9.2 Arithmetic Sequences and Partial Sums

What you should learn

- Recognize, write, and find the *n*th terms of arithmetic sequences.
- Find *n*th partial sums of arithmetic sequences.
- Use arithmetic sequences to model and solve real-life problems.

Why you should learn it

Arithmetic sequences have practical real-life applications. For instance, in Exercise 83 on page 660, an arithmetic sequence is used to model the seating capacity of an auditorium.

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Arithmetic Sequences

A sequence whose consecutive terms have a common difference is called an **arithmetic sequence.**

Definition of Arithmetic Sequence

A sequence is **arithmetic** if the differences between consecutive terms are the same. So, the sequence

$$a_1, a_2, a_3, a_4, \ldots, a_n, \ldots$$

is arithmetic if there is a number d such that

$$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots = d.$$

The number d is the **common difference** of the arithmetic sequence.

Example 1 Examples of Arithmetic Sequences

a. The sequence whose nth term is $4n + 3$ is arithmetic. For this sequence, the common difference between consecutive terms is 4.

$$7, 11, 15, 19, \ldots, 4n + 3, \ldots \qquad \text{Begin with } n = 1.$$

$$11 - 7 = 4$$

b. The sequence whose nth term is $7 - 5n$ is arithmetic. For this sequence, the common difference between consecutive terms is -5.

$$2, -3, -8, -13, \ldots, 7 - 5n, \ldots \qquad \text{Begin with } n = 1.$$

$$-3 - 2 = -5$$

c. The sequence whose nth term is $\frac{1}{4}(n + 3)$ is arithmetic. For this sequence, the common difference between consecutive terms is $\frac{1}{4}$.

$$1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, \ldots, \frac{n+3}{4}, \ldots \qquad \text{Begin with } n = 1.$$

$$\frac{5}{4} - 1 = \frac{1}{4}$$

✓CHECKPOINT Now try Exercise 1.

The sequence 1, 4, 9, 16, . . . , whose nth term is n^2, is *not* arithmetic. The difference between the first two terms is

$$a_2 - a_1 = 4 - 1 = 3$$

but the difference between the second and third terms is

$$a_3 - a_2 = 9 - 4 = 5.$$

In Example 1, notice that each of the arithmetic sequences has an nth term that is of the form $dn + c$, where the common difference of the sequence is d. An arithmetic sequence may be thought of as a linear function whose domain is the set of natural numbers.

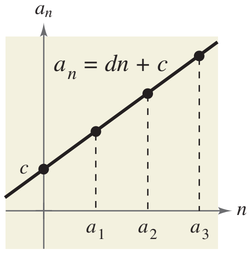


FIGURE 9.3

The nth Term of an Arithmetic Sequence

The nth term of an arithmetic sequence has the form

$$a_n = dn + c \qquad \text{Linear form}$$

where d is the common difference between consecutive terms of the sequence and $c = a_1 - d$. A graphical representation of this definition is shown in Figure 9.3. Substituting $a_1 - d$ for c in $a_n = dn + c$ yields an alternative *recursion* form for the nth term of an arithmetic sequence.

$$a_n = a_1 + (n - 1)\,d \qquad \text{Alternative form}$$

STUDY TIP

The alternative recursion form of the nth term of an arithmetic sequence can be derived from the pattern below.

$a_1 = a_1$ — 1st term

$a_2 = a_1 + d$ — 2nd term

$a_3 = a_1 + 2d$ — 3rd term

$a_4 = a_1 + 3d$ — 4th term

$a_5 = a_1 + 4d$ — 5th term

1 less

⋮

$a_n = a_1 + (n - 1)\,d$ — nth term

1 less

Example 2 Finding the nth Term of an Arithmetic Sequence

Find a formula for the nth term of the arithmetic sequence whose common difference is 3 and whose first term is 2.

Solution

Because the sequence is arithmetic, you know that the formula for the nth term is of the form $a_n = dn + c$. Moreover, because the common difference is $d = 3$, the formula must have the form

$$a_n = 3n + c. \qquad \text{Substitute 3 for } d.$$

Because $a_1 = 2$, it follows that

$$c = a_1 - d$$
$$= 2 - 3 \qquad \text{Substitute 2 for } a_1 \text{ and 3 for } d.$$
$$= -1.$$

So, the formula for the nth term is

$$a_n = 3n - 1.$$

The sequence therefore has the following form.

$$2, 5, 8, 11, 14, \ldots, 3n - 1, \ldots$$

CHECKPOINT Now try Exercise 21.

Another way to find a formula for the nth term of the sequence in Example 2 is to begin by writing the terms of the sequence.

a_1	a_2	a_3	a_4	a_5	a_6	a_7	...
2	2 + 3	5 + 3	8 + 3	11 + 3	14 + 3	17 + 3	...
2	5	8	11	14	17	20	...

From these terms, you can reason that the nth term is of the form

$$a_n = dn + c = 3n - 1.$$

As an aid to learning the formula for the nth term of an arithmetic sequence, consider having your students intuitively find the nth term of each of the following sequences.

1. $5, 8, 11, 14, 17, \ldots$

   Answer: $3n + 2$

2. $a, a + 2, a + 4, a + 6, \ldots$

   Answer: $2n + a - 2$

STUDY TIP

You can find a_1 in Example 3 by using the alternative recursion form of the nth term of an arithmetic sequence, as follows.

$$a_n = a_1 + (n - 1)d$$
$$a_4 = a_1 + (4 - 1)d$$
$$20 = a_1 + (4 - 1)5$$
$$20 = a_1 + 15$$
$$5 = a_1$$

Example 3 Writing the Terms of an Arithmetic Sequence

The fourth term of an arithmetic sequence is 20, and the 13th term is 65. Write the first 11 terms of this sequence.

Solution

You know that $a_4 = 20$ and $a_{13} = 65$. So, you must add the common difference d nine times to the fourth term to obtain the 13th term. Therefore, the fourth and 13th terms of the sequence are related by

$$a_{13} = a_4 + 9d. \qquad a_4 \text{ and } a_{13} \text{ are nine terms apart.}$$

Using $a_4 = 20$ and $a_{13} = 65$, you can conclude that $d = 5$, which implies that the sequence is as follows.

a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11} . . .
5	10	15	20	25	30	35	40	45	50	55 . . .

CHECKPOINT Now try Exercise 37.

If you know the nth term of an arithmetic sequence *and* you know the common difference of the sequence, you can find the $(n + 1)$th term by using the *recursion formula*

$$a_{n+1} = a_n + d. \qquad \text{Recursion formula}$$

With this formula, you can find any term of an arithmetic sequence, *provided* that you know the preceding term. For instance, if you know the first term, you can find the second term. Then, knowing the second term, you can find the third term, and so on.

Example 4 Using a Recursion Formula

Find the ninth term of the arithmetic sequence that begins with 2 and 9.

Solution

For this sequence, the common difference is $d = 9 - 2 = 7$. There are two ways to find the ninth term. One way is simply to write out the first nine terms (by repeatedly adding 7).

2, 9, 16, 23, 30, 37, 44, 51, 58

Another way to find the ninth term is to first find a formula for the nth term. Because the first term is 2, it follows that

$$c = a_1 - d = 2 - 7 = -5.$$

Therefore, a formula for the nth term is

$$a_n = 7n - 5$$

which implies that the ninth term is

$$a_9 = 7(9) - 5 = 58.$$

CHECKPOINT Now try Exercise 45.

The Sum of a Finite Arithmetic Sequence

There is a simple formula for the *sum* of a finite arithmetic sequence.

STUDY TIP

Note that this formula works only for *arithmetic* sequences.

The Sum of a Finite Arithmetic Sequence

The sum of a finite arithmetic sequence with n terms is

$$S_n = \frac{n}{2}(a_1 + a_n).$$

For a proof of the sum of a finite arithmetic sequence, see Proofs in Mathematics on page 723.

Example 5 Finding the Sum of a Finite Arithmetic Sequence

Find the sum: $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$.

Solution

To begin, notice that the sequence is arithmetic (with a common difference of 2). Moreover, the sequence has 10 terms. So, the sum of the sequence is

$$S_n = \frac{n}{2}(a_1 + a_n) \qquad \text{Formula for the sum of an arithmetic sequence}$$

$$= \frac{10}{2}(1 + 19) \qquad \text{Substitute 10 for } n, \text{ 1 for } a_1, \text{ and 19 for } a_n.$$

$$= 5(20) = 100. \qquad \text{Simplify.}$$

✓CHECKPOINT Now try Exercise 63.

Example 6 Finding the Sum of a Finite Arithmetic Sequence

Find the sum of the integers (a) from 1 to 100 and (b) from 1 to N.

Solution

a. The integers from 1 to 100 form an arithmetic sequence that has 100 terms. So, you can use the formula for the sum of an arithmetic sequence, as follows.

$$S_n = 1 + 2 + 3 + 4 + 5 + 6 + \cdot \cdot \cdot + 99 + 100$$

$$= \frac{n}{2}(a_1 + a_n) \qquad \text{Formula for sum of an arithmetic sequence}$$

$$= \frac{100}{2}(1 + 100) \qquad \text{Substitute 100 for } n, \text{ 1 for } a_1, \text{ 100 for } a_n.$$

$$= 50(101) = 5050 \qquad \text{Simplify.}$$

b. $S_n = 1 + 2 + 3 + 4 + \cdot \cdot \cdot + N$

$$= \frac{n}{2}(a_1 + a_n) \qquad \text{Formula for sum of an arithmetic sequence}$$

$$= \frac{N}{2}(1 + N) \qquad \text{Substitute } N \text{ for } n, \text{ 1 for } a_1, \text{ and } N \text{ for } a_n.$$

✓CHECKPOINT Now try Exercise 65.

The Granger Collection

Historical Note

A teacher of Carl Friedrich Gauss (1777–1855) asked him to add all the integers from 1 to 100. When Gauss returned with the correct answer after only a few moments, the teacher could only look at him in astounded silence. This is what Gauss did:

$$\begin{aligned} S_n &= 1 + 2 + 3 + \cdots + 100 \\ S_n &= 100 + 99 + 98 + \cdots + 1 \\ \hline 2S_n &= 101 + 101 + 101 + \cdots + 101 \end{aligned}$$

$$S_n = \frac{100 \times 101}{2} = 5050$$

The sum of the first n terms of an infinite sequence is the *nth partial sum*. The nth partial sum can be found by using the formula for the sum of a finite arithmetic sequence.

Example 7 Finding a Partial Sum of an Arithmetic Sequence

Find the 150th partial sum of the arithmetic sequence

$$5, 16, 27, 38, 49, \ldots.$$

Solution

For this arithmetic sequence, $a_1 = 5$ and $d = 16 - 5 = 11$. So,

$$c = a_1 - d = 5 - 11 = -6$$

and the nth term is $a_n = 11n - 6$. Therefore, $a_{150} = 11(150) - 6 = 1644$, and the sum of the first 150 terms is

$$S_{150} = \frac{n}{2}(a_1 + a_{150}) \qquad \text{nth partial sum formula}$$

$$= \frac{150}{2}(5 + 1644) \qquad \text{Substitute 150 for n, 5 for a_1, and 1644 for a_{150}.}$$

$$= 75(1649) \qquad \text{Simplify.}$$

$$= 123{,}675. \qquad \text{nth partial sum}$$

CHECKPOINT Now try Exercise 69.

Applications

Example 8 Prize Money

In a golf tournament, the 16 golfers with the lowest scores win cash prizes. First place receives a cash prize of \$1000, second place receives \$950, third place receives \$900, and so on. What is the total amount of prize money?

Solution

The cash prizes awarded form an arithmetic sequence in which the common difference is $d = -50$. Because

$$c = a_1 - d = 1000 - (-50) = 1050$$

you can determine that the formula for the nth term of the sequence is $a_n = -50n + 1050$. So, the 16th term of the sequence is $a_{16} = -50(16) + 1050 = 250$, and the total amount of prize money is

$$S_{16} = 1000 + 950 + 900 + \cdots + 250$$

$$S_{16} = \frac{n}{2}(a_1 + a_{16}) \qquad \text{nth partial sum formula}$$

$$= \frac{16}{2}(1000 + 250) \qquad \text{Substitute 16 for n, 1000 for a_1, and 250 for a_{16}.}$$

$$= 8(1250) = \$10{,}000. \qquad \text{Simplify.}$$

CHECKPOINT Now try Exercise 89.

Example 9 Total Sales

A small business sells \$10,000 worth of skin care products during its first year. The owner of the business has set a goal of increasing annual sales by \$7500 each year for 9 years. Assuming that this goal is met, find the total sales during the first 10 years this business is in operation.

Solution

The annual sales form an arithmetic sequence in which $a_1 = 10{,}000$ and $d = 7500$. So,

$$c = a_1 - d$$
$$= 10{,}000 - 7500$$
$$= 2500$$

and the nth term of the sequence is

$$a_n = 7500n + 2500.$$

This implies that the 10th term of the sequence is

$$a_{10} = 7500(10) + 2500$$
$$= 77{,}500. \qquad \text{See Figure 9.4.}$$

The sum of the first 10 terms of the sequence is

$$S_{10} = \frac{n}{2}(a_1 + a_{10}) \qquad n\text{th partial sum formula}$$
$$= \frac{10}{2}(10{,}000 + 77{,}500) \qquad \text{Substitute 10 for } n\text{, 10,000 for } a_1\text{, and 77,500 for } a_{10}.$$
$$= 5(87{,}500) \qquad \text{Simplify.}$$
$$= 437{,}500. \qquad \text{Simplify.}$$

So, the total sales for the first 10 years will be \$437,500.

CHECKPOINT Now try Exercise 91.

Activities

1. Determine which of the following are arithmetic sequences.
   (a) 3, 5, 7, 9, 11, . . .
   (b) 3, 6, 12, 24, 48, . . .
   (c) −3, 6, −9, 12, −15, . . .
   (d) 5, 0, −5, −10, −15, . . .
   (e) 1, 3, 6, 10, 15, 21, . . .
   Answer: (a) and (d)
2. Find the first five terms of the arithmetic sequence with $a_1 = 13$ and $d = -4$.
   Answer: 13, 9, 5, 1, −3
3. Find the sum.
   $\sum_{n=1}^{100} (2 + 3n)$
   Answer: 15,350

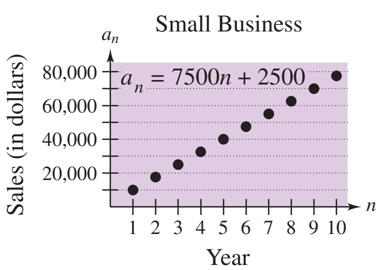


FIGURE 9.4

WRITING ABOUT MATHEMATICS

Numerical Relationships Decide whether it is possible to fill in the blanks in each of the sequences such that the resulting sequence is arithmetic. If so, find a recursion formula for the sequence.

a. −7, ___, ___, ___, ___, ___, 11
b. 17, ___, ___, ___, ___, ___, ___, ___, ___, 71
c. 2, 6, ___, ___, 162
d. 4, 7.5, ___, ___, ___, ___, ___, ___, ___, ___, 39
e. 8, 12, ___, ___, ___, 60.75

9.2 Exercises

VOCABULARY CHECK: Fill in the blanks.

1. A sequence is called an ________ sequence if the differences between two consecutive terms are the same. This difference is called the ________ difference.

2. The nth term of an arithmetic sequence has the form ________.

3. The formula $S_n = \frac{n}{2}(a_1 + a_n)$ can be used to find the sum of the first n terms of an arithmetic sequence, called the ________ of a ________ ________ ________.

PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–10, determine whether the sequence is arithmetic. If so, find the common difference.

1. 10, 8, 6, 4, 2, . . .

2. 4, 7, 10, 13, 16, . . .

3. 1, 2, 4, 8, 16, . . .

4. 80, 40, 20, 10, 5, . . .

5. $\frac{9}{4}, 2, \frac{7}{4}, \frac{3}{2}, \frac{5}{4}, \ldots$

6. $3, \frac{5}{2}, 2, \frac{3}{2}, 1, \ldots$

7. $\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{6}, \ldots$

8. 5.3, 5.7, 6.1, 6.5, 6.9, . . .

9. ln 1, ln 2, ln 3, ln 4, ln 5, . . .

10. $1^2, 2^2, 3^2, 4^2, 5^2, \ldots$

In Exercises 11–18, write the first five terms of the sequence. Determine whether the sequence is arithmetic. If so, find the common difference. (Assume that *n* begins with 1.)

11. $a_n = 5 + 3n$

12. $a_n = 100 - 3n$

13. $a_n = 3 - 4(n - 2)$

14. $a_n = 1 + (n - 1)4$

15. $a_n = (-1)^n$

16. $a_n = 2^{n-1}$

17. $a_n = \frac{(-1)^n 3}{n}$

18. $a_n = (2^n)n$

In Exercises 19–30, find a formula for a_n for the arithmetic sequence.

19. $a_1 = 1, d = 3$

20. $a_1 = 15, d = 4$

21. $a_1 = 100, d = -8$

22. $a_1 = 0, d = -\frac{2}{3}$

23. $a_1 = x, d = 2x$

24. $a_1 = -y, d = 5y$

25. $4, \frac{3}{2}, -1, -\frac{7}{2}, \ldots$

26. $10, 5, 0, -5, -10, \ldots$

27. $a_1 = 5, a_4 = 15$

28. $a_1 = -4, a_5 = 16$

29. $a_3 = 94, a_6 = 85$

30. $a_5 = 190, a_{10} = 115$

In Exercises 31–38, write the first five terms of the arithmetic sequence.

31. $a_1 = 5, d = 6$

32. $a_1 = 5, d = -\frac{3}{4}$

33. $a_1 = -2.6, d = -0.4$

34. $a_1 = 16.5, d = 0.25$

35. $a_1 = 2, a_{12} = 46$

36. $a_4 = 16, a_{10} = 46$

37. $a_8 = 26, a_{12} = 42$

38. $a_3 = 19, a_{15} = -1.7$

In Exercises 39–44, write the first five terms of the arithmetic sequence. Find the common difference and write the *n*th term of the sequence as a function of *n*.

39. $a_1 = 15, \quad a_{k+1} = a_k + 4$

40. $a_1 = 6, \quad a_{k+1} = a_k + 5$

41. $a_1 = 200, \quad a_{k+1} = a_k - 10$

42. $a_1 = 72, \quad a_{k+1} = a_k - 6$

43. $a_1 = \frac{5}{8}, \quad a_{k+1} = a_k - \frac{1}{8}$

44. $a_1 = 0.375, \quad a_{k+1} = a_k + 0.25$

In Exercises 45–48, the first two terms of the arithmetic sequence are given. Find the missing term.

45. $a_1 = 5, a_2 = 11, a_{10} =$ ▢

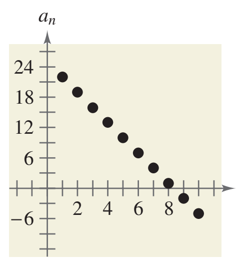
46. $a_1 = 3, a_2 = 13, a_9 =$ ▢

47. $a_1 = 4.2, a_2 = 6.6, a_7 =$ ▢

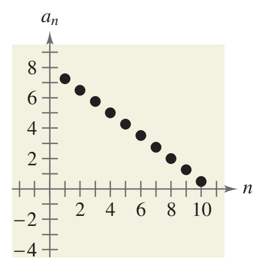
48. $a_1 = -0.7, a_2 = -13.8, a_8 =$ ▢

In Exercises 49–52, match the arithmetic sequence with its graph. [The graphs are labeled (a), (b), (c), and (d).]

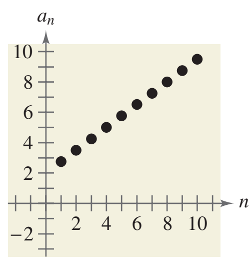
(a)



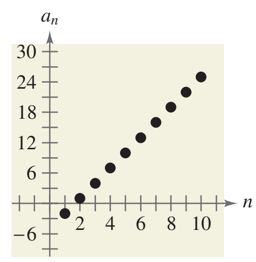
(b)



(c)



(d)



49. $a_n = -\frac{3}{4}n + 8$

50. $a_n = 3n - 5$

51. $a_n = 2 + \frac{3}{4}n$

52. $a_n = 25 - 3n$

In Exercises 53–56, use a graphing utility to graph the first 10 terms of the sequence. (Assume that *n* begins with 1.)

53. $a_n = 15 - \frac{3}{2}n$

54. $a_n = -5 + 2n$

55. $a_n = 0.2n + 3$

56. $a_n = -0.3n + 8$

In Exercises 57–64, find the indicated *n*th partial sum of the arithmetic sequence.

57. 8, 20, 32, 44, . . . , $n = 10$

58. 2, 8, 14, 20, . . . , $n = 25$

59. 4.2, 3.7, 3.2, 2.7, . . . , $n = 12$

60. 0.5, 0.9, 1.3, 1.7, . . . , $n = 10$

61. 40, 37, 34, 31, . . . , $n = 10$

62. 75, 70, 65, 60, . . . , $n = 25$

63. $a_1 = 100$, $a_{25} = 220$, $n = 25$

64. $a_1 = 15$, $a_{100} = 307$, $n = 100$

65. Find the sum of the first 100 positive odd integers.

66. Find the sum of the integers from -10 to 50.

In Exercises 67–74, find the partial sum.

67. $\sum_{n=1}^{50} n$

68. $\sum_{n=1}^{100} 2n$

69. $\sum_{n=10}^{100} 6n$

70. $\sum_{n=51}^{100} 7n$

71. $\sum_{n=11}^{30} n - \sum_{n=1}^{10} n$

72. $\sum_{n=51}^{100} n - \sum_{n=1}^{50} n$

73. $\sum_{n=1}^{400} (2n - 1)$

74. $\sum_{n=1}^{250} (1000 - n)$

In Exercises 75–80, use a graphing utility to find the partial sum.

75. $\sum_{n=1}^{20} (2n + 5)$

76. $\sum_{n=0}^{50} (1000 - 5n)$

77. $\sum_{n=1}^{100} \frac{n + 4}{2}$

78. $\sum_{n=0}^{100} \frac{8 - 3n}{16}$

79. $\sum_{i=1}^{60} \left(250 - \frac{8}{3}i\right)$

80. $\sum_{j=1}^{200} (4.5 + 0.025j)$

Job Offer **In Exercises 81 and 82, consider a job offer with the given starting salary and the given annual raise.**

(a) Determine the salary during the sixth year of employment.

(b) Determine the total compensation from the company through six full years of employment.

	Starting Salary	*Annual Raise*
81.	\$32,500	\$1500
82.	\$36,800	\$1750

83. ***Seating Capacity*** Determine the seating capacity of an auditorium with 30 rows of seats if there are 20 seats in the first row, 24 seats in the second row, 28 seats in the third row, and so on.

84. ***Seating Capacity*** Determine the seating capacity of an auditorium with 36 rows of seats if there are 15 seats in the first row, 18 seats in the second row, 21 seats in the third row, and so on.

85. ***Brick Pattern*** A brick patio has the approximate shape of a trapezoid (see figure). The patio has 18 rows of bricks. The first row has 14 bricks and the 18th row has 31 bricks. How many bricks are in the patio?

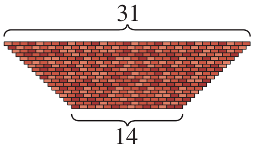


FIGURE FOR 85

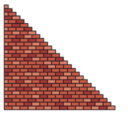


FIGURE FOR 86

86. ***Brick Pattern*** A triangular brick wall is made by cutting some bricks in half to use in the first column of every other row. The wall has 28 rows. The top row is one-half brick wide and the bottom row is 14 bricks wide. How many bricks are used in the finished wall?

87. ***Falling Object*** An object with negligible air resistance is dropped from a plane. During the first second of fall, the object falls 4.9 meters; during the second second, it falls 14.7 meters; during the third second, it falls 24.5 meters; during the fourth second, it falls 34.3 meters. If this arithmetic pattern continues, how many meters will the object fall in 10 seconds?

88. ***Falling Object*** An object with negligible air resistance is dropped from the top of the Sears Tower in Chicago at a height of 1454 feet. During the first second of fall, the object falls 16 feet; during the second second, it falls 48 feet; during the third second, it falls 80 feet; during the fourth second, it falls 112 feet. If this arithmetic pattern continues, how many feet will the object fall in 7 seconds?

89. ***Prize Money*** A county fair is holding a baked goods competition in which the top eight bakers receive cash prizes. First places receives a cash prize of $200, second place receives $175, third place receives $150, and so on.

(a) Write a sequence a_n that represents the cash prize awarded in terms of the place n in which the baked good places.

(b) Find the total amount of prize money awarded at the competition.

90. ***Prize Money*** A city bowling league is holding a tournament in which the top 12 bowlers with the highest three-game totals are awarded cash prizes. First place will win $1200, second place $1100, third place $1000, and so on.

(a) Write a sequence a_n that represents the cash prize awarded in terms of the place n in which the bowler finishes.

(b) Find the total amount of prize money awarded at the tournament.

91. ***Total Profit*** A small snowplowing company makes a profit of $8000 during its first year. The owner of the company sets a goal of increasing profit by $1500 each year for 5 years. Assuming that this goal is met, find the total profit during the first 6 years of this business. What kinds of economic factors could prevent the company from meeting its profit goal? Are there any other factors that could prevent the company from meeting its goal? Explain.

92. ***Total Sales*** An entrepreneur sells $15,000 worth of sports memorabilia during one year and sets a goal of increasing annual sales by $5000 each year for 9 years. Assuming that this goal is met, find the total sales during the first 10 years of this business. What kinds of economic factors could prevent the business from meeting its goals?

93. ***Borrowing Money*** You borrowed $2000 from a friend to purchase a new laptop computer and have agreed to pay back the loan with monthly payments of $200 plus 1% interest on the unpaid balance.

(a) Find the first six monthly payments you will make, and the unpaid balance after each month.

(b) Find the total amount of interest paid over the term of the loan.

94. ***Borrowing Money*** You borrowed $5000 from your parents to purchase a used car. The arrangements of the loan are such that you will make payments of $250 per month plus 1% interest on the unpaid balance.

(a) Find the first year's monthly payments you will make, and the unpaid balance after each month.

(b) Find the total amount of interest paid over the term of the loan.

Model It

95. ***Data Analysis: Personal Income*** The table shows the per capita personal income a_n in the United States from 1993 to 2003. (Source: U.S. Bureau of Economic Analysis)

Year	Per capita personal income, a_n
1993	$21,356
1994	$22,176
1995	$23,078
1996	$24,176
1997	$25,334
1998	$26,880
1999	$27,933
2000	$29,848
2001	$30,534
2002	$30,913
2003	$31,633

(a) Find an arithmetic sequence that models the data. Let n represent the year, with $n = 3$ corresponding to 1993.

(b) Use the *regression* feature of a graphing utility to find a linear model for the data. How does this model compare with the arithmetic sequence you found in part (a)?

(c) Use a graphing utility to graph the terms of the finite sequence you found in part (a).

(d) Use the sequence from part (a) to estimate the per capita personal income in 2004 and 2005.

(e) Use your school's library, the Internet, or some other reference source to find the actual per capita personal income in 2004 and 2005, and compare these values with the estimates from part (d).

96. ***Data Analysis: Revenue*** The table shows the annual revenue a_n (in millions of dollars) for Nextel Communications, Inc. from 1997 to 2003. (Source: Nextel Communications, Inc.)

Year	Revenue, a_n
1997	739
1998	1847
1999	3326
2000	5714
2001	7689
2002	8721
2003	10,820

(a) Construct a bar graph showing the annual revenue from 1997 to 2003.

(b) Use the *linear regression* feature of a graphing utility to find an arithmetic sequence that approximates the annual revenue from 1997 to 2003.

(c) Use summation notation to represent the *total* revenue from 1997 to 2003. Find the total revenue.

(d) Use the sequence from part (b) to estimate the annual revenue in 2008.

Synthesis

True or False? **In Exercises 97 and 98, determine whether the statement is true or false. Justify your answer.**

97. Given an arithmetic sequence for which only the first two terms are known, it is possible to find the nth term.

98. If the only known information about a finite arithmetic sequence is its first term and its last term, then it is possible to find the sum of the sequence.

99. ***Writing*** In your own words, explain what makes a sequence arithmetic.

100. ***Writing*** Explain how to use the first two terms of an arithmetic sequence to find the nth term.

101. ***Exploration***

(a) Graph the first 10 terms of the arithmetic sequence $a_n = 2 + 3n$.

(b) Graph the equation of the line $y = 3x + 2$.

(c) Discuss any differences between the graph of

$a_n = 2 + 3n$

and the graph of

$y = 3x + 2$.

(d) Compare the slope of the line in part (b) with the common difference of the sequence in part (a). What can you conclude about the slope of a line and the common difference of an arithmetic sequence?

102. ***Pattern Recognition***

(a) Compute the following sums of positive odd integers.

$1 + 3 = \square$

$1 + 3 + 5 = \square$

$1 + 3 + 5 + 7 = \square$

$1 + 3 + 5 + 7 + 9 = \square$

$1 + 3 + 5 + 7 + 9 + 11 = \square$

(b) Use the sums in part (a) to make a conjecture about the sums of positive odd integers. Check your conjecture for the sum

$1 + 3 + 5 + 7 + 9 + 11 + 13 = \square$.

(c) Verify your conjecture algebraically.

103. ***Think About It*** The sum of the first 20 terms of an arithmetic sequence with a common difference of 3 is 650. Find the first term.

104. ***Think About It*** The sum of the first n terms of an arithmetic sequence with first term a_1 and common difference d is S_n. Determine the sum if each term is increased by 5. Explain.

Skills Review

In Exercises 105–108, find the slope and y-intercept (if possible) of the equation of the line. Sketch the line.

105. $2x - 4y = 3$

106. $9x + y = -8$

107. $x - 7 = 0$

108. $y + 11 = 0$

In Exercises 109 and 110, use Gauss-Jordan elimination to solve the system of equations.

109. $\begin{cases} 2x - y + 7z = -10 \\ 3x + 2y - 4z = 17 \\ 6x - 5y + z = -20 \end{cases}$

110. $\begin{cases} -x + 4y + 10z = 4 \\ 5x - 3y + z = 31 \\ 8x + 2y - 3z = -5 \end{cases}$

111. **Make a Decision** To work an extended application analyzing the median sales price of existing one-family homes in the United States from 1987 to 2003, visit this text's website at *college.hmco.com*. *(Data Source: National Association of Realtors)*