## What You'll Learn

- To multiply binomials using FOIL
- To multiply trinomials by binomials
... And Why
To find the area of a geometric figure, as in Example 3


## Check Skills You'll Need

Find each product.

1. $4 r(r-1) 4 r^{2}-4 r$
2. $6 h\left(h^{2}+8 h-3\right)$

Simplify. Write each answer in standard form. 4-9. See margin p. 507.
4. $\left(x^{3}+3 x^{2}+x\right)+\left(5 x^{2}+x+1\right)$
5. $\left(3 t^{3}-6 t+8\right)+\left(5 t^{3}+7 t-2\right)$
6. $w(w+1)+4 w(w-7)$
7. $6 b(b-2)-b(8 b+3)$
8. $m\left(4 m^{2}-6\right)+3 m^{2}(m+9)$
9. $3 d^{2}\left(d^{3}-6\right)-d^{3}\left(2 d^{2}+4\right)$

## 1 <br> Multiplying Two Binomials

You can use an area model to multiply two binomials. The diagram below shows $(2 x+3)(x+4)$.


You can also use the Distributive Property to find the product of two binomials.


Simplify $(2 x+3)(x+4)$.

$$
\begin{aligned}
(2 x+3)(x+4) & =2 x(x+4)+3(x+4) & & \text { Distribute } x+4 . \\
& =2 x^{2}+8 x+3 x+12 & & \text { Now distribute } 2 x \text { and } 3 . \\
& =2 x^{2}+11 x+12 & & \text { Simplify. }
\end{aligned}
$$

Simplify each product.

$$
\text { a. }(6 h-7)(2 h+3)
$$

$$
12 h^{2}+4 h-21
$$

b. $(5 m+2)(8 m-1)$
$40 m^{2}+11 m-2$
c. $(9 a-8)(7 a+4)$ $63 a^{2}-20 a-32$

## DIfierentiated Instruction Solutions for All Learners

## Special Needs <br> L1

Have students who have difficulty in using the Distributive Property, or in drawing area models to multiply two binomials, use algebra tiles to build and explain their own solutions.

## Below Level $\quad 12$

Suggest that students use one method to multiply a trinomial by a binomial, and then use the other method to check their work.

## 1. Plan

## Objectives

1 To multiply binomials using FOIL
2 To multiply trinomials by binomials

## Examples

1 Using the Distributive Property
2 Multiplying Using FOIL
3 Applying Multiplication of Polynomials
4 Multiplying a Trinomial and a Binomial

## Math Background

Finding the product of two binomials by the FOIL method is a shortcut for distributing the first binomial to each term of the second, and then distributing again to find these individual products. However, no corresponding shortcut exists when one factor is a trinomial.

More Math Background: p. 492C

## Lesson Planning and Resources

See p. 492E for a list of the resources that support this lesson.

## Check Skills You'll Need

For intervention, direct students to:

## Multiplying and Factoring

## Lesson 9-2: Example 1

Extra Skills and Word
Problem Practice, Ch. 9

## 2. Teach

## Guided Instruction

## 1) 玉xanple Tactile Learners

Encourage students to draw an area model showing multiplication of the two binomials. Students can easily see there are four products when multiplying binomials.

## Exanple Auditory Learners

On the board, write an expression that multiplies two binomials. Call on students to say aloud the different parts of FOIL for the expression. Repeat with new expressions until all students have responded.

## Example Visual Learners

Some students may not understand why you subtract the area of the inner rectangle. Cut a rectangle out of a sheet of construction paper to represent the example.

## Additional Examples

(1) Simplify $(2 y-3)(y+2)$. $2 y^{2}+y-6$
(2) Simplify $(4 x+2)(3 x-6)$.
$12 x^{2}-18 x-12$
3) Find the area of the shaded region. Simplify. $5 x^{2}-2 x-2$


One way to organize multiplying two binomials is to use FOIL, which stands for "First, Outer, Inner, Last." The term FOIL is a memory device for applying the Distributive Property to the product of two binomials.

Video Tutor Help Visit: PHSchool.com Web Code: ate-0775

(2) EXAMPLE Multiplying Using FOIL

Simplify $(3 x-5)(2 x+7)$.


The product is $6 x^{2}+11 x-35$.
Simplify each product using FOIL.
a. $(3 x+4)(2 x+5) 6 x^{2}+23 x+20 \quad$ b. $(3 x-4)(2 x+5) 6 x^{2}+7 x-20$
c. $(3 x+4)(2 x-5) 6 x^{2}-7 x-20$
d. $(3 x-4)(2 x-5) 6 x^{2}-23 x+20$

## 3 EXANPLE Applying Multiplication of Polynomials

Multiple Choice Which expression best describes the area of the shaded region?
(A) $x^{2}+2 x$
(B) $6 x^{2}+17 x+5$
(C) $5 x^{2}+15 x+5$
(D) $7 x^{2}+9 x+5$

area of outer rectangle $=(3 x+1)(2 x+5)$
area of hole $=x(x+2)$
area of shaded region
$=$ area of outer rectangle - area of hole
$=(3 x+1)(2 x+5) \quad-x(x+2) \quad$ Substitute.
$=6 x^{2}+15 x+2 x+5-x^{2}-2 x$ Use FOIL to simplify $(3 x+1)(2 x+5)$ and the
Distributive Property to simplify $-x(x+2)$.
$=6 x^{2}-x^{2}+15 x+2 x-2 x+5 \quad$ Group like terms.
$=5 x^{2}+15 x+5 \quad$ Simplify.
So C is the correct answer.
© Quick Check
Find an expression for the area of each shaded region. Simplify.
a.

b.


Chapter 9 Polynomials and Factoring

## Differentiated Instruction Solutions for All Learners

## Advanced Learners [4]

Ask students to explain the statement, "The degree of the product of two nonzero polynomials is the sum of the degrees of the polynomials."

English Language Learners ELL
Explain that the term FOIL is a memory tool when multiplying binomials. Make sure students relate each letter to the word that identifies each part of the process (first, outer, inner, and last).

FOIL works when you multiply two binomials, but it is not helpful when multiplying a trinomial and a binomial. You can use the vertical method or the horizontal method to distribute each term in such factors.

## 4 Exadiple <br> Multiplying a Trinomial and a Binomial

Simplify the product $\left(4 x^{2}+x-6\right)(2 x-3)$.
Method 1 Multiply using the vertical method.

| $4 x^{2}+$$x-6$ <br> $2 x-3$ |
| ---: |
| $-12 x^{2}-3 x+18$ <br> $8 x^{3}+2 x^{2}-12 x$$\quad$Multiply by -3. <br> Multiply by $2 x$. Add like terms. |

Method 2 Multiply using the horizontal method.


The product is $8 x^{3}-10 x^{2}-15 x+18$
Quick CheckSimplify $(6 n-8)\left(2 n^{2}+n+7\right)$ using both methods shown in Example 4. $12 n^{3}-10 n^{2}+34 n-56$

## EXERCISES

For more exercises, see Extra Skill and Word Problem Practice.

## Practice and Problem Solving

(A) Practice by Example

Example 1
(page 505)

## Copy and fill in each blank.

1. $(5 a+2)(6 a-1)=\square a^{2}+7 a-230$ 2. $(3 c-7)(2 c-5)=6 c^{2}-29 c+\square$
2. $(z-4)(2 z+1)=2 z^{2}-\square z-47$
3. $(2 x+9)(x+2)=2 x^{2}+\square x+18$

Simplify each product using the Distributive Property. 5-10. See margin.
5. $(x+2)(x+5)$
6. $(h+3)(h+4)$
7. $(k+7)(k-6)$
8. $(a-8)(a-9)$
9. $(2 x-1)(x+2)$
10. $(2 y+5)(y-3)$

Example 2 (page 506)

Simplify each product using FOIL. 11-16. See margin.
11. $(r+6)(r-4)$
12. $(y+4)(5 y-8)$
13. $(x+6)(x-7)$
14. $(m-6)(m-9)$
15. $(4 b-2)(b+3)$
16. $(8 w+2)(w+5)$
17. $(x-7)(x+9)$
18. $(a+11)(a+5)$
19. $(p-1)(p+10)$
$x^{2}+2 x-63$
$a^{2}+16 a+55$
$p^{2}+9 p-10$

Lesson 9-3 Multiplying Binomials

## example Teaching Tip

Help students understand the vertical method for multiplying polynomials by relating it to multiplying whole numbers. Write $312 \times 23$ in vertical format on the board and ask volunteers to help you solve it. Stress how partial products are lined up according to place value.

## Additional Example

Simplify the product
$\left(3 x^{2}-2 x+3\right)(2 x+7)$.
$6 x^{3}+17 x^{2}-8 x+21$

## Resources

- Daily Notetaking Guide 9-3 L3
- Daily Notetaking Guide 9-3Adapted Instruction


## Closure

Ask students what the letters in FOIL represent. F: first terms, O: outer terms, I: inner terms, L: last terms
page 505 Check Skills You'll Need
4. $x^{3}+8 x^{2}+2 x+1$
5. $8 t^{3}+t+6$
6. $5 w^{2}-27 w$
7. $-2 b^{2}-15 b$
8. $7 m^{3}+27 m^{2}-6 m$
9. $d^{5}-4 d^{3}-18 d^{2}$

## pages 507-510 Exercises

5. $x^{2}+7 x+10$
6. $h^{2}+7 h+12$
7. $k^{2}+k-42$
8. $a^{2}-17 a+72$
9. $2 x^{2}+3 x-2$
10. $2 y^{2}-y-15$
11. $r^{2}+2 r-24$
12. $5 y^{2}+12 y-32$
13. $x^{2}-x-42$
14. $m^{2}-15 m+54$
15. $4 b^{2}+10 b-6$
16. $8 w^{2}+42 w+10$

## 3. Practice

Example 3

## Assignment Guide

1 а в $1-21,30-35,42-46$
2 A B $\quad 22-29,36-41$
C Challenge 47-55

| Test Prep | $56-60$ |
| :--- | :--- |
| Mixed Review | $61-86$ |

## Homework Quick Check

To check students' understanding of key skills and concepts, go over Exercises 8, 20, 36, 39, 41.

Exercises 3,4 Remind students to combine the "Outer" and "Inner" products to determine the middle term of the answer.

## Error Prevention!

Exercises 22-25 Suggest that students write the binomial on the second line when using the vertical method.

## Connection to Geometry

Exercise 43 Remind students that the bases of a trapezoid are the two parallel sides.


B
Apply Your Skills Web Code: ate-0903
40. Answers may vary. Sample:
$(x+2)\left(x^{2}+3 x+4\right) ;$
$x^{3}+5 x^{2}+10 x+8$
41. Answers may vary. Sample: vertical method, so you can keep terms aligned

## GO for Help

For a guide to solving Exercise 42, see p. 511.

44a. $x^{2}+2 x+1,121$
$x^{2}+3 x+2,132$
$x^{2}+4 x+3,143$
b. For $a x^{2}+b x+c$, a corresponds to the hundreds place, $b$ to the tens place, and $c$ to the ones place.

## Geometry Find an expression for the area of each shaded region. Simplify.

20. 


$8 x+6$
21.

$-2 x^{2}+5 x+48$

Simplify. Use the vertical method. 22-25. See margin.
22. $(x+9)\left(x^{2}-4 x+1\right)$
23. $(a-4)\left(a^{2}-2 a+1\right)$
24. $(g-3)\left(2 g^{2}+3 g+3\right)$
25. $(k+8)\left(3 k^{2}-5 k+7\right)$

Simplify. Use the horizontal method. 26-29. See margin.
26. $\left(x^{2}+2 x+1\right)(9 x-3)$
27. $\left(t^{2}-6 t+3\right)(2 t-5)$
28. $\left(7 p^{2}+5 p-1\right)(8 p+9)$
29. $\left(12 w^{2}-w-1\right)(4 w-2)$

Simplify each product. Write in standard form. 30-38. See margin.
30. $(p-7)(p+8)$
31. $(-7+p)(8+p)$
32. $\left(p^{2}-7\right)(p+8)$
33. $(5 c-9)(5 c+1)$
34. $\left(n^{2}+3\right)(n+11)$
35. $\left(3 k^{2}+2\right)\left(k+5 k^{2}\right)$
36. $(6 h-1)\left(4 h^{2}+h+3\right)$
37. $\left(9 y^{2}+2\right)\left(y^{2}-y-1\right)$
38. $(8 q-4)\left(6 q^{2}+q+1\right)$
39. Construction You are planning a rectangular garden. Its length is twice its

GPS width $x$. You want a walkway 2 ft wide around the garden.
a. Write an expression for the area of the garden and walk. $2 x^{2}+12 x+16$
b. Write an expression for the area of the walk only. $12 x+16$
c. You have enough gravel to cover $76 \mathrm{ft}^{2}$ and want to use it all on the walk. How big should you make the garden? 10 ft by 5 ft
40. Open-Ended Write a binomial and a trinomial. Find their product. See left.
41. Writing Which method do you prefer for multiplying a binomial and a trinomial? Explain. See left.

Geometry Write an expression for the area of each shaded region. Write your answer in simplest form.
42.

43.

44. a. Simplify each pair of products. a-b. See left.
i. $(x+1)(x+1)$
ii. $(x+1)(x+2)$
$11 \cdot 12$
iii. $(x+1)(x+3)$ $11 \cdot 11$
$11 \cdot 13$
b. Critical Thinking What are the similarities between the two answers in each pair of products?

508 Chapter 9 Polynomials and Factoring
pages 507-510 Exercises
22. $x^{3}+5 x^{2}-35 x+9$
23. $a^{3}-6 a^{2}+9 a-4$
24. $2 g^{3}-3 g^{2}-6 g-9$
25. $3 k^{3}+19 k^{2}-33 k+56$
26. $9 x^{3}+15 x^{2}+3 x-3$
27. $2 t^{3}-17 t^{2}+36 t-15$
28. $56 p^{3}+103 p^{2}+37 p-9$
29. $48 w^{3}-28 w^{2}-2 w+2$
30. $p^{2}+p-56$
31. $p^{2}+p-56$
32. $p^{3}+8 p^{2}-7 p-56$
45. Geometry Use the formula $V=\ell w h$ to write a polynomial in standard form for the volume of the box. $n^{3}+15 n^{2}+56 n$
46. Multiple Choice If $n$ represents an even number, which expression represents the product of the next two even numbers? D

(A) $n^{2}+3 n+2$
(B) $2 n+$
(C) $2 n+6$
(D) $n^{2}+6 n+8$

For Exercises 47-49, each expression represents the side length of a cube. Write an expression in standard form for the surface area of each cube.
47. $x+36 x^{2}+36 x+54$ 48. $4 t+196 t^{2}+48 t+6$ 49. $2 w^{2}+7$ $24 w^{4}+168 w^{2}+294$


Real-World Connection
In 2003, the U.S. consumption of fresh tomatoes was 15.4 lb per person.
50. a. Vegetable Consumption Multiply the expressions on the right side of each equation to create a model for the total number of pounds of fresh vegetables $V(t)$ consumed in a year in the United States.
$V(t)=10.56 t^{2}+1318.1 t+39,250$ $C(t)=3.2 t+157$ the U.S. annual per capita consumption of fresh vegetables, in pounds, from 1990 to 1997

$$
P(t)=3.3 t+250 \quad \text { the U.S. population, in millions, from } 1990 \text { to } 1997
$$

b. Evaluate the equation you found in part (a) with $t=5$ to find the total vegetable consumption for 1995. $(t=0$ corresponds to the year 1990.) 3 51. Financial Planning Suppose you deposit $\$ 2000$ for college in a savings account that has an annual interest rate $r$. At the end of three years, the value of your account will be $2000(1+r)^{3}$ dollars. a. $2000 r^{3}+6000 r^{2}+6000 r+2000$ a. Rewrite the expression $2000(1+r)^{3}$ by finding the product $2000(1+r)(1+r)(1+r)$. Write your answer in standard form.
b. Find the amount of money in the account if the interest rate is $3 \% . \$ 2185.45$

For Exercises 52-55, find each product using lattice multiplication, which is explained below.
Lattice multiplication probably originated in India in the twelfth century. It came into use in Italy in the fourteenth century.


> This example shows $27 \cdot 39$. Each number is treated as a binomial. The four products are placed in the small, diagonally split squares. The product of 2 and 3 , shown in red, is 6 . The first square shows $0 / 6$, which indicates 6 . The product of 7 and 3 is 21 . The second square shows $2 / 1$.

The products are totaled diagonally. For the diagonal shaded blue, the tens place of the sum $1+6+8$ is carried into the diagonal above and added into that diagonal: $1+(2+6+1)$. The product 1053 appears down the left side of the lattice and across the bottom.
52. $14 \cdot 721008$
53. 53 - 874611
54. $91 \cdot 645824$
55. 38 - 642432

## 4. Assess \& Reteach

## Lesson Quiz

Simplify each product using any method.

1. $(x+3)(x-6) x^{2}-3 x-18$
2. $(2 b-4)(3 b-5)$
$6 b^{2}-22 b+20$
3. $(3 x-4)\left(3 x^{2}+x+2\right)$
$9 x^{3}-9 x^{2}+2 x-8$
4. Find the area of the shaded region. $2 x^{2}+3 x-1$


## Alternative Assessment

Group students in pairs. Instruct one student to write a binomial and the other to write a trinomial. Then have both students multiply the pair of polynomials independently. Have students check their results with each other and discuss any discrepancies.
33. $25 c^{2}-40 c-9$
34. $n^{3}+11 n^{2}+3 n+33$
35. $15 k^{4}+3 k^{3}+10 k^{2}+2 k$
36. $24 h^{3}+2 h^{2}+17 h-3$
37. $9 y^{4}-9 y^{3}-7 y^{2}-2 y-2$
38. $48 q^{3}-16 q^{2}+4 q-4$

## Test Prep

## Resources

For additional practice with a variety of test item formats:

- Standardized Test Prep, p. 545
- Test-Taking Strategies, p. 540
- Test-Taking Strategies with Transparencies

Exercise 58 Remind students that only the product of odd numbers is odd. Ask them to consider which product's factors are both odd numbers.

## $\sigma^{-}$Crechpoint Quix

Use this Checkpoint Quiz to check students' understanding of the skills and concepts of Lessons 9-1 through 9-3.

## Resources

Grab \& Go

- Checkpoint Quiz 1
pages 507-510 Exercises

61. $20 v^{2}-28 v$
62. $3 c^{2}-27 c$
63. $8 t^{3}+48 t^{2}$
64. $3 y^{2}-10 y$
65. $-5 x^{3}+55 x^{2}$
66. $-6 t^{4}+t^{3}$
67. $-4 r^{6}+12 r$
68. $9 b^{5}+18 b^{3}$
69. $5(w+9)$
70. $x(3 x-11)$
71. $4 a(a+3)$
72. $9 n^{2}(1-n)$

Multiple Choice

Short Response

Extended Response

$$
(4 v-1)\left(2 v^{2}+v+1\right), \text { and simplify. } \text { See back of book. }
$$

60. Find an expression for the area of the shaded region. Show your work.

See back of book.
A. $n^{2}-5 n+4$
B. $n^{2}-3 n+4$
C. $n^{2}+3 n+4$
D. $n^{2}-5 n-5$
57. $(8 k-3)\left(k^{2}-k+1\right)$ is equivalent to which expression? $\mathbf{H}$
F. $8 k^{3}+11 k^{2}-11 k-3$
G. $9 k^{3}-8 k^{2}+8 k-2$
H. $8 k^{3}-11 k^{2}+11 k-3$
J. $9 k^{3}-3 k^{2}+3 k-3$
58. Which of the following products is always odd for integer values of $n$ ? $\mathbf{B}$
A. $(n+1)(n+1)$
B. $(2 n-1)(2 n+1)$
C. $(2 n-1)(n+1)$
D. $(2 n+1)(n-1)$
59. Explain how to find the product of


## Mixed Review



Lesson 8-5
Simplify each product. 61-68. See margin.
61. $4 v(5 v-7)$
62. $(c-9) 3 c$
63. $8 t^{2}(t+6)$
64. $y(3 y-10)$
65. $5 x^{2}(11-x)$
66. $-t^{3}(6 t-1)$
67. $4 r\left(3-r^{5}\right)$
68. $9 b^{2}\left(b^{3}+2 b\right)$

Factor. 69-72. See margin.
69. $5 w+45$
70. $3 x^{2}-11 x$
71. $4 a^{2}+12 a$
72. $9 n^{2}-n^{3}$
73. $34 t-51$
74. $63 v^{2}+45 v$
75. $25 m-60 m^{3}$
76. $11 k+77 k^{6}$ 17(2t-3) $9 v(7 v+5)$ $5 m\left(5-12 m^{2}\right)$ $11 k\left(1+7 k^{5}\right)$

Simplify each expression.
77. $\frac{3^{5}}{3^{2}} 27$
78. $\frac{3^{2}}{3^{5}} \frac{1}{27}$
79. $\frac{y^{12}}{y^{8}} y^{4}$
80. $\frac{2 w^{-3}}{6 w^{2}} \frac{1}{3 w^{5}}$
81. $\frac{x^{-8}}{2 x^{3}} \frac{1}{2 x^{11}}$
82. $\left(\frac{5}{3}\right)^{-1} \frac{3}{5}$
83. $\left(\frac{5}{3}\right)^{-2} \frac{9}{25}$
84. $\left(\frac{5}{3}\right)^{0} 1$
85. $\left(\frac{4 x}{7}\right)^{-2} \frac{49}{16 x^{2}}$
86. $\left(\frac{y^{-2}}{8}\right)^{-2} 64 y^{4}$

## Checkpoint Quiz 1

Lessons 9-1 through 9-3
Simplify each expression. 1-7. See margin.

1. $\left(4 x^{2}+x+3\right)+\left(5 x^{2}+9 x-2\right)$
2. $\left(7 b^{2}-5 b+3\right)-\left(b^{2}+8 b-6\right)$
3. $3 w(12 w-1)-8 w$
4. $6 k\left(4 k+k^{2}\right)+9 k\left(2 k-6 k^{2}\right)$
5. $(x+3)(x-5)$
6. $\left(2 n^{3}-5\right)\left(6 n^{2}+n\right)$
7. $\left(g^{2}+4\right)\left(4 g^{2}+8 g-9\right)$

Factor each polynomial.
8. $12 y^{2}-102\left(6 y^{2}-5\right)$
9. $5 t^{6}+25 t^{3}-10 t$
10. $18 v^{4}+27 v^{3}+36 v^{2}$
$9 v^{2}\left(2 v^{2}+3 v+4\right)$

510
Chapter 9 Polynomials and Factoring

## Checkpoint Quiz 1

1. $9 x^{2}+10 x+1$
2. $6 b^{2}-13 b+9$
3. $36 w^{2}-11 w$
4. $12 n^{5}+2 n^{4}-30 n^{2}-5 n$
5. $-48 k^{3}+42 k^{2}$
6. $4 g^{4}+8 g^{3}+7 g^{2}+32 g-36$
7. $x^{2}-2 x-15$
