Multiplying Binomials

What You'll Learn

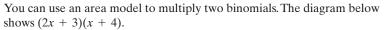
- To multiply binomials using FOIL
- To multiply trinomials by binomials

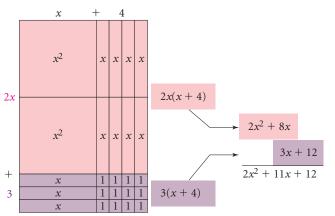
... And Why

To find the area of a geometric figure, as in Example 3

of Check Skills You'll Need	GO for Help Lesson 9-2
Find each product.	$\begin{array}{c} 6h^3 + 48h^2 - 18h \\ 2. \ 6h(h^2 + 8h - 3) \end{array} \qquad \begin{array}{c} 2y^5 - 7y^2 \\ 3. \ y^2(2y^3 - 7) \end{array}$
1. 4r(r - 1) 4r ² - 4r	2. $6h(h^2 + 8h - 3)$ 3. $y^2(2y^3 - 7)$
Simplify. Write each answer	in standard form. 4–9. See margin p. 507.
4. $(x^3 + 3x^2 + x) + (5x^2 + x)$	$x + 1$) 5. $(3t^3 - 6t + 8) + (5t^3 + 7t - 2)$
6. $w(w + 1) + 4w(w - 7)$	7. $6b(b-2) - b(8b+3)$
8. $m(4m^2 - 6) + 3m^2(m + 6)$	9) 9. $3d^2(d^3-6) - d^3(2d^2+4)$







You can also use the Distributive Property to find the product of two binomials.

EXAMPLE Using the Distributive Property Simplify (2x + 3)(x + 4). (2x + 3)(x + 4) = 2x(x + 4) + 3(x + 4)Distribute x + 4. $= 2x^2 + 8x + 3x + 12$ Now distribute 2x and 3. $= 2x^2 + 11x + 12$ Simplify. **Quick Check 1** Simplify each product. **a.** (6h - 7)(2h + 3)**b.** (5m + 2)(8m - 1)c. (9a - 8)(7a + 4) $12h^2 + 4h - 21$ $40m^2 + 11m - 2$ $63a^2 - 20a - 32$

Lesson 9-3 Multiplying Binomials 505

Differentiated Instruction Solutions for All Learners

Special Needs1BetHave students who have difficulty in using the
Distributive Property, or in drawing area models to
multiply two binomials, use algebra tiles to build and
explain their own solutions.But
Sug
trime
met

Below Level

Suggest that students use one method to multiply a trinomial by a binomial, and then use the other method to check their work.

9-3

1. Plan

Objectives

- 1 To multiply binomials using FOIL
- 2 To multiply trinomials by binomials

Examples

- 1 Using the Distributive Property
- 2 Multiplying Using FOIL
- **3** Applying Multiplication of Polynomials
- 4 Multiplying a Trinomial and a Binomial



Finding the product of two binomials by the FOIL method is a shortcut for distributing the first binomial to each term of the second, and then distributing again to find these individual products. However, no corresponding shortcut exists when one factor is a trinomial.

More Math Background: p. 492C

Lesson Planning and Resources

See p. 492E for a list of the resources that support this lesson.



Check Skills You'll Need For intervention, direct students to:

Multiplying and Factoring Lesson 9-2: Example 1 Extra Skills and Word Problem Practice, Ch. 9

2. Teach

Guided Instruction

1 EXAMPLE Tactile Learners

Encourage students to draw an area model showing multiplication of the two binomials. Students can easily see there are four products when multiplying binomials.

2 EXAMPLE Auditory Learners

On the board, write an expression that multiplies two binomials. Call on students to say aloud the different parts of FOIL for the expression. Repeat with new expressions until all students have responded.

3 EXAMPLE Visual Learners Some students may not understand why you subtract the area of the inner rectangle. Cut a rectangle out of a sheet of construction paper to represent the example.

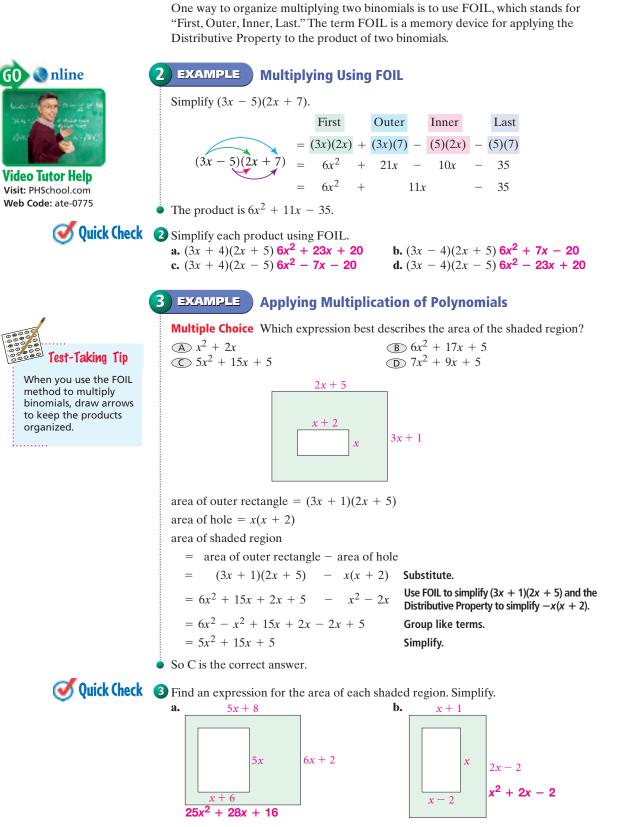


1 Simplify (2y - 3)(y + 2). $2y^2 + y - 6$

2 Simplify (4x + 2)(3x - 6). $12x^2 - 18x - 12$

3 Find the area of the shaded region. Simplify. $5x^2 - 2x - 2$

3x + 2 x + 3 x 2x - 1



506 Chapter 9 Polynomials and Factoring

Differentiated Instruction Solutions for All Learners

Advanced Learners

Ask students to explain the statement, "The degree of the product of two nonzero polynomials is the sum of the degrees of the polynomials."

English Language Learners ELL

Explain that the term FOIL is a memory tool when multiplying binomials. Make sure students relate each letter to the word that identifies each part of the process (first, outer, inner, and last).

learning style: verbal

Multiplying a Trinomial and a Binomial



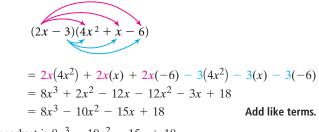
FOIL works when you multiply two binomials, but it is not helpful when multiplying a trinomial and a binomial. You can use the vertical method or the horizontal method to distribute each term in such factors.

EXAMPLE Multiplying a Trinomial and a Binomial Simplify the product $(4x^2 + x - 6)(2x - 3)$.

Method 1 Multiply using the vertical method.

 $4x^2 + x - 6$ $\frac{2x - 3}{-12x^2 - 3x + 18}$ $\frac{8x^3 + 2x^2 - 12x}{8x^3 - 10x^2 - 15x + 18}$ Multiply by 2x.
Add like terms.

Method 2 Multiply using the horizontal method.



• The product is $8x^3 - 10x^2 - 15x + 18$.

Quick Check (a) Simplify $(6n - 8)(2n^2 + n + 7)$ using both methods shown in Example 4. $12n^3 - 10n^2 + 34n - 56$

EXERCISES

For more exercises, see Extra Skill and Word Problem Practice.

Practice and Problem Solving

Practice	by Example	Copy and fill in each bla	nk.	35
	Example 1	1. $(5a + 2)(6a - 1) =$	$a^2 + 7a - 2$ 30 2. (3c -	$7)(2c - 5) = 6c^2 - 29c + \blacksquare$
GO for Help	(page 505)	3. $(z - 4)(2z + 1) = 2$	$2z^2 - \mathbf{I}z - 4$ 7 4. (2x +	$9)(x + 2) = 2x^2 + \blacksquare x + 18$ 13
- neth		Simplify each product us	sing the Distributive Propert	y. 5–10. See margin.
		5. $(x + 2)(x + 5)$	6. $(h + 3)(h + 4)$	7. $(k + 7)(k - 6)$
		8. (<i>a</i> - 8)(<i>a</i> - 9)	9. $(2x - 1)(x + 2)$	10. $(2y + 5)(y - 3)$
Example 2		Simplify each product us	sing FOIL. 11–16. See mar	gin.
	(page 506)	11. $(r + 6)(r - 4)$	12. $(y + 4)(5y - 8)$	13. $(x + 6)(x - 7)$
		14. (<i>m</i> - 6)(<i>m</i> - 9)	15. $(4b - 2)(b + 3)$	16. $(8w + 2)(w + 5)$
		17. $(x - 7)(x + 9)$ $x^2 + 2x - 63$	18. $(a + 11)(a + 5)$ $a^2 + 16a + 55$	19. $(p - 1)(p + 10)$ $p^2 + 9p - 10$

Lesson 9-3 Multiplying Binomials 507

4 EXAMPLE Teaching Tip

Help students understand the vertical method for multiplying polynomials by relating it to multiplying whole numbers. Write 312×23 in vertical format on the board and ask volunteers to help you solve it. Stress how partial products are lined up according to place value.



4 Simplify the product $(3x^2 - 2x + 3)(2x + 7)$. $6x^3 + 17x^2 - 8x + 21$

Resources

- Daily Notetaking Guide 9-3 13
- Daily Notetaking Guide 9-3— L1 Adapted Instruction

Closure

c

Ask students what the letters in FOIL represent. F: first terms, O: outer terms, I: inner terms, L: last terms

bage 505 Check Skills You'll Need
4. $x^3 + 8x^2 + 2x + 1$
5. $8t^3 + t + 6$
6. $5w^2 - 27w$
7. $-2b^2 - 15b$
8. $7m^3 + 27m^2 - 6m$
9. $d^5 - 4d^3 - 18d^2$
bages 507–510 Exercises
5. $x^2 + 7x + 10$
6. $h^2 + 7h + 12$
7. $k^2 + k - 42$
8. a ² – 17a + 72
9. $2x^2 + 3x - 2$
10. 2 <i>y</i> ² – <i>y</i> – 15
1. $r^2 + 2r - 24$
12. $5y^2 + 12y - 32$
13. $x^2 - x - 42$
14. $m^2 - 15m + 54$
$15. 4b^2 + 10b - 6$

16. $8w^2 + 42w + 10$

3. Practice

Assignment Guide

1.10	· _ ·	_			
v	Α	B	1-21	30-35,	42-46
- -		_		20 22,	12 10

УАВ	22-29, 36-4	1
C Challenge	2	47-55
Test Prep		56-60

Homework Quick Check

Mixed Review

To check students' understanding of key skills and concepts, go over Exercises 8, 20, 36, 39, 41.

61-86

Exercises 3, 4 Remind students to combine the "Outer" and "Inner" products to determine the middle term of the answer.

Error Prevention!

Exercises 22–25 Suggest that students write the binomial on the second line when using the vertical method.

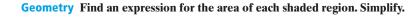
Connection to Geometry

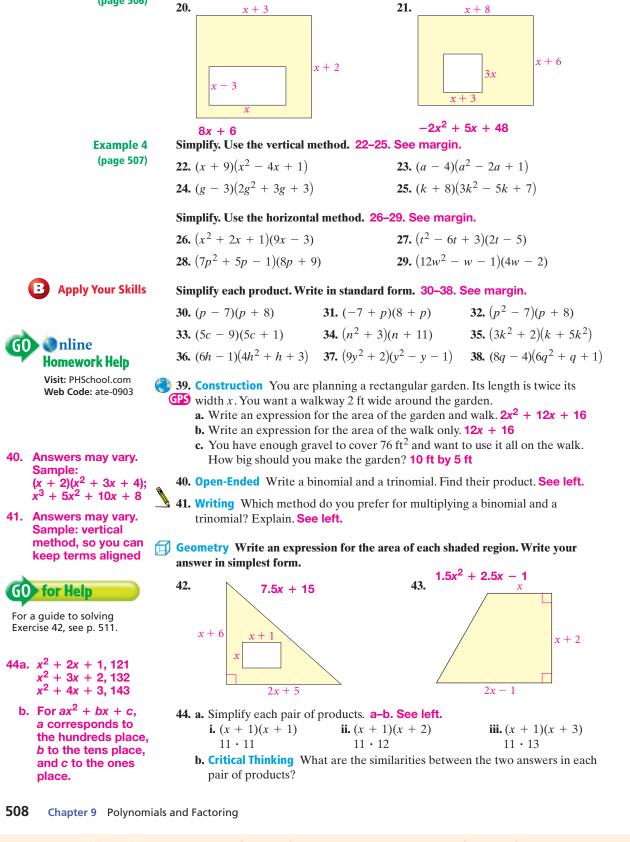
Exercise 43 Remind students that the bases of a trapezoid are the two parallel sides.

|--|

GPS Guide	d Problem S	Solving L3
Enrichment		L4
Reteaching	g	L2
Adapted Pra	actice	L1
Practice		L3
Practice 9-3 Simplify each product. Write		Multiplying Binomials
1. $(x + 3)(2x - 5)$ 4. $(x + 5)(x + 4)$	2. $(x^2 + x - 1)(x + 1)$ 5. $(2b - 1)(b^2 - 3b + 4)$	3. $(3w + 4)(2w - 1)$ 6. $(a - 11)(a + 5)$
7. $(2g - 3)(2g^2 + g - 4)$ 10. $(x + 6)(x^2 - 4x + 3)$	8. $(3s - 4)(s - 5)$ 11. $(5x - 3)(4x + 2)$	 (4x + 3)(x - 7) (3y + 7)(4y + 5)
13. $(3x + 7)(x + 5)$ 16. $(a - 6)(a + 8)$ 19. $(x - 2)(x^2 + 4x + 4)$	14. $(5x - 2)(x + 3)$ 17. $(x + 2)(2x^2 - 3x + 2)$	15. $(3m^2 - 7m + 8)(m - 2)$ 18. $(a^2 + a + 1)(a - 1)$
19. $(x - 2)(x^2 + 4x + 4)$ 22. $(2n - 3)(n^2 - 2n + 5)$ 25. $(2x^2 - 5x + 2)(4x - 3)$	 (2r + 1)(3r - 1) (p - 4)(2p + 3) (x + 7)(x + 5) 	21. $(k + 4)(3k - 4)$ 24. $(3x + 1)(4x^2 - 2x + 1)$ 27. $(6x - 11)(x + 2)$
28. $(2x + 1)(4x + 3)$ 31. $(n - 7)(n + 4)$	29. $(3x + 4)(3x - 4)$ 32. $(3x - 1)(2x + 1)$	30. $(6x - 5)(3x + 1)$ 33. $(d + 9)(d - 11)$
34. $(2x^2 + 5x - 3)(2x + 1)$ 37. $(3x + 5)(5x - 7)$ 40. $(2x^2 + 5x - 4)(2x + 7)$	35. $(b + 8)(2b - 5)$ 38. $(x - 5)(2x^2 - 7x - 2)$ 41. $(x^2 + 6x + 11)(3x + 5)$	36. $(2x - 5)(x + 4)$ 39. $(2x^2 - 9x + 11)(2x + 1)$ 42. $(5x + 7)(7x + 3)$
43. $(4x - 7)(2x - 5)$	 41. (x + ux + 11)(3x + 3) 44. (x - 9)(3x + 5) painting is 3 in. more than twice the I 	45. $(2x - 1)(x^2 - 7x + 1)$
A frame that is 2.5 in. wide		
the painting is 12 in.	ind the combined area when the heig	phrof Service
the painting is 15 in. 47. The Robertsons put a recta	ngular pool with a stone walkway are length of the pool and walkway is 3	ound it
a. Write an expression fob. Find the area of the po	, r the area of the pool. ol when the total width is 10 ft.	0 I→→→2x – 3→→1
 The Cutting Edge frame sh inside of a rectangular boar 	ol when the total width is 9 ft. op makes a mat by cutting out the d. Use the diagram to find the jual board if the area of the mat	x+4 x+8

Example 3 (page 506)





pages 507–510 Exercises	25. 3 <i>k</i> ³ + 19 <i>k</i> ² - 33 <i>k</i> + 56	$29. \ 48w^3 - 28w^2 - 2w + 2$
22. $x^3 + 5x^2 - 35x + 9$	$26. \ 9x^3 + 15x^2 + 3x - 3$	30. $p^2 + p - 56$
23. a ³ - 6a ² + 9a - 4	27. $2t^3 - 17t^2 + 36t - 15$	31. $p^2 + p - 56$
24. $2g^3 - 3g^2 - 6g - 9$	28. $56p^3 + 103p^2 + 37p - 9$	32. $p^3 + 8p^2 - 7p - 56$

5	0	8
-	-	

- 45. Geometry Use the formula $V = \ell wh$ to write a polynomial in standard form for the volume of the box. $n^3 + 15n^2 + 56n$
 - **46.** Multiple Choice If *n* represents an even number, which expression represents the product of the next two even numbers? D (A) $n^2 + 3n + 2$ (B) 2n + 3 (C) 2n + 6

C(t) = 3.2t + 157

expression in standard form for the surface area of each cube.

47. $x + 36x^2 + 36x + 5448$. $4t + 196t^2 + 48t + 649$. $2w^2 + 7$

vegetables V(t) consumed in a year in the United States.

a. Rewrite the expression $2000(1 + r)^3$ by finding the product

2000(1 + r)(1 + r)(1 + r). Write your answer in standard form.

For Exercises 47–49, each expression represents the side length of a cube. Write an

50. a. Vegetable Consumption Multiply the expressions on the right side of each

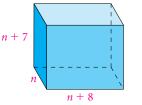
equation to create a model for the total number of pounds of fresh

b. Evaluate the equation you found in part (a) with t = 5 to find the total vegetable consumption for 1995. (t = 0 corresponds to the year 1990.)

51. Financial Planning Suppose you deposit \$2000 for college in a savings account

that has an annual interest rate r. At the end of three years, the value of your account will be $2000(1 + r)^3$ dollars. **a.** $2000r^3 + 6000r^2 + 6000r + 2000$

b. Find the amount of money in the account if the interest rate is 3%. **\$2185.45**



 $\bigcirc n^2 + 6n + 8$

 $24w^4 + 168w^2 + 294$

46,104.5 million lb

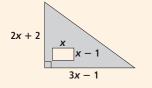
PowerPoint

Lesson Quiz

Simplify each product using any method.

4. Assess & Reteach

- 1. $(x + 3)(x 6) x^2 3x 18$
- 2. (2b 4)(3b 5) $6b^2 - 22b + 20$
- 3. $(3x 4)(3x^2 + x + 2)$ $9x^3 - 9x^2 + 2x - 8$
- 4. Find the area of the shaded region. $2x^2 + 3x 1$



Alternative Assessment

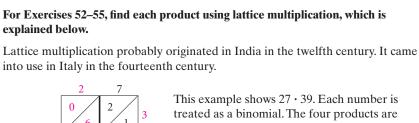
Group students in pairs. Instruct one student to write a binomial and the other to write a trinomial. Then have both students multiply the pair of polynomials independently. Have students check their results with each other and discuss any discrepancies.

Challenge



Real-World 🜏 Connection

In 2003, the U.S. consumption of fresh tomatoes was 15.4 lb per person.



This example shows $27 \cdot 39$. Each number is treated as a binomial. The four products are placed in the small, diagonally split squares. The product of 2 and 3, shown in red, is 6. The first square shows 0/6, which indicates 6. The product of 7 and 3 is 21. The second square shows 2/1.

Lesson 9-3 Multiplying Binomials

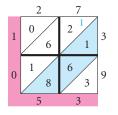
 $V(t) = 10.56t^2 + 1318.1t + 39,250$ the U.S. annual per capita consumption of fresh

vegetables, in pounds, from 1990 to 1997

P(t) = 3.3t + 250 the U.S. population, in millions, from 1990 to 1997

The products are totaled diagonally. For the diagonal shaded blue, the tens place of the sum 1 + 6 + 8 is carried into the diagonal above and added into that diagonal: 1 + (2 + 6 + 1). The product 1053 appears down the left side of the lattice and across the bottom.

52. 14 • 72 1008	53. 53 • 87 4611
54. 91 • 64 5824	55. 38 · 64 2432



509

nline lesson quiz, PHSchool.com, Web Code: ata-0903

33. $25c^2 - 40c - 9$ 334. $n^3 + 11n^2 + 3n + 33$ 335. $15k^4 + 3k^3 + 10k^2 + 2k$ 3

36. $24h^3 + 2h^2 + 17h - 3$ 37. $9y^4 - 9y^3 - 7y^2 - 2y - 2$ 38. $48q^3 - 16q^2 + 4q - 4$

Test Prep

Resources

For additional practice with a variety of test item formats:

- Standardized Test Prep, p. 545
- Test-Taking Strategies, p. 540
- Test-Taking Strategies with Transparencies

Exercise 58 Remind students that only the product of odd numbers is odd. Ask them to consider which product's factors are both odd numbers.

Checkpoint Quiz

Use this Checkpoint Quiz to check students' understanding of the skills and concepts of Lessons 9-1 through 9-3.

Resources

Grab & Go • Checkpoint Quiz 1

pages 507–510 Exercises

61. $20v^2 - 28v$ 62. $3c^2 - 27c$ 63. $8t^3 + 48t^2$ 64. $3y^2 - 10y$ 65. $-5x^3 + 55x^2$ 66. $-6t^4 + t^3$ 67. $-4r^6 + 12r$ 68. $9b^5 + 18b^3$

69. 5(w + 9)70. x(3x - 11)71. 4a(a + 3)

72. $9n^2(1 - n)$

عفو		
Multiple Choice	56. $(n - 1)(n - 4)$ is equivalent to white A. $n^2 - 5n + 4$ C. $n^2 + 3n + 4$	ch expression? A B. $n^2 - 3n + 4$ D. $n^2 - 5n - 5$
	57. $(8k - 3)(k^2 - k + 1)$ is equivalent F. $8k^3 + 11k^2 - 11k - 3$ H. $8k^3 - 11k^2 + 11k - 3$	
	58. Which of the following products is A. $(n + 1)(n + 1)$ C. $(2n - 1)(n + 1)$	always odd for integer values of n ? B B . $(2n - 1)(2n + 1)$ D . $(2n + 1)(n - 1)$
Short Response	59. Explain how to find the product of $(4v - 1)(2v^2 + v + 1)$, and simplify See back	<i>.</i>
Extended Response	60. Find an expression for the area of the shaded region. Show your work. See back	the $n+2$ $5n-2$
		2n + 6

Mixed Review

Test Prep

a for	Lesson 9-2	Simplify each product. 61–68. See margin.				
GO for Help		61. 4 <i>v</i> (5 <i>v</i> - 7)	62. (c - 9)	3c 63	3. $8t^2(t+6)$	64. <i>y</i> (3 <i>y</i> - 10)
		65. $5x^2(11 - x)$	66. -t ³ (6t -	- 1) 6	7. $4r(3 - r^5)$	68. $9b^2(b^3 + 2b)$
		Factor. 69–72. S	See margin.			
		69. 5 <i>w</i> + 45	70. $3x^2 - 3$	11 <i>x</i> 71	1. $4a^2 + 12a$	72. $9n^2 - n^3$
	Losson Q F	73. 34 <i>t</i> - 51 17(2t - 3)	9v(7v -		5. 25m - 60m ³ 5m(5 - 12m ²)	
	Lesson 8-5	Simplify each ex	-	v ¹²	n =3	
		77. $\frac{3^5}{3^2}$ 27	78. $\frac{5^2}{3^5}$ 1 / 27	79. $\frac{y}{y^8}$ y ⁴	80. $\frac{2W}{6w^2}$ 3	$\frac{1}{\sqrt{5}}$ 81. $\frac{x^{-8}}{2x^3} \frac{1}{2x^{11}}$
		82. $\left(\frac{5}{3}\right)^{-1} \frac{3}{5}$	83. $\left(\frac{5}{3}\right)^{-2} \frac{9}{25}$	84. $\left(\frac{5}{3}\right)^0$ 1	85. $\left(\frac{4x}{7}\right)^{-2}$	$\frac{49}{16x^2} 86. \ \left(\frac{y^{-2}}{8}\right)^{-2} \qquad 64y^4$

🛛 🚺 Checkpoint Quiz 1

Lessons 9-1 through 9-3

		Simplify each expression.	1–7. See margin.		
		1. $(4x^2 + x + 3) + (5x^2)$	2. $(7b^2 - 5b + 3) - (b^2 + 8b - 6)$		
		3. $3w(12w - 1) - 8w$	4. 6k(4k -	4. $6k(4k + k^2) + 9k(2k - 6k^2)$	
		5. $(x + 3)(x - 5)$	6. $(2n^3 - 5)(6n^2 + n)$	7. $(g^2 + 4)(4g^2 + 8g - 9)$	
		Factor each polynomial.			
		8. 12y ² - 10 2(6y² - 5)	9. $5t^6 + 25t^3 - 10t$ $5t(t^5 + 5t^2 - 2)$	10. $18v^4 + 27v^3 + 36v^2$ 9v²(2v² + 3v + 4)	
510	Chapter 9 Polynomial	s and Factoring			
		0	5 4		

Checkpoint Quiz 1	Checkpoint Quiz 1			
1. $9x^2 + 10x + 1$				
2. $6b^2 - 13b + 9$				

1	3. 36w ² – 11w	6. $12n^5 + 2n^4 - 30n^2 - 5n$
	4. $-48k^3 + 42k^2$	7. $4g^4 + 8g^3 + 7g^2 + 32g - 36$
	5. $x^2 - 2x - 15$	