = 8100 & 9^2 = 81 \\ 100^2 = 10000 & 10^2 = 100 \end{array}$$

Next, look at the ones digit to determine the ones digit of your square root.

## Square Root of Large Numbers

Examples:

$$\sqrt{2809}$$

$$\sqrt{7744}$$

List of  
Squares

1 Find.

$$\sqrt{784}$$

List of  
Squares

2 Find.

$$\sqrt{1764}$$

List of  
Squares

3 Find.

$$\sqrt{4225}$$

List of  
Squares

4 Find.

$$\sqrt{2304}$$

List of  
Squares

5 Find.

$$\sqrt{6241}$$

List of  
Squares

6 Find.

$$\sqrt{9801}$$

List of  
Squares

7 Find.

$$\sqrt{3481}$$

List of  
Squares

8 Find.

$$\sqrt{2209}$$

List of  
Squares

9 Find.

$$\sqrt{10201}$$

List of  
Squares

## Simplifying Perfect Square Radical Expressions

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## Two Roots for Even Powers

If we square 4, we get 16.

If we take the square root of 16, we get two answers: -4 and +4.

That's because, any number raised to an even power, such as 2, 4, 6, etc., becomes positive, even if it started out being negative.

So,  $(-4)^2 = (-4)(-4) = 16$     AND     $(4)^2 = (4)(4) = 16$

This can be written as  $\sqrt{16} = \pm 4$ , meaning positive or negative 4.

This is not an issue with odd powers, just even powers.

## Is there a difference between...

$$-\sqrt{25} \quad \& \quad \sqrt{-25} \quad ?$$

Which expression has no real roots?

Evaluate the expressions:

$$\sqrt{36} \qquad \sqrt{-36} \qquad -\sqrt{36} \qquad \pm\sqrt{36}$$

$$-\sqrt{49} \qquad \sqrt{49} \qquad \pm\sqrt{49} \qquad \sqrt{-49}$$

## Evaluate the Expressions

$$\sqrt{4} = \boxed{\phantom{00}}$$

$$\sqrt{49} = \boxed{\phantom{00}}$$

$$\sqrt{64} = \boxed{\phantom{00}}$$

$$\sqrt{1} = \boxed{\phantom{00}}$$

$$\sqrt{0} = \boxed{\phantom{00}}$$

$$\sqrt{-16} = \boxed{\phantom{00}}$$

10  $\sqrt{36} = ?$

- A 6
- B -6
- C is not real

11  $\sqrt{-81} = ?$

- A 9
- B -9
- C is not real


12  $\sqrt{400} = ?$

- A 20
- B -20
- C is not real

14  $\sqrt{5^2} = ?$

16  $(\sqrt{5})^2 = ?$

- A 25
- B 5
- C  $\sqrt{5}$
- D  $\sqrt{25}$

13 Ashley and Brandon have different methods for finding square roots. (Problem from )**Ashley's Method**

To find the square root of  $x$ , find a number so that the product of the number and itself is  $x$ . For example,  $2 \cdot 2 = 4$ , so the square root of 4 is 2.

**Brandon's Method**

To find the square root of  $x$ , multiply  $x$  by  $\frac{1}{2}$ . For example,  $4 \cdot \frac{1}{2} = 2$ , so the square root of 4 is 2.

Which student's method is not correct?

- A Ashley's Method
- B Brandon's Method

On your paper, explain why the method you selected is not correct.

15  $\sqrt{11^2} = ?$

17  $\sqrt{-9} = ?$

- A 3
- B -3
- C No real roots

18 The expression equal to  $\sqrt{54-b}$   
is equivalent to a positive integer when b is

- A -10  
 B 64  
 C 16  
 D 4

From the New York State Education Department, Office of Assessment Policy, Development and Administration. Internet. Available from [www.nysedregents.org/IntegratedAlgebra](http://www.nysedregents.org/IntegratedAlgebra); accessed 17, June, 2011.

### Square Roots of Fractions

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \quad b \neq 0$$

$$\sqrt{\frac{16}{49}} = \frac{\sqrt{16}}{\sqrt{49}} = \frac{4}{7}$$

19  $\sqrt{\frac{49}{100}} =$

- A  $\frac{7}{10}$                        C  $\frac{7}{5}$   
 B  $\frac{7}{50}$                        D no real solution

20  $\sqrt{\frac{16}{100}} =$

- A  $\frac{2}{5}$                        C  $\frac{4}{10}$   
 B  $\frac{8}{10}$                        D no real solution

22  $\sqrt{\frac{-16}{25}} =$

A  $\frac{4}{10}$

C  $\frac{4}{5}$

B  $\frac{8}{10}$

 D no real solution

23  $\sqrt{\frac{36}{121}} =$

A  $\frac{6}{12}$

C  $\frac{6}{11}$

B  $\frac{6}{13}$

 D no real solution**Square Roots of Decimals**

To find the square root of a decimal, convert the decimal to a fraction first. Follow your steps for square roots of fractions.

$\sqrt{.04} =$

$\sqrt{.0025} =$

$\sqrt{.09} =$

24 Evaluate  $\sqrt{.49}$

A  $\frac{7}{100}$

B  $\frac{7}{25}$

C  $\frac{7}{10}$

 D no real solution

25 Evaluate  $\sqrt{.36}$

A .06

B .6

C 6

 D no real solution

26 Evaluate  $\sqrt{.0121}$

A 0.11

B 11

C 1.1

 D no real solution

27 Evaluate  $\sqrt{.0064}$ 

- A 0.8             B 0.08  
 C  $\frac{4}{5}$              D no real solution

## Approximating Square Roots

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## Perfect Square

All of the examples so far have  
been from perfect squares.

What does it mean to be a perfect square?

## Non-Perfect Squares

You know how to find the square root of a perfect square.

What happens if the number is not a perfect square?

Does it have a square root?

What would the square root look like?

## Non-Perfect Squares

<u>Square Root</u>	<u>Perfect Square</u>
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121
12	144
13	169
14	196
15	225

Think about the square root of 50.

Where would it be on this chart?

What can you say about the square  
root of 50?

50 is between the perfect squares 49  
and 64 but closer to 49.

So the square root of 50 is between 7  
and 8 but closer to 7.



## Approximating Non-Perfect Squares

Square Root	Perfect Square
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121
12	144
13	169
14	196
15	225

When approximating square roots of numbers, you need to determine:

- Between which two perfect squares it lies (and therefore which 2 square roots).

- Which perfect square it is closer to (and therefore which square root).

Example:  $\sqrt{110}$

Lies between 100 & 121, closer to 100.

So  $\sqrt{110}$  is between 10 & 11, closer to 10.

## Approximating Non-Perfect Squares

Square Root	Perfect Square
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121
12	144
13	169
14	196
15	225

Approximate the following:

$$\sqrt{30}$$

$$\sqrt{200}$$

$$\sqrt{215}$$

## Approximating Non-Perfect Squares

Approximate  $\sqrt{38}$  to the nearest integer

$$\sqrt{36} < \sqrt{38} < \sqrt{49} \quad \text{Identify perfect squares closest to 38}$$

$$6 < \sqrt{38} < 7 \quad \text{Take square root}$$

Answer: Because 38 is closer to 36 than to 49,  $\sqrt{38}$  is closer to 6 than to 7. So, to the nearest integer,  $\sqrt{38} \approx 6$ .

## Approximating Non-Perfect Squares

Approximate  $\sqrt{70}$  to the nearest integer

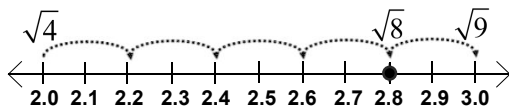
$$\sqrt{\quad} < \sqrt{70} < \sqrt{\quad} \quad \text{Identify perfect squares closest to 70}$$

$$\quad < \sqrt{70} < \quad \quad \text{Take square root}$$

Identify nearest integer

## Approximating a Square Root

Approximate  $\sqrt{8}$  on a number line.



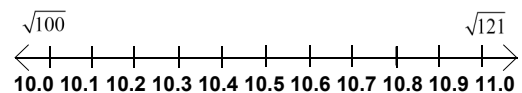
Since 8 is closer to 9 than to 4,  $\sqrt{8}$  is closer to 3 than to 2;

$$\text{so } \sqrt{8} \approx 2.8$$

## Approximating a Square Root

Another way to think about it is to use a number line.

Example: Approximate  $\sqrt{115}$



$$115 \text{ lies between } \sqrt{\quad} \text{ and } \sqrt{\quad}$$

So, it lies between whole numbers  $\quad$  and  $\quad$ .

Imagine the square roots between these numbers, and picture where  $\sqrt{115}$  lies.

29 The square root of 40 falls between which two perfect squares?

- A 3 and 4
- B 5 and 6
- C 6 and 7
- D 7 and 8

30 Which integer is  $\sqrt{40}$  closest to?

$$\sqrt{\quad} < \sqrt{\quad} < \sqrt{\quad} \quad \text{Identify perfect squares closest to 40}$$

$$\quad < \sqrt{\quad} < \quad \quad \text{Take square root}$$

Identify nearest integer

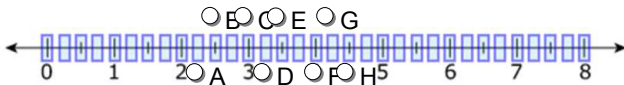
31 The square root of 110 falls between which two perfect squares?

- A 36 and 49
- B 49 and 64
- C 64 and 84
- D 100 and 121

32 Estimate to the nearest integer.

$$\sqrt{110}$$

33 Select the point on the number line that best approximates the location of  $\sqrt{14}$ .



34 Estimate to the nearest integer.

$$\sqrt{219}$$

35 Estimate to the nearest integer.

$$\sqrt{90}$$

36 Approximate  $\sqrt{29}$  to the nearest integer.

37 Approximate  $\sqrt{96}$  to the nearest integer.

38 Approximate  $\sqrt{167}$  to the nearest integer.

39 Approximate  $\sqrt{140}$  to the nearest integer.

40 Approximate  $\sqrt{40}$  to the nearest integer.

41 The expression  $\sqrt{93}$  is a number between:

- A 3 and 9
- B 8 and 9
- C 9 and 10
- D 46 and 47

From the New York State Education Department, Office of Assessment Policy, Development and Administration. Internet. Available from [www.nysedregents.org/IntegratedAlgebra](http://www.nysedregents.org/IntegratedAlgebra); accessed 17, June, 2011.

42 For what integer  $x$  is  $\sqrt{x}$  closest to 6.25?

Derived from engage<sup>ny</sup>

43 For what integer  $y$  is  $\sqrt{y}$  closest to 4.5?

Derived from engage<sup>ny</sup>

44 Between which two positive integers does  $\sqrt{56}$  lie?

- |                            |                             |
|----------------------------|-----------------------------|
| <input type="checkbox"/> 1 | <input type="checkbox"/> 6  |
| <input type="checkbox"/> 2 | <input type="checkbox"/> 7  |
| <input type="checkbox"/> 3 | <input type="checkbox"/> 8  |
| <input type="checkbox"/> 4 | <input type="checkbox"/> 9  |
| <input type="checkbox"/> 5 | <input type="checkbox"/> 10 |

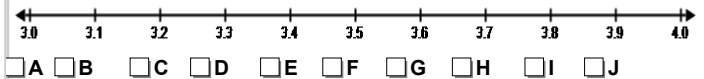
Derived from engage<sup>ny</sup>

45 Between which two positive integers does  $\sqrt{80}$  lie?

- |                            |                             |
|----------------------------|-----------------------------|
| <input type="checkbox"/> 1 | <input type="checkbox"/> 6  |
| <input type="checkbox"/> 2 | <input type="checkbox"/> 7  |
| <input type="checkbox"/> 3 | <input type="checkbox"/> 8  |
| <input type="checkbox"/> 4 | <input type="checkbox"/> 9  |
| <input type="checkbox"/> 5 | <input type="checkbox"/> 10 |

Derived from engage<sup>ny</sup>

46 Between which two labeled points on the number line would  $\sqrt{10}$  be located?



Derived from engage<sup>ny</sup>

## Rational & Irrational Numbers

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### Irrational Numbers

Just as subtraction led us to zero and negative numbers.

And division led us to fractions.

Finding the root leads us to **irrational numbers**.

Irrational numbers complete the set of Real Numbers.

Real numbers are the numbers that exist on the number line.

### Irrational Numbers

Irrational Numbers are real numbers that cannot be expressed as a ratio of two integers.

In decimal form, they extend forever and never repeat.

There are an infinite number of irrational numbers between any two integers (think of all the square roots, cube roots, etc. that don't come out evenly).

Then realize you can multiply them by an other number or add any number to them and the result will still be irrational.

### Rational & Irrational Numbers

$\sqrt{9}$  is **rational**.

This is because the **radicand** (number under the radical) is a perfect square. ( $3^2 = 9$ )

If a radicand is not a perfect square, the root is said to be irrational.

Ex:  $\sqrt{12}$

### Irrational Numbers

Irrational numbers were first discovered by Hippasus about the about 2500 years ago.

He was a Pythagorean, and the discovery was considered a problem since Pythagoreans believed that "All was Number," by which they meant the natural numbers (1, 2, 3...) and their ratios.

Hippasus was trying to find the length of the diagonal of a square with sides of length 1. Instead, he proved, that the answer was not a natural or rational number.

For a while this discovery was suppressed since it violated the beliefs of the Pythagorean school.

Hippasus had proved that  $\sqrt{2}$  is an irrational number.

### Irrational Numbers

An infinity of irrational numbers emerge from trying to find the root of a rational number.

Hippasus proved that  $\sqrt{2}$  is irrational goes on forever without repeating.

Some of its digits are shown on the next slide.

## Square Root of 2

Here are the first 1000 digits, but you can find the first 10 million digits on the Internet. The numbers go on forever, and never repeat in a pattern:

1.414213562373095048801688724209698078569671875376948073176679  
7379907324784621070388503875343276415727350138462309122970249  
2483605585073721264412149709993583141322266592750559275579995  
0501152782060571470109559971605970274534596862014728517418640  
8891986095523292304843087143214508397626036279952514079896872  
5339654633180882964062061525835239505474575028775996172983557  
5220337531857011354374603408498847160386899970699004815030544  
0277903164542478230684929369186215805784631115966687130130156  
1856898723723528850926486124949771542183342042856860601468247  
2077143585487415565706967765372022648544701585880162075847492  
2657226002085584466521458398893944370926591800311388246468157  
0826301005948587040031864803421948972782906410450726368813137  
3985525611732204024509122770022694112757362728049573810896750  
4018369868368450725799364729060762996941380475654823728997180  
3268024744206292691248590521810044598421505911202494413417285  
3147810580360337107730918286931471017111168391658172688941975  
871658215212822951848847208969...

## Principal Roots

Since you can't write out all the digits of  $\sqrt{2}$ , or use a bar to indicate a pattern, the simplest way to write that number is  $\sqrt{2}$ .

But when solving for the square root of 2, there are two answers:  $+\sqrt{2}$  or  $-\sqrt{2}$ .

These are in two different places on the number line.

To avoid confusion, it was agreed that the positive value would be called the principal root and would be written  $\sqrt{2}$ .

The negative value would be written as  $-\sqrt{2}$ .

## Transcendental Numbers

The other set of irrational numbers are the transcendental numbers.

These are also irrational. No matter how many decimals you look at, they never repeat.

But these are not the result of solving a polynomial equation with rational coefficients, so they are not due to an inverse operation.

Some of these numbers are real, and some are complex.

But, this year, we will only be working with the real transcendental numbers.

## Roots of Numbers are Often Irrational

Soon, thereafter, it was proved that many numbers have irrational roots.

We now know that the roots of most numbers to most powers are irrational.

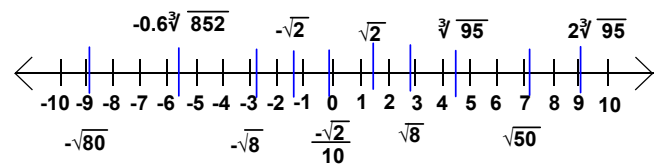
These are called algebraic irrational numbers.

In fact, there are many more irrational numbers than rational numbers.

## Algebraic Irrational Numbers

There are an infinite number of irrational numbers.

Here are just a few that fall between -10 and +10.



## Pi

We have learned about Pi in Geometry. It is the ratio of a circle's circumference to its diameter. It is represented by the symbol  $\pi$



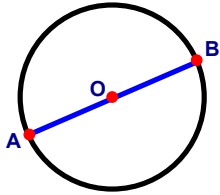
$$\pi \approx 3.14$$

Discuss why this is an approximation at your table.

Is this number rational or irrational?

### # is a Transcendental Number

The most famous of the transcendental numbers is #.



# is the ratio of the circumference to the diameter of a circle.

People tried to find the value of that ratio for millennia.

Only in the mid 1800's was it proven that there was no rational solution.

Since # is irrational (it's decimals never repeat) and it is not a solution to a equation...it is transcendental.

Some of its digits are on the next page.

<http://bobchoat.files.wordpress.com/2013/06/pi-day04.jpg>

```

3. 141592653589793238462643383279502884197169399375105820974944592
30781640628620899862803482534211706798214808651328230664709384460
9550522317253594081284811174502841027019385211055596446229489549
30381964428810975665933446128475648233786783165271201909145648566
92346034861045432664821339360726024914127372458700660631558817488
1520920962829254091715... 38204665213841469
5194151160943305727... 11793105118548074
46237962749567351... 33367336244066664
30860213949463952... 179860943 7705392171762931767523846
74818467669405132... 0568127 326356082 3771342757789609173637178
7214684409012249... 43014654 853710507 7968925892354201995611212
90219608640344181598136297 771309360 707211349999983729780499
5105973173281609631859502 943534690 264252230825334468503261
9311881710100031378387528 87533208 12061717766914730359825349
0428755468731159562863882 -78759375 77818577805321712268066130
0192787661119590921642015 80952572 35485863278865936153381827
968230301952035301852968 773622599 392497217752347913151557
485724245415069595082953 586172785 30750983817546374649393192
55060400927701671139009 240128583 13563707660104710181942955
59619894676783744944825 774726847 94753464620804668425906949
1293313677028989152104 205696602 0381501 55112533824300355
876402474964732639141 4042699227 95478 36009341721641219
92458631503028618297 6749838505 899569092721079750
9302955321165344987 60236480665 347977535663698074
2654252786255181841 28909777279 7050016145249192173
21721477235014144197 548161361157352 4751418494684385232
3907394143334547762416862518983569485562099219222184272502542568
87671790494601653466804988627232791786085784383827967976681454100
95388378636095068006422512520511739298489608412848862694560424196
52850222106611863067442786220391949450471237137865909563643719172
874677646573962413890865832645995813390478027590099465764078951
26946839835259570982582262052248940772671947826848260147699090264
01363944374553050682034962524517493996514314298091906592509372216
96461515709858387410597885959729754989...
    
```

<http://bobchoat.files.wordpress.com/2013/06/pi-day04.jpg>

### Other Transcendental Numbers

There are many more transcendental numbers.

Another famous one is "e", which you will study in Algebra II as the base of natural logarithms.

In the 1800's Georg Cantor proved there are as many transcendental numbers as real numbers.

And, there are more algebraic irrational numbers than there are rational numbers.

The integers and rational numbers with which we feel more comfortable are like islands in a vast ocean of irrational numbers.

Sorry, this element requires Flash, which is not currently supported in PDFs.

Please refer to the original Notebook file.

Sort by the square root being rational or irrational.



47 Rational or Irrational?

$$\frac{4}{5}$$

- A Rational       B Irrational

48 Rational or Irrational?

$$\sqrt{27}$$

- A Rational       B Irrational

49 Rational or Irrational?

0.141414....

- A Rational       B Irrational

51 Rational or Irrational?

1.222...

- A Rational       B Irrational

53 Which is a rational number?

- A  $\sqrt{8}$   
 B  $\pi$   
 C  $5\sqrt{9}$   
 D  $6\sqrt{2}$

50 Rational or Irrational?

 $\sqrt{81}$ 

- A Rational       B Irrational

52 Rational or Irrational?

 $5\pi$ 

- A Rational       B Irrational

54 Given the statement: "If x is a rational number, then  $\sqrt{x}$  is irrational." Which value of x makes the statement false?

- A  $\frac{3}{2}$   
 B 2  
 C 3  
 D 4



(Problem from )

55 A student made this conjecture and found two examples to support the conjecture.

If a rational number is not an integer, then the square root of the rational number is irrational.

For example,  $\sqrt{3.6}$  is irrational and  $\sqrt{\frac{1}{2}}$  is irrational.

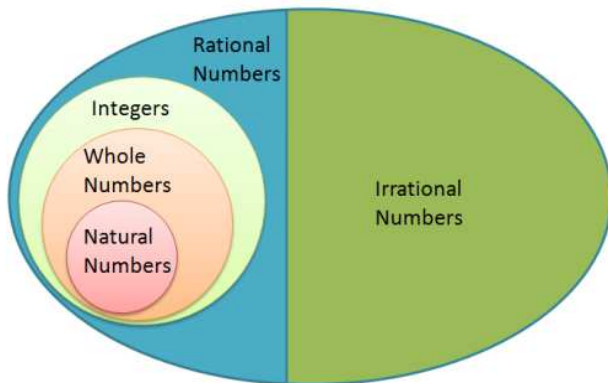
Provide an example of a non-integer rational number that shows that the conjecture is false.

## Real Numbers

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## Real Numbers

**Real numbers** are numbers that exist on a number line.



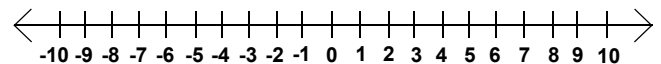
## Real Numbers

The Real Numbers are all the numbers that can be found on a number line.

That may seem like all numbers, but later in Algebra II we'll be talking about numbers which are not real numbers, and cannot be found on a number line.

As you look at the below number line, you'll see certain numbers indicated that are used to label the line.

Those are not all the numbers on the line, they just help us find all the other numbers as well...numbers not shown below.



56 What type of number is 25? Select all that apply.

- Irrational
- Rational
- Integer
- Whole Number
- Natural Number
- Real Number

57 What type of number is - ? Select all that apply.

- Irrational
- Rational
- Integer
- Whole Number
- Natural Number
- Real Number

58 What type of number is  $\sqrt{8}$ ? Select all that apply.

- Irrational
- Rational
- Integer
- Whole Number
- Natural Number
- Real Number

59 What type of number is  $-\sqrt{64}$ ? Select all that apply.

- Irrational
- Rational
- Integer
- Whole Number
- Natural Number
- Real Number

60 What type of number is  $\sqrt{2.25}$ ? Select all that apply.

- Irrational
- Rational
- Integer
- Whole Number
- Natural Number
- Real Number

## Properties of Exponents

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### Powers of Integers

Just as multiplication is repeated addition,

Exponents is repeated multiplication.

For example,  $3^5$  read as "3 to the fifth power" =  $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

In this case "3" is the base and "5" is the exponent.

The base, 3, is multiplied by itself 5 times.

### Powers of Integers

Make sure when you are evaluating exponents of negative numbers, you keep in mind the meaning of the exponent and the rules of multiplication.

For example,  $(-3)^2 = (-3) \cdot (-3) = 9$ , which is the same as  $3^2$ .

However,  $(-3)^2 = (-3)(-3) = 9$ , and  $-3^2 = -(3)(3) = -9$

which are not the same.

Similarly,  $3^3 = 3 \cdot 3 \cdot 3 = 27$

and  $(-3)^3 = (-3) \cdot (-3) \cdot (-3) = -27$ ,

which are not the same.

61 Evaluate:  $4^3$ 62 Evaluate:  $(-2)^7$ 63 Evaluate:  $(-3)^4$ 

### Properties of Exponents

The properties of exponents follow directly from expanding them to look at the repeated multiplication they represent.

Don't memorize properties, just work to understand the process by which we find these properties and if you can't recall what to do, just repeat these steps to confirm the property.

We'll use 3 as the base in our examples, but the properties hold for any base. We show that with base a and powers b and c.

We'll use the facts that:

$$(3^2) = (3 \cdot 3)$$

$$(3^3) = (3 \cdot 3 \cdot 3)$$

$$(3^5) = (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)$$

### Properties of Exponents

We need to develop all the properties of exponents so we can discover one of the inverse operations of raising a number to a power...finding the root of a number to that power.

That will emerge from the final property we'll explore.

But, getting to that property requires understanding the others first.

### Multiplying with Exponents

**When multiplying numbers with the same base, add the exponents.**

$$(a^b)(a^c) = a^{(b+c)}$$

$$(3^2)(3^3) = 3^5$$

$$(3 \cdot 3)(3 \cdot 3 \cdot 3) = (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)$$

$$(3^2)(3^3) = 3^5$$

$$(3^2)(3^3) = 3^{(2+3)}$$

### Dividing with Exponents

When dividing numbers with the same base, subtract the exponent of the denominator from that of the numerator.

$$(a^b) \div (a^c) = a^{(b-c)}$$

$$\frac{(3^3)}{(3^2)} = 3^1$$

$$\frac{\cancel{3} \cdot \cancel{3} \cdot 3}{\cancel{3} \cdot \cancel{3}} = 3$$

$$(3^3) \div (3^2) = 3^{(3-2)}$$

$$(3^3) \div (3^2) = 3^1$$

64

- 5<sup>2</sup>
- 5<sup>3</sup>
- 5<sup>6</sup>
- 5<sup>8</sup>

65

- 7<sup>1.5</sup>
- 7<sup>2</sup>
- 7<sup>10</sup>
- 7<sup>24</sup>

67

- 8<sup>2</sup>
- 8<sup>3</sup>
- 8<sup>9</sup>
- 8<sup>18</sup>

68

- 5<sup>2</sup>
- 5<sup>3</sup>
- 5<sup>6</sup>
- 5<sup>8</sup>

69 Simplify:  $4^3 (4^5)$ 

- A  $4^{15}$
- B  $4^8$
- C  $4^2$
- D  $4^7$

70 Simplify:  $5^7 \div 5^3$ 

- A  $5^2$
- B  $5^{10}$
- C  $5^{21}$
- D  $5^4$

### An Exponent of Zero

Any base raised to the power of zero is equal to 1.

$$a^0 = 1$$

Based on the multiplication rule:

$$(3^0)(3^3) = (3^{3+0})$$

$$(3^0)(3^3) = (3^3)$$

Any number times 1 is equal to itself

$$(1)(3^3) = (3^3)$$

$$(3^0)(3^3) = (3^3)$$

Comparing these two equations, we see that

$$(3^0) = 1$$

This is not just true for base 3, it's true for all bases.

71 Simplify:  $5^0 =$ 72 Simplify:  $8^0 + 1 =$ 73 Simplify:  $(7)(30^0) =$

**Negative Exponents**

A negative exponent moves the number from the numerator to denominator, and vice versa.

$$(a^{-b}) = \frac{1}{a^b} \quad a^b = \frac{1}{(a^{-b})}$$

Based on the multiplication rule and zero exponent rules:

$$(3^{(-1)})(3^{(1)}) = (3^{(-1+1)})$$

$$(3^{(-1)})(3^{(1)}) = (3^{(0)})$$

$$(3^{(-1)})(3^{(1)}) = 1$$

But, any number multiplied by its inverse is 1, so

$$\frac{1}{3^1} (3^{(1)}) = 1$$

$$(3^{(-1)})(3^{(1)}) = 1$$

Comparing these two equations, we see that

$$(3^{(-1)}) = \frac{1}{3^1}$$

**Negative Exponents**

By definition:

$$x^{-1} = \frac{1}{x}, \quad x \neq 0$$

74

- A  $4^2$
- B  $\frac{1}{4^2}$

75

- A  $4^2$
- B  $\frac{1}{4^2}$

76

- A - -
- B - -
- C  $\frac{x}{-}$
- D  $\frac{1}{-}$

77

- A - -
- B  $\frac{b}{a}$  -
- C  $\frac{a}{b}$  -
- D  $\frac{1}{a}$

78 Which expression is equivalent to  $x^4$ ?

- A  $\frac{1}{x^4}$
- B  $x^4$
- C  $-4x$
- D 0

From the New York State Education Department, Office of Assessment Policy, Development and Administration. Internet. Available from [www.nysedregents.org/integratedAlgebra](http://www.nysedregents.org/integratedAlgebra); accessed 17, June, 2011.

79 What is the value of  $2^{-3}$ ?

- A  $\frac{1}{6}$
- B  $\frac{1}{8}$
- C -6
- D -8

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80 Which expression is equivalent to  $x^{-1} \cdot y^2$ ?

- A  $xy^2$        C  $\frac{y^2}{x}$
- B  $\frac{x}{y^2}$        D  $xy^{-2}$

From the New York State Education Department, Office of Assessment Policy, Development and Administration. Internet. Available from [www.nysedregents.org/integratedAlgebra](http://www.nysedregents.org/integratedAlgebra); accessed 17, June, 2011.

81 a) Write an exponential expression for the area of a rectangle with a length of  $7^{-2}$  meters and a width of  $7^{-5}$  meters.

b) Evaluate the expression to find the area of the rectangle.

When you finish answering both parts, enter your answer to Part b) in your responder.

82 Which expressions are equivalent to  $\frac{3^{-8}}{3^{-4}}$ ?

- A  $3^{-12}$
- B  $3^{-4}$
- C  $3^2$
- D  $\frac{1}{3^2}$
- E  $\frac{1}{3^4}$
- F  $\frac{1}{3^{12}}$

From PARCC EOY sample test non-calculator #13

83 Which expressions are equivalent to  $3^2 \cdot 3^{-5}$ ?

- A  $3^3$
- B  $3^{-3}$
- C  $3^{-10}$
- D  $\frac{1}{27}$
- E  $\frac{1}{3^3}$
- F  $\frac{1}{3^{-10}}$

**Raising Exponents to Higher Powers**

When raising a number with an exponent to a power,  
multiply the exponents.

$$(a^b)^c = a^{(bc)}$$

$$(3^2)^3 = 3^6$$

$$(3^2)(3^2)(3^2) = 3^{(2+2+2)}$$

$$(3 \cdot 3)(3 \cdot 3)(3 \cdot 3) = (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)$$

$$(3^2)^3 = 3^6$$

84

- $2^{1.75}$
- $2^3$
- $2^{11}$
- $2^{28}$

85

- $g^{27}$
- $g^{12}$
- $g^6$
- $g^3$

86

- 
- 
- 
- 

88 The expression  $(x^2z^3)(xy^2z)$  is equivalent to:

- A  $x^2y^2z^3$
- B  $x^3y^2z^4$
- C  $x^3y^3z^4$
- D  $x^4y^2z^5$



## Slide 145 / 156

89 The expression  $\frac{(10w^3)^2}{5w}$  is equivalent to:

- A  $2w^5$
- B  $2w^8$
- C  $20w^8$
- D  $20w^5$

From the New York State Education Department, Office of Assessment Policy, Development and Administration. Internet. Available from [www.nysedregents.org/IntegratedAlgebra](http://www.nysedregents.org/IntegratedAlgebra); accessed 17, June, 2011.

## Slide 146 / 156

90 When  $-9x^6$  is divided by  $-3x$ ,  $x \neq 0$ , the quotient is

- A  $3x^2$
- B  $-3x^2$
- C  $-27x^{15}$
- D  $27x^8$

From the New York State Education Department, Office of Assessment Policy, Development and Administration. Internet. Available from [www.nysedregents.org/IntegratedAlgebra](http://www.nysedregents.org/IntegratedAlgebra); accessed 17, June, 2011.

## Slide 147 / 156

91 Lenny the Lizard's tank has the dimensions  $b^5$  by  $3c^2$  by  $2c^3$ . What is the volume of Larry's tank?

- A  $6b^7c^3$
- B  $6b^5c^5$
- C  $5b^5c^5$
- D  $5b^5c^6$

## Slide 148 / 156

92 A food company which sells beverages likes to use exponents to show the sales of the beverage in  $a^2$  days. If the daily sales of the beverage is  $5a^4$ , what is the total sales in  $a^2$  days?

- A  $a^6$
- B  $5a^8$
- C  $5a^6$
- D  $5a^3$

## Slide 149 / 156

93 A rectangular backyard is  $5^5$  inches long and  $5^3$  inches wide. Write an expression for the area of the backyard as a power of 5.

- A  $5^{15} \text{ in}^2$
- B  $8^5 \text{ in}^2$
- C  $25^8 \text{ in}^2$
- D  $5^8 \text{ in}^2$

## Slide 150 / 156

94 Express the volume of a cube with a length of  $4^3$  units as a power of 4.

- A  $4^9 \text{ units}^3$
- B  $4^6 \text{ units}^3$
- C  $12^6 \text{ units}^3$
- D  $12^9 \text{ units}^3$

## Glossary & Standards

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## Irrational Numbers

A number that cannot be expressed as a ratio of integers.

A radical of a non perfect square.

$\sqrt{2}$ 1.41421...	$\pi$ 3.14159...	$e$ 2.71828...
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## Radicand

The value inside the radical sign.  
The value you want to take the root of.

$\sqrt{9} = 3$	$\sqrt{29} = 5.3851\dots$	$\sqrt{144} = 12$
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## Rational Numbers

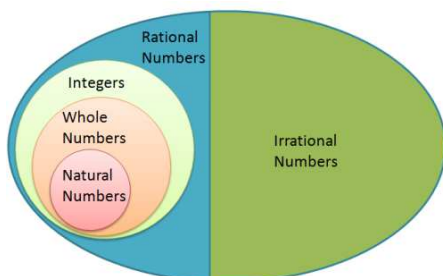
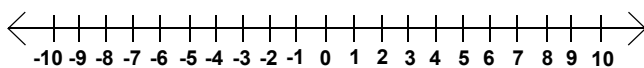
A number that can be expressed as a fraction.

$\frac{1}{4}$	2	$5.\bar{3}$	$\mathbb{Q}$ symbol for rational numbers
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## Real Numbers

All the numbers that can be found on a number line.



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## Standards for Mathematical Practice

- MP1 Making sense of problems & persevere in solving them.
- MP2 Reason abstractly & quantitatively.
- MP3 Construct viable arguments and critique the reasoning of others.
- MP4 Model with mathematics.
- MP5 Use appropriate tools strategically.
- MP6 Attend to precision.
- MP7 Look for & make use of structure.
- MP8 Look for & express regularity in repeated reasoning.

Click on each standard to bring you to an example of how to meet this standard within the unit.

