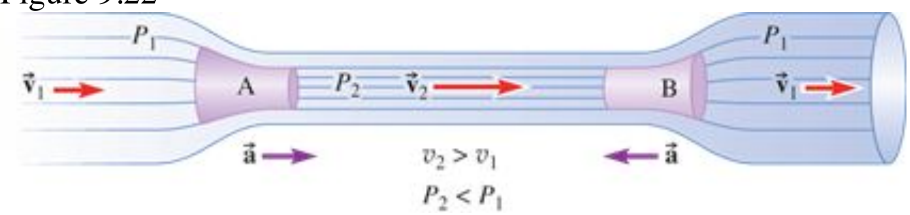
9.8 BERNOULLI'S EQUATION

The continuity equation relates the flow velocities of an ideal fluid at two different points, based on the change in cross-sectional area of the pipe. According to the continuity equation, the fluid must speed up as it enters a constriction (Fig. 9.22) and then slow down to its original speed when it leaves the constriction. Using energy ideas, we will show that the pressure of the fluid in the constriction (P_2) cannot be the same as the pressure before or after the constriction (P_1). For horizontal flow *the speed is higher where the pressure is lower*. This principle is often called the **Bernoulli effect.**

The Bernoulli effect: Fluid flows faster where the pressure is lower.

Figure 9.22



A small volume of fluid speeds up as it moves into a constriction (position A) and then slows down as it moves out of the constriction (position B).

The Bernoulli effect can seem counterintuitive at first; isn't rapidly moving fluid at *high* pressure? For instance, if you were hit with the fast-moving water out of a firehose, you would be knocked over easily. The force that knocks you over is indeed due to fluid pressure; you would justifiably conclude that the pressure was high. However, the pressure is not high *until you slow down the water* by getting in its way. The rapidly moving water in the jet is, in fact, approximately at atmospheric pressure (zero gauge pressure), but when you *stop* the water, its pressure increases dramatically.

Let's find the quantitative relationship between pressure changes and flow speed changes for an ideal fluid. In Fig. 9.23, the shaded volume of fluid flows to the right. If the left end moves a distance Δx_1, then the right end moves a distance Δx_2. Since the fluid is incompressible,

$$A_1 \Delta x_1 = A_2 \Delta x_2 = V$$

Work is done by the neighboring fluid during this flow. Fluid behind (to the left) pushes forward, doing positive work, while fluid ahead pushes backward, doing negative work. The total work done on the shaded volume by neighboring fluid is

$$W = P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2 = (P_1 - P_2)V$$

Since no dissipative forces act on an ideal fluid, the work done is equal to the total change in kinetic and gravitational potential energy. The net effect of the displacement is to move a volume V of fluid from height y_1 to height y_2 and to change its speed from v_1 to v_2. The energy change is therefore

$$\Delta E = \Delta K + \Delta U = \tfrac{1}{2}m(v_2^2 - v_1^2) + mg(y_2 - y_1)$$

where the +y-direction is up. Substituting $m = \rho V$ and equating the work done on the fluid to the change in its energy yields

$$(P_1 - P_2)\, V = \tfrac{1}{2}\, \rho V(v_2^2 - v_1^2) + \rho V g(y_2 - y_1)$$

Dividing both sides by *V* and rearranging yields Bernoulli's equation, named after Swiss mathematician Daniel Bernoulli (1700-1782), but first derived by fellow Swiss mathematician Leonhard Euler (pronounced like *oiler,* 1707-1783).

Bernoulli's equation (for ideal fluid flow):

(9-14)

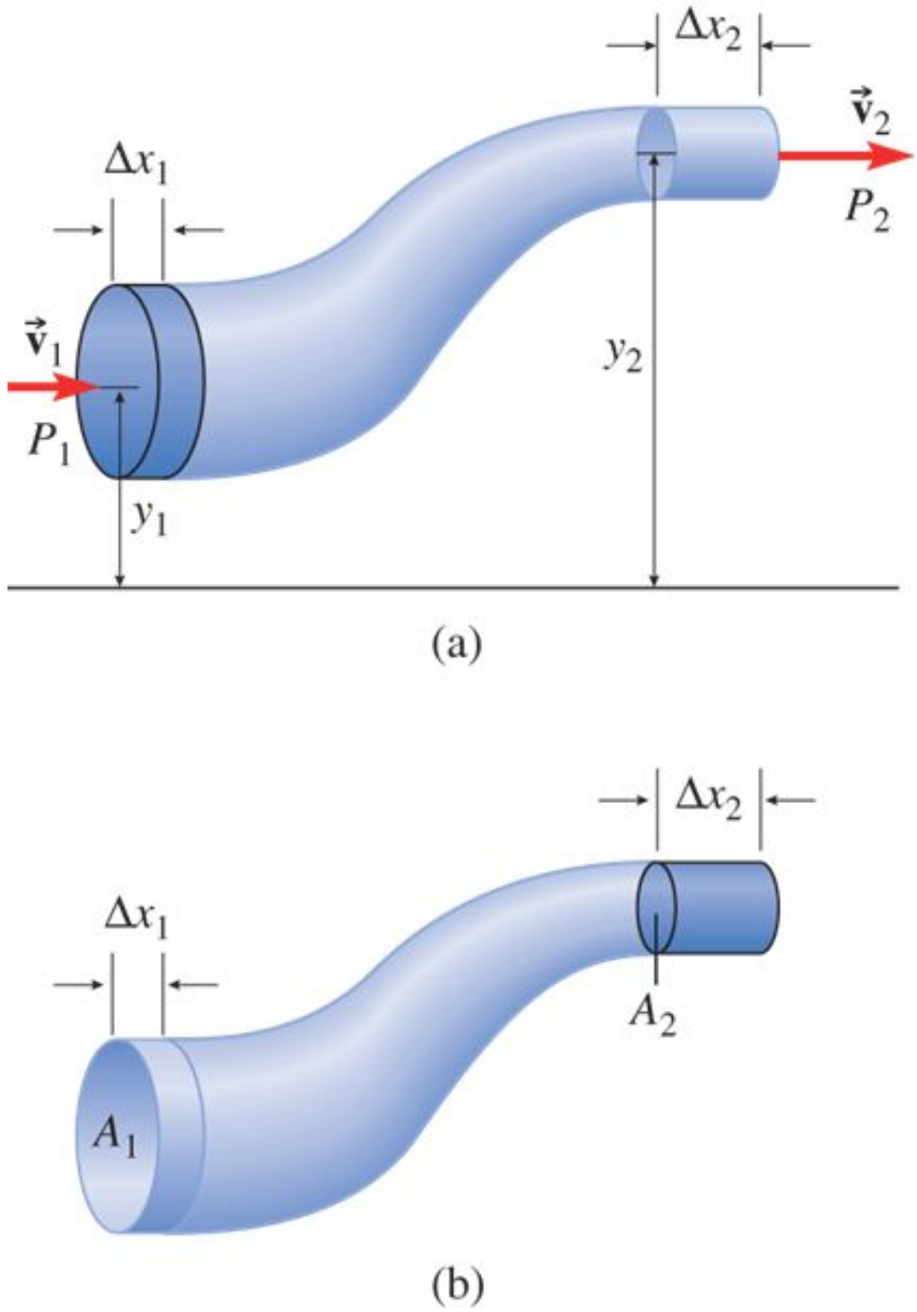
$$P_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2$$
$$(\text{or } P + \rho g y + \tfrac{1}{2}\rho v^2 = \text{constant})$$

Bernoulli's equation relates the pressure, flow speed, and height at two points in an ideal fluid. Although we derived Bernoulli's equation in a relatively simple situation, it applies to the flow of any ideal fluid as long as points 1 and 2 are on the same streamline.

CONNECTION:

Bernoulli's equation is a restatement of the principle of energy conservation applied to the flow of an ideal fluid.

Figure 9.23



Applying conservation of energy to the flow of an ideal fluid. The shaded volume of fluid in (a) is flowing to the right; (b) shows the same volume of fluid a short time later.
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Each term in Bernoulli's equation has units of pressure, which in the SI system is Pa or N/m^2. Since a joule is a newton-meter, the pascal is also equal to a joule per cubic meter (J/m^3). Each term represents work or energy per unit volume. The pressure is the work done by the fluid on the fluid ahead of it per unit volume of flow. The kinetic energy per unit volume is $\frac{1}{2}\rho v^2$ and the gravitational potential energy per unit volume is $\varrho g y$.

Tutorial: Energies (parts a-c)

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CHECKPOINT 9.8

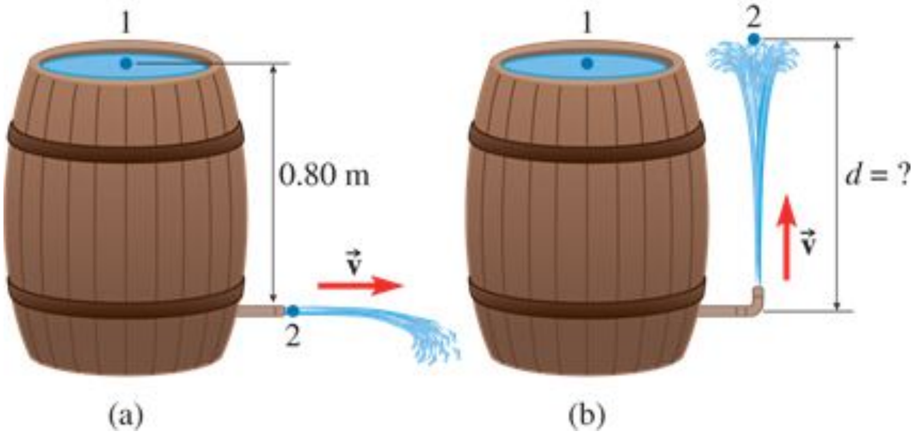
Discuss Bernoulli's equation in two special cases: (a) horizontal flow ($y_1 = y_2$) and (b) a static fluid ($v_1 = v_2 = 0$).

Example 9.10 Torricelli's Theorem

A barrel full of rainwater has a spigot near the bottom, at a depth of 0.80 m beneath the water surface. (a) When the spigot is directed horizontally (Fig. 9.24a) and is opened, how fast does the water come out? (b) If the opening points upward (Fig. 9.24b), how high does the resulting “fountain” go?

Tutorial: Energies (part d)

Figure 9.24



Full barrel of rainwater with open spigot (a) horizontal and (b) upward.
Strategy

The water at the surface is at atmospheric pressure. The water emerging from the spigot is *also* at atmospheric pressure since it is in contact with the air. If the pressure of the emerging water were different than that of the air, the stream would expand or contract until the pressures were equal. We apply Bernoulli's equation to two points: point 1 at the water surface and point 2 in the emerging stream of water.

Solution

- (a) Since $P_1 = P_2$, Bernoulli's equation is

  $$\rho g y_1 + \tfrac{1}{2}\rho v_1^2 = \rho g y_2 + \tfrac{1}{2}\rho v_2^2$$

  Point 1 is 0.80 m above point 2, so

  $$y_1 - y_2 = 0.80 \text{ m}$$

  The speed of the emerging water is v_2. What is v_1, the speed of the water at the surface? The water at the surface is moving slowly, since the barrel is draining. The continuity equation requires that

  $$v_1 A_1 = v_2 A_2$$

  Since the cross-sectional area of the spigot A_2 is much smaller than the area of the top of the barrel A_1, the speed of the water at the surface v_1 is negligibly small compared with v_2. Setting $v_1 = 0$, Bernoulli's equation reduces to

$$\rho g y_1 = \rho g y_2 + \tfrac{1}{2}\rho v_2^2$$

After dividing through by ϱ, we solve for v_2:

$$g(y_1 - y_2) = \tfrac{1}{2}v_2^2$$

$$v_2 = \sqrt{2g(y_1 - y_2)} = 4.0 \text{ m/s}$$

- (b) Now take point 2 to be at the top of the fountain. Then $v_2 = 0$ and Bernoulli's equation reduces to

$$\rho g y_1 = \rho g y_2$$

The "fountain" goes right back up to the top of the water in the barrel!

Discussion

The result of part (b) is called Torricelli's theorem. In reality, the fountain does not reach as high as the original water level; some energy is dissipated due to viscosity and air resistance.

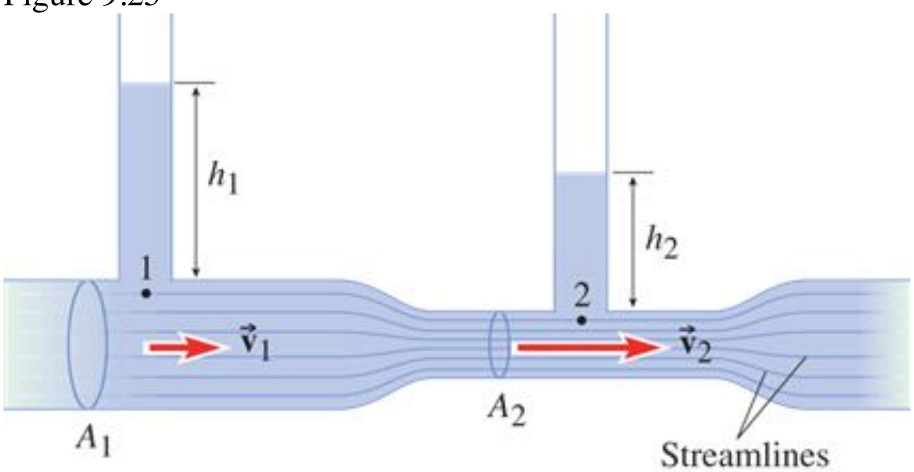
Practice Problem 9.10 Fluid in Free Fall

Verify that the speed found in part (a) is the same as if the water just fell 0.80 m straight down. That shouldn't be too surprising since Bernoulli's equation is an expression of energy conservation.

Example 9.11 The Venturi Meter

A *Venturi meter* (Fig. 9.25) measures fluid speed in a pipe. A constriction (of cross-sectional area A_2) is put in a pipe of normal cross-sectional area A_1. Two vertical tubes, open to the atmosphere, rise from two points, one of which is in the constriction. The vertical tubes function like manometers, enabling the pressure to be determined. From this information the flow speed in the pipe can be determined.

Figure 9.25



Venturi meter.

Suppose that the pipe in question carries water, $A_1 = 2.0A_2$, and the fluid heights in the vertical tubes are h_1 = 1.20 m and h_2 = 0.80 m. (a) Find the ratio of the flow speeds v_2/v_1. (b) Find the gauge pressures P_1 and P_2.

(c) Find the flow speed v_1 in the pipe.

Strategy

Neither of the two flow speeds is given. We need more than Bernoulli's equation to solve this problem. Since we know the ratio of the areas, the continuity equation gives us the ratio of the speeds. The height of the water in the vertical tubes enables us to find the pressures at points 1 and 2. The fluid pressure at the bottom of each vertical tube is the same as the pressure of the moving fluid just beneath each tube—otherwise, water would flow into or out of the vertical tubes until the pressure equalized. The water in the vertical tubes is static, so the gauge pressure at the bottom is $P = \rho gd$. Once we have the ratio of the speeds and the pressures, we apply Bernoulli's equation.

Solution

- (a) From the continuity equation, the product of flow speed and area must be the same at points 1 and 2. Therefore,

  $$\frac{v_2}{v_1} = \frac{A_1}{A_2} = 2.0$$

  The water flows twice as fast in the constriction as in the rest of the pipe.

- (b) The gauge pressures are:

  $$P_1 = \rho g h_1 = 1000\ \text{kg/m}^3 \times 9.80\ \text{N/kg} \times 1.20\ \text{m} = 11.8\ \text{kPa}$$
  $$P_2 = \rho g h_2 = 1000\ \text{kg/m}^3 \times 9.80\ \text{N/kg} \times 0.80\ \text{m} = 7.8\ \text{kPa}$$

- (c) Now we apply Bernoulli's equation. We can use gauge pressures as long as we do so on both sides—in effect we are just subtracting atmospheric pressure from both sides of the equation:

  $$P_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2$$

  Since the tube is horizontal, $y_1 \approx y_2$ and we can ignore the small change in gravitational potential energy density ρgy. Then

  $$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

  We are trying to find v_1, so we can eliminate v_2 by substituting $v_2 = 2.0\ v_1$:

  $$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho (2.0 v_1)^2$$

  Simplifying,

$$P_1 - P_2 = 1.5\rho v_1^2$$

$$v_1 = \sqrt{\frac{11\,800\text{ Pa} - 7800\text{ Pa}}{1.5 \times 1000\text{ kg/m}^3}} = 1.6\text{ m/s}$$

Discussion

The assumption that $y_1 \approx y_2$ is fine as long as the pipe radius is small compared with the difference between the static water heights (40 cm). Otherwise, we would have to account for the different y values in Bernoulli's equation.

One subtle point: recall that we assumed that the fluid pressure at the bottom of the vertical tubes was the same as the pressure of the moving fluid just beneath. Does that contradict Bernoulli's equation? Since there is an abrupt change in fluid speed, shouldn't there be a significant difference in the pressures? No, because these points are *not on the same streamline*.

Practice Problem 9.11 Garden Hose

Water flows horizontally through a garden hose of radius 1.0 cm at a speed of 1.4 m/s. The water shoots horizontally out of a nozzle of radius 0.25 cm. What is the gauge pressure of the water inside the hose?

Application of Bernoulli's Principle: Arterial Flutter and Aneurisms

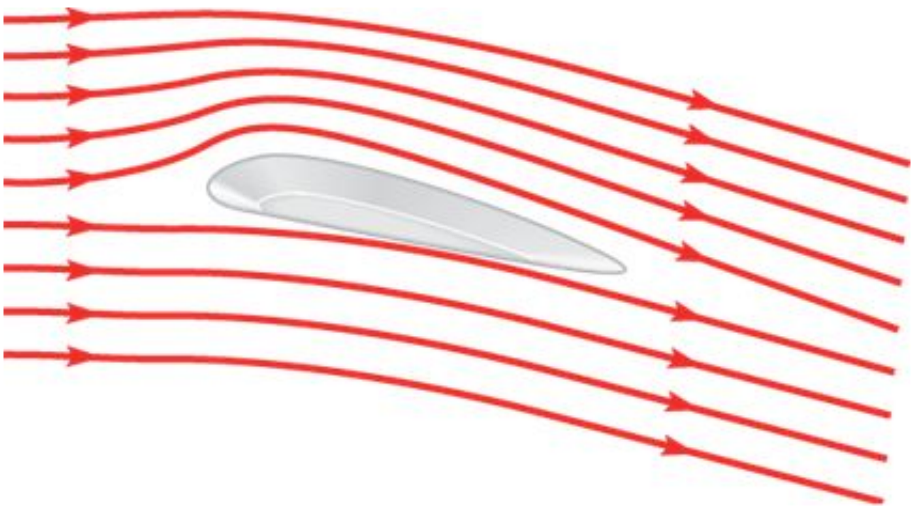
Suppose an artery is narrowed due to buildup of plaque on its inner walls. The flow of blood through the constriction is similar to that shown in Fig. 9.22. Bernoulli's equation tells us that the pressure P_2 in the constriction is lower than the pressure elsewhere. The arterial walls are elastic rather than rigid, so the lower pressure allows the arterial walls to contract a bit in the constriction. Now the flow velocity is even higher and the pressure even lower. Eventually the artery wall collapses, shutting off the flow of blood. Then the pressure builds up, reopens the artery, and allows blood to flow. The cycle of arterial flutter then begins again.

The opposite may happen where the arterial wall is weak. Blood pressure pushes the artery walls outward, forming a bulge called an aneurism. The lower flow speed in the bulge is accompanied by a higher blood pressure, which enlarges the aneurism even more (see Problem 88). Ultimately the artery may burst from the increased pressure.

Application of Bernoulli's Principle: Airplane Wings

How does an airplane wing generate lift? Figure 9.26 is a sketch of some streamlines for air flowing past an airplane wing in a wind tunnel. The streamlines bend, showing that the wing deflects air downward. By Newton's third law (or conservation of momentum), if the wing pushes downward on the air, the air also pushes upward on the wing. This upward force on the wing is lift. However, the situation is not as simple as air “bouncing” off the bottom of the wing—note that air passing above the wing is also deflected downward.

Figure 9.26



Streamlines showing the airflow past an airplane wing in a wind tunnel.

We can use Bernoulli's equation to get more insight into the generation of lift. (Bernoulli's equation applies in an approximate way to moving air. Even though air is not incompressible, for subsonic flight the density changes are small enough to be ignored.) If the air exerts a net upward force on the wing, the air pressure must be lower above the wing than beneath the wing. In Fig. 9.26, the streamlines above the wing are closer together than beneath the wing, showing that the flow speed above the wing is faster than it is beneath. This observation confirms that the pressure is lower above the wing, because where the pressure is lower, the flow speed is faster.

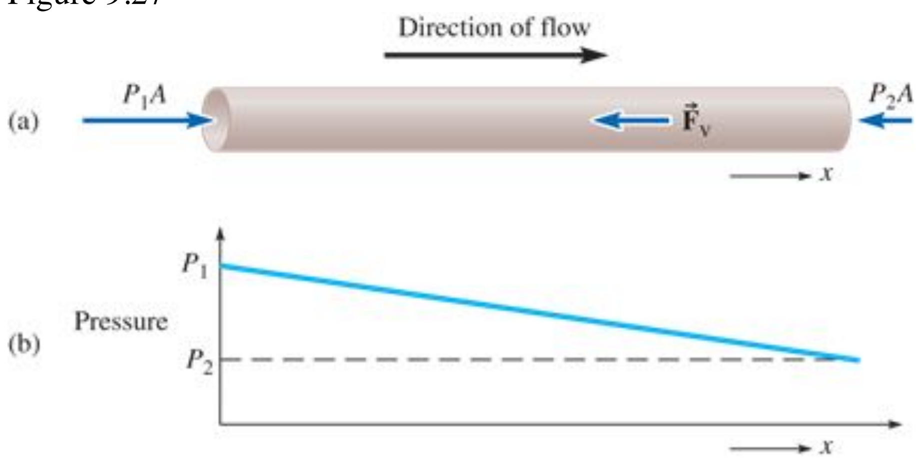
9.9 VISCOSITY

Bernoulli's equation ignores viscosity (fluid friction). According to Bernoulli's equation, an ideal fluid can continue to flow in a horizontal pipe at constant velocity on its own, just as a hockey puck would slide across frictionless ice at constant velocity without anything pushing it along. However, all real fluids have some viscosity; to maintain flow in a viscous fluid, we have to apply an external force since viscous forces oppose the flow of the fluid (Fig. 9.27). A *pressure difference* between the ends of the pipe must be maintained to keep a real liquid moving through a horizontal pipe. The pressure difference is important—in everything from blood flowing through arteries to oil pumped through a pipeline.

CONNECTION:

Kinetic friction makes a sliding object slow down unless an applied force balances the force of friction. Similarly, viscous forces oppose the flow of a fluid. Steady flow of a viscous fluid requires an applied force to balance the viscous forces. The applied force is due to the pressure difference.

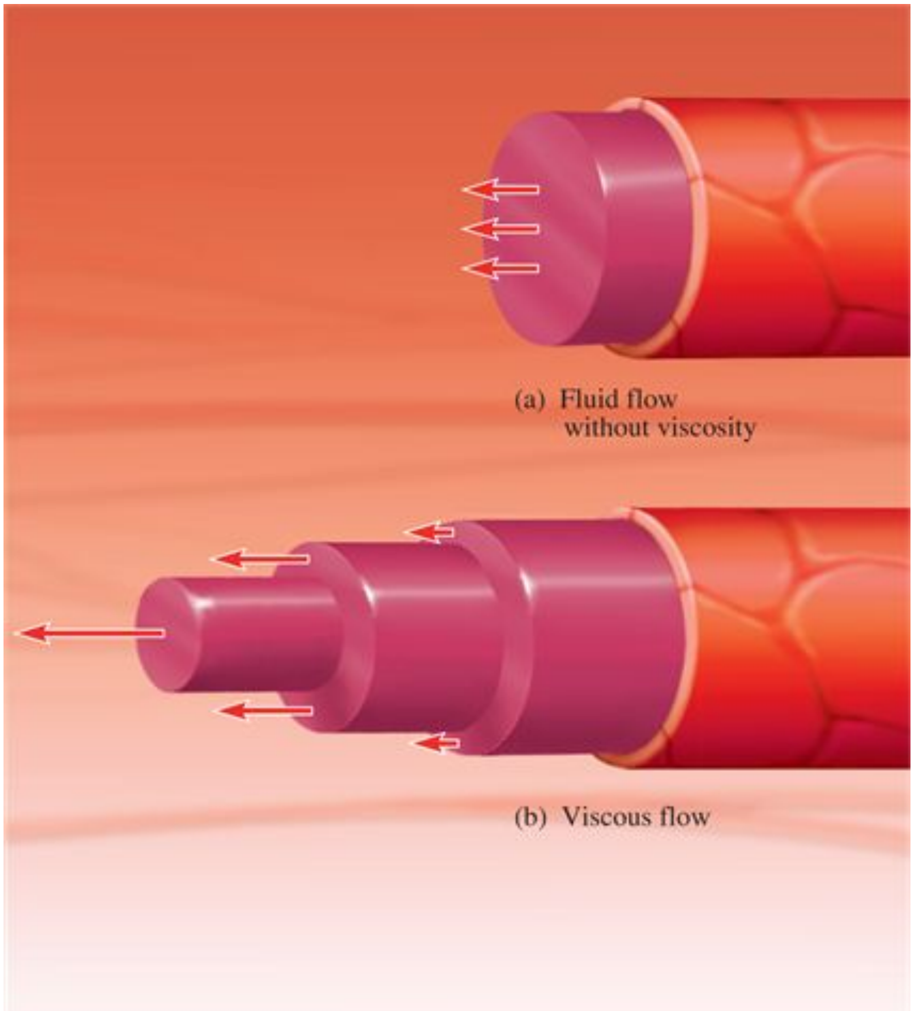
Figure 9.27



(a) To maintain viscous flow, a net force due to fluid pressure $(P_1 - P_2)A$ must be applied in the direction of flow to balance the viscous force F_v due to the pipe, which opposes flow. (b) The pressure in the fluid decreases from P_1 at the left end to P_2 at the right end.

To visualize viscous flow in a tube of circular cross section, imagine the fluid to flow in cylindrical layers, or shells. If there were no viscosity, all the layers would move at the same speed (Fig. 9.28a). In viscous flow, the fluid speed depends on the distance from the tube walls (Fig. 9.28b). The fastest flow is at the center of the tube. Layers closer to the wall of the tube move more slowly. The outermost layer of fluid, which is in contact with the tube, does not move. Each layer of fluid exerts viscous forces on the neighboring layers; these forces oppose the relative motion of the layers. The outermost layer exerts a viscous force on the tube.

Figure 9.28



(a) In nonviscous flow through a tube, the flow speed is the same everywhere. (b) In viscous flow, the flow speed depends on distance from the tube wall. This simplified sketch shows layers of fluid each moving at a different speed, but in reality the flow speed increases continuously from zero for the outermost "layer" to a maximum speed at the center.

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A liquid is more viscous if the cohesive forces between molecules are stronger. The viscosity of a liquid decreases with increasing temperature because the molecules become less tightly bound. A decrease in the temperature of the human body is dangerous because the viscosity of the blood increases and the flow of blood through the body is hindered. Gases, on the other hand, have an increase in viscosity for an increase in temperature. At higher temperatures the gas molecules move faster and collide more often with each other.

The coefficient of viscosity (or simply the *viscosity*) of a fluid is written as the Greek letter eta (η) and has units of pascal-seconds (Pa · s) in SI. Other viscosity units in common use are the poise (pronounced *pwäz,* symbol P; 1 P = 0.1 Pa · s) and the centipoise (1 cP = 0.01 P = 0.001 Pa · s). Table 9.2 lists the viscosities of some common fluids.

Table 9.2
Viscosities of Some Fluids

Substance	**Temperature (°C)**	**Viscosity (Pa · s)**
Gases		

Water vapor	100	1.3×10^{-5}
Air	0	1.7×10^{-5}
	20	1.8×10^{-5}
	30	1.9×10^{-5}
	100	2.2×10^{-5}
Liquids		
Acetone	30	0.30×10^{-3}
Methanol	30	0.51×10^{-3}
Ethanol	30	1.0×10^{-3}
Water	0	1.8×10^{-3}
	20	1.0×10^{-3}
	30	0.80×10^{-3}
	40	0.66×10^{-3}
	60	0.47×10^{-3}
	80	0.36×10^{-3}
	100	0.28×10^{-3}
Blood plasma	37	1.3×10^{-3}

Blood, whole	20	3.0×10^{-3}
	37	2.1×10^{-3}
Glycerin	20	0.83
	30	0.63
SAE 5W-30 motor oil	-30	≤ 6.6
	150	$\geq 2.9 \times 10^{-3}$

Poiseuille's Law

The volume flow rate $\Delta V/\Delta t$ for laminar flow of a viscous fluid through a horizontal, cylindrical pipe depends on several factors. First of all, the volume flow rate is proportional to the *pressure drop per unit length* ($\Delta P/L$)—also called the pressure gradient. If a pressure drop ΔP maintains a certain flow rate in a pipe of length L, then a similar pipe of length $2L$ needs twice the pressure drop to maintain the same flow rate (ΔP across the first half and another ΔP across the second half). Thus, the flow rate ($\Delta V/\Delta t$) must be proportional to the pressure drop per unit length ($\Delta P/L$).

Next, the flow rate is inversely proportional to the viscosity of the fluid. The more viscous the fluid, the smaller the flow rate, if all other factors are equal.

The only other consideration is the radius of the pipe. In the nineteenth century, during a study of flow in blood vessels, French physician Jean-Léonard Marie Poiseuille (1799-1869) discovered that the flow rate is proportional to the *fourth power* of the pipe radius:

Poiseuille's Law (for Viscous Flow)
(9-15)

$$\frac{\Delta V}{\Delta t} = \frac{\pi}{8} \frac{\Delta P/L}{\eta} r^4$$

where $\Delta V/\Delta t$ is the volume flow rate, ΔP is the pressure difference between the ends of the pipe, r and L are the inner radius and length of the pipe, respectively, and η is the viscosity of the fluid. Poiseuille's name is pronounced *pwahzoy,* in a rough English approximation.

It isn't often that we encounter a *fourth-power* dependence. Why such a strong dependence on radius? First of all, if fluids are flowing through two different pipes at the *same speed,* the volume flow rates are proportional to radius squared (flow rate = speed multiplied by cross-sectional area). But, in viscous flow, the average flow speed is larger for wider pipes; fluid farther away from the walls can flow faster. It turns out that the average flow speed for a given pressure gradient is also proportional to radius squared, giving the overall fourth power dependence on the pipe radius of Poiseuille's law.

□**Application of viscous flow:** high blood pressure

The strong dependence of flow rate on radius is important in blood flow. A person with cardiovascular disease has arteries narrowed by plaque deposits. To maintain the necessary blood flow to keep the body functioning, the blood pressure increases. If the diameter of an artery narrows to $\frac{1}{2}$ of its original value due to plaque deposits, the blood flow rate would decrease to $\frac{1}{16}$ of its original value if the pressure drop across it were to stay the same. To compensate for some of this decrease in blood flow, the heart pumps harder, increasing the blood pressure. High blood pressure is not good either; it introduces its own set of health problems, not least of which is the increased demands placed on the heart muscle.

Example 9.12 Arterial Blockage
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A cardiologist reports to her patient that the radius of the left anterior descending artery of the heart has narrowed by 10.0%. What percent increase in the blood pressure drop across the artery is required to maintain the normal blood flow through this artery?

Strategy

We assume that the viscosity of the blood has not changed, nor has the length of the artery. To maintain normal blood flow, the volume flow rate must stay the same:

$$\frac{\Delta V_1}{\Delta t} = \frac{\Delta V_2}{\Delta t}$$

Solution

If r_1 is the normal radius and r_2 is the actual radius, a 10.0% reduction in radius means $r_2 = 0.900r_1$. Then, from Poiseuille's law,

$$\frac{\pi \Delta P_1 r_1^4}{8\eta L} = \frac{\pi \Delta P_2 r_2^4}{8\eta L}$$

$$r_1^4 \Delta P_1 = r_2^4 \Delta P_2$$

We solve for the ratio of the pressure drops:

$$\frac{\Delta P_2}{\Delta P_1} = \frac{r_1^4}{r_2^4} = \frac{1}{(0.900)^4} = 1.52$$

Discussion

A factor of 1.52 means there is a 52% increase in the blood pressure difference across that artery. The increased pressure must be provided by the heart. If the normal pressure drop across the artery is 10 mm Hg, then it is now 15.2 mm Hg. The person's blood pressure either must increase by 5.2 mm Hg or there will be a reduction in blood flow through this artery. The heart is under greater strain as it works harder, attempting to maintain an adequate flow of blood.

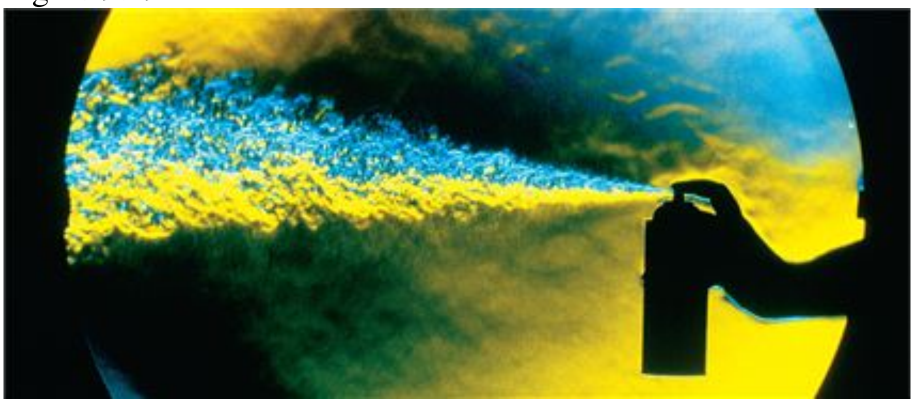
Practice Problem 9.12 New Water Pipe

The town water supply is operating at nearly full capacity. The town board decides to replace the water main with a bigger one to increase capacity. If the maximum flow rate is to increase by a factor of 4.0, by what factor should they increase the radius of the water main?

Turbulence

When the fluid velocity at a given point changes, the flow is *unsteady*. **Turbulence** is an extreme example of unsteady flow (Fig. 9.29). In turbulent flow, swirling vortices—whirlpools of fluid—appear. The vortices are not stationary; they move with the fluid. The flow velocity at any point changes erratically; prediction of the direction or speed of fluid flow under turbulent conditions is difficult.

Figure 9.29



Turbulent flow of gas emerging from the nozzle of an aerosol can.

9.10 VISCOUS DRAG

When an object moves through a fluid, the fluid exerts a drag force on it. When the relative velocity between the object and the fluid is low enough for the flow around the object to be laminar, the drag force derives from viscosity and is called **viscous drag.** The viscous drag force is proportional to the speed of the object. For larger relative speeds, the flow becomes turbulent and the drag force is proportional to the square of the object's speed.

Viscous drag: $F_D \propto v$

Turbulent drag: $F_D \propto v^2$

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The viscous drag force depends also on the shape and size of the object. For a spherical object, the viscous drag force is given by Stokes's law:

Stokes's Law (viscous drag on a sphere)
(9-16)

$$F_D = 6\pi\eta r v$$

where r is the radius of the sphere, η is the viscosity of the fluid, and v is the speed of the object with respect to the fluid.

CHECKPOINT 9.10

Compare and contrast the viscous drag force with the kinetic frictional force.

An object's **terminal velocity** is the velocity that produces just the right drag force so that the net force is zero. An object falling at its terminal velocity has zero acceleration, so it continues moving at that constant velocity. Using Stokes's law, we can find the terminal velocity of a spherical object falling through a viscous fluid. When the object moves at terminal velocity, the net force acting on it is zero. If $\rho_o > \rho_f$, the object sinks; the terminal velocity is downward and the viscous drag force acts upward to oppose the motion. For an object, such as an air bubble in oil, that rises rather than sinks ($\rho_o < \rho_f$), the terminal velocity is *upward* and the drag force is *downward*.

Example 9.13 Falling Droplet

In an experiment to measure the electric charge of the electron, a fine mist of oil droplets is sprayed into the air and observed through a telescope as they fall. These droplets are so tiny that they soon reach their terminal velocity. If the radius of the droplets is 2.40 μm and the average density of the oil is 862 kg/m^3, find the terminal speed of the droplets. The density of air is 1.20 kg/m^3 and the viscosity of air is 1.8×10^{-5} Pa · s.

Strategy

When the droplets fall at their terminal velocity, the net force on them is zero. We set the net force equal to zero and use Stokes's law for the drag force.

Solution

We set the sum of the forces equal to zero when $v = v_t$.

$$\sum F_y = +F_D + F_B - W = 0$$

If m_{air} is the mass of displaced air, then

$$6\pi\eta r v_t + m_{air} g - m_{oil} g = 0$$

Solving for v_t,

$$v_t = \frac{g(m_{oil} - m_{air})}{6\pi\eta r}$$

$$= \frac{\frac{4}{3}\pi r^3 g(\rho_{oil} - \rho_{air})}{6\pi\eta r}$$

After dividing the numerator and denominator by πr, we substitute numerical values:

$$v_t = \frac{\frac{4}{3}(2.40 \times 10^{-6}\ \text{m})^2\ (9.80\ \text{N/kg})\ (862\ \text{kg/m}^3 - 1.20\ \text{kg/m}^3)}{6 \times 1.8 \times 10^{-5}\ \text{Pa·s}}$$

$$= 6.0 \times 10^{-4}\ \text{m/s} = 0.60\ \text{mm/s}$$

Discussion

We should check the units in the final expression:

$$\frac{\text{m}^2 \cdot (\text{N/kg}) \cdot \text{kg/m}^3}{\text{Pa·s}} = \frac{\text{N/m}}{\text{N/m}^2 \times \text{s}} = \text{m/s}$$

Stokes's law was applied in this way by Robert Millikan (1868-1953) in his experiments in 1909-1913 to measure the charge of the electron. Using an atomizer, Millikan produced a fine spray of oil droplets. The droplets picked up electric charge as they were sprayed through the atomizer. Millikan kept a droplet suspended without falling by applying an upward electric force. After removing the electric force, he measured the terminal speed of the droplet as it fell through the air. He calculated the mass of the droplet from the terminal speed and the density of the oil using Stokes's law. By setting the magnitude of the electric force equal to the weight of a suspended droplet, Millikan calculated the electric charge of the droplet. He measured the charges of hundreds of different droplets and found that they were all multiples of the same quantity—the charge of an electron.

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Practice Problem 9.13 Rising Bubble

Find the terminal velocity of an air bubble of 0.500 mm radius in a cup of vegetable oil. The specific gravity of the oil is 0.840 and the viscosity is 0.160 Pa · s. Assume the diameter of the bubble does not change as it rises.

PHYSICS AT HOME

A demonstration of terminal velocity can be done at home. Climb up a small stepladder, or lean over an upstairs balcony, and drop two objects at the same time: a coin and two or three nested cone-shaped paper coffee filters. You will see the effects of viscous drag on the coffee filters as they fall with a constant terminal velocity. Enlist the help of a friend so you can get a side view of the two objects falling. Why do the coffee filters work so well?

Application of viscous drag: sedimentation velocity and the centrifuge

For small particles falling in a liquid, the terminal velocity is also called the *sedimentation velocity*. The sedimentation velocity is often small for two reasons. First, if the particle isn't much more dense than the fluid, then the vector sum of the gravitational and buoyant forces is small. Second, notice that the terminal velocity is proportional to r^2; viscous drag is most important for small particles. Thus, it can take a long time for the particles to sediment out of solution. Because the sedimentation velocity is proportional to g, it can be increased by the use of a centrifuge, a rotating container that creates artificial gravity of magnitude $g_{\text{eff}} = \omega^2 r$ [see Eq. (5-12) and Section 5.7]. Ultracentrifuges are capable of rotating at 10^5 rev/min and produce artificial gravity approaching a million times g.

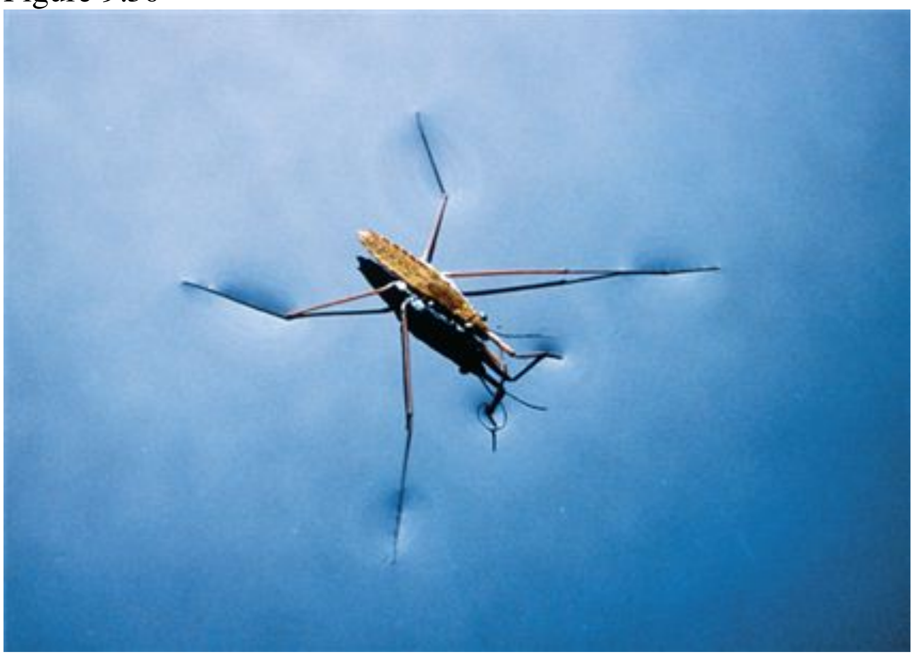
9.11 SURFACE TENSION

The surface of a liquid has special properties not associated with the interior of the liquid. The surface acts like a stretched membrane under tension. The **surface tension** (symbol γ, the Greek letter gamma) of a liquid is the force per unit *length* with which the surface pulls *on its edge*. The direction of the force is tangent to the surface at its edge. Surface tension is caused by the cohesive forces that pull the molecules toward each other.

The high surface tension of water enables water striders and other small insects to walk on the surface of a pond. The foot of the insect makes a small indentation in the water surface (Fig. 9.30); the deformation of the surface enables it to push upward on the foot as if the water surface were a thin sheet of rubber. Visually it looks similar to a person walking across the mat of a trampoline. Other small water creatures, such as mosquito larvae and planaria, hang from the surface of water, using surface tension to hold themselves up. In plants, surface tension aids in the transport of water from the roots to the leaves.

□**Application of surface tension:** how insects can walk on the surface of a pond

Figure 9.30



A water strider.

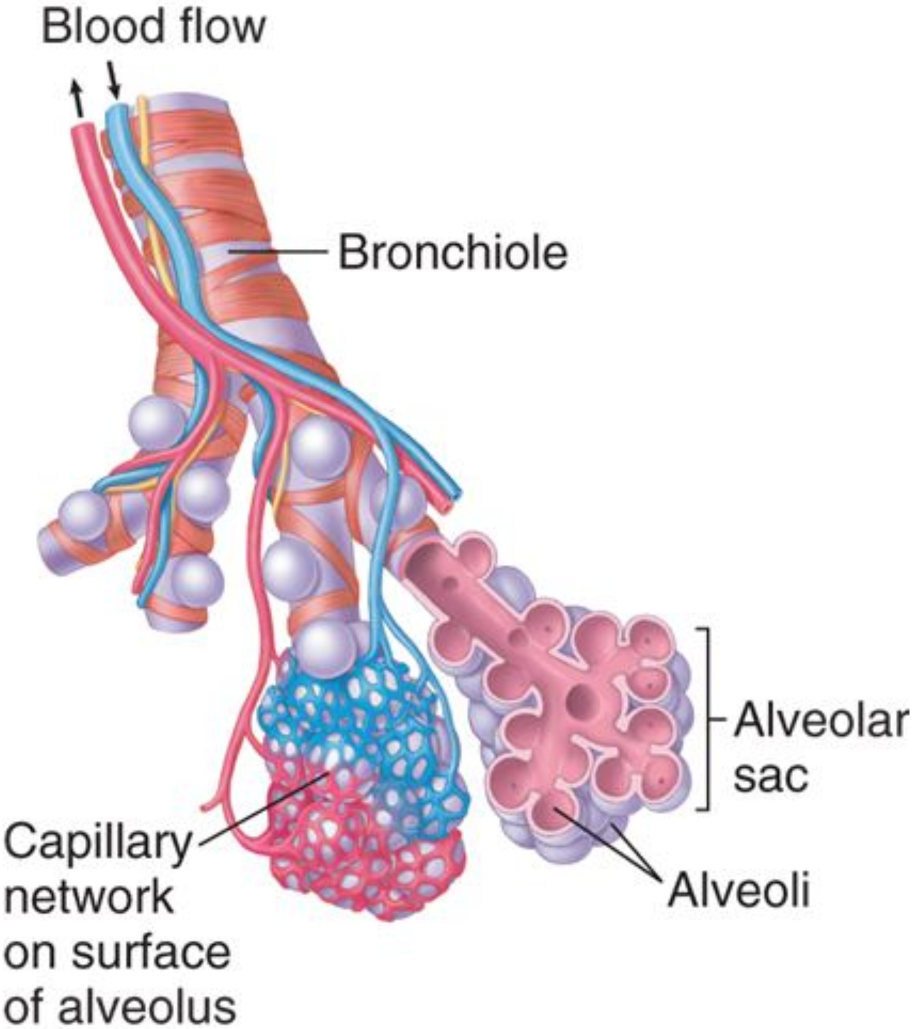
PHYSICS AT HOME

Place a needle (or a flat plastic-coated paper clip) gently on the surface of a glass of water. It may take some practice, but you should be able to get it to “float” on top of the water. Now add some detergent to the water and try again. The detergent reduces the surface tension of the water so it is unable to support the needle. Soaps and detergents are surfactants—substances that reduce the surface tension of a fluid. The reduced surface tension allows the water to spread out more, wetting more of a surface to be cleaned.

The high surface tension of water is a hindrance in the lungs. The exchange of oxygen and carbon dioxide between inspired air and the blood takes place in tiny sacs called *alveoli,* 0.05 to 0.15 mm in radius, at the end of the bronchial tubes (Fig. 9.31). If the mucus coating the alveoli had the same surface tension as other body fluids, the pressure difference between the inside and outside of the alveoli would not be great enough for them to expand and fill with air. The alveoli secrete a surfactant that decreases the surface tension in their mucous coating so they can inflate during inhalation.

□**Application of surface tension:** surfactant in the lungs

Figure 9.31



In the human lung, millions of tiny sacs called alveoli are inflated with each breath. Gas is exchanged between the air and the blood through the walls of the alveoli. The total surface area through which gas exchange takes place is about 80 m^2—about 40 times the surface area of the body.

Bubbles

In an underwater air bubble, the surface tension of the water surface tries to contract the bubble while the pressure of the enclosed air pushes outward on the surface. In equilibrium, the air pressure inside the bubble must be larger than the water pressure outside so that the net outward force due to pressure balances the inward force due to surface tension. The excess pressure $\Delta P = P_{in} - P_{out}$ depends both on the surface tension and the size of the bubble. In Problem 72, you can show that the excess pressure is

(9-17)

$$\Delta P = \frac{2\gamma}{r}$$

Look closely at a glass of champagne and you can see strings of bubbles rising, originating from the same points in the liquid. Why don't bubbles spring up from random locations? A very small bubble would require an insupportably large excess pressure. The bubbles need some sort of nucleus—a small dust particle, for instance—on which to form so they can start out larger, with excess pressures that aren't so large. The strings of bubbles in the glass of champagne are showing where suitable nuclei have been “found.”

Example 9.14 Lung Pressure

During inhalation the gauge pressure in the alveoli is about –400 Pa to allow air to flow in through the bronchial tubes. Suppose the mucous coating on an alveolus of initial radius 0.050 mm had the same surface tension as water (0.070 N/m). What lung pressure outside the alveoli would be required to begin to inflate the alveolus?

Strategy

We model an alveolus as a sphere coated with mucus. Due to the surface tension of the mucus, the alveolus must have a lower pressure outside than inside, as for a bubble.

Solution

The excess pressure is

$$\Delta P = \frac{2\gamma}{r} = \frac{2 \times 0.070\ \text{N/m}}{0.050 \times 10^{-3}\ \text{m}} = 2.8\ \text{kPa}$$

Thus, the pressure inside the alveolus would be 2.8 kPa higher than the pressure outside. The gauge pressure inside is –400 Pa, so the gauge pressure outside would be

$$P_{out} = -0.4\ \text{kPa} - 2.8\ \text{kPa} = -3.2\ \text{kPa}$$

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Discussion

The *actual* gauge pressure outside the alveoli is about –0.5 kPa rather than –3.2 kPa; then $\Delta P = P_{in} - P_{out}$ = –0.4 kPa –(-0.5 kPa) = 0.1 kPa rather than 2.8 kPa. Here the surfactant comes to the rescue; by decreasing the surface tension in the mucus, it decreases ΔP to about 0.1 kPa and allows the expansion of the alveoli to take place. For a newborn baby, the alveoli are initially collapsed, making the required pressure difference about 4 kPa. That first breath is as difficult an event as it is significant.

Practice Problem 9.14 Champagne Bubbles

A bubble in a glass of champagne is filled with CO_2. When it is 2.0 cm below the surface of the champagne, its radius is 0.50 mm. What is the gauge pressure inside the bubble? Assume that champagne has the same average density as water and a surface tension of 0.070 N/m.