

9.3 Quotient of Powers Property

Essential Question How can you divide two powers that have the same base?

1 ACTIVITY: Finding Quotients of Powers

Work with a partner.

- a. Copy and complete the table.

Quotient	Repeated Multiplication Form	Power
$\frac{2^4}{2^2}$	$\frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 2 \cdot 2}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}}}$	2^2
$\frac{(-4)^5}{(-4)^2}$	$\frac{\overset{1}{\cancel{(-4)}} \cdot \overset{1}{\cancel{(-4)}} \cdot (-4) \cdot (-4) \cdot (-4)}{\underset{1}{\cancel{(-4)}} \cdot \underset{1}{\cancel{(-4)}}}$	$(-4)^3$
$\frac{7^7}{7^3}$		
$\frac{8.5^9}{8.5^6}$		
$\frac{10^8}{10^5}$		
$\frac{3^{12}}{3^4}$		
$\frac{(-5)^7}{(-5)^5}$		
$\frac{11^4}{11^1}$		

- b. **INDUCTIVE REASONING** Describe the pattern in the table. Then write a rule for dividing two powers that have the same base.

$$\frac{a^m}{a^n} = a^{\quad}$$

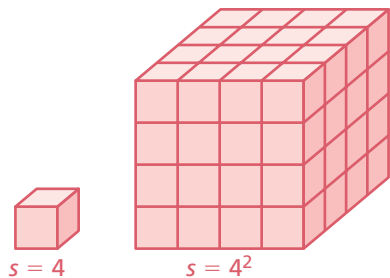
- c. Use your rule to simplify the quotients in the first column of the table above. Does your rule give the results in the third column?

2 ACTIVITY: Comparing Volumes

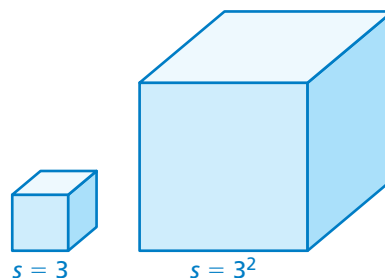
Work with a partner.

How many of the smaller cubes will fit inside the larger cube? Record your results in the table. Describe the pattern in the table.

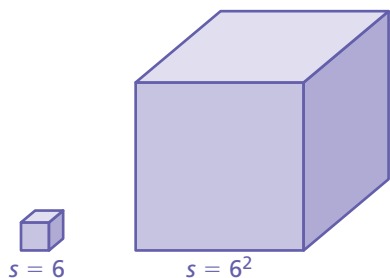
a. Sample:



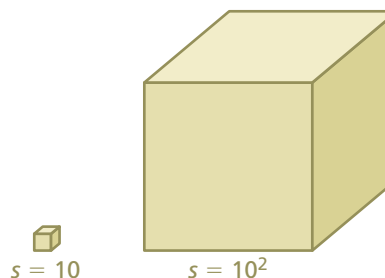
b.



c.



d.



	Volume of Smaller Cube	Volume of Larger Cube	$\frac{\text{Larger Volume}}{\text{Smaller Volume}}$	Answer
a.	4^3	$(4^2)^3 = 4^6$	$\frac{4^6}{4^3}$	4^3
b.				
c.				
d.				

What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you divide two powers that have the same base? Give two examples of your rule.

Practice

Use what you learned about the Quotient of Powers Property to complete Exercises 3–6 on page 366.

Key Idea
Quotient of Powers Property

Words To divide powers with the same base, subtract their exponents.

Numbers $\frac{4^5}{4^2} = 4^{5-2} = 4^3$

Algebra $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$

EXAMPLE 1 Dividing Powers with the Same Base

a. $\frac{2^6}{2^4} = 2^{6-4}$ The base is 2. Subtract the exponents.
 $= 2^2$ Simplify.

b. $\frac{(-7)^9}{(-7)^3} = (-7)^{9-3}$ The base is -7 . Subtract the exponents.
 $= (-7)^6$ Simplify.

c. $\frac{h^7}{h^6} = h^{7-6}$ The base is h . Subtract the exponents.
 $= h^1 = h$ Simplify.

Common Error

When dividing powers, do not divide the bases.

$$\frac{2^6}{2^4} = 2^2, \text{ not } 1^2.$$

On Your Own

Simplify the expression. Write your answer as a power.

Now You're Ready
Exercises 7–14

1. $\frac{9^7}{9^4}$ 2. $\frac{4.2^6}{4.2^5}$ 3. $\frac{(-8)^8}{(-8)^4}$ 4. $\frac{x^8}{x^3}$

EXAMPLE 2 Simplifying an Expression

Simplify $\frac{3^4 \cdot 3^2}{3^3}$. Write your answer as a power.

The numerator is a product of powers.

$$\frac{3^4 \cdot 3^2}{3^3} = \frac{3^{4+2}}{3^3}$$

Add the exponents in the numerator.

$$= \frac{3^6}{3^3}$$

Simplify.

$$= 3^{6-3}$$

The base is 3. Subtract the exponents.

$$= 3^3$$

Simplify.

EXAMPLE 3 Simplifying an Expression

Study Tip

You can also simplify the expression in Example 3 as follows.

$$\begin{aligned}\frac{a^{10}}{a^6} \cdot \frac{a^7}{a^4} &= \frac{a^{10} \cdot a^7}{a^6 \cdot a^4} \\ &= \frac{a^{17}}{a^{10}} \\ &= a^{17-10} \\ &= a^7\end{aligned}$$

Simplify $\frac{a^{10}}{a^6} \cdot \frac{a^7}{a^4}$. Write your answer as a power.

$$\begin{aligned}\frac{a^{10}}{a^6} \cdot \frac{a^7}{a^4} &= a^{10-6} \cdot a^{7-4} && \text{Subtract the exponents.} \\ &= a^4 \cdot a^3 && \text{Simplify.} \\ &= a^{4+3} && \text{Add the exponents.} \\ &= a^7 && \text{Simplify.}\end{aligned}$$

On Your Own

Simplify the expression. Write your answer as a power.

 **Now You're Ready**
Exercises 16–21

5. $\frac{2^{15}}{2^3 \cdot 2^5}$

6. $\frac{d^5}{d} \cdot \frac{d^9}{d^8}$

EXAMPLE 4 Real-Life Application

The projected population of Tennessee in 2030 is about $5 \cdot 5.9^8$. Predict the average number of people per square mile in 2030.

Use a model to solve the problem.

$$\text{People per square mile} = \frac{\text{Population in 2030}}{\text{Land area}}$$

Land Area:
about 5.9^6 mi²

$$= \frac{5 \cdot 5.9^8}{5.9^6} \quad \text{Substitute.}$$

$$= 5 \cdot \frac{5.9^8}{5.9^6} \quad \text{Rewrite.}$$

$$= 5 \cdot 5.9^2 \quad \text{Subtract the exponents.}$$

$$= 174.05 \quad \text{Evaluate.}$$

∴ There will be about 174 people per square mile in Tennessee in 2030.

On Your Own

 **Now You're Ready**
Exercises 23–28

7. The projected population of Alabama in 2020 is about $2.25 \cdot 2^{21}$. The land area of Alabama is about 2^{17} square kilometers. Predict the average number of people per square kilometer in 2020.

Vocabulary and Concept Check

- WRITING** Explain in your own words what it means to divide powers.
- WHICH ONE DOESN'T BELONG?** Which quotient does *not* belong with the other three? Explain your reasoning.

$$\frac{(-10)^7}{(-10)^2} \quad \frac{6^3}{6^2} \quad \frac{(-4)^8}{(-3)^4} \quad \frac{5^6}{5^3}$$

Practice and Problem Solving

Simplify the expression. Write your answer as a power.

3. $\frac{6^{10}}{6^4}$

4. $\frac{8^9}{8^7}$

5. $\frac{(-3)^4}{(-3)^1}$

6. $\frac{4.5^5}{4.5^3}$

1 7. $\frac{5^9}{5^3}$

8. $\frac{64^4}{64^3}$

9. $\frac{(-17)^5}{(-17)^2}$

10. $\frac{(-7.9)^{10}}{(-7.9)^4}$


11. $\frac{(-6.4)^8}{(-6.4)^6}$

12. $\frac{\pi^{11}}{\pi^7}$

13. $\frac{b^{24}}{b^{11}}$

14. $\frac{n^{18}}{n^7}$

15. **ERROR ANALYSIS** Describe and correct the error in simplifying the quotient.

 $\frac{6^{15}}{6^5} = 6^{\frac{15}{5}}$
 $= 6^3$

Simplify the expression. Write your answer as a power.

2 3 16. $\frac{7^5 \cdot 7^3}{7^2}$

17. $\frac{2^{19} \cdot 2^5}{2^{12} \cdot 2^3}$

18. $\frac{(-8.3)^8 \cdot (-8.3)^4}{(-8.3)^7 \cdot (-8.3)^3}$

19. $\frac{\pi^{30}}{\pi^{18} \cdot \pi^4}$

20. $\frac{c^{22}}{c^8 \cdot c^9}$

21. $\frac{k^{13} \cdot k^{17}}{k^5 \cdot k^{11}}$

22. **SOUND INTENSITY** The sound intensity of a normal conversation is 10^6 times greater than the quietest noise a person can hear. The sound intensity of a jet at takeoff is 10^{14} times greater than the quietest noise a person can hear. How many times more intense is the sound of a jet at takeoff than the sound of a normal conversation?



Simplify the expression.

4 23. $\frac{x \cdot 4^8}{4^5}$

24. $\frac{6^3 \cdot w}{6^2}$

25. $\frac{a^3 \cdot b^4 \cdot 5^4}{b^2 \cdot 5}$

26. $\frac{5^{12} \cdot c^{10} \cdot d^2}{5^9 \cdot c^9}$

27. $\frac{x^{15}y^9}{x^8y^3}$

28. $\frac{m^{10}n^7}{m^1n^6}$

MP3 Player	Memory (GB)	Price
A	2^1	\$70
B	2^2	\$120
C	2^3	\$170
D	2^4	\$220
E	2^5	\$270

29. **MEMORY** The memory capacities and prices of five MP3 players are shown in the table.

- How many times more memory does MP3 Player D have than MP3 Player B?
- Do the differences in price between consecutive sizes reflect a constant rate of change?



30. **CRITICAL THINKING** Consider the equation $\frac{9^m}{9^n} = 9^2$.

- Find two numbers m and n that satisfy the equation.
- Are there any other pairs of numbers that satisfy the equation? Explain.



Milky Way Galaxy
 $10 \cdot 10^{10}$ stars

31. **STARS** There are about 10^{24} stars in the Universe. Each galaxy has approximately the same number of stars as the Milky Way Galaxy. About how many galaxies are in the Universe?

32. **Number Sense** Find the value of x that makes $\frac{8^{3x}}{8^{2x+1}} = 8^9$ true. Explain how you found your answer.



Fair Game Review what you learned in previous grades & lessons

Subtract.

33. $-4 - 5$

34. $-23 - (-15)$

35. $33 - (-28)$

36. $18 - 22$

37. **MULTIPLE CHOICE** What is the value of x ?

(A) 20

(B) 30

(C) 45

(D) 60

