9-4 Arithmetic Series

An arithmetic series is an indicated sum of the terms of an arithmetic sequence.

A finite series has a first term and a last term. $2+4+6$

An infinite series continues without end. $2+4+6+\ldots$

To find the sum of a finite arithmetic series use . . .

$$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right] \text{ or } S_n = \frac{n}{2}(a_1 + a_n)$$

where *n* is the number of terms.

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$$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right] \text{ or } S_n = \frac{n}{2}(a_1 + a_n)$$

Example 1. (p. 588) Got It? 1a
What is the sum of the finite arithmetic series
4 + 9 + 14 + 19 + 24 + . . . + 99?

$a_1 = 4$

How to find *n* (the number of terms)

$d = 9 - 4 = 5$ 1. Find common difference (d)
$99 - 4 = 95$ 2. Subtract last term - first term
$95 \div 5 = 19$ 3. Divide by d
$19 + 1 = 20$ 4. Add 1 (to count first term)

$n = 20$

$S_{20} = \frac{20}{2}(4 + 99)$

$S_{20} = 1030$

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$$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right] \text{ or } S_n = \frac{n}{2}(a_1 + a_n)$$

Example 2
What is the sum of the finite arithmetic series
14 + 17 + 20 + 23 + . . . + 116?

$d = 17 - 14 = 3$

You can also use the formula for the *n*th term in an arithmetic sequence to find *n*.
$a_n = a_1 + (n-1)d$

$116 = 14 + (n-1)(3)$

$116 = 14 + 3n - 3$

$116 = 11 + 3n$

$105 = 3n$

$35 = n$

$S_{35} = \frac{35}{2}(14 + 116)$

$S_{35} = 2275$

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$$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right] \text{ or } S_n = \frac{n}{2}(a_1 + a_n)$$ $a_n = a_1 + (n-1)d$

Example 3 (p. 588) Got It? 2.
A company pays a $5,000 bonus if a new salesperson makes 10 sales in the first week and then improves by one sale per week each week thereafter. One salesperson qualified for this bonus with the minimum number of sales. How many sales did the salesperson make in week 50? In all 50 weeks?

$n = 50$

$a_1 = 10$

$d = 1$

$a_{50} = 10 + (50-1)1$

$a_{50} = 10 + 49$

$a_{50} = 59$ sales in 50^{th} week

$S_{50} = \frac{50}{2}(10 + 59)$

$S_{50} = 1725$ total sales

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Sigma Notation
(Denotes the sum of a series)

$$3+6+9+12+\ldots+30=\sum_{n=1}^{10} 3n$$

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(p. 589)

You can use the Greek capital letter sigma, Σ, to indicate a sum. With it, you use *limits* to indicate how many terms you are adding. **Limits** are the least and greatest values of n in the series. You write the limits below and above the Σ to indicate the first and last terms of the series.

For example, you can write the series $3^2 + 4^2 + 5^2 + \cdots + 108^2$ as $\sum_{n=3}^{108} n^2$.

Upper limit: the series ends with $n = 108$.

$$\sum_{n=3}^{108} n^2$$

The explicit formula for each term is n^2.

Lower limit: the series begins with $n = 3$.

For an infinite series, summation notation shows ∞ as the upper limit.

To find the number of terms in a series written in Σ form, subtract the lower limit from the upper limit and add 1.

The number of terms in the series above is $108 - 3 + 1 = 106$.

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Example 4 (p. 589) Got It? 3

What is summation notation for the series?

$a_n = a_1 + (n-1)d$

a. $-5 + 2 + 9 + 16 + \ldots + 261 + 268$

$d = 2-(-5) = 7$

$\sum_{n=1}^{40} (-12+7n)$

$a_n = -5+(n-1)7$

$a_n = -5+7n-7$

$a_n = -12+7n$

$268 = -12+7n$

$280 = 7n$

$40 = n$

b. $500 + 490 + 480 + \ldots + 20 + 10$

$d = 490-500 = -10$

$a_n = 500+(n-1)(-10)$

$a_n = 500-10n+10$

$a_n = 510-10n$

$10 = 510-10n$

$10n = 500$

$n = 50$

$\sum_{n=1}^{50} (510-10n)$

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Example 5

Evaluate

$$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right] \text{ or } S_n = \frac{n}{2}(a_1 + a_n)$$

a. $\sum_{k=3}^{10} (2k+1)$

first term $= 2(3)+1 = 7$

last term $= 2(10)+1 = 21$

$n = 10-3+1 = 8$

$S_8 = \frac{8}{2}(7+21)$

$S_8 = 112$

b. $\sum_{t=19}^{23} (5t-3)$

first term $= 5(19)-3 = 92$

last term $= 5(23)-3 = 112$

$n = 23-19+1 = 5$

$S_5 = \frac{5}{2}(92+112)$

$S_5 = 510$

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(p. 589)

take note **Key Concept** Summation Notation and Linear Functions

If the explicit formula for the nth term in summation notation is a *linear* function of n, then the series is arithmetic. The slope of the linear function is the common difference between terms of the series.

If the explicit formula is not linear, then you can evaluate the sum by adding the terms.

Example 6 (p. 590) Got It? 4b-4c

b. $\sum_{n=1}^{4} n^3$

$(1)^3 + (2)^3 + (3)^3 + (4)^3$

$1 + 8 + 27 + 64$

100

c. $\sum_{n=0}^{100} (-1)^n = 1$

$(-1)^0 + (-1)^1 + (-1)^2 + (-1)^3 + \ldots$

$1 + (-1) + 1 + (-1) + \ldots$

last term $(-1)^{100} = 1$

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9-5 Geometric Series

A geometric series is an indicated sum of the terms of a geometric sequence.

To find the sum of a finite geometric series use . . .

$$S_n = \frac{a_1 - a_n r}{1 - r} \text{ or } S_n = \frac{a_1(1 - r^n)}{1 - r}$$

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Example 1
Evaluate the sum of the finite geometric series

a. (p. 599) 8. 1 + 2 + 4 + 8 + + 128

$r = \frac{2}{1} = 2$

Nth term in a Geometric Sequence

$a_n = a_1 r^{n-1}$

$$S_n = \frac{a_1 - a_n r}{1-r} \text{ or } S_n = \frac{a_1(1-r^n)}{1-r}$$

$128 = 2 \cdot 64 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$128 = 1(2^{n-1})$

$128 = 2^{n-1}$

$2^7 = 2^{n-1}$

$7 = n-1$

$8 = n$

$S_8 = \frac{1(1-2^8)}{1-2}$

$S_8 = 255$

b. (p. 599) 10. 3 + 6 + 12 + 24 + 48 + . . . + 768

$r = \frac{6}{3} = 2$

$768 = 3(2^{n-1})$

$256 = 2^{n-1}$

$2^8 = 2^{n-1}$

$8 = n-1$

$9 = n$

9 terms

$S_9 = \frac{3(1-2^9)}{1-2}$

$\boxed{S_9 = 1533}$

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Example 2 (p. 596) Got It? 1b

What is the sum of the finite geometric series?

b. $\sum_{n=1}^{10} 5 \cdot (-2)^{n-1}$

$$S_n = \frac{a_1 - a_n r}{1-r} \text{ or } S_n = \frac{a_1(1-r^n)}{1-r}$$

first term $= 5(-2)^{1-1} = 5$

Second term $= 5(-2)^{2-1} = -10$

$r = \frac{-10}{5} = -2$

$n = 10 - 1 + 1 = 10$

$S_{10} = \frac{5(1-(-2)^{10})}{1-(-2)}$

$\boxed{S_{10} = -1705}$

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Example 3 (p. 597) Got It? 2

Got It? **2.** To save money for a vacation, you set aside $100. For each month thereafter, you plan to set aside 10% more than the previous month. How much money will you save in 12 months?

$$S_n = \frac{a_1 - a_n r}{1-r} \text{ or } S_n = \frac{a_1(1-r^n)}{1-r}$$

$a_1 = 100$

$r = 1.10$

$$S_{12} = \frac{100(1-1.10^{12})}{1-1.10}$$

$$S_{12} \approx \$2138.43$$

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Infinite Geometric Series

- An infinite geometric series is the sum of a geometric series with | r | < 1 (that is, -1<r<1) $S = \dfrac{a_1}{1-r}$

- An infinite geometric series for which | r | > 1 (that is, r > 1 or r < -1), does not have a sum (the sum does not exist)

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Find the sum of the infinitie geometric series, if it exists.

1. $a_1=4$, $r=5/7$

2. 1+2/3+4/9+…

3. $\sum_{n=1}^{\infty} 3(0.5)^{n-1}$

4. The sum of an infinite geometric series is 81, and its common ration is 2/3. Find the first three terms of the series.

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Repeating Decimals

Write 0.16666666… as a fraction.

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