Essential Question How can you use the distances between each data value and the mean of a data set to measure the spread of a data set?

## The Meaning of a Word Deviate

When you deviate from something,
you stray or depart from the normal course of action.


1 ACJIV/JY: Finding Distances From the Mean
Work with a partner. The table shows the exam scores of 14 students in your class.

Statistics
In this lesson, you will

- understand the meaning of mean absolute deviation.
- find the mean absolute deviation of data sets.
Learning Standards 6.SP. 2
6.SP. 3
6.SP.5c

| Exam Scores |  |  |  |
| :--- | :--- | :--- | :--- |
| Ben | 89 | Mike | 95 |
| Emma | 86 | Hong | 96 |
| Jeremy | 80 | Rob | 92 |
| Pete | 80 | Amy | 90 |
| Ryan | 96 | Sue | 76 |
| Dan | 94 | Kim | 84 |
| Lucy | 89 | Heather | 85 |


a. What is the mean exam score?
b. Make a dot plot of the data. Place an " X " on the number line to represent the mean.
c. Is the number of exam scores that are greater than the mean equal to the number of exam scores that are less than the mean? Explain.
d. Which exam score deviates the most from the mean? Which exam score deviates the least from the mean? Explain how you found your answers.
e. Overall, do you think the exam scores are close to the mean or far away from the mean? Explain your reasoning.

## 2 ACIIVIJY: Using Distances from the Mean

## Math Practice

 Use Operations What operation can you use to find the distance from the mean? Explain.
## Work with a partner. Use the information in Activity 1.

a. Complete the table below. Add rows if needed. Be sure to find the sum of the values in the last column of the table.

| Student with Score <br> Less Than the Mean | Exam Score | Distance from <br> the Mean |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

b. Create a table similar to the one above for students with scores greater than the mean.
c. LOGIC What do you notice about the sums you found in your tables? Why do you think this happens?

## 3 ACJIVIJY: Interpreting Distances from the Mean

## Work with a partner.

a. LOGIC Add the sums you found in your tables in Activity 2. Divide that amount by the total number of students. Round your result to the nearest tenth.
In your own words, what do you think this value represents?
b. REASONING In a data set, what do you think it means when the value you found in part (a) is close to 0 ? Explain.

## What Is Your Answer?

4. IN YOUR OWN WORDS How can you use the distances between each data value and the mean of a data set to measure the spread of a data set?
5. REASONING Find the range and the interquartile range of the data set in Activity 1 . What do you think it means when these values are close to 0 ? Explain.

## Key Vocabulary

mean absolute
deviation, p. 420

Another measure of variation is the mean absolute deviation. The mean absolute deviation is an average of how much data values differ from the mean.

## GO Key Idea

Finding the Mean Absolute Deviation (MAD)
Step 1: Find the mean of the data.
Step 2: Find the distance between each data value and the mean.
Step 3: Find the sum of the distances in Step 2.
Step 4: Divide the sum in Step 3 by the total number of data values.

## EXAMPLE (1) Finding the Mean Absolute Deviation

You record the numbers of raisins in 8 scoops of cereal. Find and interpret the mean absolute deviation of the data.

$$
1,2,2,2,4,4,4,5
$$

Step 1: $\quad$ Mean $=\frac{1+2+2+2+4+4+4+5}{8}=\frac{24}{8}=3$
Step 2: You can use a dot plot to organize the data. Replace each dot with its distance from the mean.


Step 3: The sum of the distances is $2+1+1+1+1+1+1+2=10$.
Step 4: The mean absolute deviation is $\frac{10}{8}=1.25$.
$\therefore$ So, the data values differ from the mean by an average of 1.25 raisins.

## On Your Own

Now You're Ready
Exercises 5-8

1. Find and interpret the mean absolute deviation of the data.

$$
5,8,8,10,13,14,16,22
$$

The smartphones show the numbers of runs allowed
 by two pitchers in their last $\mathbf{1 0}$ starts.
a. Find the mean, median, and mean absolute deviation of the numbers of runs allowed for each pitcher.
Order the runs allowed for Mendoza:
$0,0,0,2,4,4,5,6,6,8$.
Mean $=\frac{35}{10}=3.5$

$$
\text { Median }=\frac{4+4}{2}=4
$$

Mean absolute deviation:


The mean absolute deviation is $\frac{24}{10}=2.4$
Order the runs allowed for Rodriguez: 0, 2, 2, 3, 4, 4, 4, 5, 5, 6 .
Mean $=\frac{35}{10}=3.5$

$$
\text { Median }=\frac{4+4}{2}=4
$$

Mean absolute deviation:


The mean absolute deviation is $\frac{14}{10}=1.4$.
b. Which measure can you use to distinguish the data? What can you conclude about the pitchers from this measure?

## Study Tip

The greater the mean absolute deviation, the greater the variation of the data.

You cannot use the measures of center to distinguish the data because they are the same for each data set. The measure of variation, MAD, is 2.4 for Mendoza and 1.4 for Rodriguez. This indicates that the data for Rodriguez has less variation.
$\therefore \quad$ Using the MAD to distinguish the data, you can conclude that Rodriguez is more consistent than Mendoza.

## On Your Own

2. WHAT IF? Mendoza allows 4 runs in the next game. How would you expect the mean absolute deviation to change? Explain.

## Vocabulary and Concept Check

1. REASONING Describe a data set that has a mean absolute deviation of 0 .
2. WHICH ONE DOESN'T BELONG? Which one does not belong with the other three? Explain your reasoning.
range interquartile range mean mean absolute deviation

## Practice and Problem Solving

Find the average distance each data value in the set is from the mean. Round your answer to the nearest tenth, if necessary.
3. Model years of used cars on a lot: 2010, 2002, 2005, 2007, 2001
4. Prices of kites at a shop: $\$ 7, \$ 20, \$ 9, \$ 35, \$ 12, \$ 15, \$ 7, \$ 10, \$ 20, \$ 25$

Find and interpret the mean absolute deviation of the data. Round your answer to the nearest tenth, if necessary.
5.

| Prices of Microphones <br> (dollars) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 25 | 28 | 20 | 22 | 32 |
| 28 | 35 | 34 | 30 | 36 |

6. 

| Heights of <br> Octuplets (inches) |  |  |  |
| :---: | :---: | :---: | :---: |
| 61 | 61 | 61 | 61 |
| 61 | 61 | 61 | 61 |

8. 

| Numbers of Visitors to a <br> Website During a Week |  |  |  |
| :---: | :---: | :---: | :---: |
| 103 | 115 | 124 | 125 |
| 171 | 165 | 170 |  |

9. ERROR ANALYSIS Describe and correct the error in finding the mean absolute deviation of the data set $35,40,38,32,42$, and 41 .

$$
\begin{aligned}
& \text { mean }=\frac{35+40+38+32+42+41}{6}=38 \\
& M A D=\frac{3+2+6+4+3}{5}=3.6
\end{aligned}
$$

So, the values differ from the mean by an average of 3.6 .
10. MUSEUMS The data set shows the admission prices at several museums.

$$
\$ 20, \$ 20, \$ 16, \$ 12, \$ 15, \$ 25, \$ 11
$$

Find and interpret the range, interquartile range, and mean absolute deviation of the data.

11. MENU The table shows the prices of the five most-expensive and least-expensive dishes on a menu. Find the MAD of each data set.

| Five Most-Expensive <br> Dishes |  |  |  | Five Least-Expensive <br> Dishes |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\$ 28$ | $\$ 30$ | $\$ 28$ | $\$ 39$ | $\$ 25$ | $\$ 7$ | $\$ 7$ | $\$ 10$ | $\$ 8$ | $\$ 12$ | Then compare their variations.

12. COINS The data sets show the years of the coins in two collections. Derek's collection: 1950, 1952, 1908, 1902, 1955, 1954, 1901, 1910

Paul's collection: 1929, 1935, 1928, 1930, 1925, 1932, 1933, 1920
Find the measures of center and the measures of variation for each data set. Compare the measures. What can you conclude?

13. PROBLEM SOLVING You survey students in your class about the number of movies they watched last month. The results are shown in the table.

| Movies Watched |  |  |  |
| :---: | :---: | :---: | :---: |
| 7 | 5 | 14 | 5 |
| 6 | 9 | 10 | 12 |
| 15 | 4 | 5 | 8 |
| 11 | 10 | 9 | 2 |

a. Find the measures of center and the measures of variation for the data.
b. A new student joins the class who watched 21 movies last month. Is 21 an outlier? How does including this value affect the measures of center and the measures of variation? Explain.

REASONING Which data set do you think would have the greater mean absolute deviation? Explain your reasoning.
14. guesses for number of gumballs in a jar guesses for number of baseballs in a jar
15. monthly rainfall amounts in a city monthly amounts of water used in a home
16. REASONING The MAD of a data set is considered a more reliable measure of variation than the range or the interquartile range. Why do you think this is true?
17. Arifinkal Add and subtract the MAD from the mean in the original data set in Exercise 13.
a. What percent of the values are within one MAD of the mean? two MADs of the mean? Which values are more than twice the MAD from the mean?
b. What do you notice as you get more and more MADs away from the mean? Explain.

## Fair Game Review what you learned in previous grades \& lessons

Find the mean, median, and mode(s) of the data. (Section 9.2 and Section 9.3)
18. $4,6,7,9,6,4,5,6,8,10$
19. $1.2,1.7,1.7,2.1,1.4,1.2,1.9$
20. MULTIPLE CHOICE What is the surface area of the square pyramid? (Section 8.3)
(A) $100.8 \mathrm{yd}^{2}$
(B) $147.2 \mathrm{yd}^{2}$
(C) $211.2 \mathrm{yd}^{2}$
(D) $368 \mathrm{yd}^{2}$


