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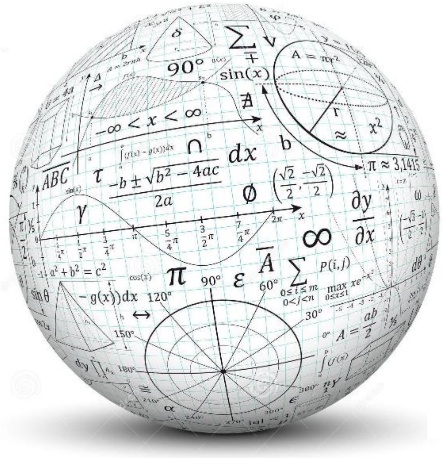
9-1 GCSE Maths Foundation & Higher Help & Revision Booklet



Lite Version - September 2016

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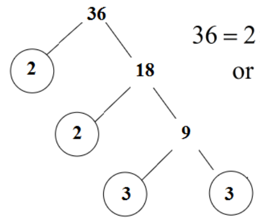
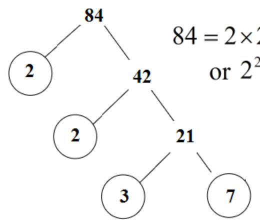
Suitable for Edexcel, AQA & OCR. The topics in *italics* are those on the Higher Tier <u>only</u>. Click on the topics for video tutorials
Click 🗎 for a worksheet. Formulae in shaded text are those <u>NOT</u> given in the exam formula booklet. You need to learn them!

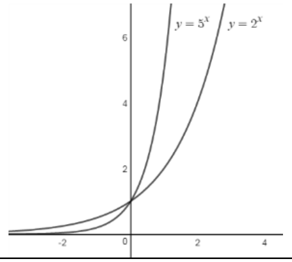
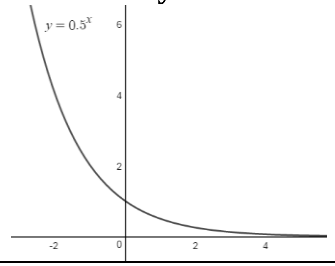
Name____ Class_________________ Target Grade 1 2 3 4 5 6 7 8 9

Number

Topic/Skill	Tips/Facts	Example
Integers	An integer is a **whole number**. It can be positive or negative.	Integers: 2, 5,100, 6345…. Non Integers: ¼ , 12.3, 0.76
Square Number	When you × a number by itself you get a square number. This number has to be an integer. Squaring a number is **NOT** the same as multiplying a number by 2.	The first 6 square numbers are: 1, 4, 9, 16, 25, 36... (a) $3^2 = 3 \times 3 = 9$ (NOT 6) (b) $5^2 = 5 \times 5 = 25$
Square Roots	This is the inverse (reverse process) of squaring a number. $\sqrt{\ }$ is used. (a) $6^2 = 36$ so $\sqrt{36} = 6$ (b) $9^2 = 81$ so $\sqrt{81} = 9$.	(a) $\sqrt{49}=7$ (b) $\sqrt{121}=11$ (c) $\sqrt{\frac{25}{4}}=\frac{\sqrt{25}}{\sqrt{4}}=\frac{5}{2}$
Cube Number/Roots	A number multiplied by itself **three times**. (The cube root $\sqrt[3]{\ }$ is the inverse).	(a) $4^3 = 4 \times 4 \times 4 = 64$ (NOT 12) (b) $2^3 = 8$ (NOT 6)
A Prime Number	A number that has only **2 factors, itself & 1**. 2 is the **only** even prime number.	2, 3, 5, 7, 11, 13, 17, 19… (1 is not a prime number!)
Rational and Irrational Numbers	Rational numbers can be written in the form $\frac{a}{b}$ where a and b are integers and Irrational numbers can't! Surds, and π are examples of irrational numbers.	Rational: $2,\ 0.4,\ \frac{1}{3},\ 0.\dot{7},\ \sqrt{36},\ -1.2,\ 4\frac{1}{5}$ Irrational: $\sqrt{3},\ \pi,\ 5\sqrt{7},\ e$
Reciprocal	The reciprocal of a number is 1 divided by that number. Often it's easier to think about turning the fraction upside down (inverting the fraction).	The reciprocal of 5 is $\frac{1}{5}$ The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$
Factors (Divisors)	The integers (whole numbers) that **go into** a number with no remainder.	Factors of 8 are 1,2,4 & 8 Factors of 12: 1,2,3,4,6 &12
Multiples	Think Times Tables. Just write out the times tables for that number.	The first six multiples of 4 are 4, 8, 12, 16, 20 and 24
Product of Prime Factors	Numbers can be made up by **multiplying prime numbers**. (2,3,5,7,11,13,17..) To find the Product of Primes start with a **factor tree**. (Shown to the right) Product means multiply so don't forget to put the × sign in between the numbers you found in your factor tree. If you are struggling with the factor tree just keep trying to divide by the prime number in order. Does it divide by 2? If so pick 2. If it doesn't divide by 2 does it divide by 3? By 5? By 7? By 11? Etc.	 $36 = 2\times2\times3\times3$ or $2^2\times3^2$  $84 = 2\times2\times3\times7$ or $2^2\times3\times7$
HCF (Highest Common Factor)	The HCF is the **largest** number that goes into 2 or more different numbers. **Method 1**: Just list the factors of each and find largest number in each list **Method 2**: Using Factor tree. Take **only** the prime numbers that appear in each list of the factors of the numbers to their **lowest** power and multiply. This method is better for less obvious examples and larger numbers. (You can use a Venn Diagram to do this too.)	**Example:** "Find the HCF of 8 and 28" **Method 1**: Factors of 8: 1,2,**4** & 8 Factors of 28: 1,2,**4**,7,14 & 28. The HCF of 8 and 28 is **4** **Method 2:** Product of Primes for 8 and 28: $8=2^3$ and $28=2^2\times7$ so you only have 2 in both lists and you take it to the lowest power giving $2^2=4$
LCM (Lowest Common Multiple)	The **lowest (or smallest) number** that 2 or more different numbers go in to. **Method 1**: Just list out the times tables of each number and see which is the **lowest number** that appears in both lists. This is the LCM **Method 2**: Using Factor tree. Take **all** the prime numbers that appear in each list of the factors to their highest power and multiply. (You can use a Venn Diagram to do this too.) Common misconception: The LCM of 2 numbers is 1. This is incorrect!	**Example:** "Find the LCM of 4 and 6" **Method 1**: Multiples of 4: 4, 8, **12**, 16, .. Multiples of 6: 6, **12**, 18.. The LCM of 4 and 6 is **12.** **Method 2**: Product of Primes for 4 and 6: $4=2^2$ and $6=2\times3$. You need both 2 and 3 to their highest power giving $2^2\times3=12$.

Rounding to 1 DP (Decimal Place)	You are rounding the number to the **nearest 10th** . Focus on the 2nd number after the decimal point. If it's 5 or more round up. If it's 4 or less round down.	(a) 2.43 = 2.4 (3 is less than 5) (b) 5.67 = 5.7 (c) 1.09 = 1.1 (9 is more than 5) (d) 2.98 = 3.0
Rounding to 2 DP	Nearest **100th**. As above but focus on the 3rd number after the decimal point.	(a) 3.562 = 3.56 (b) 0.785 = 0.79 (c) 1.499 = 1.50
Rounding to 1 SF (Significant Figures)	When reading a number from left to right the first value that is not 0 in the number is the 1st significant figure. Round the number using the same techniques as used for decimals shown above. With the number 0.043 the 4 is the first significant figure, 3 is the second.	(a) 243 to 1 SF = 200 (Rounding to the nearest 100) (b) 5.6 to 1 SF = 6 (Rounding to the nearest integer) (c) 47 to 1 SF = 50 (Rounding to the nearest 10) (d) 0.48 to 1 SF = 0.5 (Rounding to the nearest 10th)
Rounding to 2 SF	Same as before but now it's the **second** significant figure. Mind the 0's!	(a) 243 = 240 (b) 40.8 = 41 (c) 0.546 = 0.55
Estimations & Approximations	Round each number to 1 significant figure & perform the calculation. You must show workings! Estimating doesn't require the exact value. It's non calculator!	(a) 98 × 51.2 becomes 100 × 50 which = 5000 (b) 4.6 + 104.7 becomes 5+100 which = 105
Calculations using Upper and Lower Bounds	Sometimes you will get a question where the numbers or measurements given have already been rounded. You just need to work out the minimum (Lower Bound) and maximum (Upper Bound) the number could be. One way to think of it is to half the interval given subtract it from the number to find the LB and then add it to the number to get the UB. An example could be 1.4 rounded to one d.p. one d.p = 0.1. If you half this you get 0.05. This gives us a lower bound of 1.35 and an upper bound of 1.45. When you have done this work out which values are required to minimise or maximise the calculation.	**E:** A rectangle has one side length of 6cm correct to the nearest cm and an area of $24.3cm^2$ correct to 3SF. Find the greatest possible length of the missing side. **A:** **LB UB** **Length 5.5 6.5** **Area 24.25 24.35** **Max Legth =** $\dfrac{\textbf{UB Area}}{\textbf{LB Length}} = \dfrac{24.35}{5.5}$ **= 4.427....**
Intervals and Bounds and Error Intervals	You may be asked to interpret or use inequalities for upper and lower bounds. If a number has **already** been rounded, you may be asked to find the upper and lower bounds of it. One way to do this is to split the interval in half and + this amount on to the value to get the upper bound and - it for the lower bound.	**Example:** The height of a plant is 1.8m correct to 2 significant figures. Write an inequality to show this. **Answer:** $1.75 \le h < 1.85$ Be Careful with the inequality sign on the upper bound.
Fractions to Decimals	Some are obvious such as ¾ is 0.75. For those that are not simply divide the numerator by the denominator using short division **OR** SD on your Casio. Common error! 1/3 is not 0.3. £1 shared between 3 people is not 30p each.	(a) 1/8 = 0.125 (b) 3/10 = 0.3 (c) 7/100 = 0.07 (d) 43/100 = 0.43 (e) 28/1000 = 0.028 (f) 37/50 = 74/100 = 0.74
Decimals to Fractions	Some are obvious such as 0.5 = ½ or 0.75 = ¾ and 0.1 = 1/10 etc. If it's not obvious write it as a fraction over 10, 100 or 1000 and cancel down.	(a) 0.7 = 7/10 (b) 0.23 = 23/100 (c) 0.46 = 46/100 or 23/50
% to Decimals	To convert a % to a decimal ÷ by 100. To convert a decimal to a % × by 100	(a) 0.23 × 100 = 23% (b) 47% ÷ 100 = 0.47
Fractions to Percentages	A % is just a fraction out of 100. Non calculator just 'scale' the denominator up to 100 with equivalent fractions. On a calculator just × the fraction by 100.	(a)Non Calc $\frac{3}{25} = \frac{12}{100} = 12\%$ (b)Calc $\frac{9}{17} \times 100 = 52.9\%$
Simplifying Fractions	If they are not obvious like $\frac{5}{10} = \frac{1}{2}$ look for common factors to divide by.	(a) $\frac{6}{8} = \frac{3}{4}$ (divide by 2) (b) $\frac{20}{35} = \frac{4}{7}$ (divide by 5)
Mixed Numbers	See how many times the denominator goes into the numerator. This gives you the integer part and then just write the remainder over the original denominator.	(a) $\frac{9}{4} = \frac{4}{4} + \frac{4}{4} + \frac{1}{4} = 2\frac{1}{4}$ (b) $\frac{17}{5} = 3\frac{2}{5}$ (c) $\frac{5}{3} = 1\frac{2}{3}$
Ordering Fractions	Find the common denominator of the fractions given, write equivalent fractions for each and simply order the fractions by the numerators. You must use the original fractions in your answer. Ascending means smallest to largest.	Order: $\frac{3}{4}, \frac{2}{3}, \frac{5}{6}, \frac{1}{2}$ equiv $\frac{9}{12}, \frac{8}{12}, \frac{10}{12}, \frac{6}{12}$ so $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$

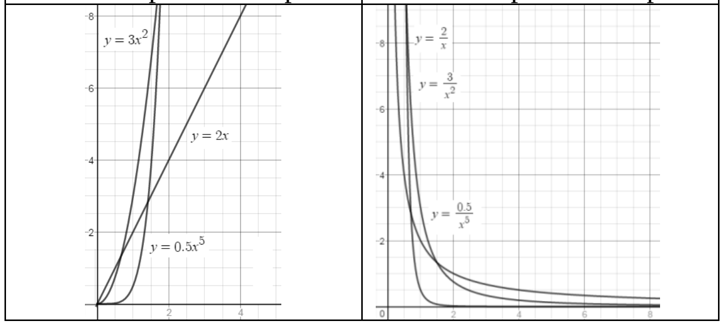
Finding a Fraction of a Quantity	'Divide by the bottom, times by the top'. If you need 3/8 of a number, divide the number in the question by 8 then multiply the answer by 3. Alternatively, use a calculator. In maths 'of' means multiply so you can just type the calculation in as shown on the right. Just × the fraction by the quantity.	**Example:** "Find 2/5 of £60" **Answer:** Start with £60 ÷ 5 = £12. Now simply multiply by two. 2 × 12 = £24. You could have simply done $\frac{2}{5}\times 60$ instead to give 24
Adding Fractions Adding Mixed Numbers	You **must have a common denominator** to add fractions. When you do, simply add the numerators. Use equivalent fractions to find common denominators. Whatever you do to the bottom, do to the top! If you have forgotten! Numerator = top, Denominator = bottom.	(a) $\frac{1}{5}+\frac{3}{5}=\frac{4}{5}$ (b) $\frac{2}{5}+\frac{3}{10}=\frac{4}{10}+\frac{3}{10}=\frac{7}{10}$ (c) $\frac{4}{5}+\frac{2}{3}=\frac{12}{15}+\frac{10}{15}=\frac{22}{15}$ (d) $\frac{3}{2}+\frac{2}{7}=\frac{21}{14}+\frac{4}{14}=1\frac{11}{14}$
Subtracting Fractions Subtracting Mixed Numbers	You **must have a common denominator** to subtract fractions. When you do, simply subtract the numerators. Use equivalent fractions to find common denominators. Please note: You can cross multiply when adding and subtracting fractions although it's a long way round for some examples.	(a) $\frac{4}{5}-\frac{1}{5}=\frac{3}{5}$ (b) $\frac{4}{5}-\frac{1}{10}=\frac{8}{10}-\frac{1}{10}=\frac{7}{10}$ (c) $\frac{3}{4}-\frac{2}{3}=\frac{9}{12}-\frac{8}{12}=\frac{1}{12}$ (d) $\frac{2}{7}-\frac{5}{6}=\frac{12}{42}-\frac{35}{42}=-\frac{23}{42}$
Multiplying Fractions	Multiply the numerators and multiply the denominators and simplify. You can cancel common factors at the start. **You do not need a common denominator.**	(a) $\frac{3}{5}\times\frac{4}{7}=\frac{12}{35}$ (b) $\frac{1}{8}\times\frac{4}{9}=\frac{4}{72}=\frac{1}{18}$ (simplified)
Dividing Fractions Multiplying & Dividing Mixed Numbers	Invert (turn upside down) the 2^{nd} fraction and multiply (as shown above). "Dividing by a fraction is the same as multiplying by its reciprocal" **You do not need a common denominator unlike adding or subtracting.** How many halves of pizza can you cut from a whole pizza? 1 ÷ ½ =2 of course!	(a) $\frac{1}{8}\div\frac{4}{9}$ is the same as $\frac{1}{8}\times\frac{9}{4}=\frac{9}{32}$ (b) $\frac{3}{4}\div\frac{5}{6}$ is the same as $\frac{3}{4}\times\frac{6}{5}=\frac{18}{20}=\frac{9}{10}$ (simplified)
Finding 10%, 5%, 1% of a quantity	To find 10% without a calculator just divide the original number by 10, to find 1% divide it by 10 again. 5% is half of 10%, 2.5% is half of that!	£36: 10% = £3.60 5% = £1.80 and 1% = £0.36 or 36p From this you could get 11% = £3.96, 12%, 16% etc.
Finding a Percentage of a Quantity using a Calculator	For harder examples just type it into a calculator. Remember, 'of' in maths means multiply. Percentage means out of 100 so you can just type the percentage in as a fraction over 100 and × by the quantity.	**E:** Find 23% of 327.5 **A:** $\frac{23}{100}\times 327.5=75.325$ *There is a % button on the Casio too. See the tutorial!*
Increase or Decrease by a %	Find the % required (see above) and add it on (increase) or take it off (decrease) If it's a calculator question just multiply the quantity by the %	**E:** Increase £30 by 10% **A:** 10% = £3 so 30+3 = £33 **E:** Decrease 20 by 40% **A:** 10% = 2,40% = 8, 20-8=12
Writing one Number as a % of Another	Write the 1^{st} number over the 2^{nd} as a fraction and × your answer by 100. It could help thinking as these like test scores. 7 as a % of 24 is 7/24 × 100.	**Example:** Write 12 as a % of 31. **A:** $\frac{12}{31}\times 100=38.7\%$
Percentage Change	You are looking at the increase or decrease as a % of the original value. 'Difference divided by the original and multiplied by 100.' **Example:** A painting was bought for £200 & sold for £250. Find the % increase in its value.	**Answer:** $\frac{50}{200}=25\%$ increase in value
Reverse Percentage	You are working out the value **BEFORE** the % increase or decrease. Use multipliers (**some** shown), set up an equation & solve working backwards. 1% increase 1.01 / 1% decrease 0.99 5% increase 1.05 / 10% decrease 0.90 10% increase 1.10 / 25% off 0.75 25% increase 1.25 / 40% of original value 0.40 **Do not** just find the % of the value in the question and take it off or add it!	**Example:** A jumper was priced at £48.60 **after** a 10% reduction. Find its original price. $J\times 0.9=48.60$ **Answer:** $J=48.60\div 0.9$ (The jumper was £54) $J=54$

Growth and Decay	Find the starting quantity, × this by the multiplier to increase or decrease the quantity and raise that to the required power. See worked example! The multiplier for growth will be greater than 1, for decay less than 1.	**E:** A bank pays 5% compound interest a year. Bob invests £3000. How much will he have after 7 years? **Answer:** $3000\times1.05^7=4221.3012....$ (about £4221.30)
Exponential Functions and their Graphs	Exponential graphs can be used to model growth and decay. Exponential graphs can be written in the form $y=a^x$. They are curves! If $a>1$ you get growth. If $0<a<1$ you have decay. In 'real life situations these graphs may be written as $y=ab^x$. An example could be the value of a car: $P=25000\times0.92^t$. This simply models the price of a car with a 'new cost' of £25000 which is losing 8% a year. An investment could be represented by $I=4000\times1.03^n$. This just shows an initial investment of £4000 and a compound rate of 3% over n years.	**Growth** $a>1$  **Decay** $0<a<1$ 
Simple and Compound Interest	**Simple Interest**: Interest calculated on ONLY the original investment. **Compound Interest**: Interest is calculated on BOTH the original investment and any interested gained over time. (This as interest on interest which is better) Be careful with two step calculations with different rates for different periods.	**Example: Simple** interest at 3% on £1000 over 4 years will give: 4 × £30 = £120. This gives a total investment of £1120. **Compound** Interest at 3% on £1000 over 4 years gives: 1000×1.03^4 = £1125.51
VAT	VAT is just 'Value Added Tax' and is a tax added to some of the goods and services we buy. The current rate for VAT in the UK is 20% All you need to do is find 20% and add it on or just use the multiplier 1.2. Careful! If the price already includes tax **divide** by 1.2 to find the pre VAT cost	**Example:** A car is priced at £2000 before tax. Find the price after VAT has been applied at 20%: **Answer:** $£20000\times1.2=£24000$ (You can find 20% and add it on if you don't want to use a multiplier)
Negative Numbers (× and ÷)	If you are **either** multiplying **or** dividing with negative numbers and the signs are the same the answer is **positive**, if they are different the answer is **negative**.	(a) -2 × 4 = -8 (b) -3 × -5 = 15 (c) 3 ÷ -3 = -1 (d) -16 ÷ -4 = 4
Negative Numbers (+ and -)	Adding a negative **decreases** the value. Subtracting a negative **increases** the value. Start on a number line and either move up of down from your start point.	(a) 2 – 4 = -2 (b) 3 - - 5 = 8 (c) -2 + - 5 = -7 (d) -4 - - 5 = 1
BODMAS/BIDMAS (Order of Operations)	**Brackets** then **Powers** (BO/BI) comes first. **Multiplication** then **Division** (DM) comes next. **Addition** then **Subtraction** (AS) comes last.	(a) 3 + 4 × 2 = 11 (do the multiplication first) (b) 3 + (4+ 1)² Brackets: (5) ² = 25 and then add 3 = 28 (c) 12 ÷ 0.5 – 3 = 21 (division first!)
Multiplying Decimals	**Method 1:** Count the **total** digits after the decimals at the start. The number you start with is the number you finish with. You may have to add 0's. **Method 2:** Consider place value. Tenths × Tenths = Hundredths.	**Example:** 0.4 × 0.2 (2 digits after the decimals in total) 4 × 2 = 8 so my answer is 0.08 as I need to finish with 2 digits after the decimals. 0.3 × 0.15 = 0.045
Dividing by a Decimal	Simply multiply both numbers by powers of 10 until the decimal you are dividing by is an integer. At this point simply divide the numbers.	(a) $4\div0.2=40\div2=20$ (b) $6\div0.03=600\div3=200$ (c) $1.5\div0.3=15\div3=5$ (d) $18\div0.06=1800\div6=300$
Standard Form	The number **must** be between 1 and $9.\dot{9}$ **and** multiplied by a power of ten. + powers of 10 for 'large numbers' and – powers of 10 for 'small numbers'	(a) $8400=8.4\times10^3$ (b) $671000=6.71\times10^5$ (c) $0.00036=3.6\times10^{-4}$ (d) $0.097=9.7\times10^{-2}$
Calculating With Standard Form	When **multiplying** numbers in SF, **multiply** the numbers and **add** the powers. When **dividing** numbers in SF, **divide** the numbers and **subtract** the powers. Make sure your answer is in standard form. You may need to adjust at the end as shown in example (c) to the right. The initial answer is not in standard form.	(a) $\left(1.2\times10^3\right)\times\left(4\times10^6\right)=8.8\times10^9$ (b) $\left(4.5\times10^5\right)\div\left(3\times10^2\right)=1.5\times10^3$ (c) $\left(4.1\times10^6\right)\times\left(3\times10^9\right)=12.3\times10^{15}=1.23\times10^{16}$

Surds (Simplifying)	$\sqrt{36}$ is a rational number as it is 6. Surds are irrational square roots. $\sqrt{2}$ is an example of a surd and we say it's an 'exact value'. Its answer is a non-terminating (keeps going!), non-recurring (its decimal part doesn't repeat) decimal. Don't be tempted to write a surd as a decimal or round it, just leave it in exact form. When simplify surds look for the largest square number that goes into the surd (highest square factor), split the roots and simplify. **Example:** Simplify $\sqrt{12}$. The largest square number that goes into 12 is 4. You can write $\sqrt{12}$ as $\sqrt{4\times 3}$. Using the rules shown below $\sqrt{4}\times\sqrt{3}$ giving $2\times\sqrt{3}=2\sqrt{3}$.	(a) $\sqrt{8}=\sqrt{4\times 2}=\sqrt{4}\times\sqrt{2}=2\sqrt{2}$ (b) $\sqrt{45}=\sqrt{9\times 5}=\sqrt{9}\times\sqrt{5}=3\sqrt{5}$ (c) $\sqrt{108}=\sqrt{36\times 3}=\sqrt{36}\times\sqrt{3}=6\sqrt{3}$ (d) (In reverse) $5\sqrt{3}=\sqrt{25}\times\sqrt{3}=\sqrt{25\times 3}=\sqrt{75}$ NB: $\sqrt{x}$ and $x^{\frac{1}{2}}$ are equivalent if you are working with both surds and the rules of indices.
Surds (Multiplying and Dividing)	Here are the rules! (1) $\sqrt{a}\times\sqrt{a}=a$ (2) $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$ (3) $\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$ Make sure you simplify your answer! $\sqrt{2}\times\sqrt{12}=\sqrt{24}=2\sqrt{6}$ for example.	(a) $\sqrt{3}\times\sqrt{3}=3$ (b) $\sqrt{3}\times\sqrt{6}=\sqrt{18}=3\sqrt{2}$ (c) $4\sqrt{3}\times 2\sqrt{5}=8\sqrt{15}$ (d) $\frac{\sqrt{10}}{\sqrt{2}}=\sqrt{\frac{10}{2}}=\sqrt{5}$
Surds (Adding and Subtracting)	You can only add and subtract like surds. In algebra $a+2a=3a$. This is true for surds. $\sqrt{5}+2\sqrt{5}=3\sqrt{5}$. Sometimes you may have to simplify first!	(a) $3\sqrt{2}+4\sqrt{2}=7\sqrt{2}$ (b) $9\sqrt{5}-6\sqrt{5}=3\sqrt{5}$ (c) $5\sqrt{2}-\sqrt{8}=5\sqrt{2}-2\sqrt{2}=3\sqrt{2}$ (Just simplify $\sqrt{8}$)
Surds (Rationalising the Denominator)	A surd is an irrational number, so if you have a surd in the denominator you rationalise the denominator. This will leave an integer value in the denominator. **Scenario 1:** No + or – sign in the denominator. In this case simply multiply the numerator and the denominator by the surd and simplify. (see Example 1) **Scenario 2:** A + or – sign in the denominator and 2 numbers (at least one being a surd). Simply multiply the numerator and the denominator to create the difference of two squares. To rationalise swap the sign between the two values in the denominator. You will need to simplify your answer. (see Example 2)	**Example 1:** $\frac{5}{\sqrt{2}}=\frac{5\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}}=\frac{5\sqrt{2}}{2}$ **Example 2:** $\frac{7}{5+\sqrt{3}}=\frac{7\times\left(5-\sqrt{3}\right)}{\left(5+\sqrt{3}\right)\times\left(5-\sqrt{3}\right)}=\frac{7\left(5-\sqrt{3}\right)}{25-5\sqrt{3}+5\sqrt{3}-9}=\frac{7\left(5-\sqrt{3}\right)}{16}$
Converting Recurring Decimals into Fractions	You know, for example, $\frac{1}{3}=0.\dot{3}$, but what if you were asked to write $0.\dot{4}$ as a fraction without a calculator? All you need to do is set up & solve an equation: Let $x=0.\dot{4}$. Simply multiply x by powers of 10 until the pattern after repeats, subtract away, solve the equation & simplify if necessary. Here is the worked answer: $x=0.\dot{4}$ so $10x=4.\dot{4}$. Now subtract to give $10x-x=4.\dot{4}-0.\dot{4}$. This will give $9x=4$ and finally simply solve the equation to write $x=\frac{4}{9}$. Be very careful to check how many digits recur after the decimal. Example (a) and (b) differ. In (a) **both** the 2 and the 3 repeat. In (b) **only** the 5 repeats. Look out for the dots! Be careful as, for example, $0.\dot{2}1\dot{5}=0.216216216...$ The dots mark the start and the end of the pattern and, of course, the 1 repeats.	(a) Write $0.\dot{2}\dot{3}$ as a fraction in its simplest form. Let $x=0.\dot{2}\dot{3}$, now $10x=2.\dot{3}\dot{2}$ now go to $100x=23.\dot{2}\dot{3}$. The pattern matches for x & $100x$ so subtract away: $100x-x=23.\dot{2}\dot{3}-0.\dot{2}\dot{3}$ so $99x=23$ and $x=\frac{23}{99}$ (b) Write $0.1\dot{5}$ as a fraction in its simplest form. Let $x=0.1\dot{5}$, now $10x=1.\dot{5}$ and $100x=15.\dot{5}$. The pattern matches for $10x$ & $100x$ so subtract away: $100x-10x=15.\dot{5}-1.\dot{5}$ so $90x=14$ and $x=\frac{7}{45}$ (Make sure you fully simplify your final answer)

Ratio, Proportion and Rates of Change

Simplifying Ratio	Simplify them like fractions by dividing by common factors if it's not obvious.	(a) 5:10 is 1:2 in its simplest form (b) 14:21 is 2:3
Ratios in the form $1:n$ / $n:1$	Divide **both** numbers in the ratio by **one of numbers** to leave one of them as 1. Be careful when it comes to which way round the answer must be! $1:n$ or $n:1$	$5:7$ would be $1:\frac{7}{5}$ as $1:n$ ($\div$ by 5) & $\frac{5}{7}:1$ as $n:1$ ($\div$ 7)
Ratio Sharing	Add the total parts. A ratio of 4:2:1 has 7 parts (**not 3 parts as 4 + 2 + 1 = 7**) Divide the amount to be shared to find the value of **one part**. Simply multiply this value by the each number in the ratio. Remember the units if applicable!	"Share £60 in a 3:2:1 ratio" 6 total parts. £60 divided by 6 = £10. Each part is worth £10 3 × £10 = **£30** 2 × £10 = **£20** 1 × £10 = **£10**
Ratios Already Shared	Sometimes a ratio is already shared and you will need to work backwards. Simply find what one part is worth and then answers the questions given. The question will give you the clue to which quantity you are dividing. In these questions just think (for example) "3 parts is worth £12, so 1 part must be worth £4" and then use this information to answer the question.	**Example:** Money was shared in the ratio 3:2:5 between Ann, Bob and Cat. Given that Bob had £16, find out how much was shared. A: Bob has 2 parts. This means £16 = 2 parts. One part will be worth £8. There are 10 parts in total so 10×8 = £80. A total of £80 was shared.
Ratios to Fractions	Add the total parts in the ratio. This becomes the denominator of the fractions. Simply write each part over that denominator. You should now be able to convert to decimals too either by simplifying or pressing SD on the calculator.	**Example:** $2:3$ has 5 parts so this would be $\frac{2}{5}$ and $\frac{3}{5}$.
Ratios to %	Write the ratios as fractions (as shown above) and then convert them into %	**Example:** $2:3$ would be 40% and 60% when shared
Best Buys	Find the unit cost by dividing the **price by the quantity**. The lowest number is the item that is the best buy. Be careful and don't round the price too early!	8 cakes for £1.28= **16p** each (this is the unit cost) 13 cakes for £2.05= **15.8p** each (so pack of 13 is better)
Basic Proportion	Find out the value of **one item** by dividing and then **multiply** your answer by the number of them you need. Some of these are recipe type questions and others are just shopping type scenarios. Just find the cost, weight or size of 1 and then multiply up. Using the units given may help you understand more.	**Example:** 3 cakes require 450g of sugar to make. Find how much sugar 5 cakes require. **Answer:** 450 ÷ 3 = 150g per cake. Now multiply this by 5 to give 750g required for 5 cakes.
Exchange Rates	£1 = $1.6. Multiply by 1.6 to go from £ to $ & divide by 1.6 to go from $ to £	**E:** Convert £12 to $ **A:** 12 × 1.6 = $19.20
Direct and Inverse Proportion	**Direct:** $y = kx$ **or** $y \propto x$ (This just reads y is directly proportional to x.) With direct proportion k is **multiplied** by x to get y. As x increases, y increases. k is known as the constant of proportionality. It's just a 'fixed value' multiplier. **Inverse:** $y = \frac{k}{x}$ **or** $y \propto \frac{1}{x}$ (This just reads y is inversely proportional to x.) With inverse proportion k is **divided** by x to get y. As x increases, y decreases. **To solve problems involving direct and inverse proportion:** (1) Pick the right equation (for either direct or inverse) and substitute the values given in the question to solve for k (the constant of proportionality) (2) Rewrite the equation with the correct value of k you have just found. (3) Substitute the 2nd given value in for x **or** y to find the required missing value. (Be careful with examples such as y is proportional to the square of x. This can be written as $y = kx^2$ instead of $y = kx$). The root of x is written as $\sqrt{x}$.	**Example 1:** "p is directly proportional to q. When $p = 12$, $q = 4$. Find p when $q = 20$" $p = kq$ $\therefore p = 3q$ **Answer:** 1st solve for k: $12 = k(4)$ now: $p = 3(20)$ $k = 3$ $p = 60$ **Example 2:** "p is inversely proportional to q. When $p = 20$, $q = 10$. Find p when $q = 4$" $p = \frac{k}{q}$ $\therefore p = \frac{200}{q}$ **Answer:** 1st solve for k: $20 = \frac{k}{10}$ now: $p = \frac{200}{4}$ $k = 200$ $p = 50$

Direct and Inverse Proportion Graphs	Graphs showing **Direct Proportion** can be written in the form $y = kx^n$ where k is the constant of proportionality. The notation $y \propto x^n$ may be used and means exactly the same thing. Direct graphs will always have the point $(0,0)$ on. The graph could be a straight line such as $y = 2x$ or a curve such as $y = 3x^2$. Graphs showing **Inverse Proportion** can be written in the form $y = \frac{k}{x^n}$ where k is the constant of proportionality. The notation $y \propto \frac{1}{x^n}$ may be used. These will not pass through the point $(0,0)$ & approach the x axis as x increases.	Direct Proportion Graphs | Inverse Proportion Graphs 

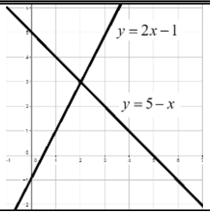
Algebra

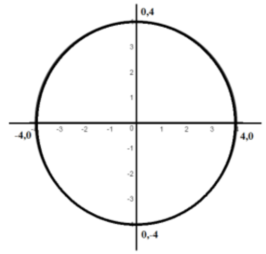
Terminology	**Expression:** A collection of **terms** (letters (unknowns/variables) and possibly numbers (constants)) **without** an equals sign. You don't solve an expression! **Equation:** A collection of terms (letters/numbers) **with** an equals sign. You can look to solve an equation for values of the unknown term (letter). **Identity:** An equation that holds true for all values. The $\equiv$ sign is often used. **Formula:** A set of symbols that expresses a rule. **Inequality:** When a two values are not equal ($\neq$).	**Expressions:** $4a+2b$, x^2-3, $1-4y$, **Equation:** $4a+2b=1$ **Identity:** $4a+2b \equiv 2(2a+b)$ **Formula:** $A=\pi r^2$ **Inequality:** $x>4$
Simplifying Expressions	Just collect the 'like terms'. Be careful with negatives! x^2 and x are not like terms. $4x+1$ is not $5x$ as 1 is not an x. It's a known as a constant.	(a) $2x+3y+4x-5y+3$ becomes $6x-2y+3$ (b) $3x+4-x^2+2x-1$ becomes $5x-x^2+3$
Identities	An identity holds true for any value you substitute in. What does that mean? $4x+10 \equiv 2(2x+5)$ is an example. Substitute 2 in for x and each side will give 14. This will be the same for all values of x. Notice the $\equiv$ instead of $=$?	(a) $x^2-2x+4 \equiv (x-1)^2+3$ (b) $x^2+x-6 \equiv (x-2)(x+3)$
x times x	Multiplying x by x gives x^2 and not $2x$. Check this with 3, 5, or 10 for example	Squaring is multiplying by itself and not by 2.
$p \times p \times p$	$p \times p \times p$ is p^3 and not $3p$. $3p$ means $3 \times p$. This is a common error!	$3\times3\times3=27$ and not 9 $5\times5\times5=125$ and not 15
$p+p+p$	This $3p$ not p^3. Just use numbers to check!	2 + 2 + 2 = 6 and not 8 4 + 4 + 4 = 12 and not 64
Basic Powers (×&÷) (Rules of Indices) $a^m \times a^n = a^{m+n}$ and $a^m \div a^n = a^{m-n}$	When **multiplying** with the same letter or number (base) just **add** the powers. When **dividing** with the same letter or number (base) just **subtract** the powers. Remember $p = p^1$ and $p^0 = 1$. The base must be the same! $a^4 \times a = a^5$ BUT $a^6 \times b^3 \neq ab^9$ (The symbol $\neq$ means doesn't equal)	(a) $p^5 \times p^3 = p^8$ (b) $p^7 \div p^4 = p^3$ (c) $p^{12} \times p = p^{13}$ (d) $2a^5 \times 3a^3 = 6a^8$ (e) $48y^5 \div 16y = 3y^4$ (f) $\frac{4m^6}{2m^2} = 2m^4$ (g) $m^2n \times m^5n^{-1} \times 2m^{-3}n^6 = 2m^4n^6$
The Zero Power (Rules of Indices) $a^0 = 1$	A number raised to the power of zero is 1. An example could be $4^0 = 1$ or $x^0 = 1$. Be careful with examples such as $3y^0$ this is not 1, it's $3\times1=3$ as the zero power is only on the y. Using the rule below though $(3y)^0 = 1$.	(a) $4^0=1$ (b) $x^0=1$ (c) $(xy)^0=1$ (d) $xy^0=x$ N.B 0^0 is undefined! Can you think or explain why?

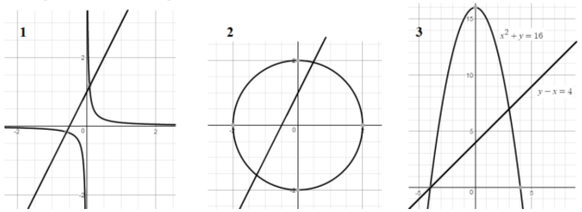
Raising to a Power (Rules of Indices) $\left(a^m\right)^n = a^{mn}$	When a number or algebraic term already raised to a power is raised to **another** power you **multiply** the powers. A common error is to multiply the bases by the powers instead of the powers by the powers. Example (c) to the right shows that 3 is to the power of 1. The common error is to write 12 instead of 3^4.	(a) $\left(x^2\right)^3 = x^6$ (b) $\left(x^4y^5\right)^{0.5} = x^2y^{2.5}$ (c) $\left(3x^4y^{\frac{1}{4}}\right)^4 = \left(3^1x^4y^{\frac{1}{4}}\right)^4 = 3^4x^{16}y^1 = 81x^{16}y$
Negative Powers (Rules of Indices) $a^{-m} = \frac{1}{a^m}$	If you have number or algebraic term raised negative power this can be written as the reciprocal of that number or term raised to the positive power. Some examples: (i) $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ (ii) $\left(\frac{2}{3}\right)^{-4} = \left(\frac{3}{2}\right)^4 = \frac{81}{16}$ (iii) $\left(\frac{x}{y}\right)^{-z} = \left(\frac{y}{x}\right)^z$	(a) $3^{-2} = \frac{1}{3^2}$ which is $\frac{1}{9}$ (b) $5^{-3} = \frac{1}{5^3}$ which is $\frac{1}{125}$ Tougher one! (c) $0.5^{-2} = 4$ (d) $0.2^{-3} = 125$
Fractional Powers (Rules of Indices) $a^{\frac{m}{n}} = \sqrt[n]{a^m}$	"Find the nth root of the number and then raise it to the power of m". This is easier to do than it is to explain! If you have $8^{\frac{1}{3}}$ you take the third (or cube) root of 8. This gives you 2. Of course $2^1 = 2$ which gives us our answer. Now, if you have $8^{\frac{2}{3}}$, you do exactly the same at the start as before but you need to raise 2 to the power of 2 this time. $2^2 = 4$ so the answer is 4. DO NOT DIVIDE THE NUMBER (BASE) BY THE POWER! Look out for negative fractional powers	(a) $16^{\frac{1}{4}}$ You need the 4th root of 16 which is 2 as $2^4 = 16$ (b) $27^{\frac{2}{3}} = \left(27^{\frac{1}{3}}\right)^2 = (3)^2 = 9$ (c) $32^{\frac{3}{5}} = \left(32^{\frac{1}{5}}\right)^3 = (2)^3 = 8$ (d) $125^{\frac{4}{3}} = \left(125^{\frac{1}{3}}\right)^4 = 625$ (e) $\left(\frac{25}{36}\right)^{-\frac{1}{2}} = \left(\frac{36}{25}\right)^{\frac{1}{2}} = \frac{6}{5}$
Expanding Single Brackets	Multiply the number or algebraic term on the outside by **each** term inside the brackets. Be careful with negatives! The question may ask you to 'multiply out'	(a) $5(3x+2) \equiv 15x+10$ A common error is $15x+5$. (b) $2x(3x-4) \equiv 6x^2-8x$
Expanding Double Brackets Expanding Triple Brackets	Multiply each term (all 4) by one another. You can use F.O.I.L & then simplify. **First, Outer, Inner, Last**. Remember to **simplify**! Don't forget $+4x-x$ is $+3x$ and x times x is x^2 not $2x$. Be careful! $(a+b)^2 = (a+b)(a+b)$ and NOT a^2+b^2	(a) $(x+2)(x-3)$ $x^2-3x+2x-6$ x^2-x-6 (b) $(2x-1)(3x+2)$ $6x^2+4x-3x-2$ $6x^2+x-2$
Factoring Single Brackets	Find the HCF of numbers &/or terms and write these on the **outside** of the bracket. **Inside** will be terms you have to × the outside by to get the original.	(a) $6x-3 \equiv 3(2x-1)$ (b) $15x+10 \equiv 5(3x+2)$ (c) $6x^2+8x \equiv 2x(3x+4)$ (d) $x^2-x^3 = x^2(1-x)$
Factoring Quadratics when $a=1$	When a quadratic expression is in the form ax^2+bx+c find the two numbers that **ADD to give** b and **MULTIPLY to give** c. Be careful with negatives. Have 2 sets of brackets with x in each and then choose the factors!	(a) $x^2+7x+10 \equiv (x+5)(x+2)$ (b) $x^2+2x-8 \equiv (x+4)(x-2)$
Factoring Quadratics when $a \neq 1$	This method is a slightly less mathematically rigorous approach but **can** make factoring easier when you have, for example $6x^2+5x-4$ to factor. When a quadratic expression is in the form ax^2+bx+c: (1) Put the value of a in the front of each of the 2 brackets. (Don't panic here!) (2) Multiply a by c (3) Find the two numbers that add to give by and multiply to give ac (4) Place these values in the brackets with the correct sign. (5) Simplify and cancel common factor. **Example:** Factor $6x^2+5x-4$	**Answer:** (1) Let's start with $(6x \quad)(6x \quad)$. (2) Multiply a by c to give $ac = -24$ (3) You need 2 numbers that add to give +5 and multiply to give -24. They will be $+8$ and -3 (4) This now gives $(6x+8)(6x-3)$. (5) At this stage take common factors out of both brackets (where applicable) and simplify: $2(3x+4)\times3(2x-1)$, Cancel to give $(3x+4)(2x-1)$

Factoring the Difference of Two Squares	An expression in the form a^2-b^2 you can factorise to give $(a+b)(a-b)$. If you look at the examples to the right, when you expand the double brackets the two middle terms cancel to just leave the first and last.	(a) $x^2-25\equiv(x+5)(x-5)$ Each term is a squared term (b) $16x^2-81\equiv(4x+9)(4x-9)$
Completing the Square for Quadratic expressions (when $a=1$).	There are times when a quadratic expression can't be factored. When a quadratic is in the form x^2+bx+c you can write this in the form $(x+p)^2+q$ (The form $(x+p)^2+q$ is found by evaluating $\left(x+\frac{b}{2}\right)^2-\left(\frac{b}{2}\right)^2+c$.) This looks quite tough but it isn't too bad! Just follow these 3 steps: (a) Have a set of brackets with x in and **half** the value of b in. (b) Square the bracket. (c) Subtract $\left(\frac{b}{2}\right)^2$ from c and tidy the expression. After a few goes it becomes easier. Try and work with fractions as your work later on in maths will require you to do examples without a calculator. There are advantages to writing an expression in the form $(x+p)^2+q$. You can gather information about the maximum or minimum of a function and the axis of symmetry. The completed square form can also allow us to solve quadratic equations of the form $ax^2+bx+c=0$ when factoring is not possible.	**Example 1:** Complete the square for x^2-6x+2 **Answer:** (a) $(x-3)$ (b) $(x-3)^2$ (c) $(x-3)^2-9+2$ which will tidy to give $(x-3)^2-7$ **Example 2:** Complete the square for x^2+5x-3 **Answer:** (a) $\left(x+\frac{5}{2}\right)$ (b) $\left(x+\frac{5}{2}\right)^2$ (c) $\left(x+\frac{5}{2}\right)^2-\frac{25}{4}-3$ which will tidy to give $\left(x+\frac{5}{2}\right)^2-\frac{37}{4}$ You can say that the minimum value of the expression in part (a) would be -7 and $-\frac{37}{4}$ in (b). We will look at this later on in more depth.
Completing the Square for Quadratic expressions (when $a\neq1$).	When a quadratic expression is in the form ax^2+bx+c where $a\neq1$ you can complete the square and write it in the form $p(x+q)^2+r$. You can use a similar technique to that above but factor out a at the start. Here is an example: Complete the square for $2x^2-12x+4$. You need to take the factor of 2 out of the **first two** terms: $2\left[x^2-6x\right]+4$. At this stage you can complete the square inside the square brackets to give: $2\left[(x-3)^2-9\right]+4$. You can now expand the square brackets to give $2(x-3)^2-18+4$ which gives $2(x-3)^2-14$. This method can be used when a is a negative number as shown to the right. You can only complete the square when the value of a is 1. In the first example you could graph the quadratic. This would open upwards (positive), have a minimum point at $(-1,-7)$ and the axis of symmetry would be the line $x=-1$. In the second example the graph would open downwards (negative), have a maximum point $\left(\frac{5}{2},\frac{21}{4}\right)$ and the axis of symmetry would be the line $x=\frac{5}{2}$	**Example 1:** Complete the square for $4x^2+8x-3$ **Answer:** Factor the 4 out $4\left[x^2+2x\right]-3$. At this stage complete the square inside the brackets to give $4\left[(x+1)^2-1\right]-3$. Now expand the square brackets to give $4(x+1)^2-4-3$. Finally tidy to $4(x+1)^2-7$ **Example 2:** Complete the square for $-x^2+5x-1$ **Answer:** Factor the -1 out to give $-\left[x^2-5x\right]-1$. Now complete the square $-\left[\left(x-\frac{5}{2}\right)^2-\frac{25}{4}\right]-1$. Expand the square brackets $-\left(x-\frac{5}{2}\right)^2+\frac{25}{4}-1$ and tidy to give the answer $-\left(x-\frac{5}{2}\right)^2+\frac{21}{4}$.

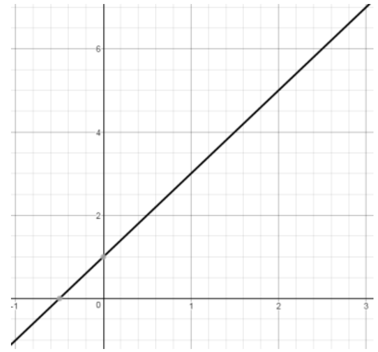
Formulae (Writing) 🗎	You may be asked to write and use a formula given a scenario. Use terms (letters) to represent the unknown quantities such as C & N and numbers to represent the constants such as +5 shown to the right which is a fixed value.	**Example:** "Bob charges £3 per window and a £5 call out charge" **Answer:** $C = 3N + 5$ with N being the number of windows cleaned and C the cost.
Formulae (Substituting into) 🗎	Substitute the numbers given into the formula or expression. Swap letters for numbers. Be careful on the order. If $x = 3$ and you need $2x^2$ square 3 first, then multiply by 2. There is a difference between $2x^2$ and $(2x)^2$. Be careful with negatives. Squaring makes it positive! Also, subtract a – means add it.	**Example:** $a = 3$, $b = 2$ and $c = 5$ Find: (i) $2a$ which is just 2(3) = 6 (ii) $3a - 2b$ so 3(3) – 2(2) = 5 (iii) $b^2 - 5$ which is $(2)^2$ - 5 = -1
Formulae/Equations (Rearranging) 🗎	Changing the subject of an equation is like solving one without a 'pretty' answer at the end. Instead of your answer being a number, it's usually an expression containing other terms (letters) and possibly numbers. Don't panic; just apply the same rules as for solving. If you have a +, subtract this value from both sides. If you have a – then add it to both sides, a × then divide both sides by this quantity and a ÷ then multiply both sides by this quantity. What you do to one side, you just do to the other! If there are no + or – subtract signs then it will be × or ÷. Remember $ut = u \times t$ and not $u + t$. Brackets means multiply too! It doesn't matter if you have your subject on the right or the left hand side!	**Example:** Make x the subject of the equation $y = \frac{2x-1}{z}$ **A:** $y = \frac{2x-1}{z}$. Start by multiplying both sides by z to give $yz = 2x - 1$. Now add 1 to both sides so $yz + 1 = 2x$ and finally divide both sides by 2 to give $\frac{yz+1}{2} = x$. You now have x as the subject.
Solving Linear Equations Unknowns on one side Unknowns on both sides 🗎	Get the x's (unknowns or letters) on one side and the numbers on the other. Use the balance method. Simply do the opposite operation to what the equation gives until you have only x's on one side and only numbers on other. If you have a +, subtract this value from both sides. If you have a – then add it to both sides, a × then divide both sides by this quantity and a ÷ then multiply both sides by this quantity. What you do to one side, you just do to the other!	(a) $2x - 3 = 7$, $2x = 10$, $x = 5$ (b) $5p + 6 = 2p + 18$, $3p + 6 = 18$, $3p = 12$, $p = 4$ (c) $\frac{y}{2} - 1 = 5$, $\frac{y}{2} = 6$, $y = 12$
Setting Up and Solving Linear Equations 🗎🗎	Find an expression for each piece of information given in the question, add them together, and simplify the expression. This will then be set equal to a value given in the question (or implied) to give you your equation. Solve the equation and then make sure you answer the original question in context!	**E:** Bob is n years old. Fred is twice his age & Sue is one year your than Bob. Their total age is 39. Set up and solve an equation to find the age of Bob. **A:** $n + 2n + n - 1 = 39$ this gives $4n - 1 = 39$ and $n = 10$
Equations with Fractions	You can think of this a couple of different ways: (1) Multiplying through by the LCM to 'clear' the fractions. (2) Cross multiplying to 'clear' the fractions. **Example 1:** (Multiplying by the LCM). Solve the equation: $\frac{x}{3} + 4 = 2 - \frac{3x}{4}$ **Answer:** Multiply both sides of the equation by 12 (which is the LCM) to leave: $4x + 48 = 24 - 9x$. At this stage you add $9x$ and subtract 48 to both sides of the equation. You can now solve to get $13x = -24$ and $x = \frac{-24}{13}$ *The section later on algebraic fractions will help for harder examples*	**Example 2** (Cross Multiplying) Solve the equation: $\frac{x-1}{4} = \frac{x}{5}$ **Answer:** Cross Multiplying to get: $5(x-1) = 4x$. Expanding to get $5x - 5 = 4x$ which gives $x = 1$. *Use either or both methods to solve equations with fractions. It's often a case of being flexible and seeing which method is quickest.*

<u>Solving Linear Simultaneous Equations (Algebraically)</u>	If you have 2 unknowns (x and y for example) you need at least two equations to find the value of both x and y. To do this you solve simultaneous equations. Either make the value in front (coefficient) of x's the same **or** the y's the same. Once they are the same (eg both 5) if the signs in front are the **same, subtract** if they are **different, add**. You will have now eliminated **one** unknown (x or y) Solve the equation you have for either x or y. (This will be a simple equation) Finally substitute that value back in to any of the other equations to solve for the other unknown. Check your answers work for both!	(a) $2x+y=7$ $3x-y=3$ Add them to get $5x=10$ & $x=2$. Substitute in: $2(2)+y=7$ so $y=3$ (b) $5x+2y=9$ $10x+3y=16$ Multiply 1st equation by 2. $10x+4y=18$. Subtract to eliminate x's to give $y=2$. Substitute in: $5x+2(2)=9$ so $x=1$
<u>Solving Linear and Non Linear Simultaneous Equations (Graphically)</u>	These equations are solved by drawing the graphs (straight lines) of the two equations given. The solutions (answer to the question) will be where the lines meet. The graph to the right shows the solutions of the simultaneous equations $y=5-x$ and $y=2x-1$. They intersect (meet) at the point with coordinates $(2,3)$. This means the solutions will be $x=2$ and $y=3$.	
<u>Solving Quadratics (the form</u> $ax^2=b$)	A quadratic equation will have a 'squared term' in such as x^2 or t^2 as its highest power. An example could be $x^2=36$. When the quadratic is in the form $ax^2=b$ simply isolate the x^2 term so you have $x^2=$ to some value and square root both sides to solve. Remember there will be a positive and a negative solution! $3\times3=9$ and $-3\times-3=9$ too. We must write both answers down.	(a) $x^2=36$ $x=\pm\sqrt{36}$ $x=\pm6$ (b) $2x^2=98$ $x^2=49$ $x=\pm\sqrt{49}$ $x=\pm7$ (c) $x^2+10=25$ $x^2=15$ $x=\pm\sqrt{15}$
<u>Solving Quadratics (the form</u> $ax^2+bx=0$)	These can be factored and set to zero as there is no constant. Here is an example: $x^2+4x=0$ now factor the x to give $x(x+4)=0$. At this stage either $x=0$ or $x+4=0$ as one or both of the factors will = 0 For the answer to be 0 either one or both of the factors must = 0. (Just think logically! $5\times0=0$, $9\times0=0$). This gives us the solutions $x=0$ or $x=-4$.	(a) $x^2-3x=0$ $x(x-3)=0$ $x=0$ or $x=3$ (b) $x^2=5x$ $x^2-5x=0$ $x(x-5)=0$ $x=0$ or $x=5$ (c) $2x^2+8x=0$ $2x(x+4)=0$ $x=0$ or $x=-4$
<u>Solving Quadratics Factoring</u> ($a=1$)	You have seen previously how to factor and expression in the form ax^2+bx+c. You can use this technique to solve equations in the form $ax^2+bx+c=0$. Once the expression is factored and set = to a value it becomes an equation and you can solve for x. Set the quadratic = 0 and solve. Here is an example: Solve the equation $x^2-x-6=0$. Using the method shown previously you can factor to give $(x-3)(x+2)=0$. This means either $(x-3)=0$ or $(x+2)=0$. Using these facts you can say $x=3$ or $x=-2$.	**Example 1:** Solve the equation $x^2+3x-10=0$ **Answer:** Factor to give $(x-2)(x+5)=0$. This means $x=2$ or $x=-5$ **Example 2:** Solve the equation $x^2+x=12$ **Answer:** First rearrange into $ax^2+bx+c=0$ to give $x^2+x-12=0$. Now factor to $(x+4)(x-3)=0$. This will give us the solutions $x=-4$ or $x=3$.
<u>Solving Quadratics Factoring</u> ($a\neq1$)	You have seen previously how to factor and expression in the form ax^2+bx+c when $a\neq1$. You can use the same method to solve an equation in the form $ax^2+bx+c=0$ as the one used in the previous section. As with all equations check that your answer is valid especially if it's in context. Some solutions may **not** be valid such as negative answers where missing lengths are involved.	**Example:** Solve the equation $2x^2+7x-4=0$ **Answer:** Factor to give $(2x-1)(x+4)=0$. This will give use the solutions $x=\frac{1}{2}$ or $x=-4$.

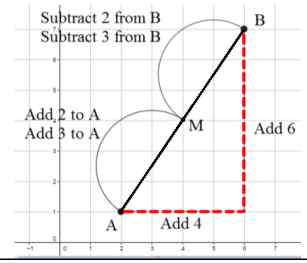
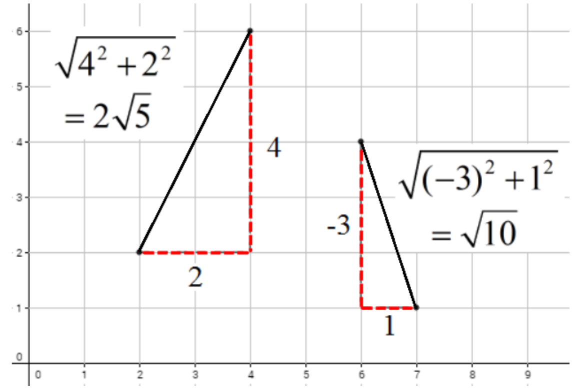
Solving Quadratics (Using the formula)	When a quadratic equation is in the form $ax^2+bx+c=0$ the solutions can be found using the quadratic equation $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$. You would use the formula if the equation doesn't factor or you can't factor it easily. Be careful with the signs on $a, b \,\&\, c$ and make sure you obtain the + and the $-$ solution using a calculator. To do this simply scroll to the $\pm$ part, start with + and then change to $-$ for the second solution. On a calculator use brackets for x and just substitute the values in to give $\frac{-(b)\pm\sqrt{(b)^2-4(a)(c)}}{2(a)}$.	**Example:** Solve the equation $3x^2+x-5=0$ **Answer:** $a=3, b=1$ and $c=-5$ Substitute in to give $x=\frac{-1\pm\sqrt{(1)^2-4(3)(-5)}}{2(3)}$ The + answer will be $x=\frac{-1+\sqrt{61}}{6}$ or 1.14 to 2 dp. The $-$ answer will be $x=\frac{-1-\sqrt{61}}{6}$ or -1.47 to 2dp
Solving Quadratics (Completing the Square)	When a quadratic equation won't factor, you have two obvious choices when it comes to solving the equation. The first is using the formula (as shown above) and the 2nd is using the completed square form. When a quadratic is in the form $ax^2+bx+c=0$ you can write this in the form $(x+p)^2+q=0$ by completing the square (as shown previously). For there to be real solutions $q<0$. Before you use this method check you can't factor it as that would be easier most of the time! Using the example given previously BUT as an equation set = 0 let's find the roots (solutions) to the equation $x^2-6x+2=0$ Here are the steps (i) $(x-3)$ (ii) $(x-3)^2$ (iii) $(x-3)^2-9+2$ which will tidy to give $(x-3)^2-7=0$. This is where we got up to before! Now (iv) add 7 to both sides of the equation to give $(x-3)^2=7$. At this stage square root both sides to give (v) $x-3=\pm\sqrt{7}$. Finally add 3 to both sides to give the 'exact answer' of $x=3\pm\sqrt{7}$. Exact means in surd form. Remember you will have two solutions $x=3+\sqrt{7}$ and $x=3-\sqrt{7}$ (or 0.35 and 5.65 to 2 d.p)	**Example 1:** Solve the equation $x^2+4x+1=0$ **Answer:** Start by completing the square: $(x+4)^2-16+1=0$. At this stage you can write $(x+4)^2=15$ followed by $x+4=\pm\sqrt{15}$. Simply subtracting the 4 from both sides gives us our exact answer of $x=-4\pm\sqrt{15}$ **Example 2:** Solve the equation $x^2-7x-2=0$ **Answer:** Start by completing the square: $\left(x-\frac{7}{2}\right)^2-\frac{49}{4}-2=0$ which will now give $\left(x-\frac{7}{2}\right)^2=\frac{53}{4}$ which in turn gives $x-\frac{7}{2}=\pm\sqrt{\frac{53}{4}}$ and further $x=\frac{7}{2}\pm\frac{\sqrt{53}}{2}$ & tidying gives $x=\frac{7\pm\sqrt{53}}{2}$
Equation of a Circle (and its graph)	The equation of a circle with its centre at the origin (0, 0) and a radius length r can be written as $x^2+y^2=r^2$. Don't panic, this is not much harder than using Pythagoras Theorem. Often you will be asked to draw one. If so just use a compass and have your centre at the origin. $x^2+y^2=16$ is shown to the right and passes through (4,0) (0,4) (-4,0) & (0,-4) $x^2+y^2=25$ would have radius 5 and pass through (5,0) (0,5) (-5,0) & (0,-5) A common error is not square rooting the radius when asked for its length! If you are given a diagram and asked for the equation, simply pick a point and substitute the x and y coordinates into $x^2+y^2=r^2$ to find the value of r^2 (or r).	**Example:** $x^2+y^2=16$ is a circle with a radius of 4. **Answer:** Use a compass set 4 units apart! 

Linear and non Linear Simultaneous Equations (Solving Algebraically)	A linear equation can be represented by a line. A non-liner by a curve or circle (for example). One of your equations can be written in the form 'Elimination' by subtraction is often not possible so the method of substitution is used for most examples. The general rule is to make either x or y the subject of the **linear** equation and substitute into the **non-linear equation**. Once you have solved the new non-linear equation for one unknown (x or y) then substitute the answer(s) back into the linear equation to find the other. Remember to solve for both x and y ! Your solutions may have to be given as coordinates as they will be the points where 2 graphs meet. The 2 graphs of the example to the right is shown below in figure 3. Figure 1 is a line & reciprocal, 2 a line & circle  	**Example:** Solve the simultaneous equations $y-x=4$ and $x^2+y=16$. **Answer:** You can rewrite the 1st equation as $y=x+4$. Now substitute this into the second equation to eliminate y to give $x^2+x+4=16$. This can be written as $x^2+x-12=0$ which factors to $(x+4)(x-3)=0$ and gives $x=-4$ or $x=3$. You now have to solve for y . Substitute the two values of x back into the linear ($y=x+4$) to solve for y . When $x=-4$, $y=-4+4$ which gives $y=0$. When $x=3$, $y=3+4$ which gives $y=7$. You have 2 solutions for x and 2 solutions for y . If these were points of intersection of two graphs the coordinates would be $(-4,0)$ and $(3,7)$.
Solving Equations by iterative methods	There will be times when it's hard to solve an equation using the techniques you have learned or could learn in maths. The equation $x=\cos(x)$ is an example. In such cases you **could** use an iterative formula to solve the equation to a certain degree of accuracy. If you have a function $\mathrm{f}(x)=0$ you can rearrange this to give $x=\mathrm{g}(x)$. This is just a new function of x using the original terms from $\mathrm{f}(x)=0$ This equation can then be used to set up the iterative formula. This can be written as $x_{n+1}=\mathrm{g}(x_n)$. This forms a sequence for values to be substituted into. You will be given a value of x_0 (starting value for the first approximation of a solution to the equation $\mathrm{f}(x)=0$) and it's simply a case of setting up the iterative formula on the calculator and finding values of x_1, x_2, x_3 and so on to locate a root. You will be given a level of accuracy to aim for or a number of iterations to produce. All that is happening is the first value (x_0) goes into the right hand side of the equation to produce a value of x on the left hand side. This value (x_1) is then taken and substituted into the right hand side again to produce a second value of x (x_2). This process continues until the sequence converges (tends to/ approaches) to a limit. This limit will represent the solution of the equation. Not all rearrangements will yield the answer you want! Be flexible when it comes to forming $x=\mathrm{g}(x)$. Some sequences may diverge!	**Example:** Use an iterative formula to find the positive root of the equation $x^2-3x-6=0$ to 3 decimal places. **Answer:** Set up an iterative formula by making x the subject of the equation $x^2-3x-6=0$. $x^2=3x+6$ $x=\pm\sqrt{3x+6}$ $x=+\sqrt{3x+6}$ Start with $x_0=4$. At this stage type in 4 ad press = on your calculator. To find x_1 type in $\sqrt{3\mathrm{Ans}+6}$. This will give $x_1=4.242640..$ To find the next value x_2, press = again. This gives $x_2=4.327576..$, press = again for x_3 which gives $x_3=4.356917..$ repeat to get $x_5=4.37047..$ for x_4 giving $x_4=4.367007..$, $x_5=4.37047..$, $x_6=4.3716604..$ $x_7=4.372068..$ $x_8=4.372208...$ At this stage both x_7 and x_8 round to 4.372. This means the iterative formula is converging to 4.372 to 3 decimal places.

<table>
<tr>
<td>Plotting Straight Line Graphs (Linear Graphs or Linear Functions)

<u>Method 1 (Table)</u>
<u>Method 2 (Gradient/Intercept Method)</u></td>
<td>Method 1: Make a table of values using the method shown to the right. The graph of $y = 2x+1$ is shown to the right for $-1 \le x \le 3$. All you have to do is substitute the values into the equation start with $x = -1$ and finishing with $x = 3$. Make sure the values are going up by 2 each time!
Method 2: Use the gradient/intercept method. The line $y = 2x+1$ is in the form $y = mx + c$. The gradient is m and the y intercept is c. This line will pass through the y axis at the point (0.1) and have a gradient of 2. That means it goes up 2 for every one it goes across (to the right as it's +). (Gradient shown below)
Make sure your line is straight. Any 'kinks' suggests your coordinates are incorrect. The values will always be going up or down by a fixed amount.
Don't worry if you get an equation such as $x + y = 5$. You can just use a table of values as before. You may even want to rearrange the equation to $y = 5 - x$ or $y = -x + 5$ if you find it easier. Finally! Make sure you draw a line!</td>
<td>
<table>
<tr><td>x</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>y</td><td>-1</td><td>1</td><td>3</td><td>5</td><td>7</td></tr>
</table>

The graph of $y = 2x+1$ for $-1 \le x \le 3$</td>
</tr>
<tr>
<td><u>Midpoint of a Line</u></td>
<td>Method 1: Add the x coordinates and divide by 2, add the y coordinates & divide by 2.

Method 2: Sketch the line (if you can) and find the values half between the two x's and two y's

Another way of thinking about this is that the midpoint is the average of the x's and the averages of the y's. The formula is $M = \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$
The numbers won't necessarily be integers and could of course be negative.</td>
<td>Example: Find the midpoint of a line through $(2,1)$ & $(6,7)$.
Answer: $\frac{2+6}{2} = 4$ and $\frac{1+7}{2} = 4$ so the midpoint is 4,4

</td>
</tr>
<tr>
<td><u>Length of a Line Segment.</u></td>
<td>To find the length of a line segment just use Pythagoras Theorem.
Method 1 (Easier Examples): Make the line the hypotenuse of a right angled triangle, count the number of squares horizontally and the number of square vertically. Use the as the lengths of the two shorter sides of a right angled triangle. At this stage apply Pythagoras Theorem.
Method 2 (Harder Examples): Use the formula $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. Don't be scared, this is Pythagoras too. It's just easier to apply if you have decimal, fractional or negative coordinates and drawing a triangle (like in method 1) is hard. It's just the 'change in x and the change in y.
Example: $A(0.5, 2)$ and $B(-3, 4.2)$. Find the length of the line segment AB.
Solution: $x_1 = 0.5, x_2 = -3, y_1 = 2$ & $y_2 = 4.2$. so $\sqrt{(0.5-(-3))^2 + (2-4.2)^2} \approx 4.1$</td>
<td>Method 1

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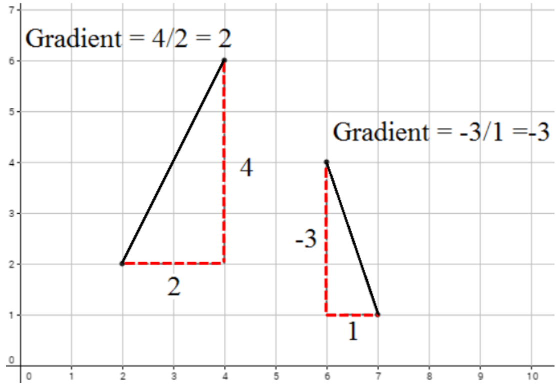
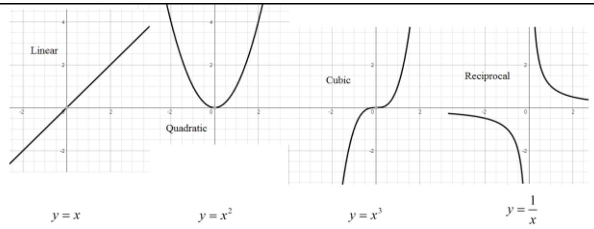
<u>The Gradient of a Line</u>	The gradient of a line is how steep the line is. The greater the number (+ or -) , the steeper the line. To find the gradient of a line **divide** the **total distance up or down** by the **total distance left or right**. Up is + and down is -. Right is + and left is -. You may be able count squares and divide as shown to the right. The gradient can be positive (sloping upwards left to right) or negative (sloping downwards from left to right). Without a graph you **could** use the formula. $m=\frac{y_1-y_2}{x_1-x_2}$ The gradient (m) of the line passing through (1,2) and (11,6) would be $m=\frac{6-2}{11-1}=\frac{4}{10}=\frac{2}{5}$. This goes up 2 units for every 5 to the right.	
<u>Find the equation of a straight line given a point and a gradient.</u>	A straight line can be written in the form $y=mx+c$ where m is the gradient and c is the point where the line crosses the y axis (c is known as the constant). To find the equation of a straight line given a point the line passes through and the gradient of the line you simply substitute the values of x , y and m into the equation $y=mx+c$. To find the value of c . Once you have the value of c simply put the equation 'back together' in the form $y=mx+c$. You may need to do this from a graph. Just find the gradient of the line and a point it passes through.	**Example:** Find the equation of the line with gradient 4 passing through the point $(2,7)$. **Answer:** In this example $m=4$ and you need to find the value of c . Simply substitute the given values in to solve for c . $y=mx+c$ which gives $7=2(4)+c$. This gives $c=-1$. The equation of the line is therefore $y=4x-1$
<u>Finding the equation of a straight line given two points</u>	If you have two points (or two sets of coordinates) then you can find the equation of the straight line passing through them. To find the equation of a straight line all you only ever need is the **gradient** and **one point** the line passes through (as shown in the example above). To find the gradient use the method shown above taking the two points you have for x_1, y_1 and x_2, y_2 . In the example to the right $x_1=6$ and $y_1=11$. $x_2=2$ and $y_2=3$. Once you have the gradient, pick one of the points you have (you can choose either) and simply substitute into $y=mx+c$ as shown in the previous section.	**Example:** Find the equation of the line passing through the points $(6,11)$ and $(2,3)$. **Answer:** First find the gradient: $m=\frac{11-3}{6-2}=\frac{8}{4}=2$. At this stage pick either one of the points the line goes through and substitute into $y=mx+c$ to give: $11=2(3)+c$. You can see $c=5$ giving us the equation $y=2x+5$.
<u>Parallel and Perpendicular Lines</u> 🗎	If two lines are **parallel** they will both have the same gradient. The two lines will never meet and stay a fixed distance apart. The value of m (the gradient) will be the same for both lines. If two lines are **perpendicular** they will be at right angles to one another. The product (×) of their gradients will always = -1 (or, if you like, the gradient of one line (m_1) is the negative reciprocal of the gradient of the other line (m_2)). This could be written as $m_1 \times m_2=-1$ ifs the lines are perpendicular OR if a line has gradient m , the line perpendicular to it will have gradient $-\frac{1}{m}$. Once you have found the gradient of the line parallel or perpendicular to the original line,	**Example 1:** Find the equation of a line parallel to the line $y=3x+2$ which passes through the point $(1,9)$. **Answer:** The gradient will be the same giving $m=3$. Now substitute into $y=mx+c$ to give $9=3(1)+c$. This means $c=6$ & the equation of the line $y=3x+6$. **Example 2:** Find the equation of a line perpendicular to the line $y=3x+2$ which passes through the point $(6,5)$. **Answer:** The gradient will be $\frac{-1}{3}$.

<table>
<tr>
<td></td>
<td>simply substitute the values of x, y and m into the equation of a straight line $y=mx+c$ to find its equation. (This method is shown previously)
Not all equations will be in the form $y=mx+c$. For example, the line $y=2x+3$ is parallel to the line $4y-8x-9=0$. Their gradients are the same.</td>
<td>Now substitute into $y=mx+c$ to give $5=\frac{-1}{3}(6)+c$
This means $c=7$ & the equation of the line $y=\frac{-1}{3}+7$</td>
</tr>
<tr>
<td><u>Graph Recognition</u></td>
<td>Linear: A straight line graph which can be written in the form $y=mx+c$
Quadratic: A parabola which is a sweeping curve in the form $y=ax^2+bx+c$
Cubic: A sweeping curve in the form $y=ax^3+bx^2+cx+d$
Reciprocal: A curve in the form $y=\frac{a}{x}$.</td>
<td>
</td>
</tr>
<tr>
<td><u>Quadratic Graphs (Plotting from a Table)</u></td>
<td>This will be a parabola which is a sweeping curve & NOT a collection of lines. Simply Fill out the table (same as linear graphs). Be careful with negatives. Squaring a negative makes it positive! Subtracting a negative will mean adding!
$y=x^2-3x+1$
<table>
<tr><td>x</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>y</td><td>11</td><td>6</td><td>1</td><td>-1</td><td>-1</td><td>1</td><td>6</td><td>11</td></tr>
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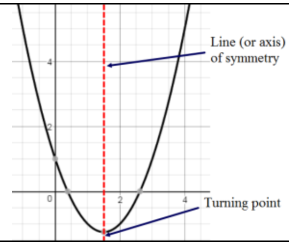
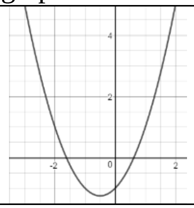
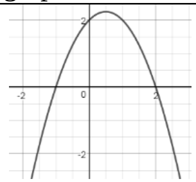
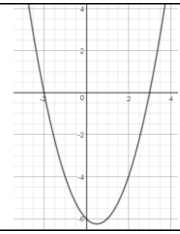
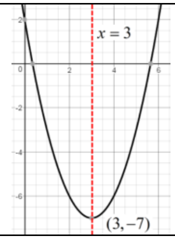
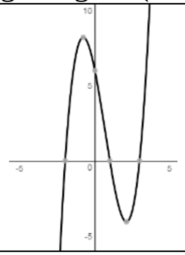
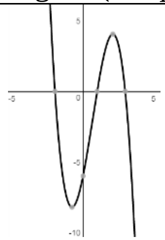
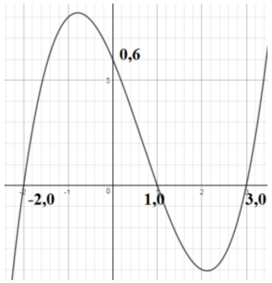
<td>
</td>
</tr>
<tr>
<td><u>Sketching Quadratic Graphs from an equation (Maximum, Minimum & Turning Points)</u></td>
<td>You may be asked to sketch a quadratic graph from its equation instead of plotting one from a table of values. This is a more complex skill.
A quadratic equation can be written in the form $y=ax^2+bx+c$. The graph is called a parabola and is a curve with either a max or min turning point.
The graph will cross the x axis when $y=0$ and cross the y axis when $x=0$.
<table>
<tr><td>Positive Quadratic Graphs
If $a>0$ the graph will have a minimum</td><td>Negative Quadratic Graphs
If $a<0$ the graph will have a maximum</td></tr>
<tr><td></td><td></td></tr>
</table>
In the Factored Form $y=(x-p)(x-q)$:
You can get the shape, find the roots (solutions) and find the y intercept easily.
In Completed Square Form $y=(x+p)^2+q$
You can get the shape, find the roots (solutions) and find the y intercept easily.
You can also find the maximum/minimum turning point & the axis of symmetry
(More information on each technique is given in previous sections)</td>
<td>Example 1: Sketch the graph of $y=x^2-x-6$ showing any points of intersection with the coordinate axis.
Answer: Factor to give $y=(x-3)(x+2)$.
<table>
<tr><td></td><td>Roots: $x=3$, $x=-2$
y intercept: $(0,-6)$</td></tr>
</table>
Example 2: Sketch the graph of $y=x^2-6x+2$ stating the coordinates of the turning point, the equation of the axis of symmetry and the roots of the equation.
Answer: In completed square form $y=(x-3)^2-7$.
<table>
<tr><td>
</td><td>Minimum point: $(3,-7)$
Axis of symmetry: $x=3$
Roots: $x=3\pm\sqrt{7}$
y intercept: $(0,2)$</td></tr>
</table></td>
</tr>
</table>

<u>Sketching Cubic Graphs</u>

Cubic equations can be written in the form $y = ax^3 + bx^2 + cx + d$. Their graphs will produce a sweeping curve that passes through the x axis up to 3 times.
The diagrams below show the difference between a positive & negative graph.

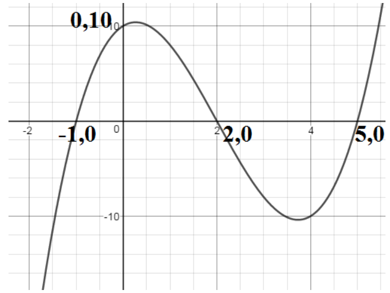
Positive Cubic Graphs +	**Negative Cubic Graphs -**
If $a > 0$ the graph will enter in the bottom left (3rd quadrant) and leave in the top right region (1st quadrant)	If $a < 0$ the graph will enter in the top left (2nd quadrant) and leave in the bottom right region (4th quadrant)
	

Cubic graphs are easier to draw when factored in the form:
$y = (x - p)(x - r)(x - q)$. Here p, q and r are just numbers!
The graph will cross the x axis when $y = 0$ and cross the y axis when $x = 0$.
If you take the equation $y = (x-3)(x+2)(x-1)$ you can sketch the graph using the information given. This example is positive cubic (if you expand the brackets the first term will be x^3 rather than $-x^3$). The shape of this graph can be seen in the table above (+ example). Now consider where it crosses the x axis. This is when $y = 0$. This gives $0 = (x-3)(x+2)(x-1)$. Solving for each factor (like you would a quadratic equation) gives $x = -2$, $x = 1$ and $x = 3$. From here you can now plot the points $(-2,0), (1,0)$ and $(3,0)$. The graph will cross the y axis when $x = 0$. Substituting in this gives $y = (0-3)(0+2)(0-1)$ which in turn gives $y = (-3)(2)(-1)$ or $y = 6$. You can now plot the point $(0,6)$. Finally draw the sketch as shown below.

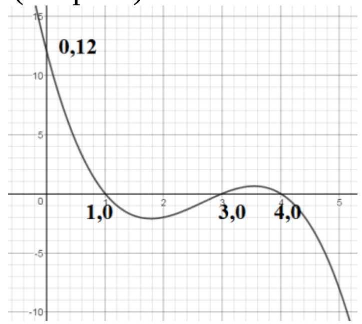


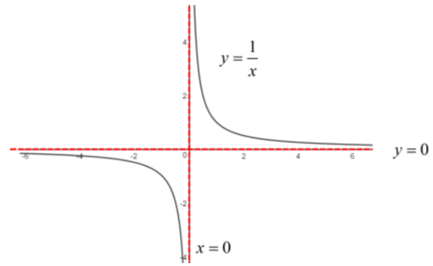
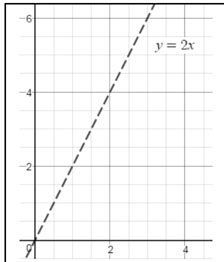
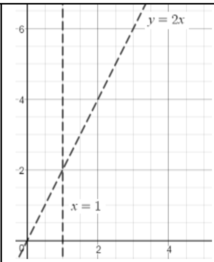
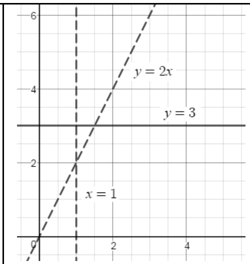
Some cubic equations have repeated roots. An example is $y = (x-3)(x+5)^2$.
The graph will touch the x axis at $(-5,0)$ and pass through at $(3,0)$

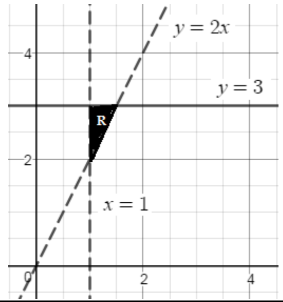
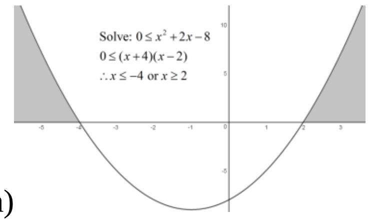
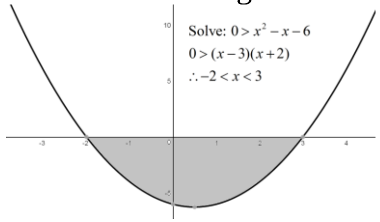
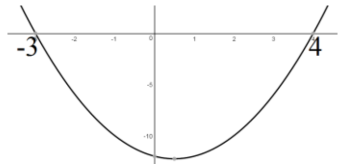
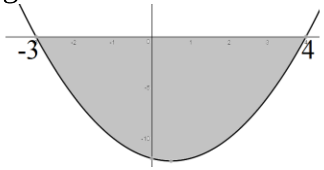
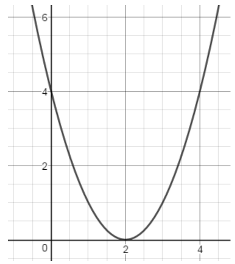
Example 1: Sketch the graph of $y = (x-5)(x-2)(x+1)$ showing any points of intersection with the coordinate axes.
Answer: The cubic is positive.
When $y = 0$ $x = -1$, $x = 2$ and $x = 5$. From here you can now plot the points $(-1,0), (2,0)$ and $(5,0)$.
When $x = 0$, $y = (0-5)(0-2)(0+1)$ which gives $y = 10$.
You can now plot the point $(0,10)$.
Finally sketch (not plot!) the curve.

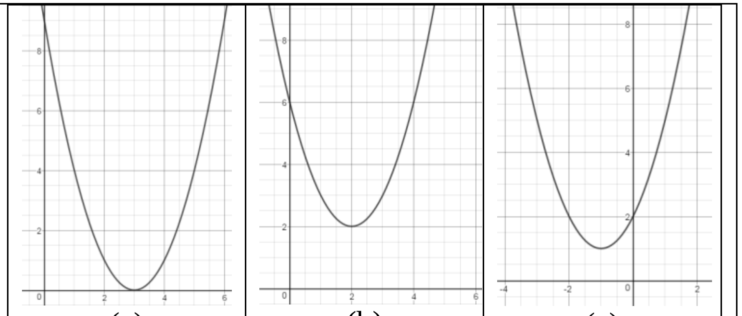
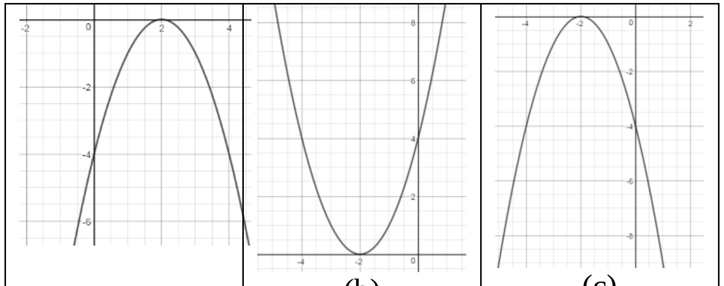
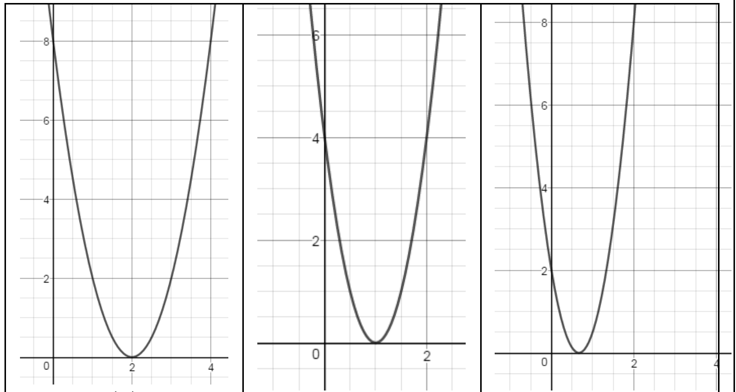


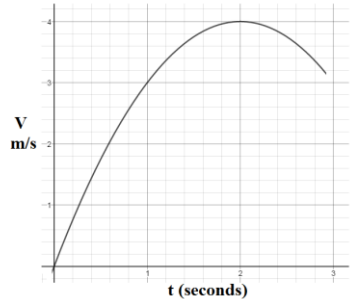
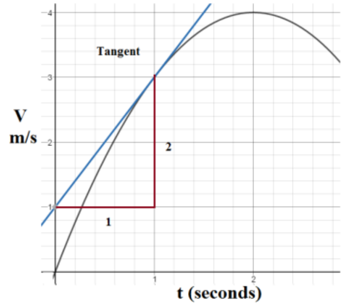
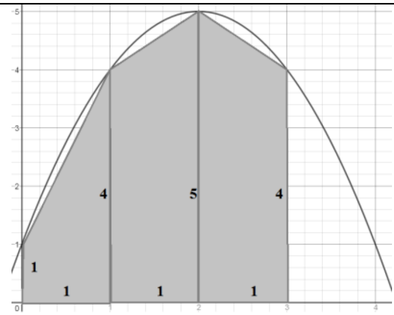
Example 2: Sketch the graph of $y = (3-x)(x-1)(x-4)$ showing any points of intersection with the coordinate axes.
Answer: The equation of cubic is negative.
When $y = 0$ $x = 3$, $x = 1$ and $x = 4$. From here you can now plot the points $(3,0), (1,0)$ and $(4,0)$.
When $x = 0$, $y = (3-0)(0-1)(0-4)$ which gives $y = 12$.
You can now plot the point $(0,12)$.
Finally sketch (not plot!) the curve.



Asymptotes	An asymptote will appear as a straight line on a graph. This broken line denotes the value(s) that the graph can never take. The asymptotes may be horizontal or vertical and the curve will approach this line but never meet or cross it. If you look at the graph $y=\frac{1}{x}$ for positive values of x, the lines $x=0$ (the y axis) and $y=0$ (the x axis) are asymptotes. As the value of x gets very large the graph will tend to 0 but never actually be zero. 1 divided by a large number is very small but will never 'disappear' As the value of x gets very small (tends to 0), the value of y becomes very large & eventually is undefined. $\frac{1}{0}$ is undefined. 1/1 = 1, 1/0.1 = 10, 1/0.01 = 100 etc	**Example:** Draw the asymptotes on the graph of $y=\frac{1}{x}$ shown below. **Answer:** The broken lines show the lines $x=0$ & $y=0$ 
Inequalities	$x>2$ "x is greater than 2" This just means the number must be bigger than 2 $x<3$ "x is less than 3" This just means the number must be smaller than 3 $x\geq 1$ "x is 1 or greater" This means the number can be **equal** to 1 **or** bigger $x\leq 6$ "x is 6 or less" This means the number can be **equal** to 6 **or** smaller $-3<x\leq 2$ "x is greater than -3 yet in turn equal to or less than 2" (-2,-1,0,1,2)	(a) State 3 **integers** that satisfy $x>4$ You could have (for example) 5, 9 & 73. (it can't be 4) (b) State 3 numbers that satisfy $x\leq 3$. 3, 2.4, -1.7 etc (c) State 3 integers that satisfy $-2<x\leq 4$ You could have (for example) -1, 3 & 4. (it can't be -2)
Set Notation for Solution Sets (Inequalities)	You can use set notation to represent inequalities as shown below. You could represent $-3<x\leq 2$ as $\{x:-3<x\leq 2\}$ You could represent $x<-5$ or $x>5$ as $\{x:x<-5\}\cup\{x:x>5\}$	$\{\ \}$ = The set of values x = For x : = such that $\cup$ = The union (and/or or both)
Solving Linear Inequalities	Use the same technique as you would for linear equations. Be careful! If × or ÷ the inequality by a negative number the inequality sign changes direction.	**E:** Solve $2x-1>7$. **A:** Add 1 to each side $2x>8$. Divide by sides by 2 to give the final answer $x>4$
Shading Regions (Linear Inequalities)	Shading inequalities allows us to find a 'region' (or set of points) that satisfy one or more linear inequalities (or constraints) given. An example might be to shade the region that satisfies both $x>5$ and $y>4$. All you need to do is draw the line of each equation given as decide which side of the line to shade. If a strict inequality is used (for example $x>2$) then you must draw a broken line. For examples such as $x\leq 6$ where 6 is included you must draw a solid line. Generally you will shade the region you **want** as the shaded area & label it 'R' You can use the method shown to the right and shade at the end. If you find it easier you can shade as you go. Different colours may help! The tricky thing is to shade the right region. For example, if you wanted to use the question to the right and you didn't know whether to shade above or below the line $y=2x$ you could test the point $(0,3)$ for example. Is 3 greater than 2 lots of 0?, yes it is. That means $(0,3)$ satisfies the inequality. This point is above the line so you would shade that area. When it comes to horizontal line ($y=1$ etc)	Shade the region that satisfies $y>2x$, $x>1$ and $y\leq 3$. Draw the line $y=2x$ using a broken line. Now add the line $x=1$ using a broken line. Finally add the line $y=3$ using a solid line. Once you have done this decide where to shade. The shading will be to the right of the line $x=1$, below the line $y=3$ and above the line $y=2x$. If you are unsure

	shading above the line is greater than and below the line is less than. For vertical lines ($x = 2$ etc) the area to the right is greater than and the area to the left is less than. When you have finished the part of the question on shading you may be asked to state/find all or some of the integer points that satisfy the inequalities. All you have to do is pick integer coordinates either inside the shaded region or on the solid lines enclosing it. **Do not** include the coordinates on any broken lines. *Do not leave these questions out just because you don't understand the equations of the straight lines! Draw a table of values, plot the points and draw a straight line through them. The technique is shown in the section on drawing straight line graphs.*	just test a point to check that it satisfies all 3 inequalities. This will give the final region shown below. 
<u>*Quadratic Inequalities*</u>	A quadratic inequality could be written in the form $ax^2 + bx + c \geq 0$. In order to find the set of points (or region) that satisfy the inequality you can factor the quadratic expression to find the critical values. The method of factoring used is the same as shown previously. Once you have factored the expression you will have the critical values. At this stage draw a little sketch of a parabola with the critical values and shade the required regions. If the expression is $> / \geq 0$ shade above the x axis (that's the line $y = 0$). If the expression is $< / \leq 0$ then shade below the x axis. Be careful with the final notation used for inequalities Here are some examples below and a worked solution to the right:  (a)  (b)	**Example:** Solve the inequality $x^2 - x - 12 < 0$ **Answer:** Factor the quadratic to give $(x+3)(x-4) < 0$ This gives us the critical values of $x = -3$ and $x = 4$ Draw a sketch with these values:  The required region is below the x axis  $\therefore$ The final answer is $-3 < x < 4$. *(3 & 4 are excluded)*
<u>*Graph Transformations (Translations)*</u>	If the graph of a function is translated it is simply moved. The graph doesn't change shape, 'size' or orientation (which was around it is). If you start with the graph of $y = \mathrm{f}(x)$ you can translate horizontally (left and right or in the x direction) or vertically (up and down or in the y direction) . **Horizontal Translations:** $y = \mathrm{f}(x - a)$ moves a units to the left. In vector form this would be a translation of $\begin{pmatrix} a \\ 0 \end{pmatrix}$. So, $\mathrm{f}(x-3)$ moves right by 3 and $\mathrm{f}(x+1)$ moves right by 1.	**Example:** The graph below shows part of the curve $y = \mathrm{f}(x)$  Sketch the graphs of: (a) $y = \mathrm{f}(x-1)$ (b) $\mathrm{f}(x) + 2$ (c) $y = \mathrm{f}(x+3) + 1$ **Answer: (next page)**

	Vertical Translations: $y = \mathrm{f}(x) + a$ moves up by a units. In vector form this would be a translation of $\begin{pmatrix} 0 \\ a \end{pmatrix}$. So, $\mathrm{f}(x)+5$ moves up by 5 and $\mathrm{f}(x)-4$ moves down by 4. Tip! If the number is on the outside, the y coordinates change. If the number is on the inside the x coordinates change! Look out for combined translations too!	 (a) (b) (c)
Graph Transformations (Reflections)	If the graph of a function is reflected, it will be mirrored in one of the coordinate axis. The shape doesn't change. If you start with the graph of $y = \mathrm{f}(x)$ you can reflect this in the x or the y axis. **Reflected in the y axis:** $y = \mathrm{f}(-x)$ is a reflection in the y axis. The y coordinates remain the same but the x coordinates become negative. **Reflected in the x axis:** $y = -\mathrm{f}(x)$ is a reflection in the x axis. The x coordinates remain the same but the y coordinates become negative. Tip! If the number is on the outside, the y coordinates change. If the number is on the inside the x coordinates change!	**Example:** Using the graph of $y = \mathrm{f}(x)$ in the section above, sketch the graphs of: (a) $y = -\mathrm{f}(x)$ (b) $y = \mathrm{f}(-x)$ (c) $y = \mathrm{f}(-x)$ **Answer:**  (a) (b) (c)
Graph Transformations (Stretches)	If the graph of a function is stretched its shape is changed. A graph can be stretched either in the y direction (vertically) or the x direction (horizontally). If you start with the graph of $y = \mathrm{f}(x)$ you can stretch this in the x or the y direction. **Stretched in the x direction:** $y = \mathrm{f}(ax)$ is a stretch, scale factor $\frac{1}{a}$ in the x direction. Simply divide the x coordinates by a. The y coordinates do not change. **Stretched in the y direction:** $y = a\,\mathrm{f}(x)$ is a stretch, scale factor a in the y direction. Simply multiply the y coordinates by a. The x coordinates do not change. Tip! If the number is on the outside, the y coordinates change. If the number is on the inside the x coordinates change!	**Example:** Using the graph of $y = \mathrm{f}(x)$ in the section above, sketch the graphs of: (a) $y = 2\,\mathrm{f}(x)$ (b) $y = \mathrm{f}(2x)$ (c) $y = 0.5\,\mathrm{f}(3x)$ **Answer:**  (a) (b) (c)

<u>Tangents to Curves (Estimating Gradients and rates of change)</u> *Chords (Gradients and Average rates of change)*	A tangent is a straight line that will touch a curve at a point. You may be asked to estimate the gradient of a tangent to a curve at a given point. You have seen in a previous section how to find the gradient of a straight line. And you are simply going to apply this to a given curve. Once the tangent has been drawn you can find the gradient by counting the squares up or down and left or right. In the example to the right you can see the tangent 'goes up 2' for every '2 it goes across to the right'. The gradient is therefore +2. Often you will be asked to state what this represents in the context of the question. In the example to the right, **the rate of change** of the velocity is the acceleration. If you have a distance/time graph the tangent will give an estimate for the velocity at a given point. This is the instantaneous rate of change. You may also be asked about the average rate of change. To do this, simply draw a chord between the two values given and estimate the gradient of the chord. This will be different from the instantaneous rate of change at a given point. A chord is a straight line that connects two sets of coordinates rather than touching the curve at a given point like a tangent. Chords and tangents on curves are the same as they are on the circles you will have met in maths!	**Example:** The graph shows the velocity of a particle. By drawing a tangent to the curve, estimate the rate at which V is increasing after 1 second  **Answer:** Drawing a tangent & estimating the gradient  You can see that the gradient from the graph is 2. This represents the acceleration which will be $2m/s^2$
<u>Area Under Curves</u>	You can **estimate** the area trapped under a curve by drawing a collection of trapezia with equal heights (the height will be the difference in the x coordinates). The more trapezia you draw, the more accurate the estimate becomes. (You will be told how many trapezia you need to draw). To start with, draw a chord (a straight line) between the x coordinates on the curve to make the top of the trapezia. Once these have been draw it's just a case of finding the area of each trapezium and adding them as shown to the right. In this example The area trapped under the curve for $0 \le x \le 3$ has been drawn with 3 trapezia. The first chord is drawn from the point on the curve where $x = 0$ and $y = 1$ to the point where $x = 1$ and $y = 4$. The second chord from where $x = 1$ and $y = 4$ to the point where $x = 2$ and $y = 5$. The final chord is drawn on the curve from the point where $x = 2$ and $y = 5$ and the point where $x = 3$ and $y = 4$.	 Area of the 1st trapezium: $\frac{1+4}{2} \times 1 = 2.5$ Area of the 2nd trapezium: $\frac{4+5}{2} \times 1 = 4.5$ Area of the 3rd trapezium: $\frac{4+5}{2} \times 1 = 4.5$ Total **estimated** area: 2.5 + 4.5 + 4.5 = 11.5

<table>
<tr>
<td>Sequences
(Number Patterns)</td>
<td>Look out for the Square Numbers
Look out for the Cube Numbers
Look out for the Fibonacci Sequence
Look out for Linear (arithmetic) sequences such as 4, 10, 16, 22…..
Look out Geometric sequences such as 2, 4, 8, 16, 32……</td>
<td>1, 4, 9, 16, 25, 36, 49…………
1, 8, 27, 64, 125, 216…….
0, 1, 1, 2, 3, 5, 8, 13, 21, 34...(add the 2 previous terms)
Increases or decreases by fixed amount. (+ or –)
Has a common ratio or constant multiplier (× or ÷)</td>
</tr>
<tr>
<td>Sequence Notation
& Finding Terms</td>
<td>A sequence may be written in the form $u_n =$. This just means the nth term of a sequence. The values of n generally start at 1. u_1 is the 1st term, u_2 the 2nd ….</td>
<td>Example: $u_n = 3n+1$. Find u_1, u_2 and u_3. Answer:
When $n=1$, $u_1 = 4$, $n=2 \therefore u_2 = 7$, $n=3 \therefore u_3 = 10$</td>
</tr>
<tr>
<td>nth Term Formula of a Linear (Arithmetic) Sequence</td>
<td>A linear or arithmetic sequence will increase or decrease by a fixed amount.
This might be 'going up by 4 each time' or 'going down by 2 each time'
To find the nth term formula of a linear (arithmetic) sequence:
(1) Find the difference. (What’s it increasing or decreasing by each time?)
(2) Multiply this number by n (Be careful with negatives)
(3) Use values of n in the table starting with $n=1$ an substitute in.
(4) Find what number you need to add or subtract to find t. Test it for all terms
This method only works if the sequence is linear!</td>
<td>Example: Find the nth term for: 3, 7, 11, 15
Answer: Create a little table with the term in:
<table>
<tr><td>n</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>t</td><td>3</td><td>7</td><td>11</td><td>15</td></tr>
</table>
The sequence is increasing by 4 each time so start by writing $4n$. Now take $n=1$ so $4n$ is 4 × 1 = 4. You need to subtract 1 to get 3 which is the required term. The nth term for the sequence is therefore $4n-1$.</td>
</tr>
<tr>
<td>Geometric Sequences</td>
<td>Geometric sequences have a common ratio (constant multiplier), or if you like the terms in the sequence increase or decrease by being multiplied by the same number each time. 3, 6, 12, 24... is a geometric sequence with first term 3 and a common ratio of 2. Each term is being multiplied by 2 to get the next. You can find the common ratio by dividing a term by the previous one. 6/3 = 2, 12/6 = 2</td>
<td>Example: Find the next 3 terms in the sequence: 4, 12, 36, 108....
Answer: The constant multiplier is 3. Simply multiply 108 by 3 Giving 324 and multiply by 3 again to give 972.</td>
</tr>
<tr>
<td>nth Term of a Quadratic Sequence</td>
<td>Quadratic sequences can be written in the form $u_n = an^2 + bn + c$.
An example of a quadratic sequence is 5, 18, 37, 62, 93….
To find the values of a, b and c you can use the technique shown below:
(i) Find the first and then the second difference between the terms:
First difference: 13, 19, 25, 31…. and Second difference: 6, 6, 6…..
(ii) Half the second difference and multiply by n^2. This gives us a. So, $3n^2$
(iii) Now set up a table to find the value of $3n^2$ for each value of n.
<table>
<tr><td>n</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>t</td><td>5</td><td>18</td><td>37</td><td>62</td><td>93</td></tr>
<tr><td>$3n^2$</td><td>3</td><td>12</td><td>27</td><td>48</td><td>75</td></tr>
</table>
(iv) Now find the linear part of the sequence $bn+c$ subtract the quadratic part from the sequence:
<table>
<tr><td>$t-3n^2$</td><td>2</td><td>6</td><td>10</td><td>14</td><td>18</td></tr>
</table>
(v) At this stage you simply need to find the nth term of a linear sequence which will give $4n-2$ using the values 2, 6, 10, 14 & 18 from the table above.
The final answer is therefore $u_n = 3n^2 + 4n - 2$</td>
<td>Example: Find a formula for the nth term of the quadratic sequence: 4, 7, 14, 25, 40….
Answer: The first difference: 3, 7, 11, 15 and the second difference: 4, 4, 4.. This gives us $a=2$ as you have to half 4 to get the quadratic part of the sequence.
<table>
<tr><td>n</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>t</td><td>4</td><td>7</td><td>14</td><td>25</td><td>40</td></tr>
<tr><td>$2n^2$</td><td>2</td><td>8</td><td>18</td><td>32</td><td>50</td></tr>
</table>
Now find the linear part of the sequence $bn+c$ subtract the quadratic part from the sequence:
<table>
<tr><td>$t-2n^2$</td><td>2</td><td>-1</td><td>-4</td><td>-7</td><td>-10</td></tr>
</table>
You just need to find the nth term of the linear sequence 2, -1, -4, -7, -10.
This is decreasing by 3 each time giving us the $bn+c$ part as $-3n+5$. Therefore the final answer is $u_n = 2n^2 - 3n + 5$. Now check it works with values!</td>
</tr>
</table>

Fibonacci Like Sequence	The original Fibonacci Sequence is: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55… It starts with 0 and forms terms by **adding** the **two numbers** before it.	**Example:** Find the next 3 terms: 4, 10, 14.... **Answer:** 24, 38, 62 (Just add the previous two terms)
Algebraic Fractions (Simplifying)	Look for the highest common factor of numbers and algebraic terms. The rules of indices and factoring expressions will help you simplify algebraic fractions.	**Example:** $\frac{x^2+x-6}{2x-4}=\frac{(x+3)(x-2)}{2(x-2)}=\frac{(x+3)}{2}$
Algebraic Fractions (Multiplying)	The main thing to remember is to cancel as many common factors as you can. The question you get may or may not require some factoring. Any easier example could be "Simplify fully $\frac{8x^4}{y^3}\times\frac{x^2y^5}{4}$". Let's start with the 8 divided by 4. This will cancel to give $\frac{2x^4}{y^3}\times\frac{x^2y^5}{1}$. Now you can combine the powers of x and write the single fraction $\frac{2x^6y^5}{y^3}$. Lastly, simply have to cancel the terms in y to give a final answer of $2x^6y^2$. A harder example may need factoring such as $\frac{x^2-x-6}{4x+8}\times\frac{x+5}{x^2-9}$. You need to factor as much as you can to give: $\frac{(x-3)(x+2)}{4(x+2)}\times\frac{(x+5)}{(x+3)(x-3)}$. Now you can cancel the common factors of $(x+2)$ & $(x-3)$ to give $\frac{1}{4}\times\frac{(x+5)}{(x+3)}$. This can be tidied, giving a final answer of $\frac{(x+5)}{4(x+3)}$ in its simplest form.	**Example 1:** Simplify fully $\frac{3p^7}{q^3r}\times\frac{4r^2}{15pq}$ **Answer:** $\frac{4p^6r}{5q^4}$ **Example 2:** Simplify fully $\frac{x^2+2x-8}{2x-6}\times\frac{x^2+x-12}{5x-10}$ **Answer:** Factor first: $\frac{(x+4)(x-2)}{2(x-3)}\times\frac{(x+4)(x-3)}{5(x-2)}$ Now cancel common factors: $\frac{(x+4)}{2}\times\frac{(x+4)}{5}$ Finally, simplify to give $\frac{(x+4)^2}{10}$. **Example 3:** Simplify fully $\frac{2x^2-11x+12}{x^2-25}\times\frac{(x+5)^2}{2x-8}$ **Answer:** Factor $\frac{(2x-3)(x-4)}{(x+5)(x-5)}\times\frac{(x+5)(x+5)}{2(x-4)}$ Cancel common factors and simplify $\frac{(2x-3)(x+5)}{2(x-5)}$ *Try and leave the final answer factored where you can.*
Algebraic Fractions (Dividing)	The technique is similar to multiplying. To start with though you must invert the second fraction and multiply. At this stage you can start cancelling down! The example $\frac{6p^5}{q^2r}\div\frac{3r^4}{p^3q^5}$ could be written as $\frac{6p^5}{q^2r}\times\frac{p^3q^5}{3r^4}$. At this stage you would use the techniques shown above in multiplying algebraic fractions. This would give a final answer of $2p^8q^3r^5$ in its simplest form. With any algebraic fraction, look out for the difference of two squares or expressions that can be factored. An example could be $25x^2-9\equiv(5x+3)(5x-3)$. This may allow you to cancel a factor that was not originally obvious. Factoring can also help reveal common factors. An example could be $10x-15\equiv5(2x-3)$. If you have the factor $(2x-3)$ somewhere else in the fraction you could look to simplify.	**Example:** "Simplify fully $\frac{4p}{5q^2}\div\frac{2p^2r}{15q^3}$" **Answer:** $\frac{4p}{5q^2}\times\frac{15q^3}{2p^2r}=\frac{2}{1}\times\frac{3q}{pr}=\frac{6q}{pr}$ **Example 2:** "Simplify fully $\frac{(p-1)}{q^2-q-6}\div\frac{p^2-1}{2q-6}$" **Answer:** $\frac{2}{(p+1)(q+2)}$ (Look out for the difference of 2 squares as above)

<table>
<tr>
<td><u>Algebraic Fractions (Adding and Subtracting)+Solving Equations with Fractions</u></td>
<td>Like with normal fractions you must have a common denominator This is simply the LCM of the expressions or terms in the denominators of each of the fractions. Let’s look at an example: “Simplify $\frac{2x}{y^3}-\frac{5y}{3z}$”. The common denominator here would be the product of the dominators which can be written as $3y^3z$. At this stage you set up a single fraction and multiply $2x$ by $3z$ and $5y$ by y^3. This will give $\frac{(2x)(3z)-(5y)(y^3)}{3y^3z}$ which will simplify to $\frac{6xz-5y^4}{3y^3z}$.
Another example could be: “Simplify $\frac{2}{(x-1)}+\frac{7}{(x+4)}$”. The common denominator will be $(x-1)(x+4)$. You can now form a single fraction starting with a single fraction $\frac{\quad}{(x-1)(x+4)}$. At this stage you need to multiply the 2 by $(x+4)$ and the 7 by $(x-1)$. This will give $\frac{2(x+4)+7(x-1)}{(x-1)(x+4)}$. Now expand the brackets and simplify: $\frac{2x+8+7x-7}{(x-1)(x+4)}$. This will simplify to $\frac{9x-1}{(x-1)(x+4)}$.
The common denominator of x and x^2 is x^2. Try it with $x=3$. You have 3 & 9.</td>
<td>Example 1: “Simplify fully $\frac{4p}{5q^2}+\frac{2r}{3}$”
Answer: $\frac{4p(3)+2r(5q^2)}{15q^2}=\frac{12p+10q^2r}{15q^2}$

Example 2: “Simplify fully $\frac{5}{(x-2)}-\frac{4}{(x-3)}$”
Answer: $\frac{5(x-3)-4(x-2)}{(x-2)(x-3)}=\frac{x-7}{(x-2)(x-3)}$

Example 3: “Simplify fully $\frac{p}{p^2-9}+\frac{p-4}{p-3}$”
Answer: $\frac{p}{(p+3)(p-3)}+\frac{p-4}{(p-3)}=\frac{p+(p+3)(p-4)}{(p+3)(p-3)}$
which gives $\frac{p^2-12}{p^2-9}$ in simplified form.
With Example 3 the difference of 2 squares was used to factor the denominator.</td>
</tr>
<tr>
<td><u>Algebraic Proof</u></td>
<td>A proof is an argument to justify a mathematical statement. When writing a proof you must show that the statement holds true for all cases not just select certain values and conclude it must be true for all values. The way to do this is to write out and manipulate algebraic expressions and identities to form your proof. Let's start with some basic expressions for numbers
<table>
<tr><td>n</td><td>$2n$</td><td>$2n+1$ or $2n-1$</td><td>$2n+2$</td><td>$2n+3$</td></tr>
<tr><td>an integer</td><td>an even integer</td><td>an odd integer</td><td>the next even integer after $2n$</td><td>the next odd integer after $2n+1$</td></tr>
</table>
Using expressions like those above to set up the expression. Expanding brackets, simplifying and refactoring is usually used to show the proof. You must include a concluding statement to end the proof. Examples are shown to the right. Simply showing isolated cases hold true by using numbers does not prove a statement is true for all values. You will not be awarded marks for doing this. The only time you can substitute numbers in is to show that a proof is not true with a counter example. You may be asked to do this.</td>
<td>Exam 1: Show that the difference between the squares of 2 consecutive odd integers is always a multiple of 8.
Answer: Let the first of the 2 numbers be $2n-1$ & the second $2n+1$. Square each to give $(2n+1)^2$ & $(2n-1)^2$.
Difference means subtract $\therefore$ $(2n+1)^2-(2n-1)^2$.
Expand brackets to give $4n^2+4n+1-(4n^2-4n+1)$.
Simplify to $8n$ by cancelling the terms. Conclude with the statement "$8n$ is a multiple of 8 $\therefore$ true for all consecutive odd integers."
Example 2: Show that product of any two odd numbers is always odd.
Answer: Let the first number be $2n-1$ and the second $2n+1$. Multiplying: $(2n-1)(2n+1)=4n^2-1$
$4n^2$ is always even as it’s a multiple of 4 $\therefore 4n^2-1$ is odd for all values of n.</td>
</tr>
</table>

Functions (Evaluating)	A function is just a rule that maps one number to another. A function will have an input (such as x) and an output (such as y). An example could be $y=2x$. Instead of writing $y=2x$ you could use function notation and write $\mathrm{f}(x)=2x$. $\mathrm{f}(x)$ just means "y is a function of x". You can evaluate functions by substituting numbers in. If $\mathrm{f}(x)=2x$ you can say $\mathrm{f}(5)=2(5)$ which of course means $\mathrm{f}(5)=10$. Evaluate simply tells you to swap x for the number(s) given. You can work backwards and find an input for functions given an output. An example using $\mathrm{f}(x)=2x$ could be: Find the value of a such that $\mathrm{f}(a)=14$. All you need to do is substitute $x=a$ in and solve: $2a=14$ which of course gives $a=7$.	**Example 1:** $\mathrm{f}(x)=3x-1$. Find (or evaluate) (a) $\mathrm{f}(5)$, (b) $\mathrm{f}(-1)$ and (c) $\mathrm{f}(p)$. **Answer:** (a) 14, (b) -4 and (c) $3p-1$ **Example 1:** $\mathrm{g}(x)=1-4x$. Given that $\mathrm{g}(t)=15$, find the value of t. **Answer:** Don't worry about it being $\mathrm{g}(x)$! Simply substitute in to give $15=1-4t$. Solving for t, $4t=-14$ and $t=-\dfrac{7}{2}$
Functions (Composite)	A composite function simply requires you to substitute one function into another. If you have two functions, for example, $\mathrm{f}(x)=x^2-1$ and $\mathrm{g}(x)=3x$ you can form the composite functions $\mathrm{fg}(x)$ or $\mathrm{gf}(x)$. (You could have $\mathrm{ff}(x)$ if you liked). $\mathrm{fg}(x)$ means "do g first and then f" whereas $\mathrm{gf}(x)$ means "do f first and then g". An example could be: Find (a) $\mathrm{fg}(2)$ and (b) $\mathrm{gf}(2)$. Answer: (a) Start with $\mathrm{g}(2)$. Using the function, $\mathrm{g}(2)=6$. Now do $\mathrm{f}(6)$. This will give $\mathrm{f}(6)=35$. For part (b) start with $\mathrm{f}(2)$. This gives $\mathrm{f}(2)=3$. Now do $\mathrm{g}(3)=9$.	**Example 1:** Using the functions $\mathrm{f}(x)=1-x$ and $\mathrm{g}(x)=x^2$ find: (a) $\mathrm{gf}(5)$ (b) $\mathrm{fg}(1)$ (c) $\mathrm{ff}(p)$ **Answer:** (a) $\mathrm{gf}(5)$ (b) $\mathrm{fg}(1)=0$ (c) $\mathrm{ff}(p)=(1-p)^2$ **Example 2:** $\mathrm{f}(x)=x+2$, $\mathrm{g}(x)=\dfrac{1}{x}$ and $\mathrm{h}(x)=x^3$. Find $\mathrm{fhg}(0.5)$ **Answer:** $\mathrm{fhg}(0.5)=10$
Functions (Inverse)	The inverse function $\mathrm{f}^{-1}(x)$ undoes the effect of the original function $\mathrm{f}(x)$. To find the inverse function of a function, write the equation out as $y=\mathrm{f}(x)$ swap the x's and y's over in the original function and then set about making y the subject by rearranging. An example could be $\mathrm{f}(x)=\sqrt{1+x^3}$. Write $y=\sqrt{1+x^3}$, now swap to give $x=\sqrt{1+y^3}$. Now rearrange to give $y=\sqrt[3]{x^2-1}$. At this stage you need to write $\mathrm{f}^{-1}(x)=\sqrt[3]{x^2-1}$. It's important you give your final answer in the form $\mathrm{f}^{-1}(x)$	**Example:** $\mathrm{f}(x)=(1-2x)^5$ Find the inverse $\mathrm{f}^{-1}(x)$. **Answer:** Let $y=(1-2x)^5$. Now swap the x's and y's over to give $x=(1-2y)^5$. Rearrange to give $y=\dfrac{1-\sqrt[5]{x}}{2}$. The final answer will be $\mathrm{f}^{-1}(x)=\dfrac{1-\sqrt[5]{x}}{2}$ *A function can only have an inverse if it's 1-2-1 for the set of values you are considering. You will study this later on.*

Using a Calculator

Using a Casio Calculator	Using a calculator effectively can really help in exams. Some basic tips: Make sure you are in Degrees and Math mode. The letter D will be at the top of the screen (Shift mode 3 gets you there) and the word Math. Find the $S \Leftrightarrow D$ button to convert from a fraction to a decimal. Shift followed by $\times 10^x = \pi$. Hit $\circ$, ,, & it will convert the answer into hours, minutes & seconds		D and Math are circled to the right!

Statistics

Mean (Basic Average)

Add **all** of the values (including 0s), divide by how many values there are.
Tip! You might need to work this backwards to find missing values in the data.

The mean of 3,4,7,6,0,4,6 is $\frac{3+4+7+6+0+4+6}{7} = 5$

Mean from a Table (Estimated and Actual)

When **grouped** data is used we get an **estimated** mean average.
(i) Find the midpoint of each class. (Shown in the 3rd column below)
(ii) Multiply Frequency by Midpoint. (Shown in the 4th column below)
(iii) Add these values up. (Shown in the Total box on the right)
(iii) Divided that total (**450**) by the sum of the frequency (**24**). (Bottom of 2nd)

Height in cm	Frequency	Midpoint	F × M
$0 < h \leq 10$	8	5	8×5=40
$10 < h \leq 30$	10	20	10×20=200
$30 < h \leq 40$	6	35	6×35=210
Total	**24**	Ignore!	**450**

Estimated Mean height: **450** ÷ **24** = 18.75cm

If the data is **not grouped** we get an **actual** mean.
(i) Multiply the frequency by given values.
(ii) Add all of these values up.
(iii) Divided this (**16**) by the sum of the frequency (**13**).

Goals in Game	Frequency	F × Goals
0	3	3×0=0
1	5	5×1=5
2	4	4×2=8
3	1	1×3=3
Total	**13**	**16**

Actual Mean goals scored per game: 16 ÷ 13 = 1.23

Median Value (another average)

The **middle** value. Put the list of numbers in order and find the middle one.
If there are two numbers in the middle find the number half way between.

Find the median: 4,5,2,3,6,7,6
in order 2,3,4,**5**,6,6,7. The Median = 5

Mode / Modal #

The number/class/item that appear most times in a list. **'Most Frequent'**

4,5,2,3,6,**4**,7,8,**4**, **Mode** = 4. Check for multiple modes!

Range

The highest value **subtract** the **lowest** value. (A measure of spread)

3, 31, 26, 102, 37, 97, 4 **Range:** 102 – 3 = 99

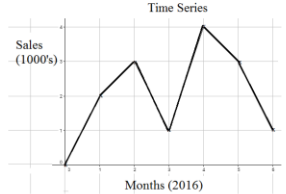
Pie Charts

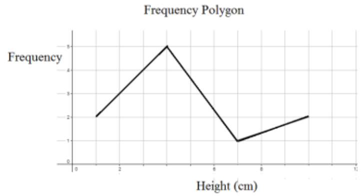
A pie chart is a circle which means there are 360°. If you are drawing one divide 360 by the total frequency. This will tell you how many degrees to use for each person or item in your table. Use a 360 degree protractor to draw one! Look out for right angles! ¼ = 90º =25% of the data. 120º is a third of the data!

If there are 40 people in a survey then each person will be worth 9° of the pie chart as 360/40 = 9. Just multiply each number in the table by 9 to work out the total degrees for each section of the table.

Time Series Graphs and Frequency Polygons and Diagrams

Time Series Graphs simply shows data collected over time. Time is plotted on the horizontal axis. Points are plotted and straight lines connect the points. These look similar to frequency polygons but are very different.
Frequency Polygons are similar to histograms and show the **frequency** for **grouped data**. They are plotted at midpoints of the class. Connect with lines.





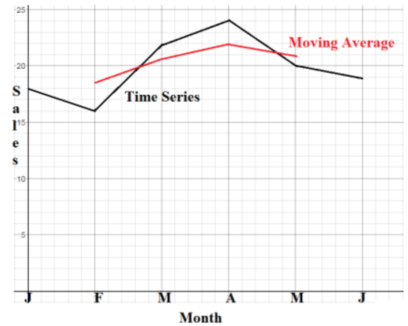
Moving Averages

A moving average gives a summary and trend for the data over time.
They are plotted at the midpoint of the 3 months (as per example)

Month	**Jan**	**Feb**	**Mar**	**Apr**	**May**	**June**
Sales	18	16	22	24	20	19

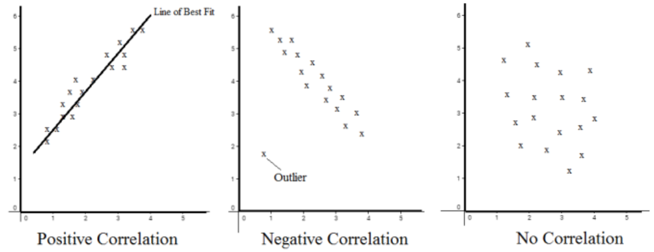
When calculating drop the first value and add the next value

Months	Jan, Feb & Mar	Feb, Mar & Apr	Mar, Apr & May	Apr, May & Jun
3 Point Moving Average	$\frac{18+16+22}{3}=18.7$	$\frac{16+22+24}{3}=20.7$	$\frac{22+24+20}{3}=22$	$\frac{24+20+19}{3}=21$



Scatter Graphs and Correlation (and Line of Best Fit)

Scatter graphs plot data in pairs (bivariate). This might be the temperature and ice cream sales or the age of a car and the value of the car.
Positive Correlation: As one value **increases**, the other **increases**.
Negative Correlation: As one value **increases** the other **decreases**.
No Correlation: There is **no linear relationship** between the two.
If you are asked to find estimates from a scatter graph you must draw a line of best fit and read up and across from it.



Line of Best Fit $\bar{x}, \bar{y}$

The line of best fit passes through $\bar{x}, \bar{y}$ where $\bar{x}$ = mean of x & $\bar{y}$ = mean of y

Find $\bar{x}, \bar{y}$ and draw the line of best fit through this point.

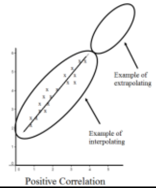
Outliers

Points on the scatter graph that **don't** follow the pattern of the other points..

An example is shown on the second graph above

Interpolation and Extrapolation

Interpolation is using the line of best fit to estimates values **within** the data set. **Extrapolation** is using the line of best fit to estimates values **outside** the data set. You must be careful when extrapolating as the estimate may not be accurate The points will either be wrongly collected or anomalies. The diagram to the right shows an example of a region where you could interpolate & extrapolate.



Pictograms

Pictograms are a convenient visual way of representing data. They are similar to bar charts. Make sure you have a key (as shown to the right). You can use half a picture but don't try and do ¼ s of thirds! Some questions will require you to work backwards and find missing values instead of drawing them. Use the key to help you with this.

Example: The pictogram below shows 10 black 12 red, 2 green & 16 others for the colours of car surveyed

Standard and Back to Back Stem and Leaf Diagrams

Standard

0	1, 4, 8
1	3, 7, 7, 9
2	2, 5, 8,
3	0, 1, 4, 6, 6, 9

Key 2|5 = 25

The study of 1 data set.

Back to Back

Key 2|0 = 20 for Boys

Boys		Girls
9, 8, 2	0	1, 4, 8
5, 3, 1	1	3, 7, 7, 9
2, 0	2	2, 5, 8,
4	3	0, 1, 4, 6, 6, 9

Key 2|5= 25 for Girls

The study **and** comparison of 2 data sets.

Example: Use the back to back stem and leaf diagram to the left to find the median score and range for the boys. **Answer:** Median = 13 and Range = 34-2=32.
Tip! Always remember your key and note how the key differs on either side of the back to back example.

Two Way Tables

Two way tables allow us to model situations where there are two variables involved. In the example to the right there is gender **and** whether the person is left or right handed. Just fill out the information step by step using the values given either in the table or the question and make sure **all** of the totals add up for each row **and** column! Often one value is given in the question. Check this as you may think you are missing some information.
You may be asked to work out some questions on probability or fractions from the table. Make sure you read the question correctly!
A question might be "One person is chosen at random. What is the probability that the person is left handed girl?" You would simply find the number of left handed girls (which is 6) and divide that by the total (which is 100). The probability would be 6/100 or you could simplify the fraction to give 3/50.
Another could be "What fraction of the boys are RH?" answer 48/58 or 24/29

Question: Complete the 2 way table below.

	Left Handed	Right Handed	**Total**
Boys	10		58
Girls			
Total		84	100

Answer: Step 1, fill out the easy parts (the totals)

	Left Handed	Right Handed	**Total**
Boys	10	*48*	58
Girls			*42*
Total	*16*	84	100

Step 2, fill out the remaining parts

	Left Handed	Right Handed	**Total**
Boys	10	*48*	58
Girls	*6*	*36*	*42*
Total	*16*	84	100

Now check each row and column adds up correctly!

Sampling (Definitions)

A **Population** is a set of items of interest.
A **Census** is a survey that covers the entire population.
A **Sample** is a small selection of items from the population. (10% + is ideal)
In a **Random Sample** each item has the same chance of being chosen.
A sample is **Biased** if individuals or groups from the population are not represented in the sample.

Example: A random sample could be picking the names of 10 people out of a hat that has 100 names in. Each person has an equally likely chance of being picked.

Simple Random Samples

In a random sample each item has an equal chance of being chosen. This may be using a table of random numbers

Example: This could be picking the names of 10 people out of a hat from a class of 30 students.

Stratified Sampling

When a stratified sample is taken it's proportional to the population it comes from. Each strata (or layer) will provide a number of items for the sample based in its size relative to the overall population.
To find out how many items from each strata are needed, simply multiply the number in the strata by the (sample size ÷ population).
When you have a value, write it in your answer and round it up or down if necessary. Make sure when you round your values you end up with the correct number in the sample!
In the example to the right the first and the last numbers have been rounded down and the middle number rounded up. This gives the 70 required. Use logic if you need to round.

$$Number\ in\ smaple = Number\ in\ layer \times \frac{Sample\ size}{Population}$$

Or

$$Number\ in\ smaple = \frac{Number\ in\ layer}{Population} \times Sample\ size$$

These two calculations are essentially the same thing.

Example: A sample stratified by year group of 70 is taken. Find out how many students from each year should be in the sample:

Year	Number of Students
Year 7	210
Year 8	230
Year 9	190

Answer: You need 70 in the sample out of a total of 630 students. ∴ multiply each strata value by $\frac{70}{630}$

Year	# of Students	Calculation	# in Sample
Year 7	210	$210 \times \frac{70}{630} = 23.33..$	23
Year 8	230	$230 \times \frac{70}{630} = 25.55..$	26
Year 9	190	$190 \times \frac{70}{630} = 21.11..$	21

Cumulative Frequency Table

Cumulative frequency is a running total. Study the frequency table below.

Age	Frequency
$0 < a \le 10$	15
$10 < a \le 40$	35
$40 < a \le 50$	10

You can add a 3rd column to show the running total (cumulative frequency).

Age	Frequency	Cumulative Frequency
$0 < a \le 10$	15	15
$10 < a \le 40$	35	15 + 35 = 50
$40 < a \le 50$	10	50 + 10 = 60

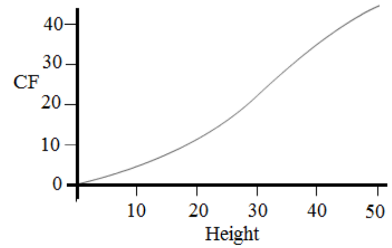
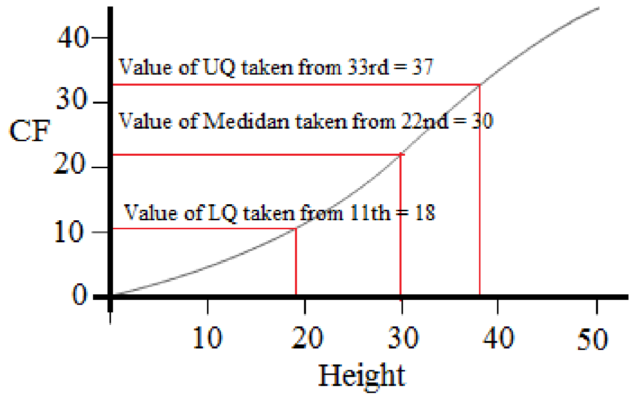
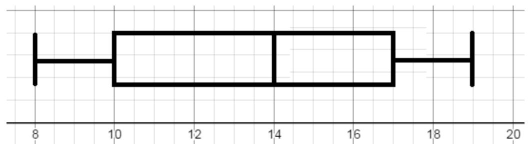
The CF shows there are 15 people aged 0-10, 50 people aged 0-40 and 60 people aged 0-50. Simply add the totals before to get the cumulative frequency.

Example: Complete the cumulative frequency table.

Height	Frequency	Cumulative Frequency
$0 < h \le 5$	4	
$5 < h \le 20$	8	
$20 < h \le 30$	9	
$30 < h \le 50$	23	

Answer:

Height	Frequency	Cumulative Frequency
$0 < h \le 5$	4	4
$5 < h \le 20$	8	12
$20 < h \le 30$	9	21
$30 < h \le 50$	23	44

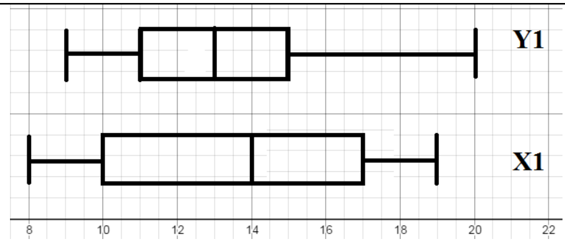
Cumulative Frequency Graph	Using a completed cumulative frequency table (as shown previously) allows you to draw a cumulative frequency **curve**. The points are plotted at the end of the class interval (upper class boundary) and a sweeping curve is drawn through the points as show to the right. The class interval is drawn on the horizontal axis and the cumulative frequency on the vertical axis. Make sure you label each axis correctly.	
Lower Quartile (Q1), Median (Q2), Upper Quartile (Q3) and IQR *(See previous and next video)*	**Lower Quartile (Q1):** 25% of the values in the data set are less than the value of the LQ. To find **an estimated** value ÷ the data set (44 here) by 4 and read across and down on the graph as shown to the right. (In the case shown it will be from the 11th value). The LQ is **not** 11. It's ~18-19 in the case to the right! **Median (Q2):** 50% of the values in the data set are less than the value of the median. To **an estimated** value ÷ the data set (44 here) by 2 and read across and down on the graph as shown to the right. (In the case shown it will be from the 22nd value). The median is **not** 22. It's about 30 in the case to the right! **Upper Quartile (Q3):** 75% of the values in the data set are less than the value of the UQ. To find **an estimated** value ÷ the data set by 4 and × 3 and read across and down on the graph as shown to the right. (In the case shown it will be from the 33rd value). The UQ is **not** 33. It's about 38 in the case shown! **Interquartile Range IQR:** represents the 'middle 50% of the data set. The IQR is found by calculating the Upper Quartile - Lower Quartile (Q3-Q1). In the case shown an estimated will be 37-18 = 19. This shows that 50% of the items in the data set were within a range of 19 units.	 *A box plot could be drawn off the bottom of this CF curve using the lines from the LQ, M and UQ* ***but*** *you will need the lowest and highest values too.*
Drawing Box Plots	A box plot is a convenient, visual way of representing the 5 main summary statistics. The **lowest value**, the **lower quartile**, the **median**, the **upper quartile** and the **highest** value are shown on a box plot. A box plot can be drawn independently or from a cumulative frequency curve **if** you are also given a maximum and minimum value either in the question or you can work it out. The first vertical line is the lowest value, the 2nd the LQ, the 3rd is the median, the 4th is the UQ and the final vertical line is the highest value in the data set. The 'box' represents the middle 50% of the data. The range can be found by calculating the higher value – lowest value. The Range and IQR are measures of spread and the median is an average.	**Example:** The students in class X1 sat maths test. The test was out of 25. The highest score was 19, the lowest score was 8, the median score was 14, the lower quartile was 10 and the upper quartile was 17. Draw a box plot to represent the information. **Answer:** 
Comparing Box Plots	Box plots allow the reader to way of compare data sets. When two or more box plots are drawn on the **same** scale or set of axis they can be compared and commented on.	**Example:** The students in class Y1 also sat the maths test. Compare the results of the two classes using the box plots below.

The two areas to focus when comparing box plots are:

(1) The **average** using the **median**.
(2) The **spread** of the data using the IQR (and range to a lesser extent).

The higher the median value the higher the average is. The lower the median the lower the average is.
The smaller the IQR the more consistent the observations are, the wider the IQR the less consistent they are. Small box = consistent, Large box = less consistent.

When comparing box plots you must compare them **in context**. You cannot simply say "The median is higher" you need to use this in context. Talk about scores, heights, minutes etc.



Answer: You need to give at least two comparisons in context. Focus on the median to begin with. X1 have a higher Median than Y1. A statement could be "On average the students in X1 were more successful in the test than in Y1 as their median score was higher by 1 mark". For the second comparison you should look at the spread of data using the IQR. A statement could be "The students in Y1 were more consistent than X1 in terms of their score as the IQR was 4 vs 7"
You could also make reference to the highest and lowest scores, the range, 25% and 75% of the data but the median and IQR are the main choices.

Drawing Histograms

Histograms look similar to bar charts. The 3 main differences between them are (1) The **area** of the bar is **proportional** to the frequency of that class. (2) The grouped continuous data is in classes of unequal widths. (3) Histograms show **frequency density** and not frequency on the vertical axis.
To draw a histogram You need the frequency density. Frequency Density is calculated by dividing the **frequency** by the **class width** (size or interval).

You can say $FD = \frac{F}{CW}$.

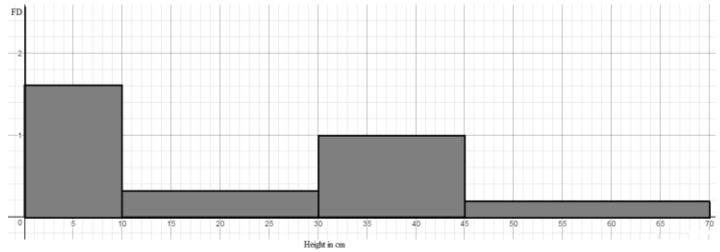
(The classes (height) below have widths of 10,20,15 and 25)
Histograms are drawn from continuous grouped data like the following table.

Height(cm)	Frequency
$0 < h \le 10$	8
$10 < h \le 30$	6
$30 < h \le 45$	15
$45 < h \le 70$	5

In the example to the right, we will draw a histogram for the data above. When drawing the histogram, make sure frequency density (NOT FREQUENCY) goes on the vertical axis and the height (or any continuous measure) goes on the horizontal axis. Make sure you use a good scale for the histogram and draw them accurately. The bars **must** be proportional to the frequency.

Example: Draw a histogram using the previous table.
Answer: You need to add two columns and find the frequency density.

Height	Frequency (F)	Class Width (CW)	Frequency Density (FD)
$0 < h \le 10$	8	5	$8 \div 5 = 1.6$
$10 < h \le 30$	6	20	$6 \div 20 = 0.3$
$30 < h \le 45$	15	15	$15 \div 15 = 1$
$45 < h \le 70$	5	25	$5 \div 25 = 0.2$

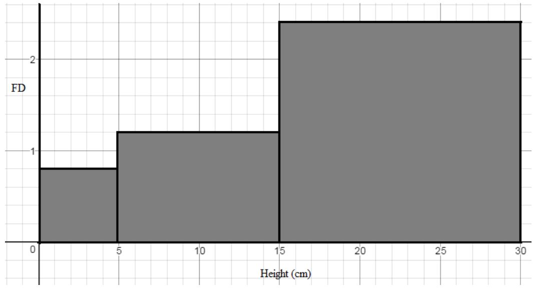


Frequency density always goes on the vertical axis and the classes on the horizontal axis.

<table>
<tr>
<td><u>Interpreting Histograms</u></td>
<td>You may be asked to interpret a histogram instead of drawing one. You may be asked to find out information about the frequency or the class widths from a histogram that has been drawn.
You can simply rearrange the formula given in the previous section to find either the frequency or the class width.

<table>
<tr><th>Frequency Density</th><th>Frequency</th><th>Class Width</th></tr>
<tr><td>$FD = \frac{F}{CW}$</td><td>$FD \times CW = F$</td><td>$CW = \frac{F}{FD}$</td></tr>
</table>

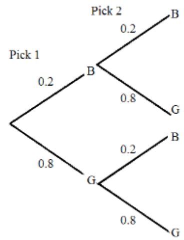
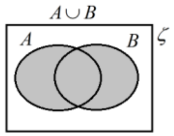
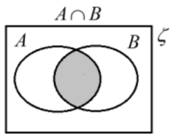
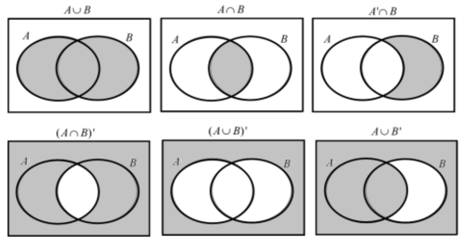
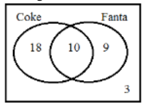
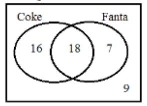
Remember, the area of the bar is proportional to the frequency of the interval.
To find the frequency you will either:
(1) Count squares
(2) Multiply the frequency density by the class width.

Check that the scale factor is 1:1. With more challenging questions there may be a scale factor enlargement on the area of k .</td>
<td>Example: The histogram below shows information about the heights of a number of plants. Given that there were 4 plants less than 5cm tall, find the number of plants that were more than 5cm tall.


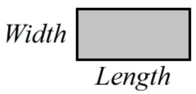
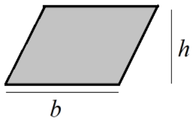
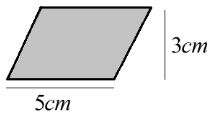
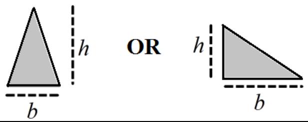
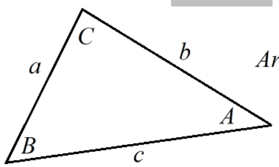
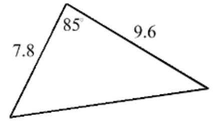
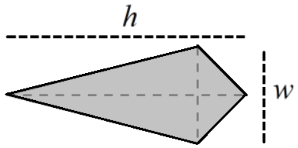
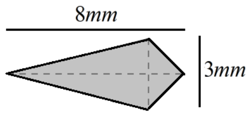
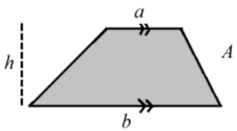
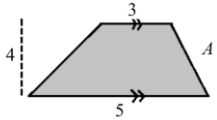
Answer: Here the FD = 0.8 for the interval $0 < h \le 5$ .Using $FD \times CW = F$ you get $0.8 \times 5 = 4$. This means there is no scale factor to deal with. To find the number of plants for $5 < h \le 15$ you calculate $1.2 \times 10 = 12$ and for $15 < h \le 30$ you calculate $2.4 \times 15 = 36$. This gives a total of $12 + 36 = 48$</td>
</tr>
</table>

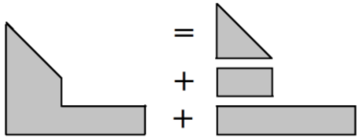
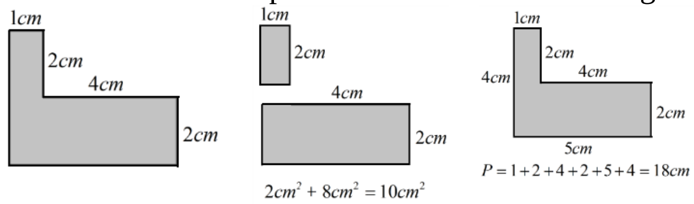
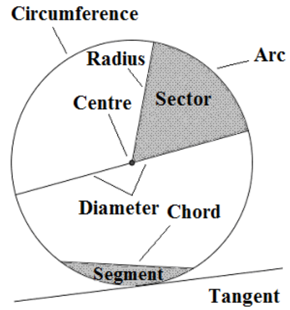
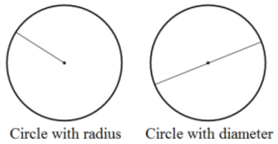
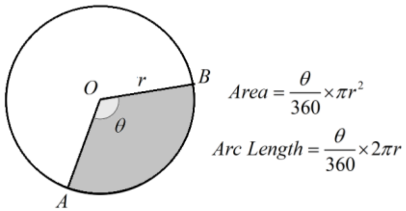
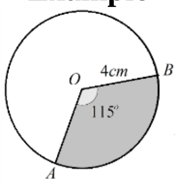
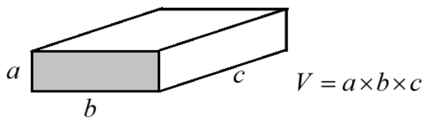
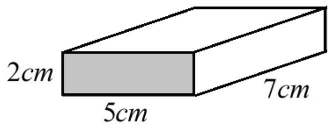
Probability

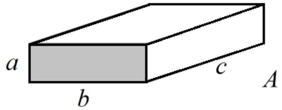
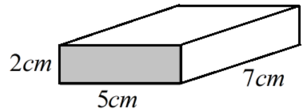
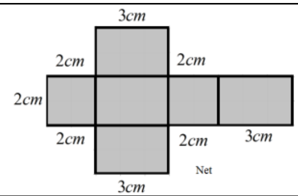
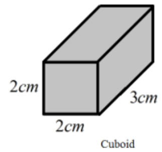
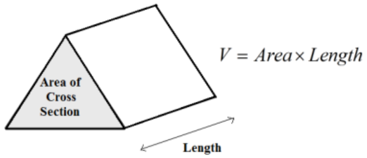
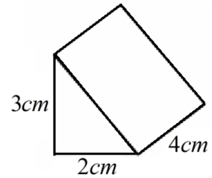
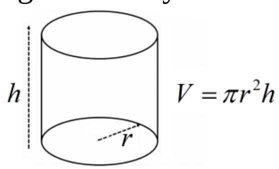
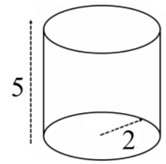
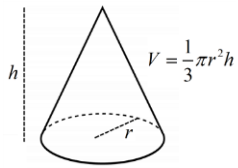
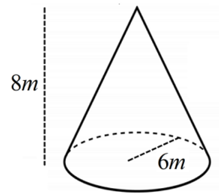
<u>Simple Probability (Theoretical)</u>	The number of things you **want to happen** divided by the number of things that **could happen**. 1 head on a fair coin, 2 sides so the probability of head = ½	Probability of rolling a 4 on a fair 6 sided die is 1/6 There is one 4 and 6 different numbers.
<u>Listing Outcomes Systematically and Sample Spaces</u>	Listing outcomes systematically makes working with probability easier. You can often just use thee the first letter of each word. Think 'combinations' here. Sample spaces show all the possible outcomes of 2 events.	**Example:** Bob picks **two** items from Coke, Sweets, Burger and Ice Cream. List the possible combinations: **Answer: CS, CB, CI, SB, SI, BI**
<u>Basic Notation</u>	$P(A)$ just means 'the probability of A happening' $P(A')$ means 'the probability of A not happening'. This is called the compliment of A and it can be read as A dashed.	**Example:** There are 8 balls in a bag. 5 are Green and the rest are Red. Find: (a) $P(Green)$ (b) $P(Green')$ (c) $P(Red)$. **Answer:** (a) 5/8 (b) 3/8 (c) 3/8
<u>Mutually Exclusive Events and the OR Rule for Addition</u>	If two events are mutually exclusive they cannot happen at the same time. For mutually exclusive events $P(A \text{ or } B) = P(A) + P(B)$ (The OR Rule). You just add the two probabilities. It looks confusing but it's easy! The probability of something **not** happening is "1 minus the probability of it happening" or more technically $P(A') = 1 - P(A)$ **The probabilities of all possible outcomes in an event will always add to 1.**	**Example:** Bob either catches the bus or the train to work. The probability of him catching the bus is 0.4. Find the probability that he doesn't catch the bus. **Answer:** 1 - 0.4 = 0.6 *The probability of picking and card that is red and a spade from a pack of cards are mutually exclusive.*
<u>Relative Frequency and Expected Outcomes</u>	This is how many times something happens ÷ by the number of trials that took place in the experiment. To find the number of expected outcomes just × the probability by the number of trials. This is different to theoretical probability.	**Example:** The probability a football team wins a game is 0.2. How many games would you expect them to win out of 40? **Answer:** 0.2 × 40 = 8, so about 8 games.

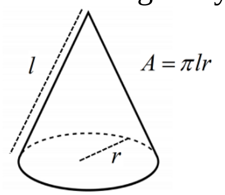
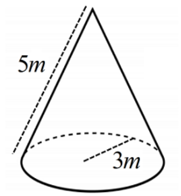
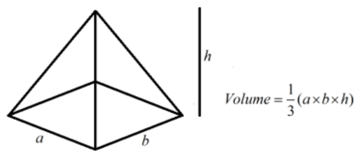
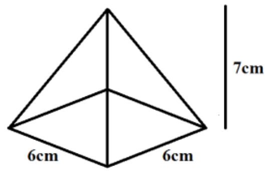
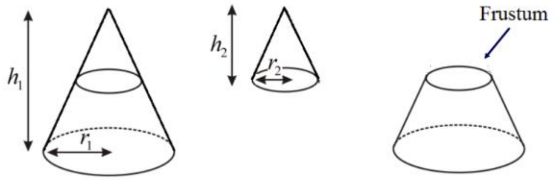
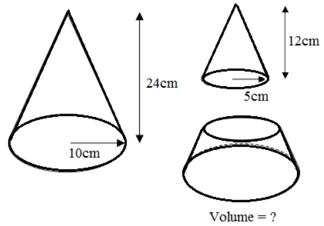
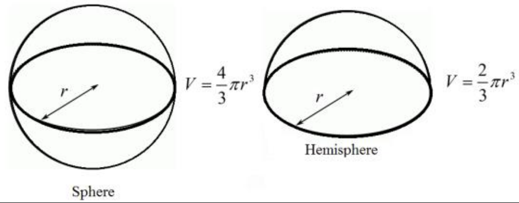
Frequency Trees	These show how many times two or more events happen. **NOT** a tree diagram!	Outcomes on branches, frequency at the end of branch!
Tree Diagrams for Probability (For video see Independent Events and Conditional Probability)	Use a tree diagram to help work out the probability of more than one event. **All branches must sum to 1 when you add downwards (0.2 + 0.8 = 1 as shown in the example to the right).** If you are modelling conditional probability check that your probabilities on the second branches reflect any changes. An example could be sweets in a bag. If you have 7 mints out of 10 in a bag of sweets on the first pick and you choose one then there will only be 6 mints left out of 9 sweets. The probabilities of independent events don't change	
Independent Events and *Conditional Probability.*	If two or more events are said to be independent, the outcome of a previous event doesn't influence the probability of the next. The question may tell you that the events are independent. Anything 'without replacement' is conditional.	Independent could be replacing a counter in a bag after picking it. Conditional would be where the counter wasn't replaced.
The **AND** and **OR** Rule in Probability (See previous videos for tutorial).	**OR** means you need to **Add** the probabilities and **AND** means **Multiply** them. Be careful with the wording. Two chocolates for example is chocolate AND chocolate. Two mints or Two Toffee means you would have to add once you have multiplied! **Check to see if the problem is independent or conditional!**	Using the Tree Diagram above: $P(B \text{ and } B) = 0.2 \times 0.2 = 0.04$ $P(G \text{ and } G) = 0.8 \times 0.8 = 0.64$ $P(1 \text{ of each}) = (0.2 \times 0.8) + (0.2 \times 0.8) = 0.32$
Set Notation	A represents the Set A and its elements. A set is just a collection of items A' (A dashed) means the elements not in Set A (the compliment or NOT A) $\in$ means 'is an element of a set'. (This is just a value in the set) $\{\ \}$ Shows the items (or numbers) in the set. ξ Universal Set (All values being considered (even if they are not in A or B)) $A \cup B$ (The Union) is A or B **or** both. $A \cap B$ (The Intersection) is both A and B $A \cap B \mid B$ is both A and B given it's already in B	**Set A are the even numbers less than 10:** $A = \{2,4,6,8\}$ **Set B are the prime numbers less than 10:** $B = \{2,3,5,7\}$ $4 \in A$ simply means 4 is in Set A $A \cup B = \{2,3,4,5,6,7,8\}$ These are in either or both! $A \cap B = \{2\}$ This is the single value in both sets!
Venn Diagrams (Shading) 2 and 3 circles.	You can use Venn diagrams to represent sets and to calculate probabilities. You **may** be asked to shade Venn Diagrams as shown below and to the right. The Union 'A or B or Both' The Intersection 'A and B'	
Venn Diagrams (Problem Solving)	**Finding the Outside Value** (1) Write the 'both' value in intersection (middle of Venn). (2) Find what's left over for the two 'only' parts and fill those in individually. (3) What's left over goes on the outside to give the 'neither' value. **Finding the Intersection Value** (1) Put the 'neither' value outside (2) Take the number of 'neither' from the total number of items. (3) Add the 2 'Coke' and 'Fanta' values together. (4) Subtract the added amount to get how many go in the 'both'	40 People Go to a Party. 28 take Coke, 19 take Fanta & 10 take **both**. How many take **neither**? The answer is 3 50 People Go to a Party. 34 take Coke, 19 take Fanta & 10 take **neither** How many take **both**? The answer is 9

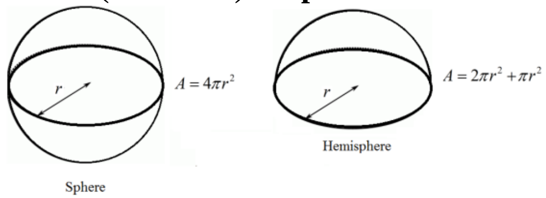
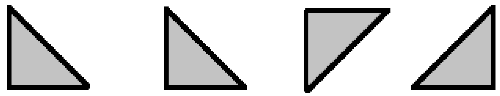
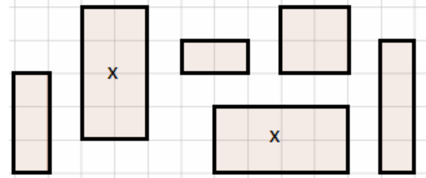
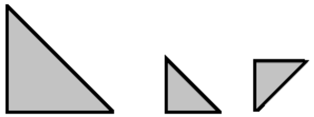
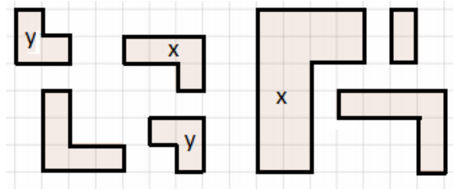
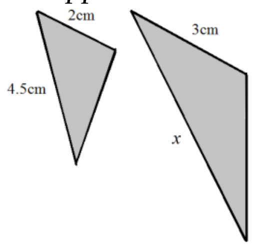
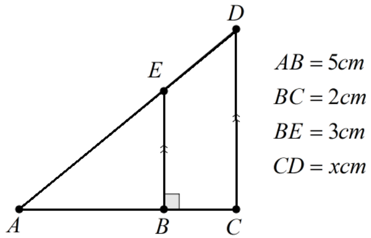
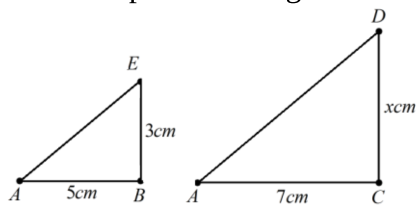
Geometry and Measures

Area of Rectangle	The **area** is the **space** inside a shape. **Multiply** the two side lengths.  $Area = Length \times Width$ $Perimeter = 2(Length + Width)$	**Example:** Find the area of a rectangle with side lengths 3cm and 5cm. **Answer:** $A = 3 \times 5$ which gives $15cm^2$. Remember the units for area are always something 'squared'
Perimeter of a Rectangle	The **perimeter** is the **distance** around the outside. **Add each** side length! Some rectangles only show two lengths, make sure you are adding all four sides!	**Example:** Find the perimeter of the rectangle above. **Answer:** $P = 3+3+5+5$ which gives $16cm$. Not $16cm^2$
Area of Parallelogram	Treat these like a rectangle. Base × Perpendicular Height. Not the slant height! It will be the same for a rhombus. A rhombus has 4 sides of equal length.  $Area = base \times height$	**Example:** Find the area of a parallelogram below.  **Answer:** $Area = 5 \times 3 = 15cm^2$ (Again units are squared)
Area of a Triangle	Multiply the base by the height **and half your answer.** Please half the answer!  $Area = \frac{base \times height}{2}$	**Example:** Find the area of a triangle with a base of 5cm and a height of 4cm. **Answer:** $Area = \frac{4 \times 5}{2}$ which gives $10cm^2$
Area of a Triangle ($Using\ \frac{1}{2}ab\sin(C)$)	To find the area of a triangle where the base and perpendicular height is given you can use the method above. If it's not possible to do this use the formula below.  $Area = \frac{1}{2}ab\sin(C)$	**Example:** Find the area of the triangle below:  **Answer:** $a = 7.8$, $b = 9.6$ and $C = 85^\circ$. Using the formula: $Area = \frac{1}{2}(7.8)(9.6)\sin(85^\circ)$ which gives $Area = 44.8u^2$ correct to 1 decimal place.
Area of a Kite	Use the same method as you do for a triangle. Height × Width and half answer. The area is half that of a rectangle with the same dimensions.  $A = \frac{h \times w}{2}$	**Example:** Find the area of the kite below.  **Answer:** $Area = \frac{8 \times 3}{2} = 12mm^2$
Area of a Trapezium	Add the two **parallel sides**, multiply it by the height and half your answer. If you can't remember this split it up into rectangles and triangles if you can.  $A = \frac{a+b}{2} \times h$	**Example:** Find the area of the parallelogram below.  $A = \frac{3+5}{2} \times 4$ **Answer:** + top & bottom, × by 4 and half it to give $16u^2$

Area/Perimeter of Compound /Composite Shapes	Simply split the shape into smaller shapes where there area or perimeter is easier to work out. Look out for missing lengths. You may need to find them! There are a number of different ways you can split most composite/compound shapes up. Use whichever method is easiest. Loads of possible splits on them!  	**Example:** Find the area & perimeter of the shape below **Area:** Split it up into 2 . **Perimeter:** Add **ALL** the lengths 
Circle Parts	**Diameter**: A line through the centre from circumference to circumference **Radius**: A line from centre to circumference. (Half the length of a diameter) **Centre**: The middle of the circle. This will be shown with an O and/or a dot. **Tangent**: A line that touches the circumference. (Doesn't go through circle) **Chord**: A line like a diameter but it doesn't go through the centre. **Arc (Major or Minor)**: A small part of the circumference. **Sector (Major or Minor)**: A part of the area of the circle (enclosed by 2 radii) **Segment (Major or Minor)**: Area trapped between chord and circumference.	
Area and Circumference of a Circle	**Area** = πr^2 which just means pi × radius × radius **Area** = Space inside. Just multiply the radius by the radius by π. The units **will** be something 'squared'.  **Circumference** = πd which is just pi × the diameter. **Circumference** = Distance around the outside. The units are **NOT** squared	**Example:** Find the area and circumference of a circle with a radius of $5cm$. **Answer:** If the radius was 5cm then $A = \pi \times 5^2 = 78.5cm^2$ If the radius was 5cm then $C = \pi \times 10 = 31.4cm$ *If you don't have π on your calculator use 3.14.*
Arc Length and Area of a Sector	The **arc** is just part of the **circumference**. The **sector** is part of the **area**. Take the angle given as a fraction of 360º & × formulae shown above for each. **Example**   	**Example:** Find (a) The minor arc length AB and (b) The area of the sector OAB from the diagram to the right. **Answer:** (a) Arc $AB = \frac{115}{360} \times \pi \times 8 = 8.03cm$ (b) Sector Area: $OAB = \frac{115}{360} \times \pi \times 16 = 16.1cm^2$
Volume of a Cuboid (Capacity)	The volume is the amount of space inside a 3D shape. To find the volume (or capacity) you simply multiply the **area of cross** section by the **length**. You can choose any cross section on a cuboid. You *could* also just multiply each length! The units will be something cubed such as cm³,m³,km³  $V = a \times b \times c$	**Example:** Find the volume of the cuboid below.  **Answer:** $V = 2 \times 5 \times 7$ which gives $V = 70cm^3$

Surface Area of a Cuboid	The surface area is the area of the outside of a 3D shape. Find the area of **each face** and add them up. Check if it is open or closed top. If it's open it will only have 5 faces. Think about a dice. You can touch all six faces. The total surface area would just be 6 times the area of each face. Units is always 'squared'  $A=(2\times a\times b)+(2\times a\times c)+(2\times b\times c)$	**Example:** Find the surface area of the cuboid below.  **Answer:** $A=2(2\times5)+2(2\times7)+2(5\times7)$ $A=118cm^2$
Sketching the Net of a Cuboid (and other 3D Shapes)	Just think what the box/cube/prism would look like if you unfolded it. Don't forget the lid if it has one. Dimensions must be accurate and have a label. The example shown to the right is a net of a 2cm × 2cm × 3cm closed cuboid. An open topped cube will have 5 faces, a closed top will have 6. There are different possible ways of drawing nets. You don't need to draw any flaps!	 
Volume of a Prism Surface Area of a Triangular Prism	This is the same as the cuboid when finding the volume. **Area** of the **cross section** × **length**. Be very careful with triangular prisms. Make sure you half your answer when finding the area of the cross section. For cylinders you will need the area of a circle. If you are already given the area simply multiply that by the length. Answer will be in something cubed such as cm³ 	**Example 1:** Find the volume of the prism below.  **Answer:** The area of the cross section is $3cm^2$ (Remember to half it!) The volume will just be $3\times4=12cm^3$
Volume of a Cylinder Surface Area of a Cylinder	A cylinder is a prism. You would use the method shown above. The cross section is simply a circle. Find the area of that circle using $A=\pi r^2$ and multiply the answer by the height of the cylinder. The formula is $V=\pi r^2 h$. 	**Example:** Find the volume of the cylinder below:  **Answer:** $V=\pi\times2^2\times5$ which can be written as $V=20\pi u^3$ or $62.8u^3$
Volume of a Cone	A cone is not a prism as it doesn't have a constant cross section. The formula used is $V=\frac{1}{3}\pi r^2 h$. Just find the volume of a cylinder and divide by 3. A cylinder is just $V=\pi r^2 h$ which is the area of a circle × the height. (A prism!) 	**Example:** Find the volume of the cone below.  **Answer:** $V=\frac{1}{3}\pi\times6^2\times8$ which gives $96\pi m^3$ or $301.6m^3$

<u>Surface Area of a Cone</u>	The **curved surface** of a cone is given as $A = \pi l r$ where r is the radius and l is the slant height. This **doesn't** include the area of the base. You may need to use Pythagoras to find the slant height if you are given the perpendicular height.  **If you have a solid cone and need the base too add the area of the end circle to your answer!**	**Example:** Find the curved surface area of the cone below:  **Answer:** $A = \pi \times 5 \times 3$ which gives $15\pi m^2$ or $48.1m^2$
<u>Volume of a Pyramid</u>	This is similar to the volume of a cone. Again a pyramid is not a prism as there is no constant cross section. Just find the area of the base, multiply by the height & ÷ by 3 or if you like: $V = \frac{1}{3}bh$. This works for triangular based too! 	**Example:** Find the volume of the pyramid below.  **Answer:** $V = \frac{1}{3} \times 6 \times 6 \times 7$ which gives $21\pi cm^3$
<u>Frustums</u>	A frustum is either a cone or a pyramid with the top removed. To find the volume simply find the volume of the original large cone or pyramid and then take away the volume of the smaller cone/pyramid you removed from the top. (1) Check you have the radius and perpendicular height. If you have the slant height you will need Pythagoras Theorem or trigonometry. (2) You may need to use similar triangles to find a missing radius or height.  Total Volume - Smaller Cone Volume = Volume of Frustum	**Example:** Find the volume of the frustum below.  **Answer:** $V = \frac{1}{3}\pi(10)^2(24) - \frac{1}{3}\pi(5)^2(12) = 700\pi cm^3$ *(This is just the volume of the large cone – small cone)* N.B You could also find the surface area if required!
<u>Volume of a Sphere</u>	A sphere is just a perfect ball! If the sphere has radius r the volume is given as $V = \frac{4}{3}\pi r^3$. Just substitute the values in. Your answer will be in units cubed. Look out for hemispheres. This is just half a sphere so half your answer. 	**Example 1:** Find the volume of a sphere with diameter 10cm. **Answer:** Radius = 5cm. $\therefore$ $V = \frac{4}{3} \times \pi \times 5^3 = \frac{500\pi}{3} cm^3$ **Example 2:** Find the volume of a hemisphere with radius 4mm. **Answer:** $V = \frac{2}{3} \times \pi \times 4^3 = \frac{128\pi}{3} mm^3$ *(You could just find the sphere and half your answer)*

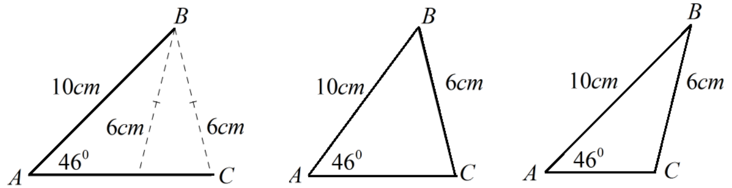
Surface Area of a Sphere	The surface area of a sphere is given as $A = 4\pi r^2$. It's the area of the outside! Just square the radius and multiply it by 4π. Look out for hemispheres. These will need the surface area ($A = 2\pi r^2$) + a **possible** additional circle on the top. 	**Example 1:** Find the surface area of a sphere with radius 3cm. **Answer:** $A = 4\pi(3)^2 = 36\pi cm^2$ (Remember cm^2 for area) **Example 2:** Find the surface area of a **solid** hemisphere with radius 6m. **Answer:** $A = 2\pi(6)^2 + \pi(6)^2 = 3\pi(6)^2 = 108\pi m^2$ *(On this example you could have just used $A = 3\pi r^2$)*
3D Shapes Solids	**Faces**=Flat Surfaces **Edges**=Where Faces Meet **Vertices**=Where Edges Meet	A Cube has 6 faces, 8 vertices and 12 edges
Congruent Shapes	Congruent shapes are identical. They are the **same shape and the same size**. Some shapes will be rotated or reflected (their orientation changed) but still congruent to another shape shown. All of the triangles below are congruent! 	**Example:** The 2 shapes marked × are congruent. 
Similar Shapes	**Same shape, different size.** The proportion is the same for each side length. Check each side length is a multiple of the other shape. Adding or subtracting a fixed value to each side length **doesn't** keep the proportion. Think scale factors 	**Example:** Below the shapes: x = similar y = congruent 
Problem Solving with Similar Shapes/Similar Triangles	To find a missing length look at the corresponding side lengths and then take one of two approaches. *(The first approach is less ideal for harder examples)*:  **Method 1**: The larger shape has side length 3 and the smaller side shape has side length of 2. Remember, these sides are in **proportion** as they are **similar shapes**. This gives a scale factor of 3/2 or 1.5. To find x, multiply 4.5 by 1.5 as the sides are proportional. x = 6.75cm. **Method 2**: Set up an equation and look at the sides in proportion: $\frac{x}{3} = \frac{4.5}{2}$ Multiply both sides by 3 to give $x = \frac{3 \times 4.5}{2}$ which gives x = 6.75cm.	**Example:** Find the value of x in the diagram below.  **Answer:** Split the triangles and use similarity.  $\frac{x}{7} = \frac{3}{5}$ which gives $x = \frac{7 \times 3}{5}$ and simplifies to $x = 4.2$ (One way of doing it!!)

Congruent Triangles

There are 4 ways you can prove that two triangles are congruent (identical).

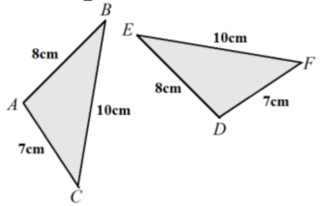
SSS (Side/Side/Side)	**RHS** (Right Angle/ Hypotenuse/ Side)	**SAS** (Side/Angle/Side)	**ASA** (Angle/Side/Angle)
All sides are of equal length for both triangles.	The hypotenuse and one side length are the same for both RA triangles.	Two side lengths are the same AND the enclosed angle for both triangles	Two angles are the same size and a corresponding side for both triangles.

There is one combination that **doesn't** prove congruency. "Don't be an ASS". Angle Side Side is not sufficient (unless it's RHS!) to prove congruency as shown below. Triangle *ABC* can be drawn 2 different ways despite having two equal sides and one equal angle.



You will often have to construct an argument using one of the 4 choices above being clear in your work. Some examples are shown to the right. Each makes references to the equal sides or angles and has a conclusion that includes either SSS, RHS, SAS or ASA.
The 4 possible scenarios are shown to the right.

Example 1:



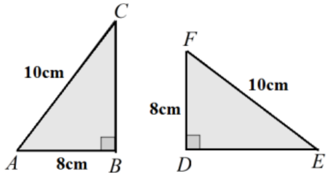
$AB = DE$

$AC = DF$

$BC = EF$

$\therefore$ The two triangles are congruent by SSS.

Example 2:



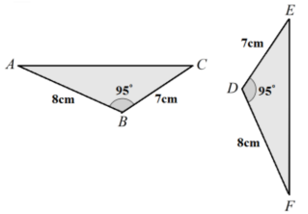
$\angle ABC = \angle EDF = 90^\circ$

$AC = EF$

$AB = DF$

$\therefore$ The two triangles are congruent by RHS

Example 3:



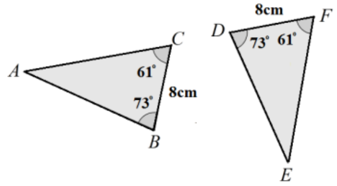
$AB = DF$

$BC = DE$

$\angle ABC = \angle EDF$

$\therefore$ The two triangles are congruent by SAS.

Example 4:



$BC = DF$

$\angle ABC = \angle EDF$

$\angle ACB = \angle EFD$

$\therefore$ The two triangles are congruent by AAS.

Similar Solids

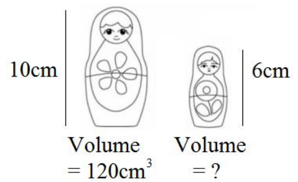
🗎

If two solids are mathematically similar they are the same <u>shape</u> **but** different <u>sizes</u>. Their proportion remains. The easiest thing to do is find either the linear, area or volume scale factor that connects the solids and put them in a table. Let's start with an easy example. Let's say Cuboid A and Cuboid B are mathematically similar. Cuboid A is twice the length of Cuboid B. Setting up a table you get:

Linear Scale Factor	2
Area Scale Factor	$2^2 = 4$
Volume Scale Factor	$2^3 = 8$

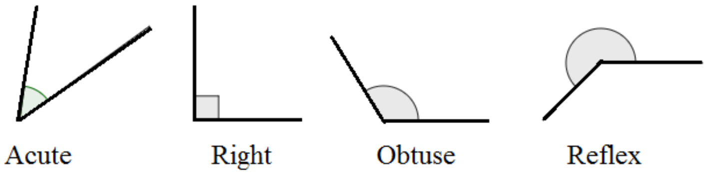
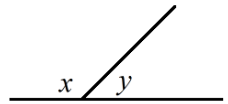
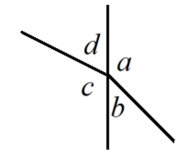
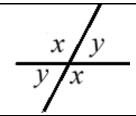
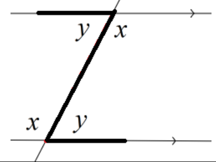
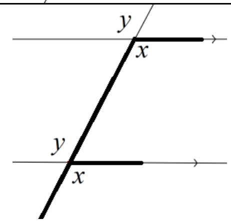
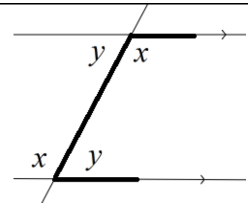
This means that the area of Cuboid A will be 4 times the area of Cuboid B and the volume of Cuboid A will be 8 times the volume of Cuboid B. If you are given the area scale factor, square root it to get the linear scale factor. If you are given the volume scale factor, cube root it to get the linear.

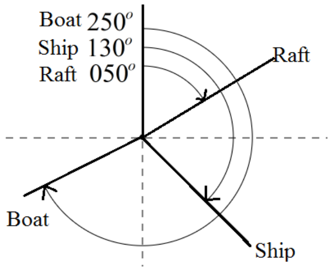
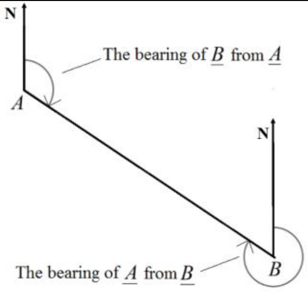
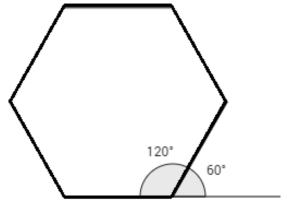
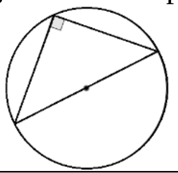
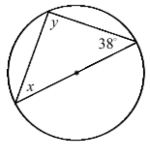
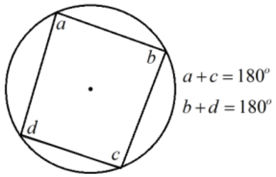
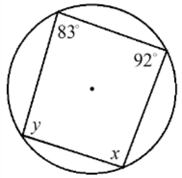
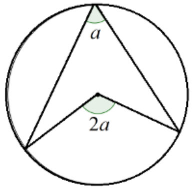
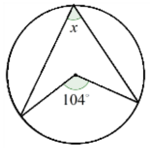
Example 1: Two mathematically similar dolls are shown below. Find the volume of the smaller doll.

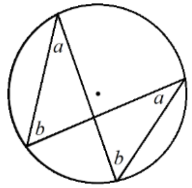
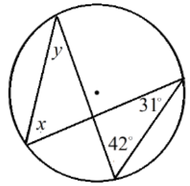
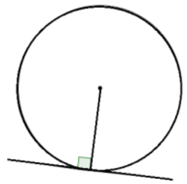
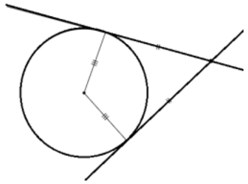
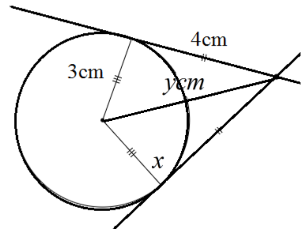
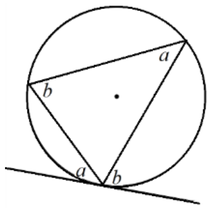
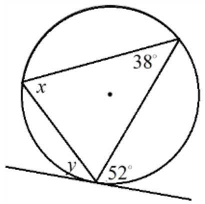


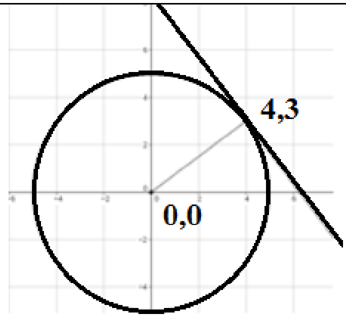
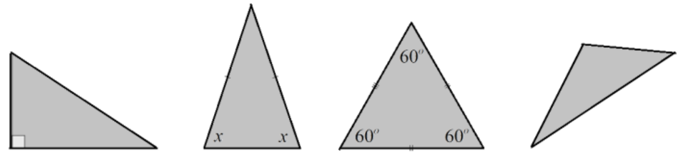
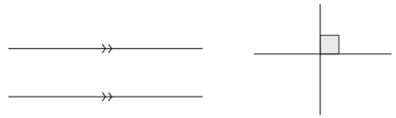
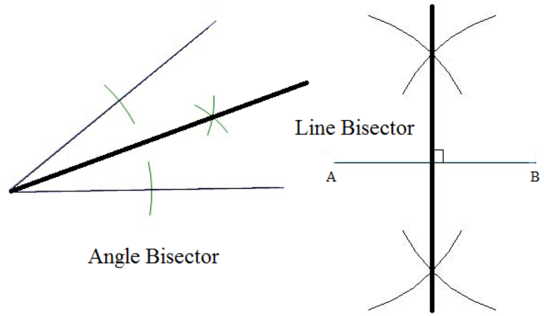
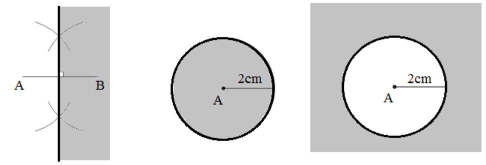
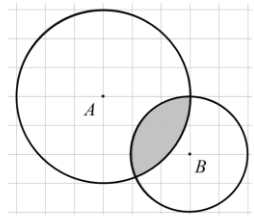
Linear Scale Factor	$6/10 = 0.6$
Area Scale Factor	0.6^2
Volume Scale Factor	0.6^3

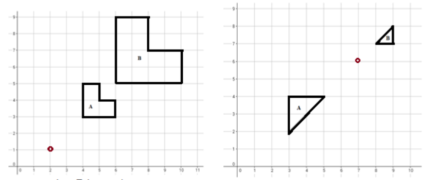
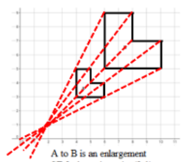
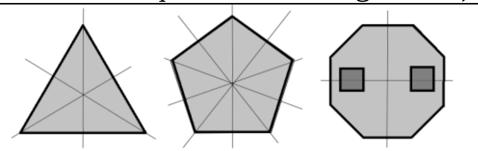
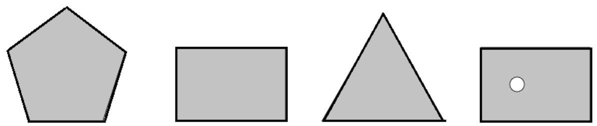
All you need to do now is multiply the volume of the larger doll by the volume scale factor to find the volume of the smaller doll. This gives $120 \times 0.6^3 = 25.92cm^3$.

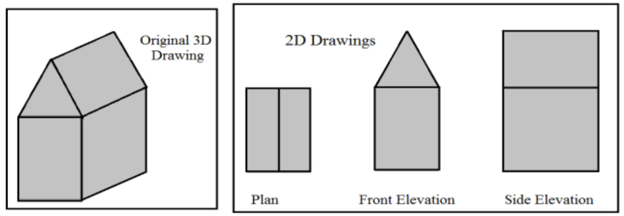
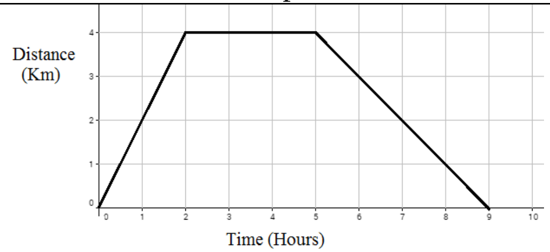
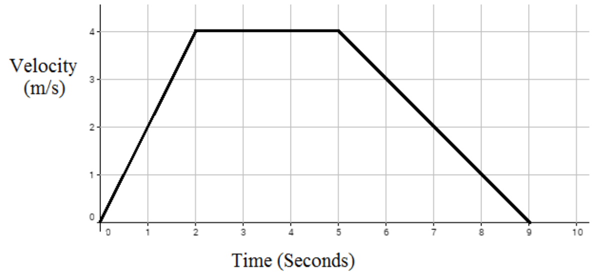
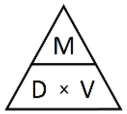
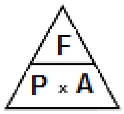
Metric Enlargements	$mm \to cm$ $\div 10$ $mm^2 \to cm^2$ $\div 10^2 = 100$ $mm^3 \to cm^3$ $\div 10^3 = 1000$ $mm \leftarrow cm$ $\times 10$ $mm^2 \leftarrow cm^2$ $\times 10^2$ $mm^3 \leftarrow cm^3$ $\times 10^3$ $cm \to m$ $\div 100$ $cm^2 \to m^2$ $\div 100^2$ $cm^3 \to m^3$ $\div 100^3$ $cm \leftarrow m$ $\times 100$ $cm^2 \leftarrow m^2$ $\times 100^2$ $cm^3 \leftarrow m^3$ $\times 100^3$ $mm \to m$ $\div 1000$ $mm^2 \to m^2$ $\div 1000^2$ $mm^3 \to m^3$ $\div 1000^3$ $mm \leftarrow m$ $\times 1000$ $mm^2 \leftarrow m^2$ $\times 1000^2$ $mm^3 \leftarrow m^3$ $\times 1000^3$	**Example :** Convert $300cm^3$ into m^3 **Answer:** $300 \div 100^3 = 0.0003m^3$ *If you are given the dimensions of a rectangle (for example) in cm and asked to find the area in m^2 you may want to convert at the start rather than at the end. The answer will be the same.*
Angle Types	**Acute angles** are less than 90°, **Right angles** are exactly 90°, (Often shown by a small square on no value) **Obtuse angles** are greater than 90° but less than 180° **Reflex angles** are greater than 180° and less than 360° *If you are asked to draw a reflex angle it may be easier to draw and acute one and then mark the larger angle round the other side!*	
Basic Angle Facts	The angles on a straight line **add to** 180°. (This may be written as ‘sum to’) The angles around a point **add to** 360°. Look out for right angles! These have a little square and often no numbers on! Some questions will need algebra to solve for an unknown angle. An example might be a line with and angle of $2x$ and an angle of $3x$. Just add them and solve: $2x+3x=180^0$ so $5x=180^0$ and $x=36^0$	 $x+y=180^o$  $a+b+c+d=360^o$
Opposite Angles	Opposite angles are **equal**. Remember also that the angles on a straight line add to 180° and angles around a point add to 360°. This will help you with some multi-step problems later on.	
Alternate Angles	Alternate angles are equal. These look like a letter Z. $x=x$ & $y=y$ of course! **(Do not use the term ‘Z angles’ in an exam. You must use Alternate)** Often you will be asked to state with a reason why you have given your answer. There is often more than one way to explain how you found the angle.	
Corresponding Angles	Corresponding angles are equal. These look like the letter F. **(Do not use the term ‘F angles’ in an exam. You must use Corresponding)** Often you will be asked to state with a reason why you have given your answer. There is often more than one way to explain how you found the angle. Some students may see two angles as corresponding instead of selecting alternate angles. As long as you show how you will get the marks.	
Co-interior Angles	Co-interior angles add to 180°. $x+y=180^o$ These look like the letter C **(Do not use the term ‘C angles’ in an exam. You must use Co-interior)** Often you will be asked to state with a reason why you have given your answer. Check your answer makes sense. Clearly $x+y$ are not the same size unless they = 90^o. Students often incorrectly just say ‘opposite angles’.	

<u>Bearings</u>	Bearings are just angles! Here are the rules you must use with bearings: (1) **Measure from North** (Draw a north line at each point to help you) (2) **Measure clockwise** (Using a protractor **or** by using angle facts) (3) **Your answer must have 3 digits (**An angle of 40° has a bearing of 040°) The first diagram shows the bearing **of** 3 water vehicles **from** a harbour. The second diagram shows how to find the bearing of one place from another. B **from** A is measured at A and A **from** B is measured at B (Use north lines) Just fill out all missing angles. Sometimes you can use angle facts above instead of measuring the angle. Co-interior angles add to 180° as shown above.	 
<u>Exterior Angles of a Regular Polygon</u>	For **regular** polygons divide 360^o by the number of sides. This will give you the size of **each exterior** angle. This is shown on the straight line to the right. d The **regular** hexagon shown has an exterior angle of 60º As 360º ÷ 6 = 60º The interior angles of 120º are also shown. This was found by subtracting the exterior angle from 180º as angles on a straight line add to 180º If you are unsure, interior is **inside** and exterior is **outside**!	
<u>Interior Angles of Regular Polygon</u>	To find an interior angle subtract the exterior angle from 180° as shown above. Add all the angles for the sum. The number of angles = number of sides.	Use the diagram above to help you! You could also use $(n-2)\times180$ to find the sum of the interior angles.
<u>*Circle Theorem 1 (Angles in a semi-circle)*</u>	Angles in a semicircle have a 90º angle at the circumference. Make sure the diameter does pass through the centre (this may have an O on). You may be asked questions involving Pythagoras in examples involving semi circles. 	**Example:** Find the value of x and y in the circle below.  **Answer:** Angles in a semicircle $\therefore y = 90^\circ$. Angles in a triangle add to $180^\circ \therefore x = 52^\circ$
<u>*Circle Theorem (Angles in a Quadrilateral)*</u>	The opposite angles in a cyclic quadrilateral sum (add) to 180^o. Make sure each vertex of the quadrilateral touches the circle. 	**Example:** Find the value of x and y in the circle below.  **Answer:** Opposite angles add to 180° $\therefore x = 97^\circ$ $y = 88^\circ$.
<u>*Circle Theorem (The Arrow)*</u>	The angle at the centre is double that at the circumference. Look for an O. 	**Example:** Find the value of x in the circle below.  **Answer:** x is half of 104° at the centre $\therefore x = 52^\circ$

<u>Circle Theorem (The Bow/Angles in the Same Segment)</u>	The angles at the top of the bow are the same. The angles at the bottom of the bow are the same. You don't need the lines to go through the centre. 	**Example:** Find the value of x and y in the circle below.  **Answer:** Using the bow, $y = 31^\circ$ and $x = 42^\circ$
<u>Circle Theorem (Tangent)</u>	When a tangent meets a radius it meets at right angles. You can say the angle between the radius and the tangent is 90^o or that they are perpendicular.   When two tangents are drawn from a point to the circle their lengths are equal.	**Example:** Find the value of x and y in the circle below.  **Answer:** $x = 90^\circ$ and using Pythagoras Theorem. $y = 5$
<u>Circle Theorem (The Alternate Segment Theorem)</u>	The angle a between the tangent and the chord is the same size as the angle a in the alternate segment. This is true for b and b too. One way to spot these is to look for the two angles 'opening to the left' which are b. These will be the same. The two angles opening to the right will both have the value of a. 	**Example:** Find the value of x and y in the circle below.  **Answer:** $y = 38^\circ$ and $x = 52^\circ$ using the alternate segment theorem. Look at the way the angles open up if you are struggling. x and 52° to the right, y and 38° to the left.
<u>All Circle Theorem</u>	*Combining theorems to solve multistep problems.*	Rotate your paper to help you spot different theorem.
<u>The Equation of a Tangent to a Circle</u>	The tangent to a circle is a straight line that touches the circle and is perpendicular (at right angles) to the radius at that given point. To find the equation of a tangent to a circle at a given point: (i) Find the gradient of the radius using the centre of the circle and the given point as your two points. *(See the section on gradient previously)* (ii) Use the negative reciprocal for the gradient of the tangent *(see parallel and perpendicular lines section)* (iii) Use the equation of a straight line ($y = mx + c$) with your gradient and the point you are given in the question. *(See the section on $y = mx + c$)*	**Example:** Find the equation of the tangent to the circle $x^2 + y^2 = 25$ at the point $(4,3)$. **Answer:** (1) The centre of the circle is $(0,0)$ and passes through the point $(4,3)$. The gradient of the radius is $\frac{3-0}{4-0} = \frac{3}{4}$. (2) The gradient of the tangent will be $-\frac{4}{3}$ as it's the negative reciprocal of the gradient of the radius.

		(3) Substituting into $y = mx + c$ with the point $(4,3)$ and a gradient of $-\frac{4}{3}$ you get: $3 = -\frac{4}{3}(4) + c$ which gives $c = \frac{25}{3}$ and as a result the equation of the tangent is $y = -\frac{4}{3}x + \frac{25}{3}$. A diagram is shown to the left.
<u>Types of Triangles</u>	**Right Angle Triangles** have one 90° angle in. Look out for the square in these **Isosceles** have 2 equal sides and 2 equal base angles. Look for notation! **Equilateral** have 3 equal sides and 3 equal angles (60°) Look for notation! **Scalene** have different size sides and angles. No notation. You can spot isosceles and equilateral by the small lines on their sides.	Right Angled Isosceles Equilateral Scalene
<u>Parallel and Perpendicular Lines</u>	**Parallel lines** never meet and have a fixed distance between them. **Perpendicular lines** are at right angles. There is a 90º angle between them. If two straight lines are parallel the value of m in the equation $y = mx + c$ will be the same for both lines. $y = 3x - 1$ and $y = 3x + 2$ as their gradients are equal.	Parallel Perpendicular
<u>Angle and Line Bisectors</u>	**Angle Bisector**: Cuts the angle in half. Open the compass up. Place the sharp end on the vertex. Mark a point on each line Without changing the compass put the compass on each point and mark a centre point. Get a ruler and draw a line through the vertex and centre point. **Line Bisector** (Perpendicular Bisector): Cuts the line in half and at right angles Put the sharp end on Point A. Open the compass up past half way on the line. Mark a point above and below the line. Without changing the compass do the same from B. Draw a straight line through the points. **You MUST leave your construction marks on all bisection questions!**	
<u>Loci and Regions</u>	A locus is just a path of points or region that follows a rule. For the locus of points **closer to** B **than** A you will create a perpendicular bisector as above and shade to the right of the line as shown to the right. For the locus of points **less than or more than a fixed distance** from A use a compass with the given radius to draw a circle. You may have to combine loci. Points Closer to B than A Points less than 2cm from A Points more than 2cm from A Look out for broken lines on strict inequalities.	**Example:** Draw the locus of points no more than 3cm from A and no more than 2cm from B. **Answer:** Draw a circle with radius 3cm from A and one with radius 2cm from B. Shade **inside** as it's no more than! (If it were more than it would have been outside!) 

Translating a Shape (A Transformation)	Translate means to move the shape. There is no change in its size or its orientation. Vectors are used to give information about the ‘movement’ The top number tells you to move right or left. Right is + and left is - . The bottom number tells you to move up or down. Up is + and down is - . *If coordinates are used for the translation just treat them like vectors.*	(a) $\begin{pmatrix}2\\3\end{pmatrix}$ is right 2 and up 3 (b) $\begin{pmatrix}-1\\2\end{pmatrix}$ is left 1 and up 2 (c) $\begin{pmatrix}3\\-5\end{pmatrix}$ is right 3 and down 5 (d) $\begin{pmatrix}0\\4\end{pmatrix}$ is just up 4
Rotating a Shape (Transformation)	The size of the shape doesn’t change. The shape is simply turned about a point. You will be given (i) A direction (ii) An angle and (iii) A centre of rotation.	(a) Rotate Shape A 90° clockwise about (0,1) (b) Rotate Shape B 270° anti clockwise about (0,0)
Reflecting a Shape (Transformation)	Think about standing looking in a mirror. Learn lines such as $x=2$ (vertical), $y=-1$ (horizontal) & $y=x$ (diagonal). Use a mirror if you are unsure.	(a) Reflect Shape A in the x axis. (b) Reflect Shape A in the line $x=2$.
Enlarging a Shape (Basic examples) (Transformation)	You will be given a **S**cale **F**actor and no centre of enlargement. **Multiply** each side length of the shape by the scale factor. A scale factor of 2 is twice as big (× by 2), **not** + 2 to each side. See below for centre of enlargement examples.	SF of 3 = 3 times larger (× EACH side length by 3) SF of ½ = half the size (÷ EACH side length by 2) SF of 1 = no change in the size of the shape
Enlargements Given a centre of Enlargement (Including Negative and Fractional)	If the scale factor is positive both shapes will be the same side of the centre of enlargement. If the SF is negative the two shapes will be either side of the centre. Negative enlargements will look like they have been rotated. One way to do this is with guidelines & the other way is to do it with vectors. SF 2 is twice as big & twice as far away from the centre of enlargement.	 A to B is an enlargement SF 2 about the point (2,1) A to B is an elargement SF -1/2 about the point (7,6)
Finding the centre of Enlargement	Draw guidelines through each corresponding vertex of the two shapes with a pencil and ruler. Each line will pass through the centre of enlargement when done accurately as shown to the right. Be careful with negative enlargements when finding the corresponding corners as the shape will be a different way round.	 A to B is an enlargement SF 2 about the point (2,1)
Combining Transformations	Perform 2 transformations and then state the single transformation that maps thee original shape to the final shape. You may need to use resultant vectors.	To find the resultant vector you can add the 2 vectors you used in the translations given.
Naming Transformations (The 4 choices)	**Rotations** will be the same size but often a different way around. (orientation) **Translations** have simply been moved. No change to size or orientation. **Reflections** will sometimes have the same orientation depending on the shape. **Enlargements** will be the same shape but either larger or smaller!	(Centre, direction and angle required for Rotations) (The vector is required for Translations) (The reflection line for Reflections) Look out for $y=x$ (The scale factor is required for Enlargements)
Line Symmetry	How many **mirror lines** can you draw on the shape? Regular shapes will have the same number of sides as they do symmetry lines and rotational symmetry. Be careful with patterns within shapes. This will change the symmetry! Parallelograms seem to catch people out too!	
Rotational Symmetry	How many times does the shape (and pattern if applicable) look **the same** when you **turn** it through **360°**? This gives us the order of rotational symmetry. Be careful with patterns. Regular shapes without patterns will have the same number of sides as their rotational symmetry. Use tracing paper if you need. A circle without a pattern will have an undefined number!	 Order = 5 (Regular) Order = 2 Order = 3 (Regular) Order = 1

Plans and Elevations	These types of drawing take 3D drawings and produce 3 different 2D drawings. **Plan View**: From above. Think 'birds eye view' **Side Elevation**: A 2D shot from the side of the object. **Front Elevation**: A 2D shot from the side of the object. You will be told which is the front and/or side. Remember to put the units on!	
Metric Units (See Video on converting units)	**Length**: mm, cm, m and km. 1km = 1000m = 100'000cm = 1'000'000mm **Mass**: mg, g, kg, tonnes 1kg = 1000g **Volume**: ml, cl, l 1 litre = 1000ml	Man's height ~ 1.8-2m, credit card ~ 0.8mm thick Adults weight 70kg, a small cake = 150g Glass of coke is about 250ml
Speed, Distance, Time	**Speed** = Distance ÷ Time **Distance** = Speed × Time **Time** = Distance ÷ Speed Remember the correct units! 	(a) Speed is 4mph, Time is 2 hours, Find the Distance. D = S × T so 4×2 = 8 miles. (b) Time is 5 hours, Distance = 12km, Find the Speed S = D ÷ T so 12÷5 = 2.4kph
Distance/Time Graphs	Distance/Time graphs show the **distance covered** and the **time taken** as shown to the right. Distance is on the vertical axis and time is on the horizontal. You can find the **speed** from the gradient of the line (Distance ÷ Time). The steeper the line, the quicker the speed. If there is a flat line (horizontal to the time axis) the object is stationary. On the example to the right the speed on the first section is 4 ÷ 2 = 2km/h, the second 0km/h and the third 4 ÷ 4 = 1km/h	
Velocity Time Graphs	Velocity/Time Graphs show the speed of an object over a given time. Velocity is on the vertical axis and time is on the horizontal. You can find the acceleration from the gradient of the line (Speed ÷ Time). The steeper the line the quicker the acceleration. If the line goes up it's acceleration, if it goes down it's deceleration and if it's flat then there is no acceleration and a constant speed. The area under the graph is the distance covered. This can be found by either finding the area of the trapezium or adding the areas of rectangles and triangles.	
Density, Mass, Volume	**Density =** Mass ÷ Volume **Mass =** Density × Volume **Volume =** Mass ÷ Density Remember the correct units. 	(a) Density is $8kg/m^3$, Mass is $2kg$, Find the Volume. V = M ÷ D so 2 ÷ 8 = ¼ m^3 (b) Volume is $20cm^3$, Mass is $30g$, Find the Density D = M ÷ V so 30 ÷ 20 = 1.5 g/cm^3
Pressure	**Pressure =** Force ÷ Area **Force =** Pressure × Area **Area =** Force ÷ Pressure *(Force is measured in Newtons (N))* 	(a) Force is 12N, Area = $3m^2$. Find the Pressure P = F ÷ A so 12 ÷ 3 = $4N/m^2$ (b) Area = $1.2m^2$, Pressure = $4.8N/m^2$. Find the Force F = P × A so 1.2 × 4.8 = $5.76N$

Kinematics (Constant Acceleration Equations SUVAT)

You can use the **contant** acceletaion equations (or SUVAT equations) to calculate the motion of an object. This could be a particle or even a car!

The equations used	Where
(1) $v = u + at$	s = displacement
(2) $s = ut + \frac{1}{2}at^2$	u = initial velocity
(3) $v^2 = u^2 + 2as$	v = final velocity
	a = acceleration
	t = time

You will be given 3 pieces of information and need to find the 4th by using the correct equation. Simply write SUVAT down the side of the page or in a table (as shown to the right), fill the given values in and calculate the missing value.

The units used generally speaking are: $s = ,\ u =,\ v = \ m/s,\ a = \ m/s^2$ & $t = s$

The notation ms^{-1} for velocity and ms^{-2} for acceleration may also be used.

Displacement is like distance but has direction too (it's a vector).

Velocity is like speed but again has direction too (it's a vector).

Acceleration is the rate of change of velocity with respect to time. This is vector too! *All values but time can be negative.*

Example 1: A car starts from rest and accelerates for at $3ms^{-2}$ for 5 seconds. Find the velocity of the car after 5 seconds.

Answer:

S	U = 0	V = ?	A = 3	T = 5

You need $v = u + at$. Substitute the 3 values in to find the 4th: $v = 0 + 3(5)$. This gives the answer of $v = 15ms^{-1}$

Example 2: A car travels 150m. The initial velocity of the car is $8ms^{-1}$ and after 150m the velocity is $12ms^{-1}$. Find the acceleration of the car.

Answer:

S = 150	U = 8	V = 12	A = ?	T

You need $v^2 = u^2 + 2as$. Substitute the 3 values in to find the 4th: This gives $12^2 = 8^2 + 2a(150)$.

Now solve for a.

$a = \dfrac{12^2 - 8^2}{2(150)}$ which simplifies to $a = 0.267ms^{-2}$

Vectors and Vector Notation, Equal Vectors, Magnitude of a Vector

$\begin{pmatrix} a \\ b \end{pmatrix}$ is a column vector. a is right(+) or left(-) and b is up(+) or down(-)

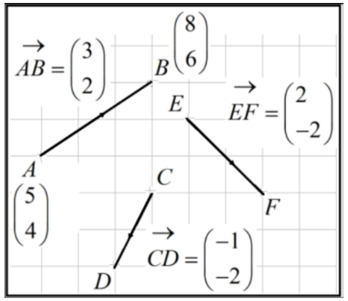
$\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ could represent a displacement of 3 metres to the right and 4 metres up.

$\overrightarrow{AB} = B - A$. To find the direction vector $\overrightarrow{AB}$ simply subtract the position vector of A from the position vector of B. It's just a line segment!

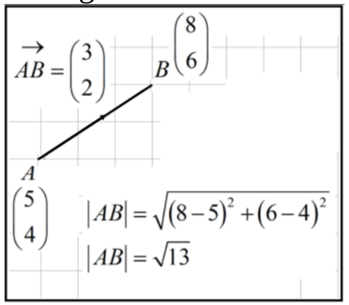
$\overrightarrow{AB} = -\overrightarrow{BA}$ (Vectors have direction and magnitude).

The magnitude (length of a vector) $\overrightarrow{AB}$ is $|AB|$. Just use Pythagoras Theorem!

Position & Direction Vectors



Magnitude of a Vector

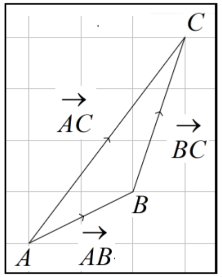


Vectors (Adding, Subtracting and Multiplying and Resultant Vectors)

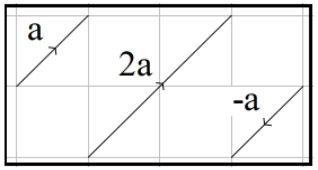
You can add $\begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 5 \\ 4 \end{pmatrix} = \begin{pmatrix} 7 \\ 5 \end{pmatrix}$ subtract $\begin{pmatrix} 8 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$ & × them $2\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$

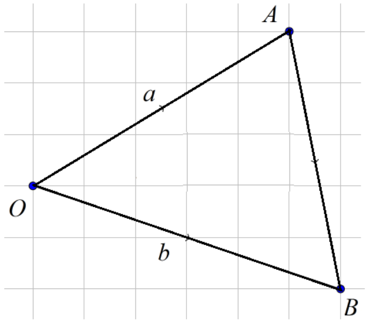
The **resultant** vector is just a direct vector between to points. In the diagram to the rightyou to get to C from A either via B or directly. This can be written as $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$. This is called the triangle laws as the 3 vectors make a triangle

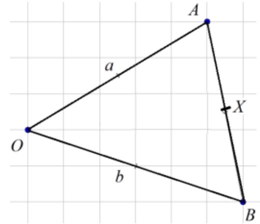
You can multiply a vector by a scalar as shown to the far right.



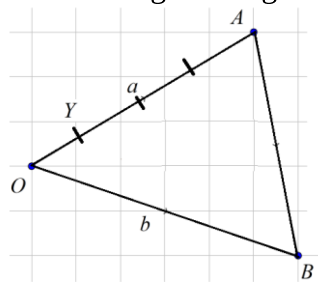
Scalar multiples



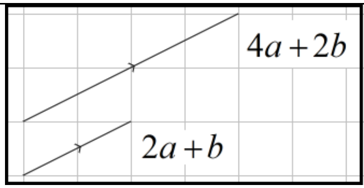
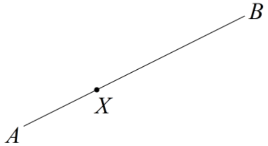
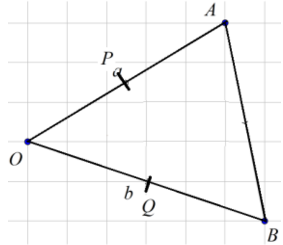
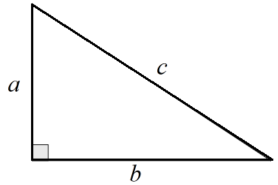
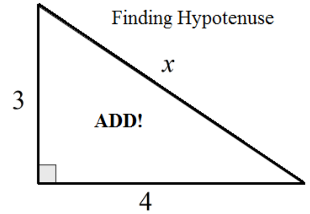
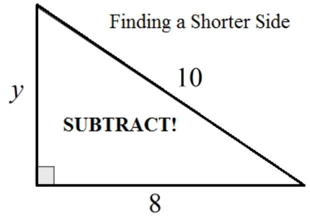
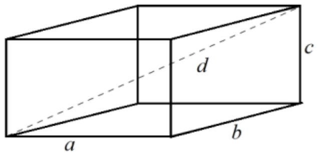
<table>
<tr>
<td><u>Vectors (Solving Geometric Problems)</u></td>
<td>You can use vectors to solve problems in geomerty. It's important to remember that vectors have both direction and magnitude (size). You must look at which way the arros on the vectors are pointing!
You can find a 'vector journey' by simply tracing your finger along the given route you want to take. The diagram below shows how to get from each point in the triangle to the others. You can see that travelling from O to A is different from A to O. The magnitude of the vector is the same but the direction is reversed to give the negative value.
We can say $\overrightarrow{OA} = a$ and $\overrightarrow{AO} = -a$
Look out for the arrows to show the direction of the vector.


$\overrightarrow{OA} = a \quad \overrightarrow{AO} = -a$
$\overrightarrow{OB} = b \quad \overrightarrow{BO} = -b$
$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -a + b = b - a$
$\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA} = -b + a = a - b$
You may be given midpoints or ratios in questions. Simply set up a vector journey and use the information given. Drawing it out will really help especially as most vectors questions don’t include the grid.
The example to the right shows this.
You will need to be able to simplify and factor basic algebraic expressions as they will often highlight parallel vectors.
An example could be $\overrightarrow{AB} = 2p - \frac{1}{2}(4q - 2p)$. This simplifies to give $p - 2q$ which is parallel to $3(p - 2q)$ for example. This is covered below.
With ratios if you have a ratio of 2:3 then the line is split into 5 parts. So one part is 2/5 of the line and the other 3/5. A ratio of 5:7 has 12 parts so it would be split as 5/12 and 7/12.</td>
<td>Example 1: X is the midpoint of AB. Find $\overrightarrow{OX}$
Answer: Draw X on the original diagram


Now build up a journey.
You could use $\overrightarrow{OX} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$.
This will give: $\overrightarrow{OX} = a + \frac{1}{2}(b - a)$.
This will simplify to $\frac{1}{2}a + \frac{1}{2}b$ or $\frac{1}{2}(a + b)$

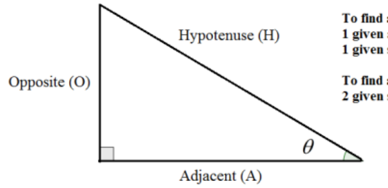
Example 2: Y is the point on OA such that the ratio $OY : YA$ is $1:3$. Find $\overrightarrow{BY}$.
Answer: Draw Y on the original diagram


If the ratio is $1:3$ split the line into four parts (quarters). and simply build a vector journey.
$\overrightarrow{BY} = \overrightarrow{BO} + \frac{1}{4}\overrightarrow{OA}$ which gives $\overrightarrow{BY} = -b + \frac{1}{4}a$.</td>
</tr>
<tr>
<td><u>Parallel and Collinear Vectors</u></td>
<td>Parallel vectors will simply be a mutliple of of each other. If you have the vector $2a + b$ and the vector $4a + 2b$ then you can say these are parallel. $4a + 2b \equiv 2(2a + b)$. This means that $4a + 2b$ is in the same direction (or parallel) to $2a + b$ but twice the length. By factoring, it's easier to see parallel vectors and will help when you construct a proof in a vectors question.</td>
<td>Example: The points P and Q are the midpoints of the lines OA and OB respectively. Show that the lines AB and PQ are parallel.
Answer: Mark the points P and Q on the diagram.</td>
</tr>
</table>

	 **Collinear** points are points on the same straight line as shown below.  $\overrightarrow{AX} = k\overrightarrow{AB}$ (where k is just a scalar enlargement) shows that line lines AX and AB are parallel. As they both pass through A it can be said that they are collinear, or if you like, are all on the same line. To show 3 points are collinear you must show that one vector is a multiple of the other **AND** **both** vectors pass through one of the points. Showing the two vectors are parallel is not enough as parallel vectors could be anywhere on 'the grid'.	 You have seen in a previous example that $\overrightarrow{AB} = b - a$. All you need to do is show that $\overrightarrow{PQ} = k(b-a)$. $\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$ which will give $\overrightarrow{PQ} = -\frac{1}{2}a + \frac{1}{2}b$ which in turn will tidy to $\overrightarrow{PQ} = \frac{1}{2}(b-a)$. This is a vector parallel to $\overrightarrow{AB}$ and half its length.
<u>Pythagoras Theorem for Right Angle Triangles</u>	**Pythagoras Theorem** is used to find **missing lengths** in **right angled triangles** when 2 side lengths are given. The triangle must be a right angled triangle. $a^2 + b^2 = c^2$  a & b are the **2 shorter sides** and c is the **hypotenuse** (longest). Make sure you label each correctly. Neither shorter side can be longer than the hypotenuse! Make sure you square root the answer to find the length. You + when you need the hypotenuse & - when you need a shorter side.	 $a = 3,\ b = 4,\ c = x$ $a^2 + b = c^2$ $9 + 16 = x^2$ $25 = x^2$ $5 = x$  $a = y,\ b = 8,\ c = 10$ $a^2 = c^2 - b^2$ $y^2 = 100 - 64$ $y^2 = 36$ $y = 6$
<u>3D Pythagoras Theorem</u>	**Pythagoras Theorem** in 3D is used to find **missing lengths** in cuboids. Use the same principles as 2D. $a^2 + b^2 + c^2 = d^2$. Here d is the diagonal of the box.  Look out for pencil in box questions. These just want the max diagonal length.	**Example:** Find the longest pencil that can fit in a pencil tin with dimensions 12cm, 13cm and 9cm. The pencil tin is in the shape of a cuboid. **Answer:** $Length = \sqrt{12^2 + 13^2 + 9^2}$ which gives $19.8cm$ correct to 1dp. *If you are given a cube the side lengths are the same. If you are working backwards set* $3x^2 =$ *to the diagonal.*

Trigonometric Ratios

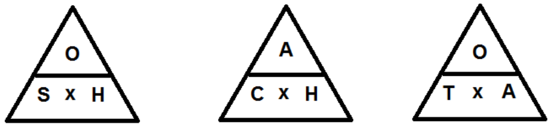
Trigonometric ratios are used to find **missing lengths and angles** in right angled triangles. You would use Pythagoras if you had 2 given sides and need to find the 3rd. The triangle shows each side length **relative** to the angle θ.



To find a missing side length you need:
1 given angle size
1 given side length

To find a missing angle you need:
2 given side lengths

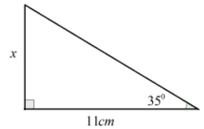
You can use the Trig Ratios below to help find missing lengths and angles.



$\sin = \frac{\text{opposite}}{\text{hypotenuse}}$ $\cos = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\tan = \frac{\text{opposite}}{\text{adjacent}}$

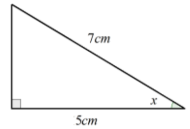
Use $\sin, \cos \,\&\, \tan$ for finding lengths and $\sin^{-1}, \cos^{-1} \,\&\, \tan^{-1}$ for finding angles. Press 'shift' on your Casio when you need the angle!

Example 1: (a) Find the value of x



Answer: You want the opposite (length) and you have the adjacent side to the given angle. You use tan here.
$x = 11 \times \tan(35^0)$ which gives $x = 7.70cm$

Example 2: (b) Find the value of x



Answer: You want the angle. You have the adjacent side and the hypotenuse. You use cos here.

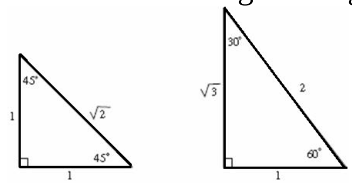
$\cos(x) = \frac{5}{7}$ which gives $x = \cos^{-1}\left(\frac{5}{7}\right)$ and $x = 44.4^o$

Special Values for Angles in Trigonometry

You will need to learn the values for special angles in the table below.

	0^o	30^o	45^o	60^o	90^o
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$ *or* $\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$ *or* $\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$ *or* $\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	

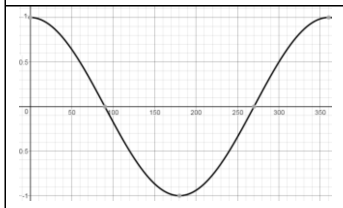
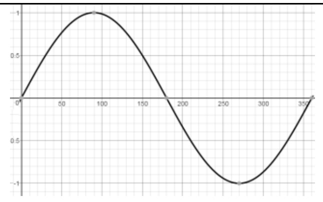
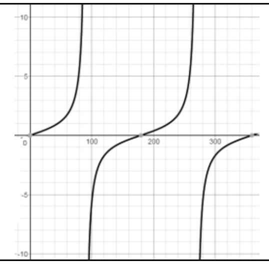
You can derive these values using the diagrams below.

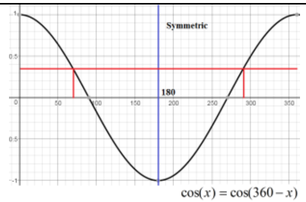


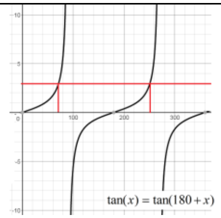
Remember: $\sin(x) = \frac{opp}{hyp}$, $\cos(x) = \frac{adj}{hyp}$ & $\tan(x) = \frac{opp}{adj}$

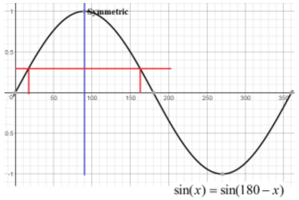
Graphs of Trig Functions (with basic equations)

There are 3 trigonometric graphs you will need to know:

$y = \cos(x)$ for $0 \le x \le 360^o$	$y = \sin(x)$ for $0 \le x \le 360^o$	$y = \tan(x)$ for $0 \le x \le 360^o$
		
Repeats every 360^o	Repeats every 360^o	Repeats every 180^o







You can find values and solve equations using symmetry and cycles. This is covered in the videos.

Transforming Trig Graphs

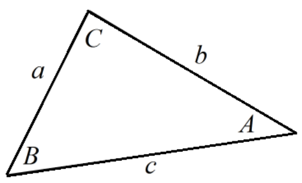
The same rules apply as shown previously for algebraic graphs. The horizontal translations will be in degrees! See the full video to help on algebraic graphs.

Translations Reflections and Stretches.

Sine Rule (Including Ambiguous Case)

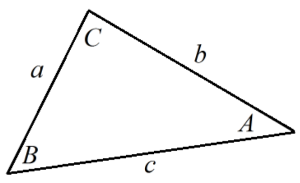
Which Rule do I Use?

Missing Lengths



$$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$$

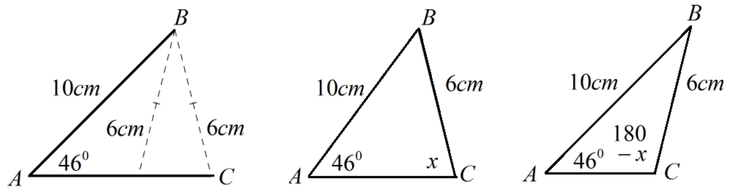
Missing Angles



$$\frac{\sin(A)}{a}=\frac{\sin(B)}{b}=\frac{\sin(C)}{c}$$

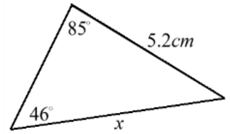
The Sine Rule allows you to find missing lengths and angles in non-right angled triangles. You can use the cosine rule in 2 situations:
(1) To find a missing length when **2 sides** are given and **1 'non-enclosed' angle.**
(2) To find a missing angle when **2 angles** are given and **1 side** is given.

Be careful! Look out for the ambiguous case of the sign rule.



A triangle could be formed two different ways when two side lengths are the same and one angle as shown above. Use the sine rule to find the acute angle and then subtract this value from 180° to get the other possible angle.

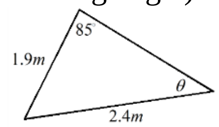
Example 1: (find a missing length): Find the value of x



Answer: Use the formula to give $\dfrac{x}{\sin(46^{\circ})}=\dfrac{5.2}{\sin(85^{\circ})}$

This gives $x=\dfrac{5.2\sin(46^{\circ})}{\sin(85^{\circ})}$ which leads to $x=3.75cm$

Example 2: (find a missing angle): Find the value of θ.



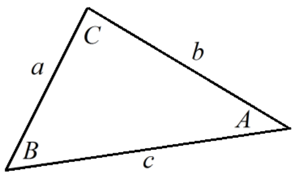
Answer: The formula to give $\dfrac{\sin(\theta)}{1.9}=\dfrac{\sin(85^{\circ})}{2.4}$ which can be written as $\sin(\theta)=\dfrac{1.9\sin(85^{\circ})}{2.4}$.

This gives $\theta=\sin^{-1}\left(\dfrac{1.9\sin(85^{\circ})}{2.4}\right)$ and $\theta=52.1^{\circ}$.

Cosine Rule

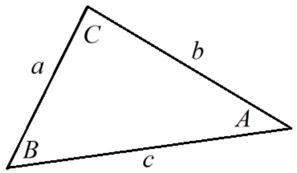
Which Rule do I Use?

Missing Lengths



$$a^2=b^2+c^2-2bc\cos(A)$$

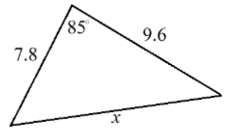
Missing Angles



$$\cos(A)=\frac{b^2+c^2-a^2}{2bc}$$

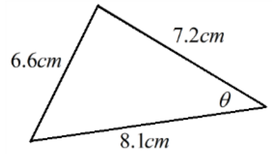
The Cosine Rule allows us to find missing lengths and angles in non-right angled triangles. You can use the cosine rule in 2 situations:
(1) To find a missing length when there is an enclosed angle and 2 given sides.
(2) To find a missing angle when 3 side lengths are given.
Make sure your calculator is in degrees mode!
Remember that you can still use Pythagoras Theorem and the trig ratios (SOH/CAH/TOA) when you have right angled triangles!

Q1: (finding a missing length): Find the value of x



A: Use the formula on the left to fins a missing length:
$x=\sqrt{7.8^2+9.6^2-2(7.8)(9.6)\cos(85^{\circ})}\quad \therefore x=11.8$

Q2: (find a missing angle): Find the value of θ.



A: $\theta=\cos^{-1}\left(\dfrac{7.2^2+8.1^2-6.6^2}{2(7.2)(8.1)}\right)$ which gives: $\theta=50.7^{\circ}$

Index

Date	Topic I need to work on	What have I done about it?	Teacher Suggestions	Update	Has progress been made?
10/1/2017	*Finding the area of a circle*	*I have watched videos and taken my notes home from lessons to practice.*	*Always have the formula to hand and substitute the values in.*	*10/2/2017*	*Yes, I can now do the basic examples and learning harder examples*