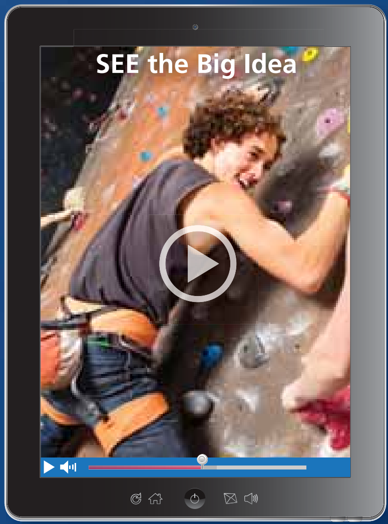
9 Right Triangles and Trigonometry

Leaning Tower of Pisa *(p. 518)*

Skiing *(p. 501)*

Washington Monument *(p. 495)*



Rock Wall *(p. 485)*

Fire Escape *(p. 473)*

Mathematical Thinking: *Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.*

Maintaining Mathematical Proficiency

Using Properties of Radicals (A.11.A)

Example 1 Simplify $\sqrt{128}$.

$\sqrt{128} = \sqrt{64 \cdot 2}$ Factor using the greatest perfect square factor.

$= \sqrt{64} \cdot \sqrt{2}$ Product Property of Radicals

$= 8\sqrt{2}$ Simplify.

Example 2 Simplify $\frac{4}{\sqrt{5}}$.

$\frac{4}{\sqrt{5}} = \frac{4}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$ Multiply by $\frac{\sqrt{5}}{\sqrt{5}}$.

$= \frac{4\sqrt{5}}{\sqrt{25}}$ Product Property of Radicals

$= \frac{4\sqrt{5}}{5}$ Simplify.

Simplify the expression.

1. $\sqrt{75}$
2. $\sqrt{270}$
3. $\sqrt{135}$
4. $\frac{2}{\sqrt{7}}$
5. $\frac{5}{\sqrt{2}}$
6. $\frac{12}{\sqrt{6}}$

Solving Proportions (7.4.D)

Example 3 Solve $\frac{x}{10} = \frac{3}{2}$.

$\frac{x}{10} = \frac{3}{2}$ Write the proportion.

$x \cdot 2 = 10 \cdot 3$ Cross Products Property

$2x = 30$ Multiply.

$\frac{2x}{2} = \frac{30}{2}$ Divide each side by 2.

$x = 15$ Simplify.

Solve the proportion.

7. $\frac{x}{12} = \frac{3}{4}$
8. $\frac{x}{3} = \frac{5}{2}$
9. $\frac{4}{x} = \frac{7}{56}$
10. $\frac{10}{23} = \frac{4}{x}$
11. $\frac{x+1}{2} = \frac{21}{14}$
12. $\frac{9}{3x-15} = \frac{3}{12}$

13. **ABSTRACT REASONING** The Product Property of Radicals allows you to simplify the square root of a product. Are you able to simplify the square root of a sum? of a difference? Explain.

Mathematical Thinking

Mathematically proficient students display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication. **(G.1.G)**

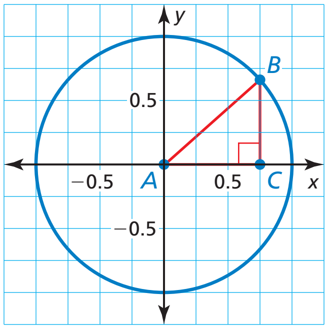
Attending to Precision

Core Concept

Standard Position for a Right Triangle

In *unit circle trigonometry*, a right triangle is in **standard position** when:

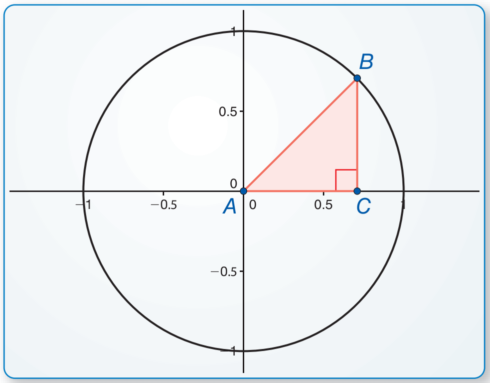
1. The hypotenuse is a radius of the circle of radius 1 with center at the origin.
2. One leg of the right triangle lies on the x-axis.
3. The other leg of the right triangle is perpendicular to the x-axis.



EXAMPLE 1 **Drawing an Isosceles Right Triangle in Standard Position**

Use dynamic geometry software to construct an isosceles right triangle in standard position. What are the exact coordinates of its vertices?

SOLUTION



Sample

Points
$A(0, 0)$
$B(0.71, 0.71)$
$C(0.71, 0)$
Segments
$AB = 1$
$BC = 0.71$
$AC = 0.71$
Angle
$m\angle A = 45°$

To determine the exact coordinates of the vertices, label the length of each leg x. By the Pythagorean Theorem, which you will study in Section 9.1, $x^2 + x^2 = 1$. Solving this equation yields

$$x = \frac{1}{\sqrt{2}}, \text{ or } \frac{\sqrt{2}}{2}.$$

▶ So, the exact coordinates of the vertices are $A(0, 0)$, $B\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$, and $C\left(\frac{\sqrt{2}}{2}, 0\right)$.

Monitoring Progress

1. Use dynamic geometry software to construct a right triangle with acute angle measures of 30° and 60° in standard position. What are the exact coordinates of its vertices?
2. Use dynamic geometry software to construct a right triangle with acute angle measures of 20° and 70° in standard position. What are the approximate coordinates of its vertices?

9.1 The Pythagorean Theorem

Essential Question How can you prove the Pythagorean Theorem?

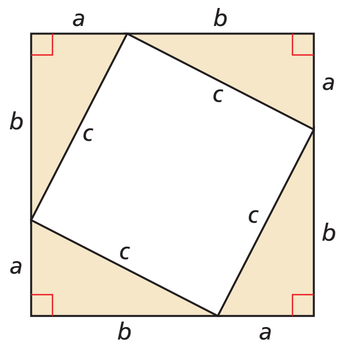
TEXAS ESSENTIAL KNOWLEDGE AND SKILLS
G.6.D
G.9.B

EXPLORATION 1 Proving the Pythagorean Theorem without Words

Work with a partner.

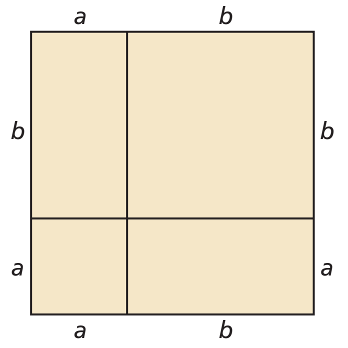
a. Draw and cut out a right triangle with legs a and b, and hypotenuse c.

b. Make three copies of your right triangle. Arrange all four triangles to form a large square, as shown.



c. Find the area of the large square in terms of a, b, and c by summing the areas of the triangles and the small square.

d. Copy the large square. Divide it into two smaller squares and two equally-sized rectangles, as shown.



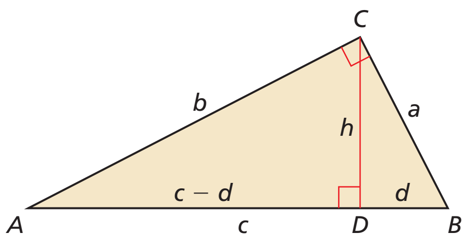
e. Find the area of the large square in terms of a and b by summing the areas of the rectangles and the smaller squares.

f. Compare your answers to parts (c) and (e). Explain how this proves the Pythagorean Theorem.

EXPLORATION 2 Proving the Pythagorean Theorem

Work with a partner.

a. Draw a right triangle with legs a and b, and hypotenuse c, as shown. Draw the altitude from C to $\overline{AB}$. Label the lengths, as shown.



REASONING
To be proficient in math, you need to know and flexibly use different properties of operations and objects.

b. Explain why $\triangle ABC$, $\triangle ACD$, and $\triangle CBD$ are similar.

c. Write a two-column proof using the similar triangles in part (b) to prove that $a^2 + b^2 = c^2$.

Communicate Your Answer

3. How can you prove the Pythagorean Theorem?

4. Use the Internet or some other resource to find a way to prove the Pythagorean Theorem that is different from Explorations 1 and 2.

9.1 Lesson

What You Will Learn

- Use the Pythagorean Theorem.
- Use the Converse of the Pythagorean Theorem.
- Classify triangles.

Core Vocabulary

Pythagorean triple, *p. 468*

Previous
right triangle
legs of a right triangle
hypotenuse

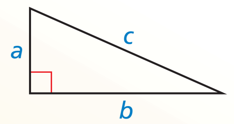
Using the Pythagorean Theorem

One of the most famous theorems in mathematics is the Pythagorean Theorem, named for the ancient Greek mathematician Pythagoras. This theorem describes the relationship between the side lengths of a right triangle.

Theorem

Theorem 9.1 Pythagorean Theorem

In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



$c^2 = a^2 + b^2$

Proof Explorations 1 and 2, p. 467; Ex. 39, p. 488

A **Pythagorean triple** is a set of three positive integers a, b, and c that satisfy the equation $c^2 = a^2 + b^2$.

STUDY TIP

You may find it helpful to memorize the basic Pythagorean triples, shown in **bold**, for standardized tests.

Core Concept

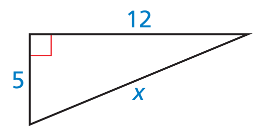
Common Pythagorean Triples and Some of Their Multiples

3, 4, 5	**5, 12, 13**	**8, 15, 17**	**7, 24, 25**
6, 8, 10	10, 24, 26	16, 30, 34	14, 48, 50
9, 12, 15	15, 36, 39	24, 45, 51	21, 72, 75
$3x, 4x, 5x$	$5x, 12x, 13x$	$8x, 15x, 17x$	$7x, 24x, 25x$

The most common Pythagorean triples are in bold. The other triples are the result of multiplying each integer in a bold-faced triple by the same factor.

EXAMPLE 1 Using the Pythagorean Theorem

Find the value of x. Then tell whether the side lengths form a Pythagorean triple.



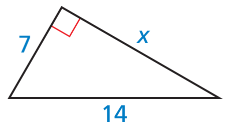
SOLUTION

$c^2 = a^2 + b^2$	Pythagorean Theorem
$x^2 = 5^2 + 12^2$	Substitute.
$x^2 = 25 + 144$	Multiply.
$x^2 = 169$	Add.
$x = 13$	Find the positive square root.

▶ The value of x is 13. Because the side lengths 5, 12, and 13 are integers that satisfy the equation $c^2 = a^2 + b^2$, they form a Pythagorean triple.

EXAMPLE 2 Using the Pythagorean Theorem

Find the value of x. Then tell whether the side lengths form a Pythagorean triple.



SOLUTION

$c^2 = a^2 + b^2$ — Pythagorean Theorem

$14^2 = 7^2 + x^2$ — Substitute.

$196 = 49 + x^2$ — Multiply.

$147 = x^2$ — Subtract 49 from each side.

$\sqrt{147} = x$ — Find the positive square root.

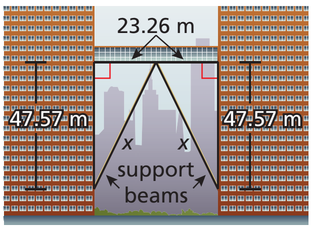
$\sqrt{49} \cdot \sqrt{3} = x$ — Product Property of Radicals

$7\sqrt{3} = x$ — Simplify.

▶ The value of x is $7\sqrt{3}$. Because $7\sqrt{3}$ is not an integer, the side lengths do not form a Pythagorean triple.

EXAMPLE 3 Solving a Real-Life Problem

The skyscrapers shown are connected by a skywalk with support beams. Use the Pythagorean Theorem to approximate the length of each support beam.



SOLUTION

Each support beam forms the hypotenuse of a right triangle. The right triangles are congruent, so the support beams are the same length.

$x^2 = (23.26)^2 + (47.57)^2$ — Pythagorean Theorem

$x = \sqrt{(23.26)^2 + (47.57)^2}$ — Find the positive square root.

$x \approx 52.95$ — Use a calculator to approximate.

▶ The length of each support beam is about 52.95 meters.

Monitoring Progress

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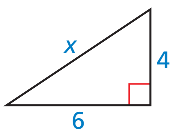
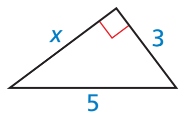
Find the value of x. Then tell whether the side lengths form a Pythagorean triple.

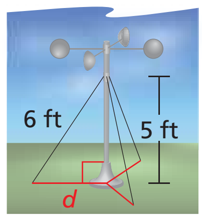
1.



2.



3. An anemometer is a device used to measure wind speed. The anemometer shown is attached to the top of a pole. Support wires are attached to the pole 5 feet above the ground. Each support wire is 6 feet long. How far from the base of the pole is each wire attached to the ground?



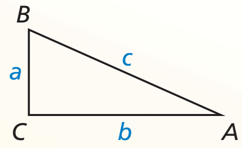
Using the Converse of the Pythagorean Theorem

The converse of the Pythagorean Theorem is also true. You can use it to determine whether a triangle with given side lengths is a right triangle.

Theorem

Theorem 9.2 Converse of the Pythagorean Theorem

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.



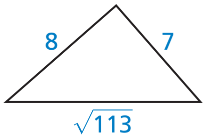
If $c^2 = a^2 + b^2$, then $\triangle ABC$ is a right triangle.

Proof Ex. 39, p. 474

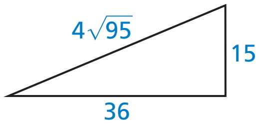
EXAMPLE 4 Verifying Right Triangles

Tell whether each triangle is a right triangle.

a.



b.



SELECTING TOOLS

Use a calculator to determine that $\sqrt{113} \approx 10.630$ is the length of the longest side in part (a).

SOLUTION

Let c represent the length of the longest side of the triangle. Check to see whether the side lengths satisfy the equation $c^2 = a^2 + b^2$.

a. $\left(\sqrt{113}\right)^2 \stackrel{?}{=} 7^2 + 8^2$

$113 \stackrel{?}{=} 49 + 64$

$113 = 113$ ✓

▶ The triangle is a right triangle.

b. $\left(4\sqrt{95}\right)^2 \stackrel{?}{=} 15^2 + 36^2$

$4^2 \cdot \left(\sqrt{95}\right)^2 \stackrel{?}{=} 15^2 + 36^2$

$16 \cdot 95 \stackrel{?}{=} 225 + 1296$

$1520 \neq 1521$ ✗

▶ The triangle is *not* a right triangle.

Monitoring Progress

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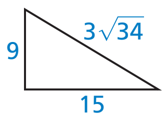
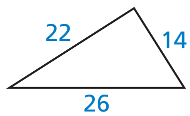
Tell whether the triangle is a right triangle.

4.



5.



Classifying Triangles

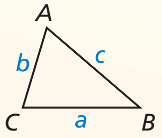
The Converse of the Pythagorean Theorem is used to determine whether a triangle is a right triangle. You can use the theorem below to determine whether a triangle is acute or obtuse.

Theorem

Theorem 9.3 Pythagorean Inequalities Theorem

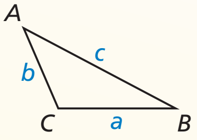
For any $\triangle ABC$, where c is the length of the longest side, the following statements are true.

If $c^2 < a^2 + b^2$, then $\triangle ABC$ is acute.



$c^2 < a^2 + b^2$

If $c^2 > a^2 + b^2$, then $\triangle ABC$ is obtuse.



$c^2 > a^2 + b^2$

Proof Exs. 42 and 43, p. 474

REMEMBER

The Triangle Inequality Theorem (Theorem 6.11) on page 343 states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

EXAMPLE 5 Classifying Triangles

Verify that segments with lengths of 4.3 feet, 5.2 feet, and 6.1 feet form a triangle. Is the triangle *acute*, *right*, or *obtuse*?

SOLUTION

Step 1 Use the Triangle Inequality Theorem (Theorem 6.11) to verify that the segments form a triangle.

$4.3 + 5.2 \overset{?}{>} 6.1$ $4.3 + 6.1 \overset{?}{>} 5.2$ $5.2 + 6.1 \overset{?}{>} 4.3$

$9.5 > 6.1$ ✓ $10.4 > 5.2$ ✓ $11.3 > 4.3$ ✓

▶ The segments with lengths of 4.3 feet, 5.2 feet, and 6.1 feet form a triangle.

Step 2 Classify the triangle by comparing the square of the length of the longest side with the sum of the squares of the lengths of the other two sides.

$c^2 \;\square\; a^2 + b^2$	Compare c^2 with $a^2 + b^2$.
$6.1^2 \;\square\; 4.3^2 + 5.2^2$	Substitute.
$37.21 \;\square\; 18.49 + 27.04$	Simplify.
$37.21 < 45.53$	c^2 is less than $a^2 + b^2$.

▶ The segments with lengths of 4.3 feet, 5.2 feet, and 6.1 feet form an acute triangle.

Monitoring Progress

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6. Verify that segments with lengths of 3, 4, and 6 form a triangle. Is the triangle *acute*, *right*, or *obtuse*?

7. Verify that segments with lengths of 2.1, 2.8, and 3.5 form a triangle. Is the triangle *acute*, *right*, or *obtuse*?

9.1 Exercises

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Vocabulary and Core Concept Check

1. **VOCABULARY** What is a Pythagorean triple?

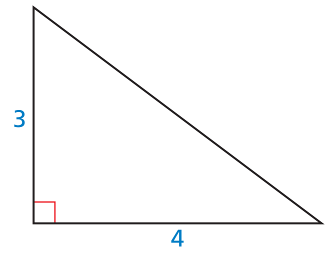
2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

Find the length of the longest side.

Find the length of the hypotenuse.

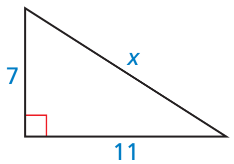
Find the length of the longest leg.

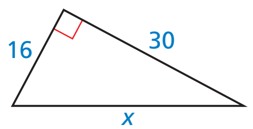
Find the length of the side opposite the right angle.

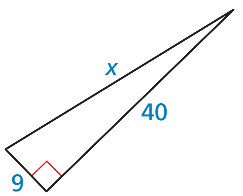


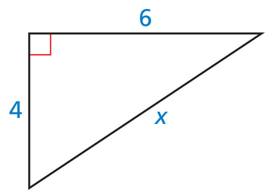
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the value of *x*. Then tell whether the side lengths form a Pythagorean triple. *(See Example 1.)*

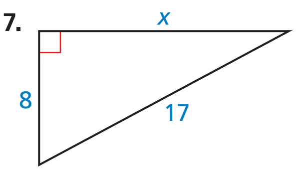
3. 

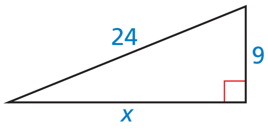
4. 

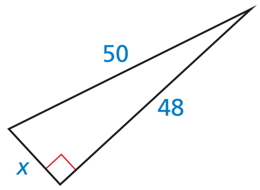
5. 

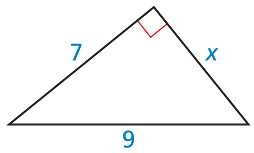
6. 

In Exercises 7–10, find the value of *x*. Then tell whether the side lengths form a Pythagorean triple. *(See Example 2.)*

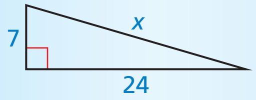
7. 

8. 

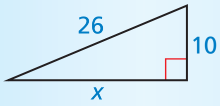
9. 

10. 

ERROR ANALYSIS **In Exercises 11 and 12, describe and correct the error in using the Pythagorean Theorem (Theorem 9.1).**

11. 

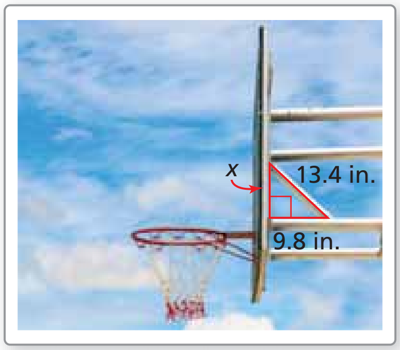
$$c^2 = a^2 + b^2$$
$$x^2 = 7^2 + 24^2$$
$$x^2 = (7 + 24)^2$$
$$x^2 = 31^2$$
$$x = 31$$

12. 

$$c^2 = a^2 + b^2$$
$$x^2 = 10^2 + 26^2$$
$$x^2 = 100 + 676$$
$$x^2 = 776$$
$$x = \sqrt{776}$$
$$x \approx 27.9$$

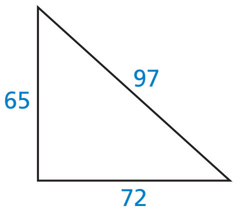
13. MODELING WITH MATHEMATICS The fire escape forms a right triangle, as shown. Use the Pythagorean Theorem (Theorem 9.1) to approximate the distance between the two platforms. *(See Example 3.)*

14. MODELING WITH MATHEMATICS The backboard of the basketball hoop forms a right triangle with the supporting rods, as shown. Use the Pythagorean Theorem (Theorem 9.1) to approximate the distance between the rods where they meet the backboard.



In Exercises 15–20, tell whether the triangle is a right triangle. *(See Example 4.)*

15.



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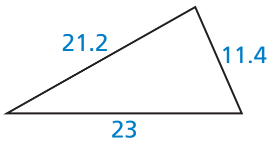

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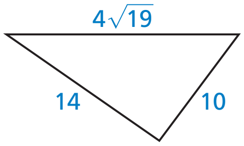
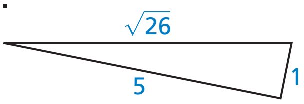


18.



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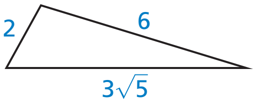

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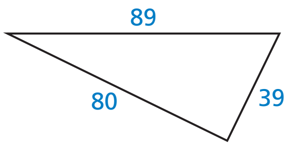


In Exercises 21–28, verify that the segment lengths form a triangle. Is the triangle *acute*, *right*, or *obtuse*? *(See Example 5.)*

21. 10, 11, and 14

22. 6, 8, and 10

23. 12, 16, and 20

24. 15, 20, and 36

25. 5.3, 6.7, and 7.8

26. 4.1, 8.2, and 12.2

27. 24, 30, and $6\sqrt{43}$

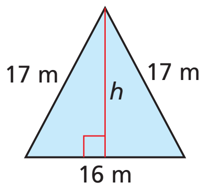
28. 10, 15, and $5\sqrt{13}$

29. MODELING WITH MATHEMATICS In baseball, the lengths of the paths between consecutive bases are 90 feet, and the paths form right angles. The player on first base tries to steal second base. How far does the ball need to travel from home plate to second base to get the player out?

30. REASONING You are making a canvas frame for a painting using stretcher bars. The rectangular painting will be 10 inches long and 8 inches wide. Using a ruler, how can you be certain that the corners of the frame are 90°?

In Exercises 31–34, find the area of the isosceles triangle.

31.



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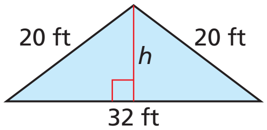

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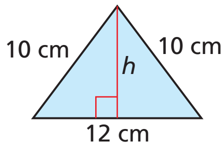

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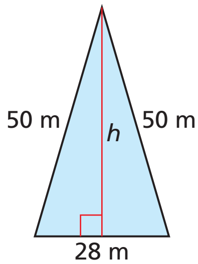
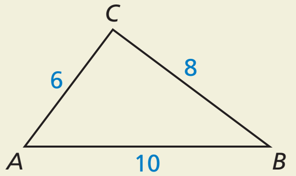
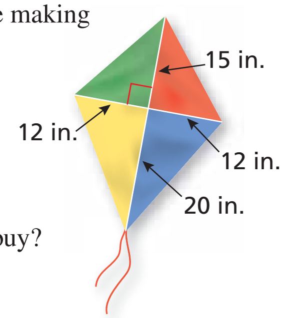


35. ANALYZING RELATIONSHIPS Justify the Distance Formula using the Pythagorean Theorem (Thm. 9.1).

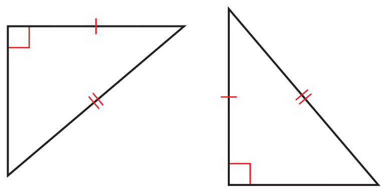
36. HOW DO YOU SEE IT? How do you know $\angle C$ is a right angle without using the Pythagorean Theorem (Theorem 9.1)?



37. PROBLEM SOLVING You are making a kite and need to figure out how much binding to buy. You need the binding for the perimeter of the kite. The binding comes in packages of two yards. How many packages should you buy?



38. PROVING A THEOREM Use the Pythagorean Theorem (Theorem 9.1) to prove the Hypotenuse-Leg (HL) Congruence Theorem (Theorem 5.9).



39. PROVING A THEOREM Prove the Converse of the Pythagorean Theorem (Theorem 9.2). (*Hint*: Draw $\triangle ABC$ with side lengths a, b, and c, where c is the length of the longest side. Then draw a right triangle with side lengths a, b, and x, where x is the length of the hypotenuse. Compare lengths c and x.)

40. THOUGHT PROVOKING Consider two integers m and n, where $m > n$. Do the following expressions produce a Pythagorean triple? If yes, prove your answer. If no, give a counterexample.

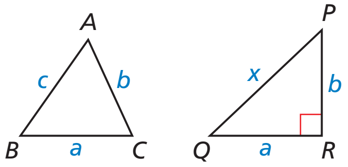
$$2mn,\ m^2 - n^2,\ m^2 + n^2$$

41. MAKING AN ARGUMENT Your friend claims 72 and 75 cannot be part of a Pythagorean triple because $72^2 + 75^2$ does not equal a positive integer squared. Is your friend correct? Explain your reasoning.

42. PROVING A THEOREM Copy and complete the proof of the Pythagorean Inequalities Theorem (Theorem 9.3) when $c^2 < a^2 + b^2$.

Given In $\triangle ABC$, $c^2 < a^2 + b^2$, where c is the length of the longest side.
$\triangle PQR$ has side lengths a, b, and x, where x is the length of the hypotenuse, and $\angle R$ is a right angle.

Prove $\triangle ABC$ is an acute triangle.



STATEMENTS	REASONS
1. In $\triangle ABC$, $c^2 < a^2 + b^2$, where c is the length of the longest side. $\triangle PQR$ has side lengths a, b, and x, where x is the length of the hypotenuse, and $\angle R$ is a right angle.	1. ______________
2. $a^2 + b^2 = x^2$	2. ______________
3. $c^2 < x^2$	3. ______________
4. $c < x$	4. Take the positive square root of each side.
5. $m\angle R = 90°$	5. ______________
6. $m\angle C < m\angle R$	6. Converse of the Hinge Theorem (Theorem 6.13)
7. $m\angle C < 90°$	7. ______________
8. $\angle C$ is an acute angle.	8. ______________
9. $\triangle ABC$ is an acute triangle.	9. ______________

43. PROVING A THEOREM Prove the Pythagorean Inequalities Theorem (Theorem 9.3) when $c^2 > a^2 + b^2$. (*Hint*: Look back at Exercise 42.)

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Simplify the expression by rationalizing the denominator. *(Skills Review Handbook)*

44. $\dfrac{7}{\sqrt{2}}$ **45.** $\dfrac{14}{\sqrt{3}}$ **46.** $\dfrac{8}{\sqrt{2}}$ **47.** $\dfrac{12}{\sqrt{3}}$

9.2 Special Right Triangles

Essential Question

What is the relationship among the side lengths of 45°-45°-90° triangles? 30°-60°-90° triangles?

Texas Essential Knowledge and Skills

G.9.B

USING PRECISE MATHEMATICAL LANGUAGE

To be proficient in math, you need to express numerical answers with a degree of precision appropriate for the problem context.

EXPLORATION 1 Side Ratios of an Isosceles Right Triangle

Work with a partner.

a. Use dynamic geometry software to construct an isosceles right triangle with a leg length of 4 units.

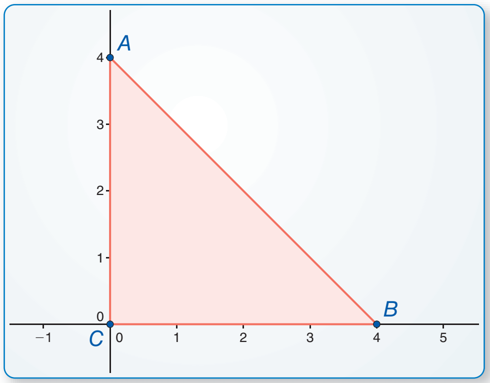
b. Find the acute angle measures. Explain why this triangle is called a 45°-45°-90° triangle.

c. Find the exact ratios of the side lengths (using square roots).

$\frac{AB}{AC} =$ ____

$\frac{AB}{BC} =$ ____

$\frac{AC}{BC} =$ ____



Sample

Points
$A(0, 4)$
$B(4, 0)$
$C(0, 0)$
Segments
$AB = 5.66$
$BC = 4$
$AC = 4$
Angles
$m\angle A = 45°$
$m\angle B = 45°$

d. Repeat parts (a) and (c) for several other isosceles right triangles. Use your results to write a conjecture about the ratios of the side lengths of an isosceles right triangle.

EXPLORATION 2 Side Ratios of a 30°-60°-90° Triangle

Work with a partner.

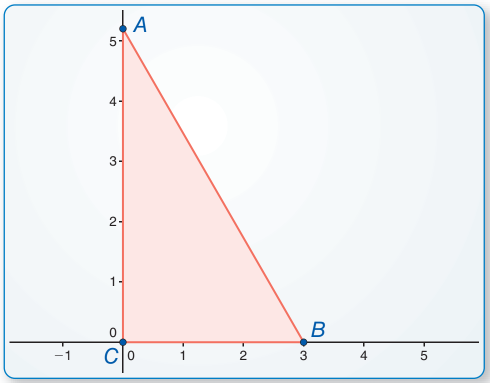
a. Use dynamic geometry software to construct a right triangle with acute angle measures of 30° and 60° (a 30°-60°-90° triangle), where the shorter leg length is 3 units.

b. Find the exact ratios of the side lengths (using square roots).

$\frac{AB}{AC} =$ ____

$\frac{AB}{BC} =$ ____

$\frac{AC}{BC} =$ ____



Sample

Points
$A(0, 5.20)$
$B(3, 0)$
$C(0, 0)$
Segments
$AB = 6$
$BC = 3$
$AC = 5.20$
Angles
$m\angle A = 30°$
$m\angle B = 60°$

c. Repeat parts (a) and (b) for several other 30°-60°-90° triangles. Use your results to write a conjecture about the ratios of the side lengths of a 30°-60°-90° triangle.

Communicate Your Answer

3. What is the relationship among the side lengths of 45°-45°-90° triangles? 30°-60°-90° triangles?

9.2 Lesson

Core Vocabulary

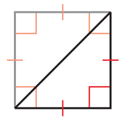
Previous
isosceles triangle

What You Will Learn

- Find side lengths in special right triangles.
- Solve real-life problems involving special right triangles.

Finding Side Lengths in Special Right Triangles

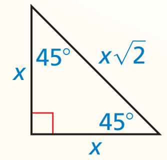
A 45°-45°-90° triangle is an *isosceles right triangle* that can be formed by cutting a square in half diagonally.



Theorem

Theorem 9.4 45°-45°-90° Triangle Theorem

In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.



hypotenuse = leg • $\sqrt{2}$

Proof Ex. 19, p. 480

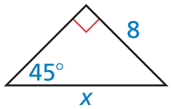
REMEMBER

A radical with index 2 is in simplest form when no radicands have perfect squares as factors other than 1, no radicands contain fractions, and no radicals appear in the denominator of a fraction.

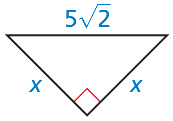
EXAMPLE 1 Finding Side Lengths in 45°-45°-90° Triangles

Find the value of x. Write your answer in simplest form.

a.



b.



SOLUTION

a. By the Triangle Sum Theorem (Theorem 5.1), the measure of the third angle must be 45°, so the triangle is a 45°-45°-90° triangle.

hypotenuse = leg • $\sqrt{2}$	45°-45°-90° Triangle Theorem
$x = 8 \cdot \sqrt{2}$	Substitute.
$x = 8\sqrt{2}$	Simplify.

▶ The value of x is $8\sqrt{2}$.

b. By the Base Angles Theorem (Theorem 5.6) and the Corollary to the Triangle Sum Theorem (Corollary 5.1), the triangle is a 45°-45°-90° triangle.

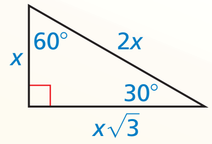
hypotenuse = leg • $\sqrt{2}$	45°-45°-90° Triangle Theorem
$5\sqrt{2} = x \cdot \sqrt{2}$	Substitute.
$\frac{5\sqrt{2}}{\sqrt{2}} = \frac{x\sqrt{2}}{\sqrt{2}}$	Divide each side by $\sqrt{2}$.
$5 = x$	Simplify.

▶ The value of x is 5.

Theorem

Theorem 9.5 30°-60°-90° Triangle Theorem

In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

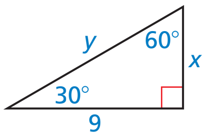


hypotenuse = shorter leg • 2
longer leg = shorter leg • $\sqrt{3}$

Proof Ex. 21, p. 480

EXAMPLE 2 Finding Side Lengths in a 30°-60°-90° Triangle

Find the values of x and y. Write your answer in simplest form.



REMEMBER

Because the angle opposite 9 is larger than the angle opposite *x*, the leg with length 9 is longer than the leg with length *x* by the Triangle Larger Angle Theorem (Theorem 6.10).

SOLUTION

Step 1 Find the value of x.

longer leg = shorter leg • $\sqrt{3}$	30°-60°-90° Triangle Theorem
$9 = x \cdot \sqrt{3}$	Substitute.
$\frac{9}{\sqrt{3}} = x$	Divide each side by $\sqrt{3}$.
$\frac{9}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = x$	Multiply by $\frac{\sqrt{3}}{\sqrt{3}}$.
$\frac{9\sqrt{3}}{3} = x$	Multiply fractions.
$3\sqrt{3} = x$	Simplify.

▶ The value of x is $3\sqrt{3}$.

Step 2 Find the value of y.

hypotenuse = shorter leg • 2	30°-60°-90° Triangle Theorem
$y = 3\sqrt{3} \cdot 2$	Substitute.
$y = 6\sqrt{3}$	Simplify.

▶ The value of y is $6\sqrt{3}$.

Monitoring Progress

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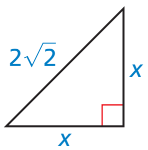
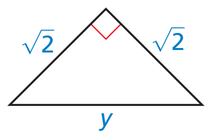
Find the value of the variable. Write your answer in simplest form.

1.



2.



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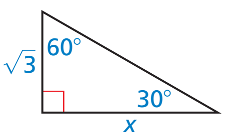

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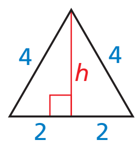


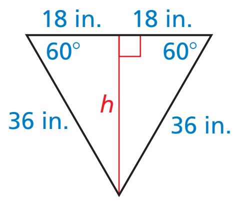
Solving Real-Life Problems

EXAMPLE 3 Modeling with Mathematics

The road sign is shaped like an equilateral triangle. Estimate the area of the sign by finding the area of the equilateral triangle.

SOLUTION

First find the height h of the triangle by dividing it into two $30°$-$60°$-$90°$ triangles. The length of the longer leg of one of these triangles is h. The length of the shorter leg is 18 inches.



$h = 18 \cdot \sqrt{3} = 18\sqrt{3}$ $\quad$ $30°$-$60°$-$90°$ Triangle Theorem

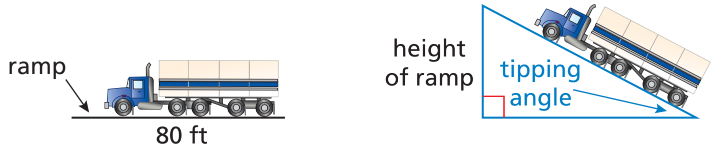
Use $h = 18\sqrt{3}$ to find the area of the equilateral triangle.

$\text{Area} = \frac{1}{2}bh = \frac{1}{2}(36)(18\sqrt{3}) \approx 561.18$

▶ The area of the sign is about 561 square inches.

EXAMPLE 4 Finding the Height of a Ramp

A tipping platform is a ramp used to unload trucks. How high is the end of an 80-foot ramp when the tipping angle is $30°$? $45°$?



SOLUTION

When the tipping angle is $30°$, the height h of the ramp is the length of the shorter leg of a $30°$-$60°$-$90°$ triangle. The length of the hypotenuse is 80 feet.

$80 = 2h$ $\quad$ $30°$-$60°$-$90°$ Triangle Theorem

$40 = h$ $\quad$ Divide each side by 2.

When the tipping angle is $45°$, the height h of the ramp is the length of a leg of a $45°$-$45°$-$90°$ triangle. The length of the hypotenuse is 80 feet.

$80 = h \cdot \sqrt{2}$ $\quad$ $45°$-$45°$-$90°$ Triangle Theorem

$\frac{80}{\sqrt{2}} = h$ $\quad$ Divide each side by $\sqrt{2}$.

$56.6 \approx h$ $\quad$ Use a calculator.

▶ When the tipping angle is $30°$, the ramp height is 40 feet. When the tipping angle is $45°$, the ramp height is about 56 feet 7 inches.

Monitoring Progress

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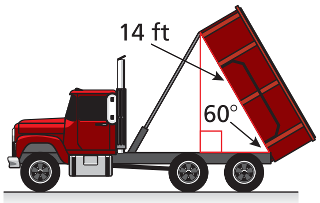

5. The logo on a recycling bin resembles an equilateral triangle with side lengths of 6 centimeters. Approximate the area of the logo.

6. The body of a dump truck is raised to empty a load of sand. How high is the 14-foot-long body from the frame when it is tipped upward by a $60°$ angle?

9.2 Exercises

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Vocabulary and Core Concept Check

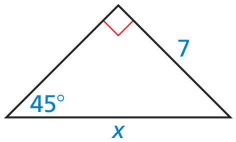
1. **VOCABULARY** Name two special right triangles by their angle measures.

2. **WRITING** Explain why the acute angles in an isosceles right triangle always measure 45°.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the value of x. Write your answer in simplest form. *(See Example 1.)*

3.



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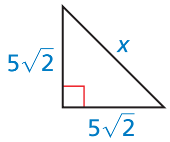
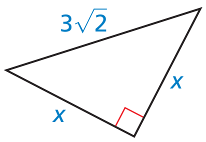


5.



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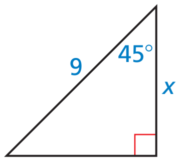
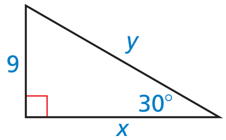


In Exercises 7–10, find the values of x and y. Write your answers in simplest form. *(See Example 2.)*

7.



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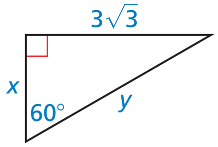

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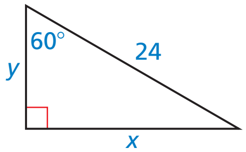

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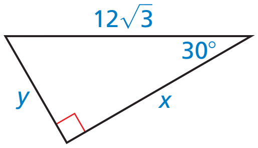
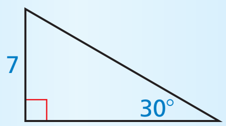


ERROR ANALYSIS In Exercises 11 and 12, describe and correct the error in finding the length of the hypotenuse.

11. ✗

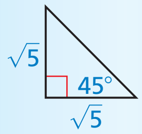


By the Triangle Sum Theorem (Theorem 5.1), the measure of the third angle must be 60°. So, the triangle is a 30°-60°-90° triangle.

hypotenuse = shorter leg • $\sqrt{3} = 7\sqrt{3}$

So, the length of the hypotenuse is $7\sqrt{3}$ units.

12. ✗



By the Triangle Sum Theorem (Theorem 5.1), the measure of the third angle must be 45°. So, the triangle is a 45°-45°-90° triangle.

hypotenuse = leg • leg • $\sqrt{2} = 5\sqrt{2}$

So, the length of the hypotenuse is $5\sqrt{2}$ units.

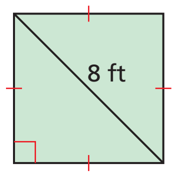
In Exercises 13 and 14, sketch the figure that is described. Find the indicated length. Round decimal answers to the nearest tenth.

13. The side length of an equilateral triangle is 5 centimeters. Find the length of an altitude.

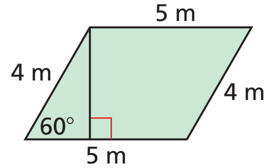
14. The perimeter of a square is 36 inches. Find the length of a diagonal.

In Exercises 15 and 16, find the area of the figure. Round decimal answers to the nearest tenth. *(See Example 3.)*

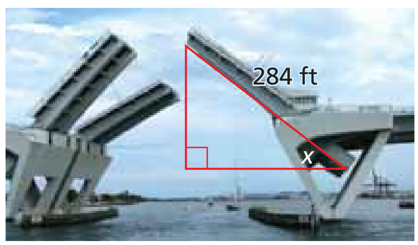
15.



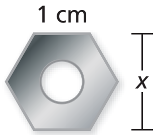
16.



17. PROBLEM SOLVING Each half of the drawbridge is about 284 feet long. How high does the drawbridge rise when x is 30°? 45°? 60°? *(See Example 4.)*



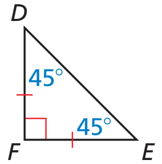
18. MODELING WITH MATHEMATICS A nut is shaped like a regular hexagon with side lengths of 1 centimeter. Find the value of x. (*Hint*: A regular hexagon can be divided into six congruent triangles.)



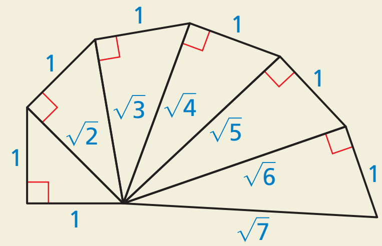
19. PROVING A THEOREM Write a paragraph proof of the 45°-45°-90° Triangle Theorem (Theorem 9.4).

Given $\triangle DEF$ is a 45°-45°-90° triangle.

Prove The hypotenuse is $\sqrt{2}$ times as long as each leg.



20. HOW DO YOU SEE IT? The diagram shows part of the *Wheel of Theodorus*.



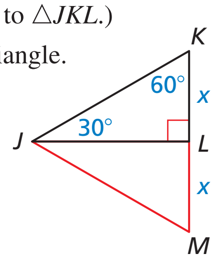
a. Which triangles, if any, are 45°-45°-90° triangles?

b. Which triangles, if any, are 30°-60°-90° triangles?

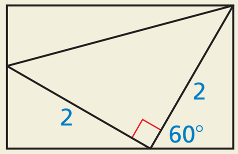
21. PROVING A THEOREM Write a paragraph proof of the 30°-60°-90° Triangle Theorem (Theorem 9.5). (*Hint*: Construct $\triangle JML$ congruent to $\triangle JKL$.)

Given $\triangle JKL$ is a 30°-60°-90° triangle.

Prove The hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

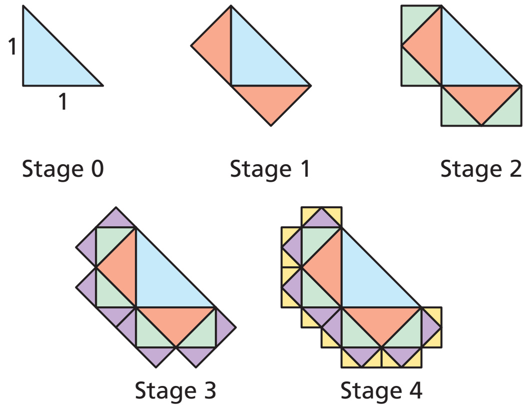


22. THOUGHT PROVOKING A special right triangle is a right triangle that has rational angle measures and each side length contains at most one square root. There are only three special right triangles. The diagram below is called the *Ailles rectangle*. Label the sides and angles in the diagram. Describe all three special right triangles.



23. WRITING Describe two ways to show that all isosceles right triangles are similar to each other.

24. MAKING AN ARGUMENT Each triangle in the diagram is a 45°-45°-90° triangle. At Stage 0, the legs of the triangle are each 1 unit long. Your brother claims the lengths of the legs of the triangles added are halved at each stage. So, the length of a leg of a triangle added in Stage 8 will be $\frac{1}{256}$ unit. Is your brother correct? Explain your reasoning.



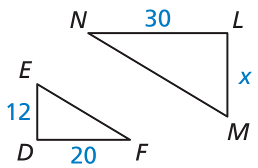
25. USING STRUCTURE $\triangle TUV$ is a 30°-60°-90° triangle, where two vertices are $U(3, -1)$ and $V(-3, -1)$, $\overline{UV}$ is the hypotenuse, and point T is in Quadrant I. Find the coordinates of T.

Maintaining Mathematical Proficiency

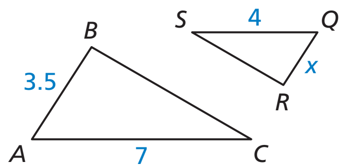
Reviewing what you learned in previous grades and lessons

Find the value of *x*. *(Section 8.1)*

26. $\triangle DEF \sim \triangle LMN$



27. $\triangle ABC \sim \triangle QRS$



9.3 Similar Right Triangles

Essential Question

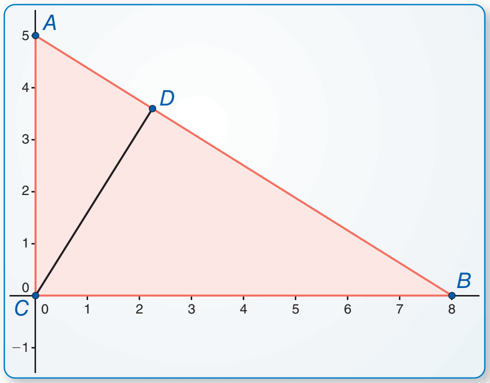
How are altitudes and geometric means of right triangles related?

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS
G.8.A
G.8.B

EXPLORATION 1 Writing a Conjecture

Work with a partner.

a. Use dynamic geometry software to construct right $\triangle ABC$, as shown. Draw $\overline{CD}$ so that it is an altitude from the right angle to the hypotenuse of $\triangle ABC$.



Points
$A(0, 5)$
$B(8, 0)$
$C(0, 0)$
$D(2.25, 3.6)$
Segments
$AB = 9.43$
$BC = 8$
$AC = 5$

MAKING MATHEMATICAL ARGUMENTS

To be proficient in math, you need to understand and use stated assumptions, definitions, and previously established results in constructing arguments.

b. The **geometric mean** of two positive numbers a and b is the positive number x that satisfies

$$\frac{a}{x} = \frac{x}{b}.$$

x is the geometric mean of a and b.

Write a proportion involving the side lengths of $\triangle CBD$ and $\triangle ACD$ so that CD is the geometric mean of two of the other side lengths. Use similar triangles to justify your steps.

c. Use the proportion you wrote in part (b) to find CD.

d. Generalize the proportion you wrote in part (b). Then write a conjecture about how the geometric mean is related to the altitude from the right angle to the hypotenuse of a right triangle.

EXPLORATION 2 Comparing Geometric and Arithmetic Means

Work with a partner. Use a spreadsheet to find the arithmetic mean and the geometric mean of several pairs of positive numbers. Compare the two means. What do you notice?

	A	B	C	D
1	a	b	Arithmetic Mean	Geometric Mean
2	3	4	3.5	3.464
3	4	5		
4	6	7		
5	0.5	0.5		
6	0.4	0.8		
7	2	5		
8	1	4		
9	9	16		
10	10	100		
11				

Communicate Your Answer

3. How are altitudes and geometric means of right triangles related?

9.3 Lesson

Core Vocabulary

geometric mean, *p. 484*

Previous

altitude of a triangle

similar figures

What You Will Learn

- Identify similar triangles.
- Solve real-life problems involving similar triangles.
- Use geometric means.

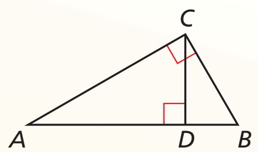
Identifying Similar Triangles

When the altitude is drawn to the hypotenuse of a right triangle, the two smaller triangles are similar to the original triangle and to each other.

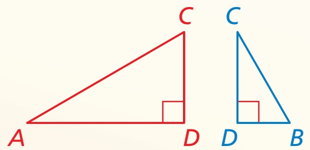
Theorem

Theorem 9.6 Right Triangle Similarity Theorem

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.



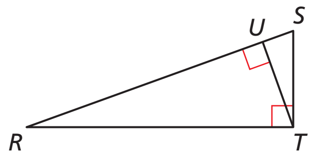
$\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$, and $\triangle CBD \sim \triangle ACD$.



Proof Ex. 45, p. 488

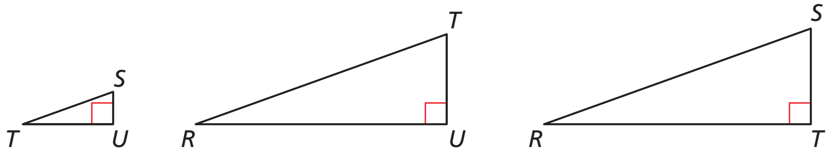
EXAMPLE 1 Identifying Similar Triangles

Identify the similar triangles in the diagram.



SOLUTION

Sketch the three similar right triangles so that the corresponding angles and sides have the same orientation.



▶ $\triangle TSU \sim \triangle RTU \sim \triangle RST$

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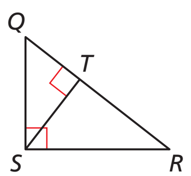
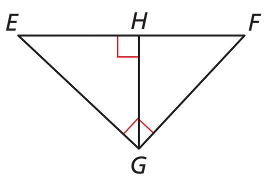
Identify the similar triangles.

1.



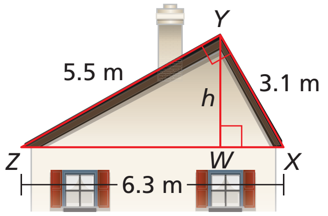
2.



Solving Real-Life Problems

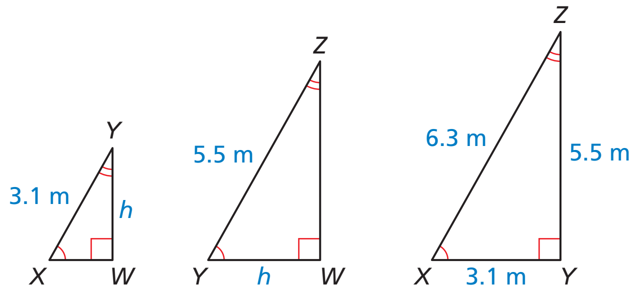
EXAMPLE 2 **Modeling with Mathematics**

A roof has a cross section that is a right triangle. The diagram shows the approximate dimensions of this cross section. Find the height h of the roof.



SOLUTION

1. **Understand the Problem** You are given the side lengths of a right triangle. You need to find the height of the roof, which is the altitude drawn to the hypotenuse.
2. **Make a Plan** Identify any similar triangles. Then use the similar triangles to write a proportion involving the height and solve for h.
3. **Solve the Problem** Identify the similar triangles and sketch them.



$\triangle XYW \sim \triangle YZW \sim \triangle XZY$

COMMON ERROR

Notice that if you tried to write a proportion using $\triangle XYW$ and $\triangle YZW$, then there would be two unknowns, so you would not be able to solve for h.

Because $\triangle XYW \sim \triangle XZY$, you can write a proportion.

$$\frac{YW}{ZY} = \frac{XY}{XZ}$$ Corresponding side lengths of similar triangles are proportional.

$$\frac{h}{5.5} = \frac{3.1}{6.3}$$ Substitute.

$$h \approx 2.7$$ Multiply each side by 5.5.

▶ The height of the roof is about 2.7 meters.

4. **Look Back** Because the height of the roof is a leg of right $\triangle YZW$ and right $\triangle XYW$, it should be shorter than each of their hypotenuses. The lengths of the two hypotenuses are $YZ = 5.5$ and $XY = 3.1$. Because $2.7 < 3.1$, the answer seems reasonable.

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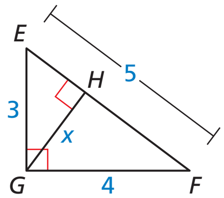
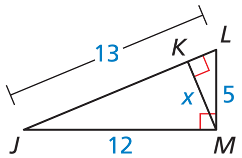
Find the value of x.

3.



4.



Using a Geometric Mean

Core Concept

Geometric Mean

The **geometric mean** of two positive numbers a and b is the positive number x that satisfies $\frac{a}{x} = \frac{x}{b}$. So, $x^2 = ab$ and $x = \sqrt{ab}$.

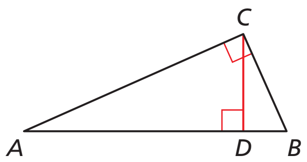
EXAMPLE 3 **Finding a Geometric Mean**

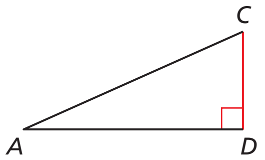
Find the geometric mean of 24 and 48.

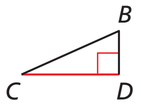
SOLUTION

$x^2 = ab$	Definition of geometric mean
$x^2 = 24 \cdot 48$	Substitute 24 for a and 48 for b.
$x = \sqrt{24 \cdot 48}$	Take the positive square root of each side.
$x = \sqrt{24 \cdot 24 \cdot 2}$	Factor.
$x = 24\sqrt{2}$	Simplify.

▶ The geometric mean of 24 and 48 is $24\sqrt{2} \approx 33.9$.







In right $\triangle ABC$, altitude $\overline{CD}$ is drawn to the hypotenuse, forming two smaller right triangles that are similar to $\triangle ABC$. From the Right Triangle Similarity Theorem, you know that $\triangle CBD \sim \triangle ACD \sim \triangle ABC$. Because the triangles are similar, you can write and simplify the following proportions involving geometric means.

$$\frac{CD}{AD} = \frac{BD}{CD} \qquad \frac{CB}{DB} = \frac{AB}{CB} \qquad \frac{AC}{AD} = \frac{AB}{AC}$$

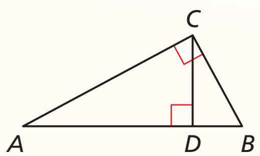
$$CD^2 = AD \cdot BD \qquad CB^2 = DB \cdot AB \qquad AC^2 = AD \cdot AB$$

Theorems

Theorem 9.7 Geometric Mean (Altitude) Theorem

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of the altitude is the geometric mean of the lengths of the two segments of the hypotenuse.



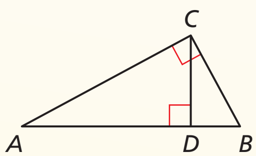
$CD^2 = AD \cdot BD$

Proof Ex. 41, p. 488

Theorem 9.8 Geometric Mean (Leg) Theorem

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.



$CB^2 = DB \cdot AB$

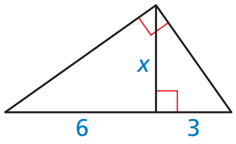
$AC^2 = AD \cdot AB$

Proof Ex. 42, p. 488

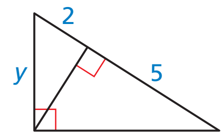
EXAMPLE 4 Using a Geometric Mean

Find the value of each variable.

a.



b.



COMMON ERROR

In Example 4(b), the Geometric Mean (Leg) Theorem gives $y^2 = 2 \cdot (5 + 2)$, not $y^2 = 5 \cdot (5 + 2)$, because the side with length y is adjacent to the segment with length 2.

SOLUTION

a. Apply the Geometric Mean (Altitude) Theorem.

$$x^2 = 6 \cdot 3$$
$$x^2 = 18$$
$$x = \sqrt{18}$$
$$x = \sqrt{9} \cdot \sqrt{2}$$
$$x = 3\sqrt{2}$$

▶ The value of x is $3\sqrt{2}$.

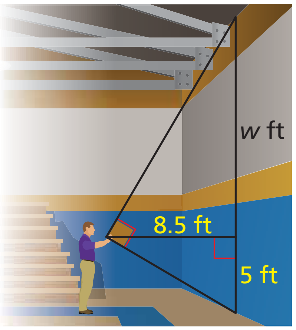
b. Apply the Geometric Mean (Leg) Theorem.

$$y^2 = 2 \cdot (5 + 2)$$
$$y^2 = 2 \cdot 7$$
$$y^2 = 14$$
$$y = \sqrt{14}$$

▶ The value of y is $\sqrt{14}$.

EXAMPLE 5 Using Indirect Measurement

To find the cost of installing a rock wall in your school gymnasium, you need to find the height of the gym wall. You use a cardboard square to line up the top and bottom of the gym wall. Your friend measures the vertical distance from the ground to your eye and the horizontal distance from you to the gym wall. Approximate the height of the gym wall.



SOLUTION

By the Geometric Mean (Altitude) Theorem, you know that 8.5 is the geometric mean of w and 5.

$8.5^2 = w \cdot 5$	Geometric Mean (Altitude) Theorem
$72.25 = 5w$	Square 8.5.
$14.45 = w$	Divide each side by 5.

▶ The height of the wall is $5 + w = 5 + 14.45 = 19.45$ feet.

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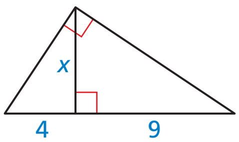
Find the geometric mean of the two numbers.

5. 12 and 27 **6.** 18 and 54 **7.** 16 and 18

8. Find the value of x in the triangle at the left.



9. WHAT IF? In Example 5, the vertical distance from the ground to your eye is 5.5 feet and the distance from you to the gym wall is 9 feet. Approximate the height of the gym wall.

9.3 Exercises

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Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and __________________.

2. **WRITING** In your own words, explain *geometric mean*.

Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, identify the similar triangles. *(See Example 1.)*

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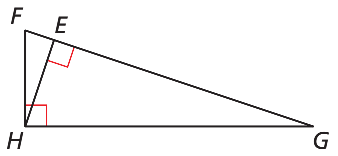

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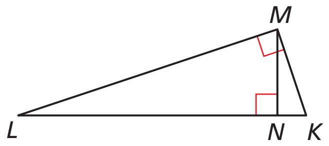

In Exercises 5–10, find the value of x. *(See Example 2.)*

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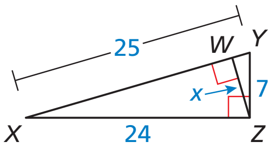

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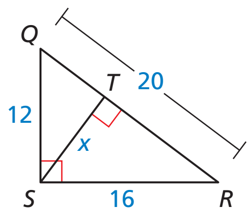

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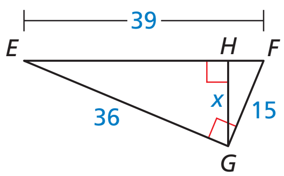

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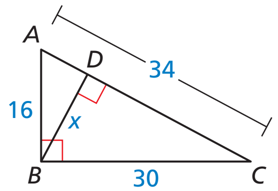

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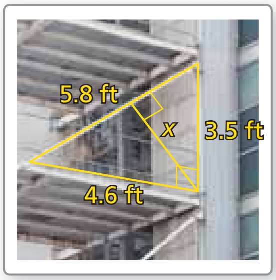

In Exercises 11–18, find the geometric mean of the two numbers. *(See Example 3.)*

11. 8 and 32
12. 9 and 16
13. 14 and 20
14. 25 and 35
15. 16 and 25
16. 8 and 28
17. 17 and 36
18. 24 and 45

In Exercises 19–26, find the value of the variable. *(See Example 4.)*

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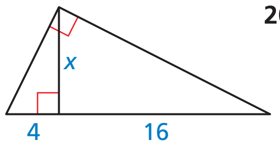

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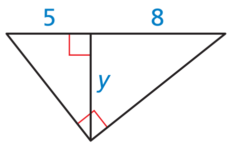

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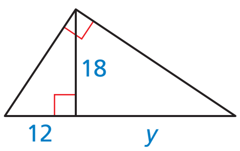

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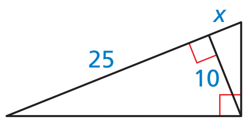

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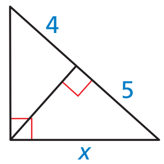

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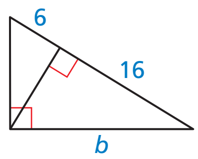

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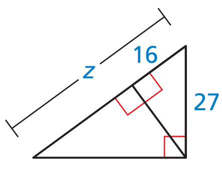

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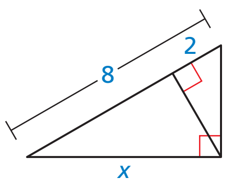
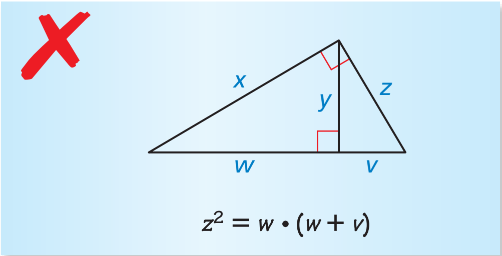
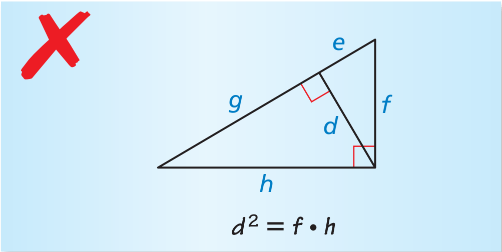


ERROR ANALYSIS **In Exercises 27 and 28, describe and correct the error in writing an equation for the given diagram.**

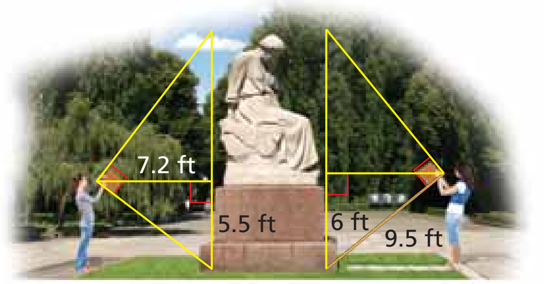
27.



28.



MODELING WITH MATHEMATICS **In Exercises 29 and 30, use the diagram.** *(See Example 5.)*



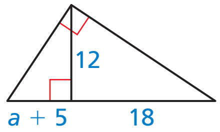
Ex. 29 Ex. 30

29. You want to determine the height of a monument at a local park. You use a cardboard square to line up the top and bottom of the monument, as shown at the above left. Your friend measures the vertical distance from the ground to your eye and the horizontal distance from you to the monument. Approximate the height of the monument.

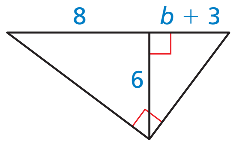
30. Your classmate is standing on the other side of the monument. She has a piece of rope staked at the base of the monument. She extends the rope to the cardboard square she is holding lined up to the top and bottom of the monument. Use the information in the diagram above to approximate the height of the monument. Do you get the same answer as in Exercise 29? Explain your reasoning.

MATHEMATICAL CONNECTIONS **In Exercises 31–34, find the value(s) of the variable(s).**

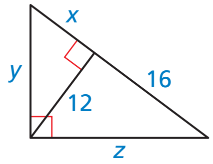
31.



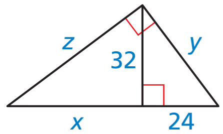
32.



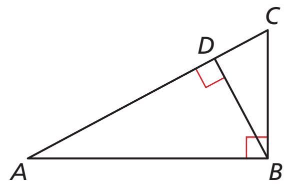
33.



34.



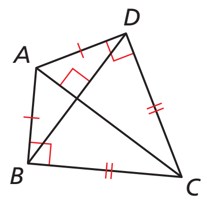
35. **REASONING** Use the diagram. Decide which proportions are true. Select all that apply.



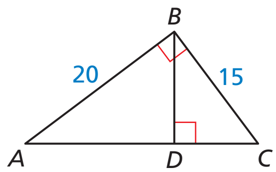
Ⓐ $\dfrac{DB}{DC} = \dfrac{DA}{DB}$ Ⓑ $\dfrac{BA}{CB} = \dfrac{CB}{BD}$

Ⓒ $\dfrac{CA}{BA} = \dfrac{BA}{CA}$ Ⓓ $\dfrac{DB}{BC} = \dfrac{DA}{BA}$

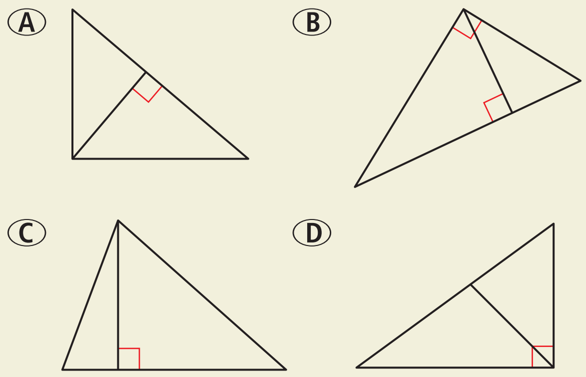
36. **ANALYZING RELATIONSHIPS** You are designing a diamond-shaped kite. You know that $AD = 44.8$ centimeters, $DC = 72$ centimeters, and $AC = 84.8$ centimeters. You want to use a straight crossbar $\overline{BD}$. About how long should it be? Explain your reasoning.



37. **ANALYZING RELATIONSHIPS** Use the Geometric Mean Theorems (Theorems 9.7 and 9.8) to find AC and BD.



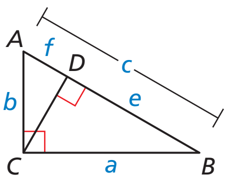
38. HOW DO YOU SEE IT? In which of the following triangles does the Geometric Mean (Altitude) Theorem (Theorem 9.7) apply?



39. PROVING A THEOREM Use the diagram of $\triangle ABC$. Copy and complete the proof of the Pythagorean Theorem (Theorem 9.1).

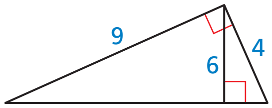
Given In $\triangle ABC$, $\angle BCA$ is a right angle.

Prove $c^2 = a^2 + b^2$



STATEMENTS	REASONS
1. In $\triangle ABC$, $\angle BCA$ is a right angle.	1. ________
2. Draw a perpendicular segment (altitude) from C to $\overline{AB}$.	2. Perpendicular Postulate (Postulate 3.2)
3. $ce = a^2$ and $cf = b^2$	3. ________
4. $ce + b^2 =$ ____ $+ b^2$	4. Addition Property of Equality
5. $ce + cf = a^2 + b^2$	5. ________
6. $c(e + f) = a^2 + b^2$	6. ________
7. $e + f =$ ____	7. Segment Addition Postulate (Postulate 1.2)
8. $c \cdot c = a^2 + b^2$	8. ________
9. $c^2 = a^2 + b^2$	9. Simplify.

40. MAKING AN ARGUMENT Your friend claims the geometric mean of 4 and 9 is 6, and then labels the triangle, as shown. Is your friend correct? Explain your reasoning.



In Exercises 41 and 42, use the given statements to prove the theorem.

Given $\triangle ABC$ is a right triangle.
Altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$.

41. PROVING A THEOREM Prove the Geometric Mean (Altitude) Theorem (Theorem 9.7) by showing that $CD^2 = AD \cdot BD$.

42. PROVING A THEOREM Prove the Geometric Mean (Leg) Theorem (Theorem 9.8) by showing that $CB^2 = DB \cdot AB$ and $AC^2 = AD \cdot AB$.

43. CRITICAL THINKING Draw a right isosceles triangle and label the two leg lengths x. Then draw the altitude to the hypotenuse and label its length y. Now, use the Right Triangle Similarity Theorem (Theorem 9.6) to draw the three similar triangles from the image and label any side length that is equal to either x or y. What can you conclude about the relationship between the two smaller triangles? Explain your reasoning.

44. THOUGHT PROVOKING The arithmetic mean and geometric mean of two nonnegative numbers x and y are shown.

$$\text{arithmetic mean} = \frac{x + y}{2}$$

$$\text{geometric mean} = \sqrt{xy}$$

Write an inequality that relates these two means. Justify your answer.

45. PROVING A THEOREM Prove the Right Triangle Similarity Theorem (Theorem 9.6) by proving three similarity statements.

Given $\triangle ABC$ is a right triangle.
Altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$.

Prove $\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$, $\triangle CBD \sim \triangle ACD$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation for x. *(Skills Review Handbook)*

46. $13 = \frac{x}{5}$ **47.** $29 = \frac{x}{4}$ **48.** $9 = \frac{78}{x}$ **49.** $30 = \frac{115}{x}$

9.1–9.3 What Did You Learn?

Core Vocabulary

Pythagorean triple, *p. 468*

geometric mean, *p. 484*

Core Concepts

Section 9.1

Theorem 9.1 Pythagorean Theorem, *p. 468*
Common Pythagorean Triples and Some of Their Multiples, *p. 468*
Theorem 9.2 Converse of the Pythagorean Theorem, *p. 470*
Theorem 9.3 Pythagorean Inequalities Theorem, *p. 471*

Section 9.2

Theorem 9.4 45°-45°-90° Triangle Theorem, *p. 476*
Theorem 9.5 30°-60°-90° Triangle Theorem, *p. 477*

Section 9.3

Theorem 9.6 Right Triangle Similarity Theorem, *p. 482*
Theorem 9.7 Geometric Mean (Altitude) Theorem, *p. 484*
Theorem 9.8 Geometric Mean (Leg) Theorem, *p. 484*

Mathematical Thinking

1. In Exercise 31 on page 473, describe the steps you took to find the area of the triangle.

2. In Exercise 23 on page 480, can one of the ways be used to show that all 30°-60°-90° triangles are similar? Explain.

3. Explain why the Geometric Mean (Altitude) Theorem (Theorem 9.7) does not apply to three of the triangles in Exercise 38 on page 488.

Study Skills

Form a Weekly Study Group, Set Up Rules

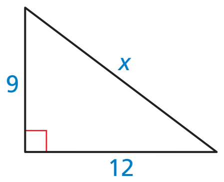
Consider using the following rules.

- Members must attend regularly, be on time, and participate.
- The sessions will focus on the key math concepts, not on the needs of one student.
- Students who skip classes will not be allowed to participate in the study group.
- Students who keep the group from being productive will be asked to leave the group.

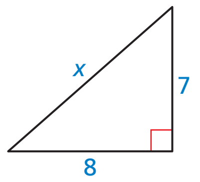
9.1–9.3 Quiz

Find the value of *x*. Tell whether the side lengths form a Pythagorean triple. *(Section 9.1)*

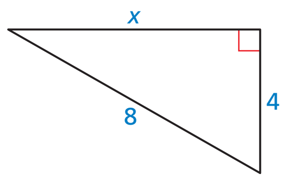
1.



2.



3.



Verify that the segment lengths form a triangle. Is the triangle *acute*, *right*, or *obtuse*? *(Section 9.1)*

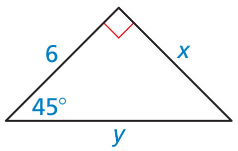
4. 24, 32, and 40

5. 7, 9, and 13

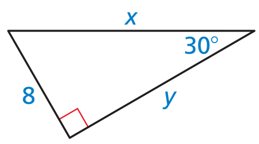
6. 12, 15, and $10\sqrt{3}$

Find the values of *x* and *y*. Write your answer in simplest form. *(Section 9.2)*

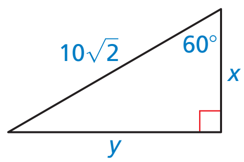
7.



8.



9.



Find the geometric mean of the two numbers. *(Section 9.3)*

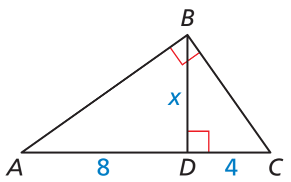
10. 6 and 12

11. 15 and 20

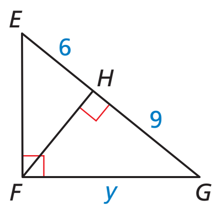
12. 18 and 26

Identify the similar right triangles. Then find the value of the variable. *(Section 9.3)*

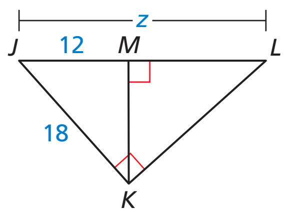
13.



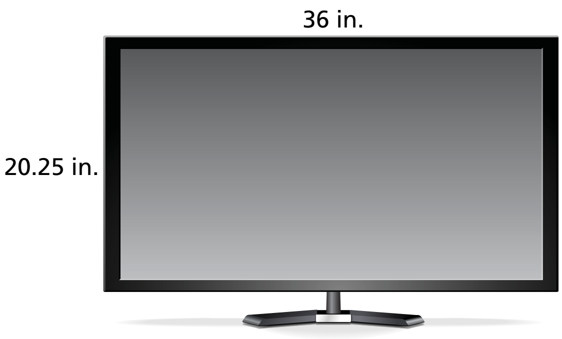
14.



15.



16. Television sizes are measured by the length of their diagonal. You want to purchase a television that is at least 40 inches. Should you purchase the television shown? Explain your reasoning. *(Section 9.1)*

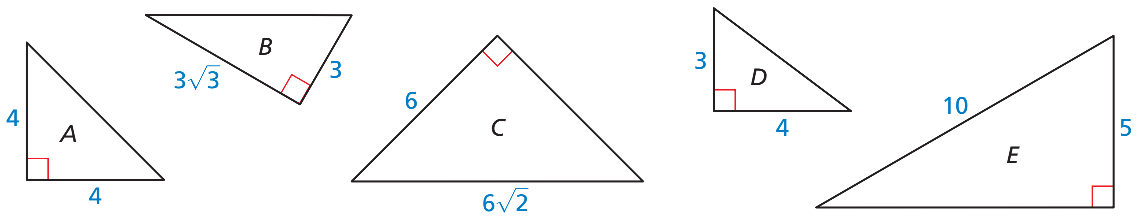


17. Each triangle shown below is a right triangle. *(Sections 9.1–9.3)*

a. Are any of the triangles special right triangles? Explain your reasoning.

b. List all similar triangles, if any.

c. Find the lengths of the altitudes of triangles *B* and *C*.



9.4 The Tangent Ratio

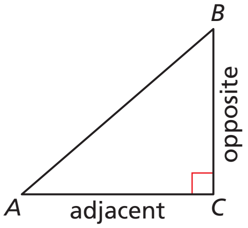
TEXAS ESSENTIAL KNOWLEDGE AND SKILLS
G.9.A
G.9.B

Essential Question

How is a right triangle used to find the tangent of an acute angle? Is there a unique right triangle that must be used?

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The *tangent* of $\angle A$ (written as $\tan A$) is defined as follows.

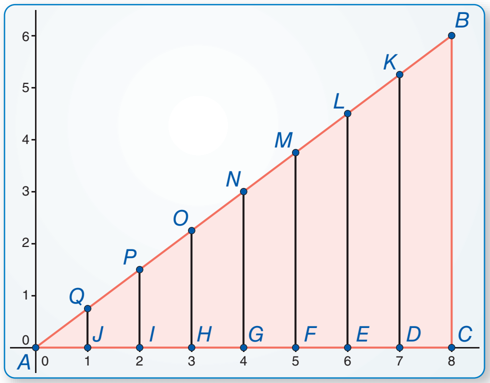
$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{BC}{AC}$$



EXPLORATION 1 Calculating a Tangent Ratio

Work with a partner. Use dynamic geometry software.

a. Construct $\triangle ABC$, as shown. Construct segments perpendicular to $\overline{AC}$ to form right triangles that share vertex A and are similar to $\triangle ABC$ with vertices, as shown.



Sample
Points
$A(0, 0)$
$B(8, 6)$
$C(8, 0)$
Angle
$m\angle BAC = 36.87°$

b. Calculate each given ratio to complete the table for the decimal value of $\tan A$ for each right triangle. What can you conclude?

Ratio	$\frac{BC}{AC}$	$\frac{KD}{AD}$	$\frac{LE}{AE}$	$\frac{MF}{AF}$	$\frac{NG}{AG}$	$\frac{OH}{AH}$	$\frac{PI}{AI}$	$\frac{QJ}{AJ}$
$\tan A$								

USING PRECISE MATHEMATICAL LANGUAGE

To be proficient in math, you need to express numerical answers with a degree of precision appropriate for the problem context.

EXPLORATION 2 Using a Calculator

Work with a partner. Use a calculator that has a tangent key to calculate the tangent of 36.87°. Do you get the same result as in Exploration 1? Explain.

Communicate Your Answer

3. Repeat Exploration 1 for $\triangle ABC$ with vertices $A(0, 0)$, $B(8, 5)$, and $C(8, 0)$. Construct the seven perpendicular segments so that not all of them intersect $\overline{AC}$ at integer values of x. Discuss your results.

4. How is a right triangle used to find the tangent of an acute angle? Is there a unique right triangle that must be used?

9.4 Lesson

What You Will Learn

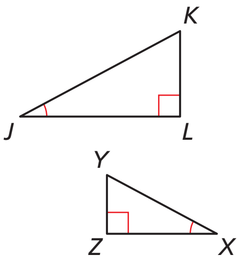
- Use the tangent ratio.
- Solve real-life problems involving the tangent ratio.

Core Vocabulary

trigonometric ratio, *p. 492*
tangent, *p. 492*
angle of elevation, *p. 494*

Using the Tangent Ratio

A **trigonometric ratio** is a ratio of the lengths of two sides in a right triangle. All right triangles with a given acute angle are similar by the AA Similarity Theorem (Theorem 8.3). So, $\triangle JKL \sim \triangle XYZ$, and you can write $\frac{KL}{YZ} = \frac{JL}{XZ}$. This can be rewritten as $\frac{KL}{JL} = \frac{YZ}{XZ}$, which is a trigonometric ratio. So, trigonometric ratios are constant for a given angle measure.



The **tangent** ratio is a trigonometric ratio for acute angles that involves the lengths of the legs of a right triangle.

READING

Remember the following abbreviations.

tangent → tan
opposite → opp.
adjacent → adj.

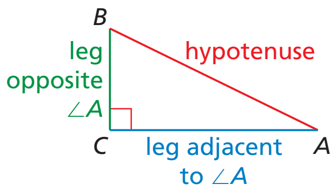
Core Concept

Tangent Ratio

Let $\triangle ABC$ be a right triangle with acute $\angle A$.

The tangent of $\angle A$ (written as tan A) is defined as follows.

$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{BC}{AC}$$



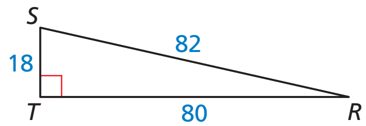
In the right triangle above, $\angle A$ and $\angle B$ are complementary. So, $\angle B$ is acute. You can use the same diagram to find the tangent of $\angle B$. Notice that the leg adjacent to $\angle A$ is the leg *opposite* $\angle B$ and the leg opposite $\angle A$ is the leg *adjacent* to $\angle B$.

USING PRECISE MATHEMATICAL LANGUAGE

Unless told otherwise, you should round the values of trigonometric ratios to four decimal places and round lengths to the nearest tenth.

EXAMPLE 1 Finding Tangent Ratios

Find tan S and tan R. Write each answer as a fraction and as a decimal rounded to four places.



SOLUTION

$$\tan S = \frac{\text{opp. } \angle S}{\text{adj. to } \angle S} = \frac{RT}{ST} = \frac{80}{18} = \frac{40}{9} \approx 4.4444$$

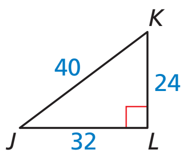
$$\tan R = \frac{\text{opp. } \angle R}{\text{adj. to } \angle R} = \frac{ST}{RT} = \frac{18}{80} = \frac{9}{40} = 0.2250$$

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Find tan J and tan K. Write each answer as a fraction and as a decimal rounded to four places.

1.



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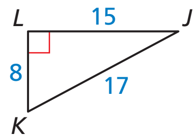
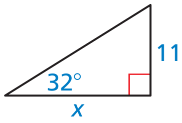


EXAMPLE 2 Finding a Leg Length

Find the value of x. Round your answer to the nearest tenth.



SOLUTION

Use the tangent of an acute angle to find a leg length.

$\tan 32^\circ = \frac{\text{opp.}}{\text{adj.}}$	Write ratio for tangent of 32°.
$\tan 32^\circ = \frac{11}{x}$	Substitute.
$x \cdot \tan 32^\circ = 11$	Multiply each side by x.
$x = \frac{11}{\tan 32^\circ}$	Divide each side by tan 32°.
$x \approx 17.6$	Use a calculator.

▶ The value of x is about 17.6.

SELECTING TOOLS

You can also use the Table of Trigonometric Ratios available at *BigIdeasMath.com* to find the decimal approximations of trigonometric ratios.

You can find the tangent of an acute angle measuring 30°, 45°, or 60° by applying what you know about special right triangles.

STUDY TIP

The tangents of all 60° angles are the same constant ratio. Any right triangle with a 60° angle can be used to determine this value.

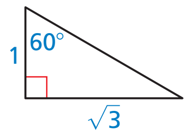
EXAMPLE 3 Using a Special Right Triangle to Find a Tangent

Use a special right triangle to find the tangent of a 60° angle.

SOLUTION

Step 1 Because all 30°-60°-90° triangles are similar, you can simplify your calculations by choosing 1 as the length of the shorter leg. Use the 30°-60°-90° Triangle Theorem (Theorem 9.5) to find the length of the longer leg.

longer leg $=$ shorter leg $\cdot \sqrt{3}$	30°-60°-90° Triangle Theorem
$= 1 \cdot \sqrt{3}$	Substitute.
$= \sqrt{3}$	Simplify.



Step 2 Find tan 60°.

$\tan 60^\circ = \frac{\text{opp.}}{\text{adj.}}$	Write ratio for tangent of 60°.
$\tan 60^\circ = \frac{\sqrt{3}}{1}$	Substitute.
$\tan 60^\circ = \sqrt{3}$	Simplify.

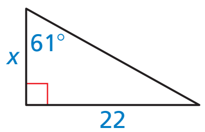
▶ The tangent of any 60° angle is $\sqrt{3} \approx 1.7321$.

Monitoring Progress

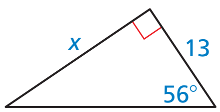
Help in English and Spanish at *BigIdeasMath.com*

Find the value of x. Round your answer to the nearest tenth.

3.



4.



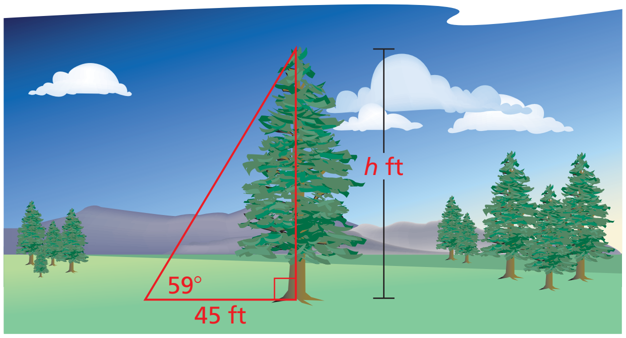
5. WHAT IF? In Example 3, the side length of the shorter leg is 5 instead of 1. Show that the tangent of 60° is still equal to $\sqrt{3}$.

Solving Real-Life Problems

The angle that an upward line of sight makes with a horizontal line is called the **angle of elevation**.

EXAMPLE 4 **Modeling with Mathematics**

You are measuring the height of a spruce tree. You stand 45 feet from the base of the tree. You measure the angle of elevation from the ground to the top of the tree to be 59°. Find the height h of the tree to the nearest foot.



SOLUTION

1. **Understand the Problem** You are given the angle of elevation and the distance from the tree. You need to find the height of the tree to the nearest foot.

2. **Make a Plan** Write a trigonometric ratio for the tangent of the angle of elevation involving the height h. Then solve for h.

3. **Solve the Problem**

$$\tan 59° = \frac{\text{opp.}}{\text{adj.}}$$ Write ratio for tangent of 59°.

$$\tan 59° = \frac{h}{45}$$ Substitute.

$$45 \cdot \tan 59° = h$$ Multiply each side by 45.

$$74.9 \approx h$$ Use a calculator.

▶ The tree is about 75 feet tall.

4. **Look Back** Check your answer. Because 59° is close to 60°, the value of h should be close to the length of the longer leg of a 30°-60°-90° triangle, where the length of the shorter leg is 45 feet.

$$\text{longer leg} = \text{shorter leg} \cdot \sqrt{3}$$ 30°-60°-90° Triangle Theorem

$$= 45 \cdot \sqrt{3}$$ Substitute.

$$\approx 77.9$$ Use a calculator.

The value of 77.9 feet is close to the value of h. ✓

Monitoring Progress

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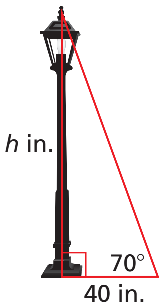

6. You are measuring the height of a lamppost. You stand 40 inches from the base of the lamppost. You measure the angle of elevation from the ground to the top of the lamppost to be 70°. Find the height h of the lamppost to the nearest inch.

9.4 Exercises

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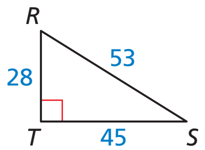
Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** The tangent ratio compares the length of _______ to the length of ________.

2. **WRITING** Explain how you know the tangent ratio is constant for a given angle measure.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the tangents of the acute angles in the right triangle. Write each answer as a fraction and as a decimal rounded to four decimal places. *(See Example 1.)*

3.


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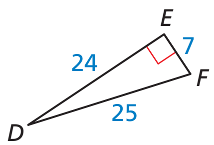

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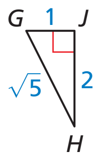

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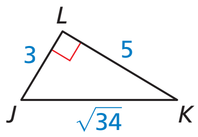

In Exercises 7–10, find the value of x. Round your answer to the nearest tenth. *(See Example 2.)*

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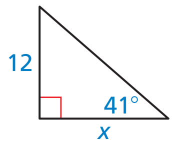

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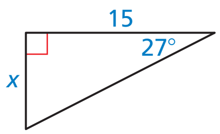

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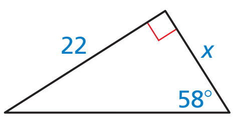

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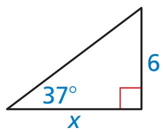
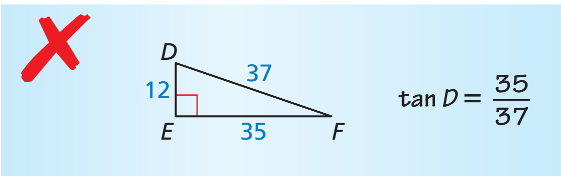


ERROR ANALYSIS In Exercises 11 and 12, describe the error in the statement of the tangent ratio. Correct the error if possible. Otherwise, write not possible.

11.


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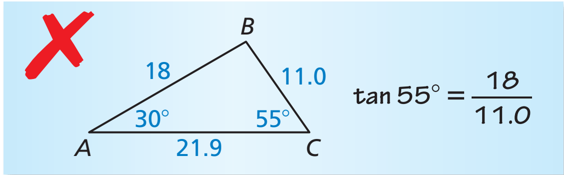


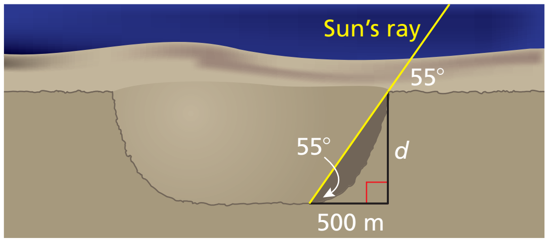
In Exercises 13 and 14, use a special right triangle to find the tangent of the given angle measure. *(See Example 3.)*

13. 45°

14. 30°

15. **MODELING WITH MATHEMATICS** A surveyor is standing 118 feet from the base of the Washington Monument. The surveyor measures the angle of elevation from the ground to the top of the monument to be 78°. Find the height h of the Washington Monument to the nearest foot. *(See Example 4.)*

16. **MODELING WITH MATHEMATICS** Scientists can measure the depths of craters on the moon by looking at photos of shadows. The length of the shadow cast by the edge of a crater is 500 meters. The angle of elevation of the rays of the Sun is 55°. Estimate the depth d of the crater.



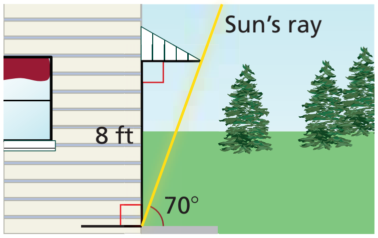
17. **USING STRUCTURE** Find the tangent of the smaller acute angle in a right triangle with side lengths 5, 12, and 13.

18. **USING STRUCTURE** Find the tangent of the larger acute angle in a right triangle with side lengths 3, 4, and 5.

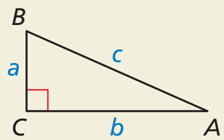
19. **REASONING** How does the tangent of an acute angle in a right triangle change as the angle measure increases? Justify your answer.

20. **CRITICAL THINKING** For what angle measure(s) is the tangent of an acute angle in a right triangle equal to 1? greater than 1? less than 1? Justify your answer.

21. **MAKING AN ARGUMENT** Your family room has a sliding-glass door. You want to buy an awning for the door that will be just long enough to keep the Sun out when it is at its highest point in the sky. The angle of elevation of the rays of the Sun at this point is 70°, and the height of the door is 8 feet. Your sister claims you can determine how far the overhang should extend by multiplying 8 by tan 70°. Is your sister correct? Explain.

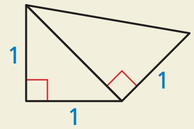


22. **HOW DO YOU SEE IT?** Write expressions for the tangent of each acute angle in the right triangle. Explain how the tangent of one acute angle is related to the tangent of the other acute angle. What kind of angle pair is $\angle A$ and $\angle B$?

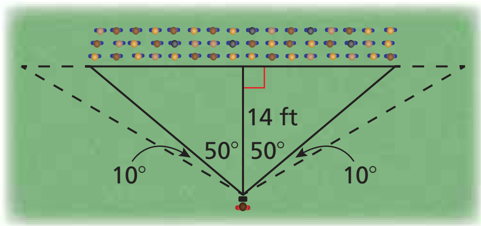


23. **REASONING** Explain why it is not possible to find the tangent of a right angle or an obtuse angle.

24. **THOUGHT PROVOKING** To create the diagram below, you begin with an isosceles right triangle with legs 1 unit long. Then the hypotenuse of the first triangle becomes the leg of a second triangle, whose remaining leg is 1 unit long. Continue the diagram until you have constructed an angle whose tangent is $\frac{1}{\sqrt{6}}$. Approximate the measure of this angle.



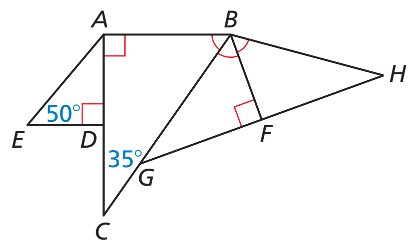
25. **PROBLEM SOLVING** Your class is having a class picture taken on the lawn. The photographer is positioned 14 feet away from the center of the class. The photographer turns 50° to look at either end of the class.



a. What is the distance between the ends of the class?

b. The photographer turns another 10° either way to see the end of the camera range. If each student needs 2 feet of space, about how many more students can fit at the end of each row? Explain.

26. **PROBLEM SOLVING** Find the perimeter of the figure, where $AC = 26$, $AD = BF$, and D is the midpoint of $\overline{AC}$.

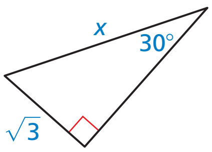


Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the value of *x*. *(Section 9.2)*

27.



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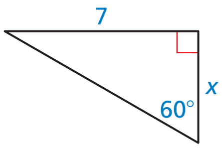

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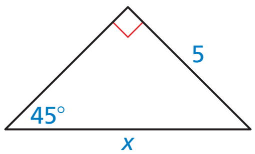


9.5 The Sine and Cosine Ratios

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS
G.9.A
G.9.B

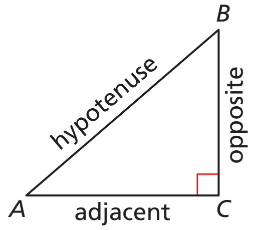
Essential Question

How is a right triangle used to find the sine and cosine of an acute angle? Is there a unique right triangle that must be used?

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The *sine* of $\angle A$ and *cosine* of $\angle A$ (written as sin A and cos A, respectively) are defined as follows.

$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{BC}{AB}$$

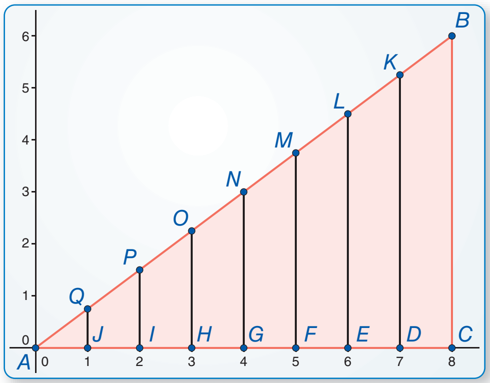
$$\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{AC}{AB}$$



EXPLORATION 1 Calculating Sine and Cosine Ratios

Work with a partner. Use dynamic geometry software.

a. Construct $\triangle ABC$, as shown. Construct segments perpendicular to $\overline{AC}$ to form right triangles that share vertex A and are similar to $\triangle ABC$ with vertices, as shown.



Sample
Points
$A(0, 0)$
$B(8, 6)$
$C(8, 0)$
Angle
$m\angle BAC = 36.87°$

b. Calculate each given ratio to complete the table for the decimal values of sin A and cos A for each right triangle. What can you conclude?

Sine ratio	$\frac{BC}{AB}$	$\frac{KD}{AK}$	$\frac{LE}{AL}$	$\frac{MF}{AM}$	$\frac{NG}{AN}$	$\frac{OH}{AO}$	$\frac{PI}{AP}$	$\frac{QJ}{AQ}$
sin *A*								
Cosine ratio	$\frac{AC}{AB}$	$\frac{AD}{AK}$	$\frac{AE}{AL}$	$\frac{AF}{AM}$	$\frac{AG}{AN}$	$\frac{AH}{AO}$	$\frac{AI}{AP}$	$\frac{AJ}{AQ}$
cos *A*								

Communicate Your Answer

ANALYZING MATHEMATICAL RELATIONSHIPS

To be proficient in math, you need to look closely to discern a pattern or structure.

2. How is a right triangle used to find the sine and cosine of an acute angle? Is there a unique right triangle that must be used?

3. In Exploration 1, what is the relationship between $\angle A$ and $\angle B$ in terms of their measures? Find sin B and cos B. How are these two values related to sin A and cos A? Explain why these relationships exist.

9.5 Lesson

Core Vocabulary

sine, *p. 498*
cosine, *p. 498*
angle of depression, *p. 501*

What You Will Learn

- Use the sine and cosine ratios.
- Find the sine and cosine of angle measures in special right triangles.
- Solve real-life problems involving sine and cosine ratios.

Using the Sine and Cosine Ratios

The **sine** and **cosine** ratios are trigonometric ratios for acute angles that involve the lengths of a leg and the hypotenuse of a right triangle.

READING

Remember the following abbreviations.

sine → sin
cosine → cos
hypotenuse → hyp.

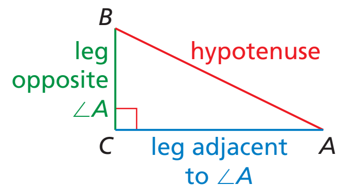
Core Concept

Sine and Cosine Ratios

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The sine of $\angle A$ and cosine of $\angle A$ (written as $\sin A$ and $\cos A$) are defined as follows.

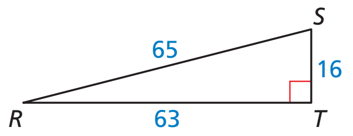
$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{BC}{AB}$$

$$\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{AC}{AB}$$



EXAMPLE 1 Finding Sine and Cosine Ratios

Find $\sin S$, $\sin R$, $\cos S$, and $\cos R$. Write each answer as a fraction and as a decimal rounded to four places.



SOLUTION

$$\sin S = \frac{\text{opp. } \angle S}{\text{hyp.}} = \frac{RT}{SR} = \frac{63}{65} \approx 0.9692 \qquad \sin R = \frac{\text{opp. } \angle R}{\text{hyp.}} = \frac{ST}{SR} = \frac{16}{65} \approx 0.2462$$

$$\cos S = \frac{\text{adj. to } \angle S}{\text{hyp.}} = \frac{ST}{SR} = \frac{16}{65} \approx 0.2462 \qquad \cos R = \frac{\text{adj. to } \angle R}{\text{hyp.}} = \frac{RT}{SR} = \frac{63}{65} \approx 0.9692$$

In Example 1, notice that $\sin S = \cos R$ and $\sin R = \cos S$. This is true because the side opposite $\angle S$ is adjacent to $\angle R$ and the side opposite $\angle R$ is adjacent to $\angle S$. The relationship between the sine and cosine of $\angle S$ and $\angle R$ is true for all complementary angles.

Core Concept

Sine and Cosine of Complementary Angles

The sine of an acute angle is equal to the cosine of its complement. The cosine of an acute angle is equal to the sine of its complement.

Let A and B be complementary angles. Then the following statements are true.

$$\sin A = \cos(90° - A) = \cos B \qquad \sin B = \cos(90° - B) = \cos A$$

$$\cos A = \sin(90° - A) = \sin B \qquad \cos B = \sin(90° - B) = \sin A$$

EXAMPLE 2 Rewriting Trigonometric Expressions

Write sin 56° in terms of cosine.

SOLUTION

Use the fact that the sine of an acute angle is equal to the cosine of its complement.

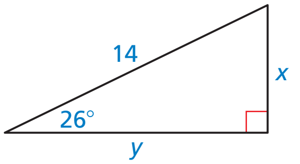
$$\sin 56° = \cos(90° - 56°) = \cos 34°$$

▶ The sine of 56° is the same as the cosine of 34°.

You can use the sine and cosine ratios to find unknown measures in right triangles.

EXAMPLE 3 Finding Leg Lengths

Find the values of x and y using sine and cosine. Round your answers to the nearest tenth.



SOLUTION

Step 1 Use a sine ratio to find the value of x.

$\sin 26° = \frac{\text{opp.}}{\text{hyp.}}$ — Write ratio for sine of 26°.

$\sin 26° = \frac{x}{14}$ — Substitute.

$14 \cdot \sin 26° = x$ — Multiply each side by 14.

$6.1 \approx x$ — Use a calculator.

▶ The value of x is about 6.1.

Step 2 Use a cosine ratio to find the value of y.

$\cos 26° = \frac{\text{adj.}}{\text{hyp.}}$ — Write ratio for cosine of 26°.

$\cos 26° = \frac{y}{14}$ — Substitute.

$14 \cdot \cos 26° = y$ — Multiply each side by 14.

$12.6 \approx y$ — Use a calculator.

▶ The value of y is about 12.6.

Monitoring Progress

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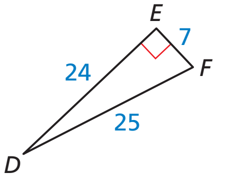
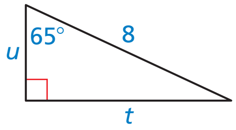
1. Find sin D, sin F, cos D, and cos F. Write each answer as a fraction and as a decimal rounded to four places.



2. Write cos 23° in terms of sine.

3. Find the values of u and t using sine and cosine. Round your answers to the nearest tenth.



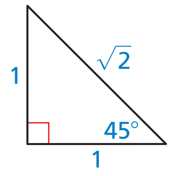
Finding Sine and Cosine in Special Right Triangles

EXAMPLE 4 **Finding the Sine and Cosine of 45°**

Find the sine and cosine of a 45° angle.

SOLUTION

Begin by sketching a 45°-45°-90° triangle. Because all such triangles are similar, you can simplify your calculations by choosing 1 as the length of each leg. Using the 45°-45°-90° Triangle Theorem (Theorem 9.4), the length of the hypotenuse is $\sqrt{2}$.



STUDY TIP

Notice that

$$\sin 45° = \cos(90 - 45)° = \cos 45°.$$

$$\sin 45° = \frac{\text{opp.}}{\text{hyp.}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \approx 0.7071$$

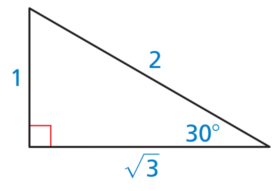
$$\cos 45° = \frac{\text{adj.}}{\text{hyp.}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \approx 0.7071$$

EXAMPLE 5 **Finding the Sine and Cosine of 30°**

Find the sine and cosine of a 30° angle.

SOLUTION

Begin by sketching a 30°-60°-90° triangle. Because all such triangles are similar, you can simplify your calculations by choosing 1 as the length of the shorter leg. Using the 30°-60°-90° Triangle Theorem (Theorem 9.5), the length of the longer leg is $\sqrt{3}$ and the length of the hypotenuse is 2.



$$\sin 30° = \frac{\text{opp.}}{\text{hyp.}} = \frac{1}{2} = 0.5000$$

$$\cos 30° = \frac{\text{adj.}}{\text{hyp.}} = \frac{\sqrt{3}}{2} \approx 0.8660$$

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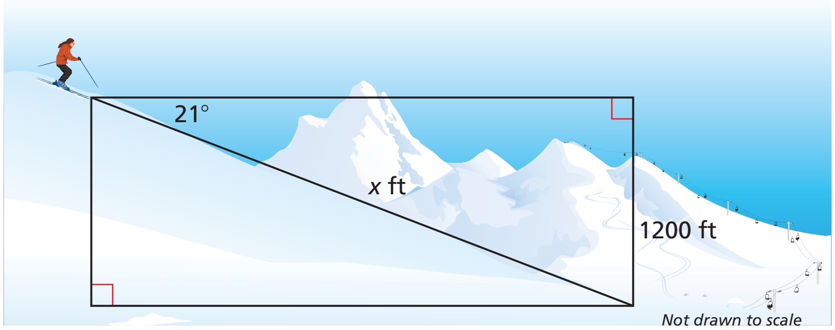
4. Find the sine and cosine of a 60° angle.

Solving Real-Life Problems

Recall from the previous lesson that the angle an upward line of sight makes with a horizontal line is called the *angle of elevation*. The angle that a downward line of sight makes with a horizontal line is called the **angle of depression**.

EXAMPLE 6 Modeling with Mathematics

You are skiing on a mountain with an altitude of 1200 feet. The angle of depression is 21°. Find the distance x you ski down the mountain to the nearest foot.



SOLUTION

1. **Understand the Problem** You are given the angle of depresion and the altitude of the mountain. You need to find the distance that you ski down the mountain.
2. **Make a Plan** Write a trigonometric ratio for the sine of the angle of depression involving the distance x. Then solve for x.
3. **Solve the Problem**

$$\sin 21° = \frac{\text{opp.}}{\text{hyp.}}$$ Write ratio for sine of 21°.

$$\sin 21° = \frac{1200}{x}$$ Substitute.

$$x \cdot \sin 21° = 1200$$ Multiply each side by x.

$$x = \frac{1200}{\sin 21°}$$ Divide each side by sin 21°.

$$x \approx 3348.5$$ Use a calculator.

▶ You ski about 3349 feet down the mountain.

4. **Look Back** Check your answer. The value of sin 21° is about 0.3584. Substitute for x in the sine ratio and compare the values.

$$\frac{1200}{x} \approx \frac{1200}{3348.5}$$

$$\approx 0.3584$$

This value is approximately the same as the value of sin 21°. ✓

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5. **WHAT IF?** In Example 6, the angle of depression is 28°. Find the distance x you ski down the mountain to the nearest foot.

9.5 Exercises

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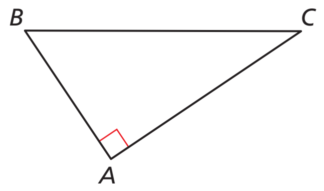
Vocabulary and Core Concept Check

1. VOCABULARY The sine ratio compares the length of ____________ to the length of _____________.

2. WHICH ONE DOESN'T BELONG? Which ratio does *not* belong with the other three? Explain your reasoning.

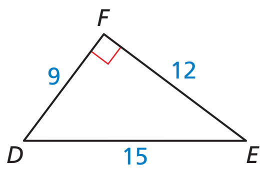
$\sin B$	$\cos C$
$\tan B$	$\frac{AC}{BC}$



Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, find $\sin D$, $\sin E$, $\cos D$, and $\cos E$. Write each answer as a fraction and as a decimal rounded to four places. *(See Example 1.)*

3.



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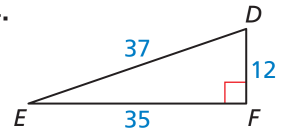

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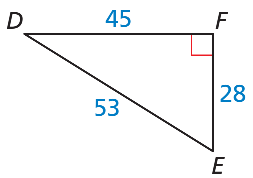

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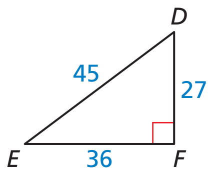

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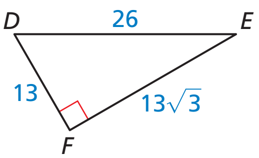

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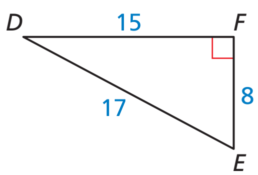


In Exercises 9–12, write the expression in terms of cosine. *(See Example 2.)*

9. $\sin 37°$

10. $\sin 81°$

11. $\sin 29°$

12. $\sin 64°$

In Exercises 13–16, write the expression in terms of sine.

13. $\cos 59°$

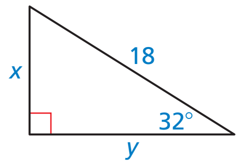
14. $\cos 42°$

15. $\cos 73°$

16. $\cos 18°$

In Exercises 17–22, find the value of each variable using sine and cosine. Round your answers to the nearest tenth. *(See Example 3.)*

17.



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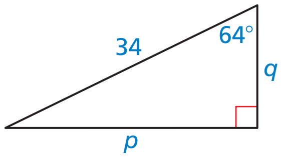

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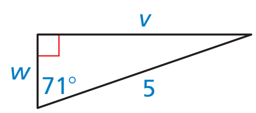

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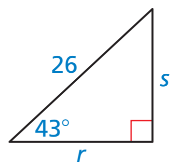

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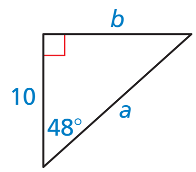

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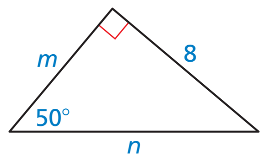
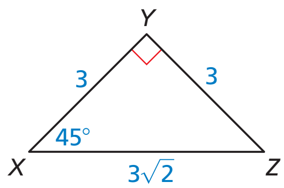
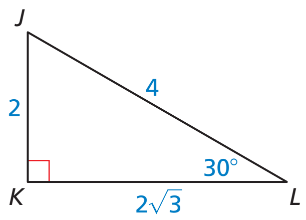


23. REASONING Which ratios are equal? Select all that apply. *(See Example 4.)*



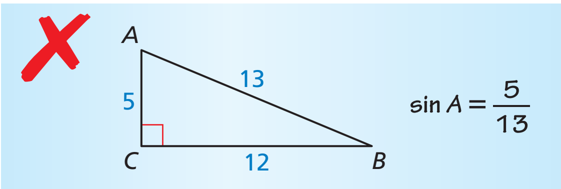
$\sin X$	$\cos X$	$\sin Z$	$\cos Z$

24. REASONING Which ratios are equal to $\frac{1}{2}$? Select all that apply. *(See Example 5.)*



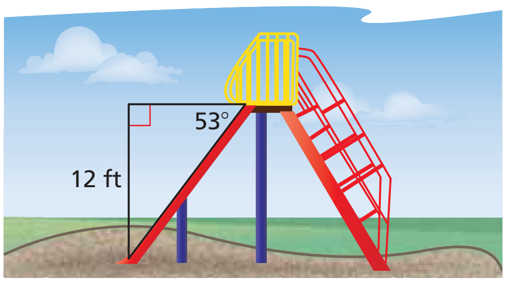
sin L | cos L | sin J | cos J

25. ERROR ANALYSIS Describe and correct the error in finding sin A.

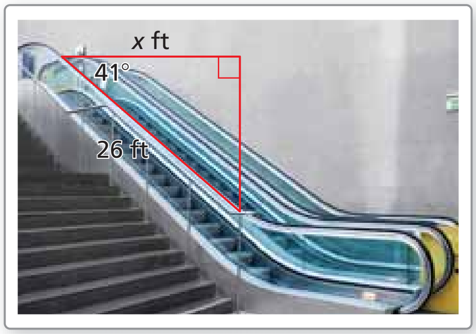


26. WRITING Explain how to tell which side of a right triangle is adjacent to an angle and which side is the hypotenuse.

27. MODELING WITH MATHEMATICS The top of the slide is 12 feet from the ground and has an angle of depression of 53°. What is the length of the slide? *(See Example 6.)*



28. MODELING WITH MATHEMATICS Find the horizontal distance x the escalator covers.

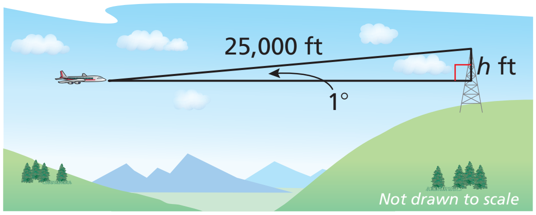


29. PROBLEM SOLVING You are flying a kite with 20 feet of string extended. The angle of elevation from the spool of string to the kite is 67°.

a. Draw and label a diagram that represents the situation.

b. How far off the ground is the kite if you hold the spool 5 feet off the ground? Describe how the height where you hold the spool affects the height of the kite.

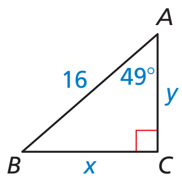
30. MODELING WITH MATHEMATICS Planes that fly at high speeds and low elevations have radar systems that can determine the range of an obstacle and the angle of elevation to the top of the obstacle. The radar of a plane flying at an altitude of 20,000 feet detects a tower that is 25,000 feet away, with an angle of elevation of 1°.



a. How many feet must the plane rise to pass over the tower?

b. Planes cannot come closer than 1000 feet vertically to any object. At what altitude must the plane fly in order to pass over the tower?

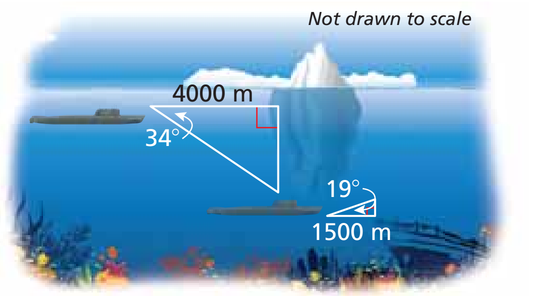
31. MAKING AN ARGUMENT Your friend uses the equation $\sin 49° = \frac{x}{16}$ to find BC. Your cousin uses the equation $\cos 41° = \frac{x}{16}$ to find BC. Who is correct? Explain your reasoning.



32. WRITING Describe what you must know about a triangle in order to use the sine ratio and what you must know about a triangle in order to use the cosine ratio.

33. MATHEMATICAL CONNECTIONS If $\triangle EQU$ is equilateral and $\triangle RGT$ is a right triangle with $RG = 2$, $RT = 1$, and $m\angle T = 90°$, show that $\sin E = \cos G$.

34. MODELING WITH MATHEMATICS Submarines use sonar systems, which are similar to radar systems, to detect obstacles. Sonar systems use sound to detect objects under water.

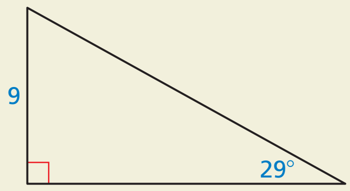


a. You are traveling underwater in a submarine. The sonar system detects an iceberg 4000 meters ahead, with an angle of depression of 34° to the bottom of the iceberg. How many meters must the submarine lower to pass under the iceberg?

b. The sonar system then detects a sunken ship 1500 meters ahead, with an angle of elevation of 19° to the highest part of the sunken ship. How many meters must the submarine rise to pass over the sunken ship?

35. ABSTRACT REASONING Make a conjecture about how you could use trigonometric ratios to find angle measures in a triangle.

36. HOW DO YOU SEE IT? Using only the given information, would you use a sine ratio or a cosine ratio to find the length of the hypotenuse? Explain your reasoning.



37. MULTIPLE REPRESENTATIONS You are standing on a cliff above an ocean. You see a sailboat from your vantage point 30 feet above the ocean.

a. Draw and label a diagram of the situation.

b. Make a table showing the angle of depression and the length of your line of sight. Use the angles 40°, 50°, 60°, 70°, and 80°.

c. Graph the values you found in part (b), with the angle measures on the x-axis.

d. Predict the length of the line of sight when the angle of depression is 30°.

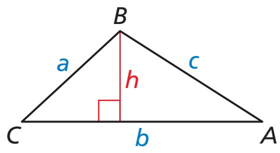
38. THOUGHT PROVOKING One of the following infinite series represents $\sin x$ and the other one represents $\cos x$ (where x is measured in radians). Which is which? Justify your answer. Then use each series to approximate the sine and cosine of $\frac{\pi}{6}$. (*Hints*: $\pi = 180°$; $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$; Find the values that the sine and cosine ratios approach as the angle measure approaches zero.)

a. $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$

b. $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

39. CRITICAL THINKING Let A be any acute angle of a right triangle. Show that (a) $\tan A = \frac{\sin A}{\cos A}$ and (b) $(\sin A)^2 + (\cos A)^2 = 1$.

40. CRITICAL THINKING Explain why the area of $\triangle ABC$ in the diagram can be found using the formula Area $= \frac{1}{2}ab \sin C$. Then calculate the area when $a = 4$, $b = 7$, and $m\angle C = 40°$.

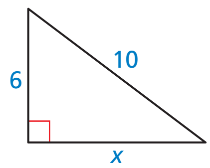


Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the value of x. Tell whether the side lengths form a Pythagorean triple. *(Section 9.1)*

41.



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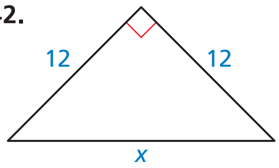

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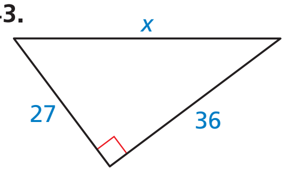

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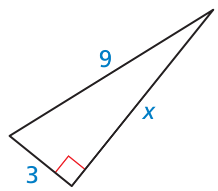


9.6 Solving Right Triangles

Texas Essential Knowledge and Skills
G.9.A
G.9.B

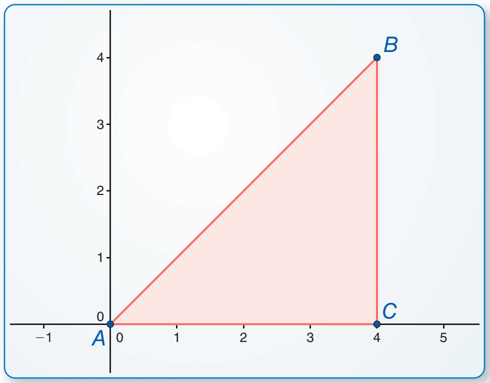
Essential Question

When you know the lengths of the sides of a right triangle, how can you find the measures of the two acute angles?

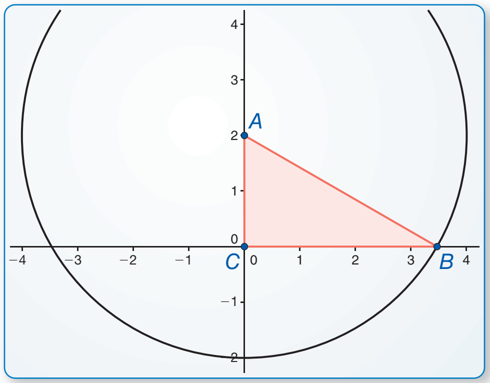
EXPLORATION 1 Solving Special Right Triangles

Work with a partner. Use the figures to find the values of the sine and cosine of $\angle A$ and $\angle B$. Use these values to find the measures of $\angle A$ and $\angle B$. Use dynamic geometry software to verify your answers.

a.



b.



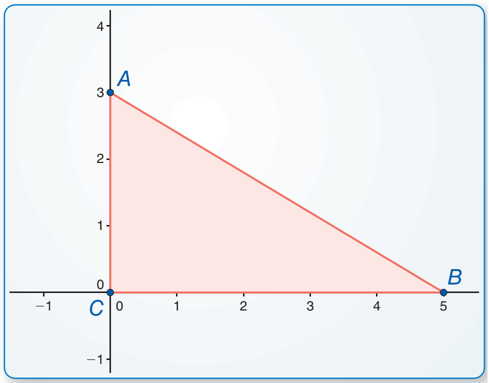
USING PRECISE MATHEMATICAL LANGUAGE

To be proficient in math, you need to calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context.

EXPLORATION 2 Solving Right Triangles

Work with a partner. You can use a calculator to find the measure of an angle when you know the value of the sine, cosine, or tangent of the angle. Use the inverse sine, inverse cosine, or inverse tangent feature of your calculator to approximate the measures of $\angle A$ and $\angle B$ to the nearest tenth of a degree. Then use dynamic geometry software to verify your answers.

a.



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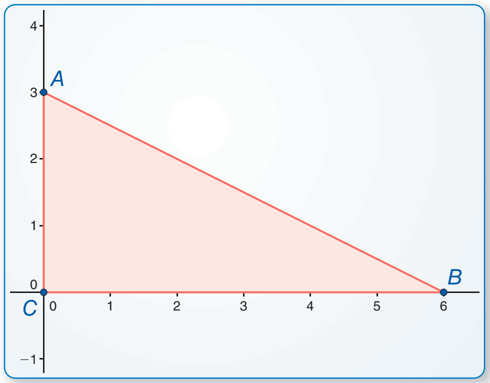


Communicate Your Answer

3. When you know the lengths of the sides of a right triangle, how can you find the measures of the two acute angles?

4. A ladder leaning against a building forms a right triangle with the building and the ground. The legs of the right triangle (in meters) form a 5-12-13 Pythagorean triple. Find the measures of the two acute angles to the nearest tenth of a degree.

9.6 Lesson

Core Vocabulary

inverse tangent, *p. 506*
inverse sine, *p. 506*
inverse cosine, *p. 506*
solve a right triangle, *p. 507*

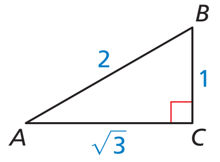
What You Will Learn

- Use inverse trigonometric ratios.
- Solve right triangles.

Using Inverse Trigonometric Ratios

EXAMPLE 1 Identifying Angles from Trigonometric Ratios

Determine which of the two acute angles has a cosine of 0.5.



SOLUTION

Find the cosine of each acute angle.

$$\cos A = \frac{\text{adj. to } \angle A}{\text{hyp.}} = \frac{\sqrt{3}}{2} \approx 0.8660 \qquad \cos B = \frac{\text{adj. to } \angle B}{\text{hyp.}} = \frac{1}{2} = 0.5$$

- The acute angle that has a cosine of 0.5 is $\angle B$.

If the measure of an acute angle is 60°, then its cosine is 0.5. The converse is also true. If the cosine of an acute angle is 0.5, then the measure of the angle is 60°. So, in Example 1, the measure of $\angle B$ must be 60° because its cosine is 0.5.

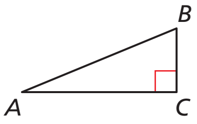
READING

The expression "$\tan^{-1} x$" is read as "the inverse tangent of *x*."

Core Concept

Inverse Trigonometric Ratios

Let $\angle A$ be an acute angle.



Inverse Tangent If $\tan A = x$, then $\tan^{-1} x = m\angle A$. $\quad \tan^{-1} \frac{BC}{AC} = m\angle A$

Inverse Sine If $\sin A = y$, then $\sin^{-1} y = m\angle A$. $\quad \sin^{-1} \frac{BC}{AB} = m\angle A$

Inverse Cosine If $\cos A = z$, then $\cos^{-1} z = m\angle A$. $\quad \cos^{-1} \frac{AC}{AB} = m\angle A$

ANOTHER WAY

You can use the Table of Trigonometric Ratios available at *BigIdeasMath.com* to approximate $\tan^{-1} 0.75$ to the nearest degree. Find the number closest to 0.75 in the tangent column and read the angle measure at the left.

EXAMPLE 2 Finding Angles Measures

Let $\angle A$, $\angle B$, and $\angle C$ be acute angles. Use a calculator to approximate the measures of $\angle A$, $\angle B$, and $\angle C$ to the nearest tenth of a degree.

a. $\tan A = 0.75$ **b.** $\sin B = 0.87$ **c.** $\cos C = 0.15$

SOLUTION

a. $m\angle A = \tan^{-1} 0.75 \approx 36.9°$

b. $m\angle B = \sin^{-1} 0.87 \approx 60.5°$

c. $m\angle C = \cos^{-1} 0.15 \approx 81.4°$

Monitoring Progress

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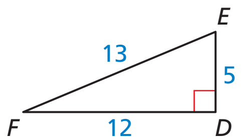

Determine which of the two acute angles has the given trigonometric ratio.

1. The sine of the angle is $\frac{12}{13}$. **2.** The tangent of the angle is $\frac{5}{12}$.

Monitoring Progress

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Let $\angle G$, $\angle H$, and $\angle K$ be acute angles. Use a calculator to approximate the measures of $\angle G$, $\angle H$, and $\angle K$ to the nearest tenth of a degree.

3. $\tan G = 0.43$ **4.** $\sin H = 0.68$ **5.** $\cos K = 0.94$

Solving Right Triangles

Core Concept

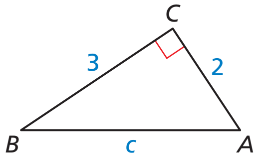
Solving a Right Triangle

To **solve a right triangle** means to find all unknown side lengths and angle measures. You can solve a right triangle when you know either of the following.

- two side lengths
- one side length and the measure of one acute angle

EXAMPLE 3 Solving a Right Triangle

Solve the right triangle. Round decimal answers to the nearest tenth.



SOLUTION

Step 1 Use the Pythagorean Theorem (Theorem 9.1) to find the length of the hypotenuse.

$c^2 = a^2 + b^2$ Pythagorean Theorem

$c^2 = 3^2 + 2^2$ Substitute.

$c^2 = 13$ Simplify.

$c = \sqrt{13}$ Find the positive square root.

$c \approx 3.6$ Use a calculator.

ANOTHER WAY

You could also have found $m\angle A$ first by finding $\tan^{-1}\frac{3}{2} \approx 56.3°$.

Step 2 Find $m\angle B$.

$m\angle B = \tan^{-1}\frac{2}{3} \approx 33.7°$ Use a calculator.

Step 3 Find $m\angle A$.

Because $\angle A$ and $\angle B$ are complements, you can write

$$\begin{aligned} m\angle A &= 90° - m\angle B \\ &\approx 90° - 33.7° \\ &= 56.3°. \end{aligned}$$

In $\triangle ABC$, $c \approx 3.6$, $m\angle B \approx 33.7°$, and $m\angle A \approx 56.3°$.

Monitoring Progress

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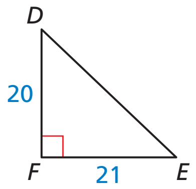
Solve the right triangle. Round decimal answers to the nearest tenth.

6.



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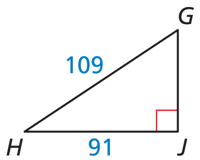
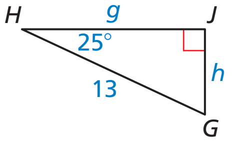


EXAMPLE 4 Solving a Right Triangle

Solve the right triangle. Round decimal answers to the nearest tenth.



SOLUTION

Use trigonometric ratios to find the values of g and h.

$$\sin H = \frac{\text{opp.}}{\text{hyp.}} \qquad \cos H = \frac{\text{adj.}}{\text{hyp.}}$$

$$\sin 25^\circ = \frac{h}{13} \qquad \cos 25^\circ = \frac{g}{13}$$

$$13 \cdot \sin 25^\circ = h \qquad 13 \cdot \cos 25^\circ = g$$

$$5.5 \approx h \qquad 11.8 \approx g$$

Because $\angle H$ and $\angle G$ are complements, you can write

$$m\angle G = 90^\circ - m\angle H = 90^\circ - 25^\circ = 65^\circ.$$

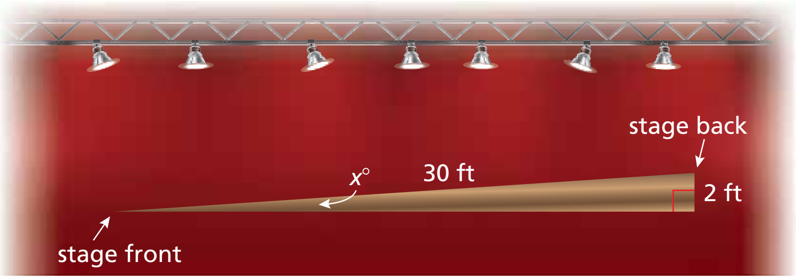
▶ In $\triangle GHJ$, $h \approx 5.5$, $g \approx 11.8$, and $m\angle G = 65^\circ$.

READING

A *raked stage* slants upward from front to back to give the audience a better view.

EXAMPLE 5 Solving a Real-Life Problem

Your school is building a *raked stage*. The stage will be 30 feet long from front to back, with a total rise of 2 feet. You want the rake (angle of elevation) to be 5° or less for safety. Is the raked stage within your desired range?



SOLUTION

Use the inverse sine ratio to find the degree measure x of the rake.

$$x \approx \sin^{-1}\frac{2}{30} \approx 3.8$$

▶ The rake is about 3.8°, so it is within your desired range of 5° or less.

Monitoring Progress

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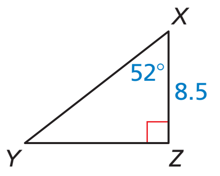
8. Solve the right triangle. Round decimal answers to the nearest tenth.



9. **WHAT IF?** In Example 5, suppose another raked stage is 20 feet long from front to back with a total rise of 2 feet. Is the raked stage within your desired range?

9.6 Exercises

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Vocabulary and Core Concept Check

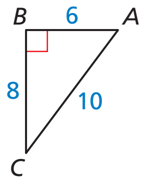
1. **COMPLETE THE SENTENCE** To solve a right triangle means to find the measures of all its _______ and _____.

2. **WRITING** Explain when you can use a trigonometric ratio to find a side length of a right triangle and when you can use the Pythagorean Theorem (Theorem 9.1).

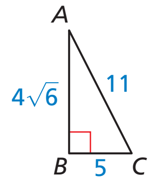
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, determine which of the two acute angles has the given trigonometric ratio. *(See Example 1.)*

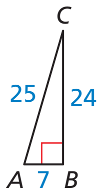
3. The cosine of the angle is $\frac{4}{5}$.



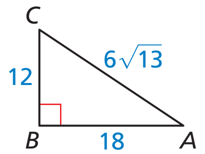
4. The sine of the angle is $\frac{5}{11}$.



5. The sine of the angle is 0.96.



6. The tangent of the angle is 1.5.



In Exercises 7–12, let $\angle D$ be an acute angle. Use a calculator to approximate the measure of $\angle D$ to the nearest tenth of a degree. *(See Example 2.)*

7. $\sin D = 0.75$

8. $\sin D = 0.19$

9. $\cos D = 0.33$

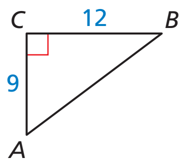
10. $\cos D = 0.64$

11. $\tan D = 0.28$

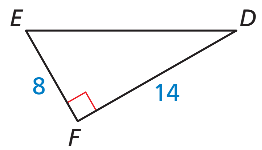
12. $\tan D = 0.72$

In Exercises 13–18, solve the right triangle. Round decimal answers to the nearest tenth. *(See Examples 3 and 4.)*

13.



14.



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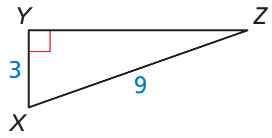

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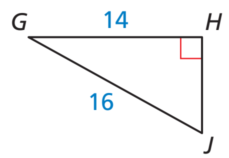

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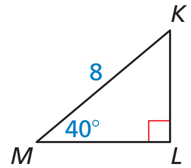

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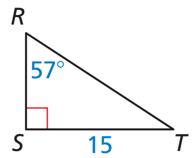
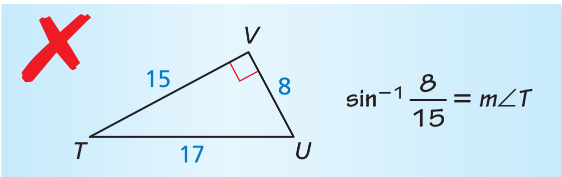
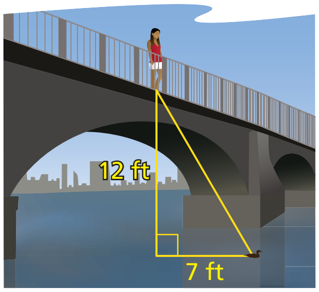


19. **ERROR ANALYSIS** Describe and correct the error in using an inverse trigonometric ratio.

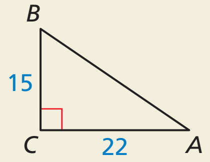


20. **PROBLEM SOLVING** In order to unload clay easily, the body of a dump truck must be elevated to at least 45°. The body of a dump truck that is 14 feet long has been raised 8 feet. Will the clay pour out easily? Explain your reasoning. *(See Example 5.)*

21. **PROBLEM SOLVING** You are standing on a footbridge that is 12 feet above a lake. You look down and see a duck in the water. The duck is 7 feet away from the footbridge. What is the angle of elevation from the duck to you?



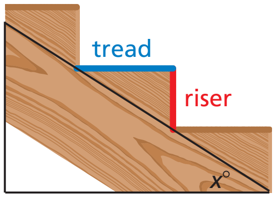
22. **HOW DO YOU SEE IT?** Write three expressions that can be used to approximate the measure of $\angle A$. Which expression would you choose? Explain your choice.



23. **MODELING WITH MATHEMATICS** The Uniform Federal Accessibility Standards specify that a wheelchair ramp may not have an incline greater than 4.76°. You want to build a ramp with a vertical rise of 8 inches. You want to minimize the horizontal distance taken up by the ramp. Draw a diagram showing the approximate dimensions of your ramp.

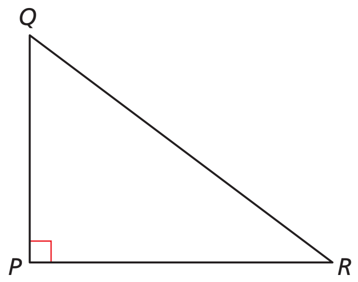
24. **MODELING WITH MATHEMATICS** The horizontal part of a step is called the *tread*. The vertical part is called the *riser*. The recommended riser-to-tread ratio is 7 inches : 11 inches.

   a. Find the value of x for stairs built using the recommended riser-to-tread ratio.



   b. You want to build stairs that are less steep than the stairs in part (a). Give an example of a riser-to-tread ratio that you could use. Find the value of x for your stairs.

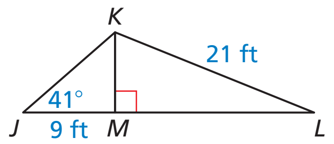
25. **USING TOOLS** Find the measure of $\angle R$ without using a protractor. Justify your technique.



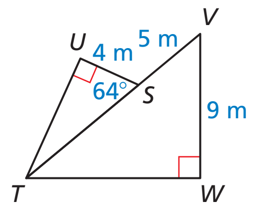
26. **MAKING AN ARGUMENT** Your friend claims that $\tan^{-1} x = \frac{1}{\tan x}$. Is your friend correct? Explain your reasoning.

USING STRUCTURE In Exercises 27 and 28, solve each triangle.

27. $\triangle JKM$ and $\triangle LKM$



28. $\triangle TUS$ and $\triangle VTW$



29. **MATHEMATICAL CONNECTIONS** Write an expression that can be used to find the measure of the acute angle formed by each line and the x-axis. Then approximate the angle measure to the nearest tenth of a degree.

   a. $y = 3x$

   b. $y = \frac{4}{3}x + 4$

30. **THOUGHT PROVOKING** Simplify each expression. Justify your answer.

   a. $\sin^{-1}(\sin x)$

   b. $\tan(\tan^{-1} y)$

   c. $\cos(\cos^{-1} z)$

31. **REASONING** Explain why the expression $\sin^{-1}(1.2)$ does not make sense.

32. **USING STRUCTURE** The perimeter of rectangle $ABCD$ is 16 centimeters, and the ratio of its width to its length is 1 : 3. Segment BD divides the rectangle into two congruent triangles. Find the side lengths and angle measures of these two triangles.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. *(Skills Review Handbook)*

33. $\frac{12}{x} = \frac{3}{2}$

34. $\frac{13}{9} = \frac{x}{18}$

35. $\frac{x}{2.1} = \frac{4.1}{3.5}$

36. $\frac{5.6}{12.7} = \frac{4.9}{x}$

9.7 Law of Sines and Law of Cosines

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS
G.6.D

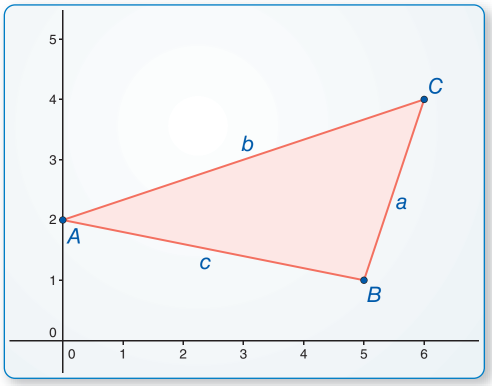
Essential Question

What are the Law of Sines and the Law of Cosines?

EXPLORATION 1 Discovering the Law of Sines

Work with a partner.

a. Copy and complete the table for the triangle shown. What can you conclude?



Sample
Segments
$a = 3.16$
$b = 6.32$
$c = 5.10$
Angles
$m\angle A = 29.74°$
$m\angle B = 97.13°$
$m\angle C = 53.13°$

$m\angle A$	a	$\frac{\sin A}{a}$	$m\angle B$	b	$\frac{\sin B}{b}$	$m\angle C$	c	$\frac{\sin C}{c}$

SELECTING TOOLS
To be proficient in math, you need to use technology to compare predictions with data.

b. Use dynamic geometry software to draw two other triangles. Copy and complete the table in part (a) for each triangle. Use your results to write a conjecture about the relationship between the sines of the angles and the lengths of the sides of a triangle.

EXPLORATION 2 Discovering the Law of Cosines

Work with a partner.

a. Copy and complete the table for the triangle in Exploration 1(a). What can you conclude?

c	c^2	a	a^2	b	b^2	$m\angle C$	$a^2 + b^2 - 2ab \cos C$

b. Use dynamic geometry software to draw two other triangles. Copy and complete the table in part (a) for each triangle. Use your results to write a conjecture about what you observe in the completed tables.

Communicate Your Answer

3. What are the Law of Sines and the Law of Cosines?

4. When would you use the Law of Sines to solve a triangle? When would you use the Law of Cosines to solve a triangle?

9.7 Lesson

Core Vocabulary

Law of Sines, *p. 513*
Law of Cosines, *p. 515*

What You Will Learn

- Find areas of triangles.
- Use the Law of Sines to solve triangles.
- Use the Law of Cosines to solve triangles.

Finding Areas of Triangles

So far, you have used trigonometric ratios to solve right triangles. In this lesson, you will learn how to solve any triangle. When the triangle is obtuse, you may need to find a trigonometric ratio for an obtuse angle.

EXAMPLE 1 Finding Trigonometric Ratios for Obtuse Angles

Use a calculator to find each trigonometric ratio. Round your answer to four decimal places.

a. $\tan 150°$ **b.** $\sin 120°$ **c.** $\cos 95°$

SOLUTION

a. $\tan 150° \approx -0.5774$ **b.** $\sin 120° \approx 0.8660$ **c.** $\cos 95° \approx -0.0872$

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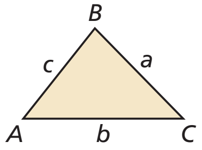
Use a calculator to find the trigonometric ratio. Round your answer to four decimal places.

1. $\tan 110°$ **2.** $\sin 97°$ **3.** $\cos 165°$

Core Concept

Area of a Triangle

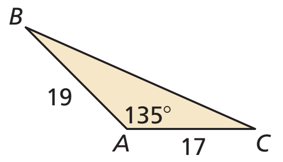
The area of any triangle is given by one-half the product of the lengths of two sides times the sine of their included angle. For $\triangle ABC$ shown, there are three ways to calculate the area.



Area $= \frac{1}{2}bc \sin A$ Area $= \frac{1}{2}ac \sin B$ Area $= \frac{1}{2}ab \sin C$

EXAMPLE 2 Finding the Area of a Triangle

Find the area of the triangle. Round your answer to the nearest tenth.



SOLUTION

$$\text{Area} = \frac{1}{2}bc \sin A = \frac{1}{2}(17)(19) \sin 135° \approx 114.2$$

- The area of the triangle is about 114.2 square units.

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Find the area of $\triangle ABC$ with the given side lengths and included angle. Round your answer to the nearest tenth.

4. $B = 60°, a = 19, c = 14$ **5.** $C = 29°, a = 38, b = 31$

Using the Law of Sines

The trigonometric ratios in the previous sections can only be used to solve right triangles. You will learn two laws that can be used to solve any triangle.

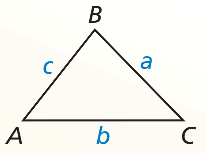
You can use the **Law of Sines** to solve triangles when two angles and the length of any side are known (AAS or ASA cases), or when the lengths of two sides and an angle opposite one of the two sides are known (SSA case).

Theorem

Theorem 9.9 Law of Sines

The Law of Sines can be written in either of the following forms for $\triangle ABC$ with sides of length a, b, and c.

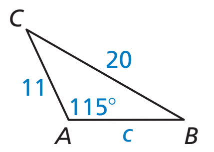
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \qquad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



Proof Ex. 51, p. 520

EXAMPLE 3 Using the Law of Sines (SSA Case)

Solve the triangle. Round decimal answers to the nearest tenth.



SOLUTION

Use the Law of Sines to find $m\angle B$.

$$\frac{\sin B}{b} = \frac{\sin A}{a} \qquad \text{Law of Sines}$$

$$\frac{\sin B}{11} = \frac{\sin 115^\circ}{20} \qquad \text{Substitute.}$$

$$\sin B = \frac{11 \sin 115^\circ}{20} \qquad \text{Multiply each side by 11.}$$

$$m\angle B \approx 29.9^\circ \qquad \text{Use a calculator.}$$

By the Triangle Sum Theorem (Theorem 5.1), $m\angle C \approx 180^\circ - 115^\circ - 29.9^\circ = 35.1^\circ$.

Use the Law of Sines again to find the remaining side length c of the triangle.

$$\frac{c}{\sin C} = \frac{a}{\sin A} \qquad \text{Law of Sines}$$

$$\frac{c}{\sin 35.1^\circ} = \frac{20}{\sin 115^\circ} \qquad \text{Substitute.}$$

$$c = \frac{20 \sin 35.1^\circ}{\sin 115^\circ} \qquad \text{Multiply each side by } \sin 35.1^\circ.$$

$$c \approx 12.7 \qquad \text{Use a calculator.}$$

▶ In $\triangle ABC$, $m\angle B \approx 29.9^\circ$, $m\angle C \approx 35.1^\circ$, and $c \approx 12.7$.

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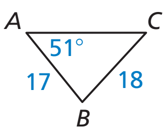
Solve the triangle. Round decimal answers to the nearest tenth.

6.



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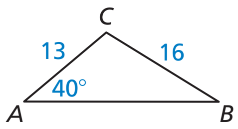
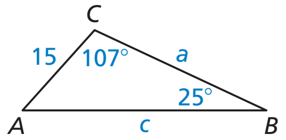


EXAMPLE 4 Using the Law of Sines (AAS Case)

Solve the triangle. Round decimal answers to the nearest tenth.



SOLUTION

By the Triangle Sum Theorem (Theorem 5.1), $m\angle A = 180° - 107° - 25° = 48°$.

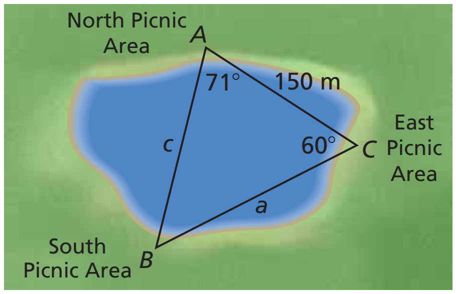
By the Law of Sines, you can write $\frac{a}{\sin 48°} = \frac{15}{\sin 25°} = \frac{c}{\sin 107°}$.

$\frac{a}{\sin 48°} = \frac{15}{\sin 25°}$	Write two equations, each with one variable.	$\frac{c}{\sin 107°} = \frac{15}{\sin 25°}$
$a = \frac{15 \sin 48°}{\sin 25°}$	Solve for each variable.	$c = \frac{15 \sin 107°}{\sin 25°}$
$a \approx 26.4$	Use a calculator.	$c \approx 33.9$

▶ In $\triangle ABC$, $m\angle A = 48°$, $a \approx 26.4$, and $c \approx 33.9$.

EXAMPLE 5 Using the Law of Sines (ASA Case)

A surveyor makes the measurements shown to determine the length of a bridge to be built across a small lake from the North Picnic Area to the South Picnic Area. Find the length of the bridge.



SOLUTION

In the diagram, c represents the distance from the North Picnic Area to the South Picnic Area, so c represents the length of the bridge.

By the Triangle Sum Theorem (Theorem 5.1), $m\angle B = 180° - 71° - 60° = 49°$.

By the Law of Sines, you can write $\frac{a}{\sin 71°} = \frac{150}{\sin 49°} = \frac{c}{\sin 60°}$.

$\frac{c}{\sin 60°} = \frac{150}{\sin 49°}$ — Write an equation involving c.

$c = \frac{150 \sin 60°}{\sin 49°}$ — Multiply each side by sin 60°.

$c \approx 172.1$ — Use a calculator.

▶ The length of the bridge will be about 172.1 meters.

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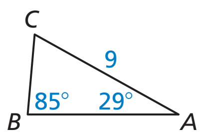
Solve the triangle. Round decimal answers to the nearest tenth.

8.



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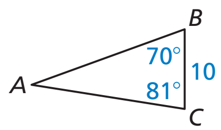


10. WHAT IF? In Example 5, what would be the length of a bridge from the South Picnic Area to the East Picnic Area?

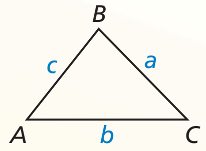
Using the Law of Cosines

You can use the **Law of Cosines** to solve triangles when two sides and the included angle are known (SAS case), or when all three sides are known (SSS case).

Theorem

Theorem 9.10 Law of Cosines

If $\triangle ABC$ has sides of length a, b, and c, as shown, then the following are true.



$$a^2 = b^2 + c^2 - 2bc \cos A$$

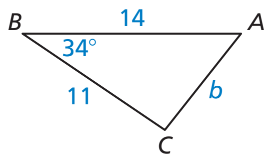
$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Proof Ex. 52, p. 520

EXAMPLE 6 Using the Law of Cosines (SAS Case)

Solve the triangle. Round decimal answers to the nearest tenth.



SOLUTION

Use the Law of Cosines to find side length b.

$b^2 = a^2 + c^2 - 2ac \cos B$ Law of Cosines

$b^2 = 11^2 + 14^2 - 2(11)(14) \cos 34°$ Substitute.

$b^2 = 317 - 308 \cos 34°$ Simplify.

$b = \sqrt{317 - 308 \cos 34°}$ Find the positive square root.

$b \approx 7.85$ Use a calculator.

ANOTHER WAY

When you know all three sides and one angle, you can use the Law of Cosines or the Law of Sines to find the measure of a second angle.

Use the Law of Sines to find $m\angle A$.

$\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$ Law of Sines

$\dfrac{\sin A}{11} = \dfrac{\sin 34°}{7.85}$ Substitute.

$\sin A = \dfrac{11 \sin 34°}{7.85}$ Multiply each side by 11.

$m\angle A \approx 51.6°$ Use a calculator.

By the Triangle Sum Theorem (Theorem 5.1), $m\angle C \approx 180° - 34° - 51.6° = 94.4°$.

COMMON ERROR

In Example 6, the smaller remaining angle is found first because the inverse sine feature of a calculator only gives angle measures from 0° to 90°. So, when an angle is obtuse, like $\angle C$ because $14^2 > (7.85)^2 + 11^2$, you will not get the obtuse measure.

▶ In $\triangle ABC$, $b \approx 7.85$, $m\angle A \approx 51.6°$, and $m\angle C \approx 94.4°$.

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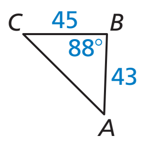
Solve the triangle. Round decimal answers to the nearest tenth.

11.



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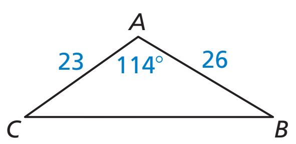
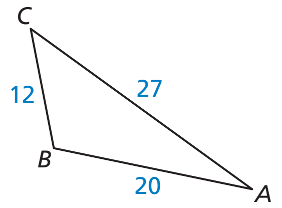


EXAMPLE 7 Using the Law of Cosines (SSS Case)

Solve the triangle. Round decimal answers to the nearest tenth.



SOLUTION

First, find the angle opposite the longest side, $\overline{AC}$. Use the Law of Cosines to find $m\angle B$.

$$b^2 = a^2 + c^2 - 2ac \cos B \quad \text{Law of Cosines}$$

$$27^2 = 12^2 + 20^2 - 2(12)(20) \cos B \quad \text{Substitute.}$$

$$\frac{27^2 - 12^2 - 20^2}{-2(12)(20)} = \cos B \quad \text{Solve for } \cos B.$$

$$m\angle B \approx 112.7° \quad \text{Use a calculator.}$$

> **COMMON ERROR**
>
> In Example 7, the largest angle is found first to make sure that the other two angles are acute. This way, when you use the Law of Sines to find another angle measure, you will know that it is between 0° and 90°.

Now, use the Law of Sines to find $m\angle A$.

$$\frac{\sin A}{a} = \frac{\sin B}{b} \quad \text{Law of Sines}$$

$$\frac{\sin A}{12} = \frac{\sin 112.7°}{27} \quad \text{Substitute for } a, b, \text{ and } B.$$

$$\sin A = \frac{12 \sin 112.7°}{27} \quad \text{Multiply each side by 12.}$$

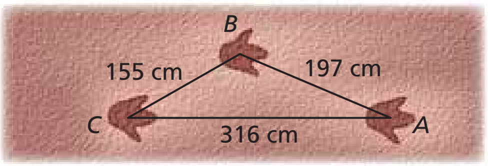
$$m\angle A \approx 24.2° \quad \text{Use a calculator.}$$

By the Triangle Sum Theorem (Theorem 5.1), $m\angle C \approx 180° - 24.2° - 112.7° = 43.1°$.

▶ In $\triangle ABC$, $m\angle A \approx 24.2°$, $m\angle B \approx 112.7°$, and $m\angle C \approx 43.1°$.

EXAMPLE 8 Solving a Real-Life Problem

An organism's step angle is a measure of walking efficiency. The closer the step angle is to 180°, the more efficiently the organism walked. The diagram shows a set of footprints for a dinosaur. Find the step angle *B*.



SOLUTION

$$b^2 = a^2 + c^2 - 2ac \cos B \quad \text{Law of Cosines}$$

$$316^2 = 155^2 + 197^2 - 2(155)(197) \cos B \quad \text{Substitute.}$$

$$\frac{316^2 - 155^2 - 197^2}{-2(155)(197)} = \cos B \quad \text{Solve for } \cos B.$$

$$127.3° \approx m\angle B \quad \text{Use a calculator.}$$

▶ The step angle *B* is about 127.3°.

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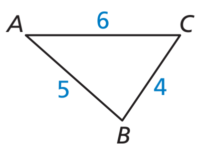
Solve the triangle. Round decimal answers to the nearest tenth.

13.



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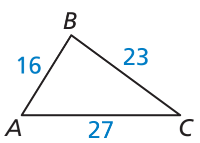

9.7 Exercises

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Vocabulary and Core Concept Check

1. **WRITING** What type of triangle would you use the Law of Sines or the Law of Cosines to solve?

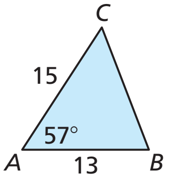
2. **VOCABULARY** What information do you need to use the Law of Sines?

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, use a calculator to find the trigonometric ratio. Round your answer to four decimal places. *(See Example 1.)*

3. sin 127°
4. sin 98°
5. cos 139°
6. cos 108°
7. tan 165°
8. tan 116°

In Exercises 9–12, find the area of the triangle. Round your answer to the nearest tenth. *(See Example 2.)*

9.


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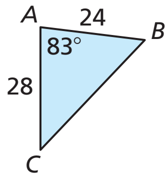

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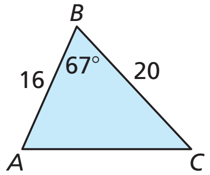

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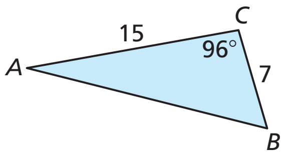

In Exercises 13–18, solve the triangle. Round decimal answers to the nearest tenth. *(See Examples 3, 4, and 5.)*

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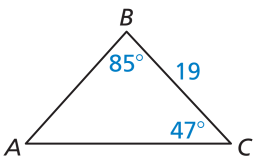

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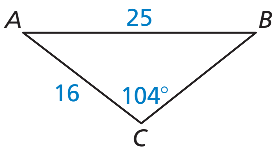

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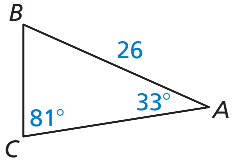

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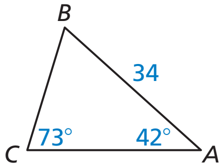

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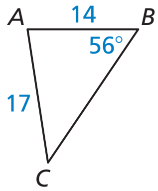

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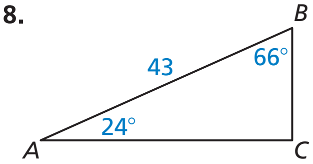
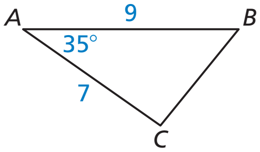


In Exercises 19–24, solve the triangle. Round decimal answers to the nearest tenth. *(See Examples 6 and 7.)*

19.


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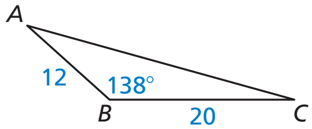

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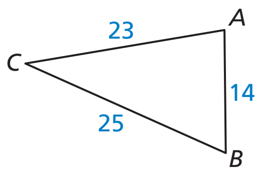

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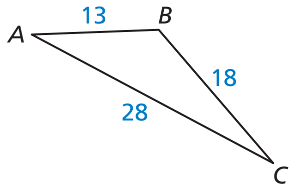

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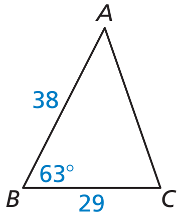

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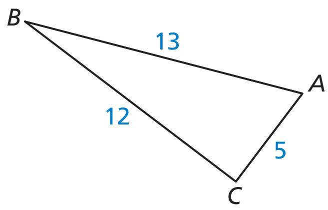
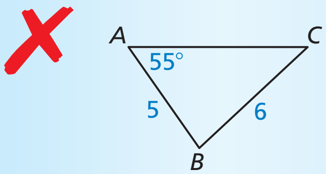


25. **ERROR ANALYSIS** Describe and correct the error in finding $m\angle C$.



$$\frac{\sin C}{6} = \frac{\sin 55^\circ}{5}$$

$$\sin C = \frac{6 \sin 55^\circ}{5}$$

$$m\angle C \approx 79.4^\circ$$

26. ERROR ANALYSIS Describe and correct the error in finding $m\angle A$ in $\triangle ABC$ when $a = 19$, $b = 21$, and $c = 11$.

✗

$$\cos A = \frac{19^2 - 21^2 - 11^2}{-2(19)(21)}$$

$$m\angle A \approx 75.4^\circ$$

COMPARING METHODS **In Exercises 27–32, tell whether you would use the Law of Sines, the Law of Cosines, or the Pythagorean Theorem (Theorem 9.1) and trigonometric ratios to solve the triangle with the given information. Explain your reasoning. Then solve the triangle.**

27. $A = 72^\circ$, $B = 44^\circ$, $b = 14$

28. $B = 98^\circ$, $C = 37^\circ$, $a = 18$

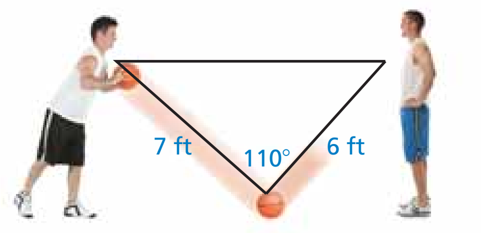
29. $C = 65^\circ$, $a = 12$, $b = 21$

30. $B = 90^\circ$, $a = 15$, $c = 6$

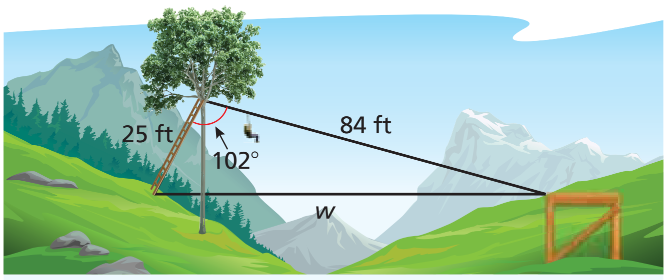
31. $C = 40^\circ$, $b = 27$, $c = 36$

32. $a = 34$, $b = 19$, $c = 27$

33. MODELING WITH MATHEMATICS You and your friend are standing on the baseline of a basketball court. You bounce a basketball to your friend, as shown in the diagram. What is the distance between you and your friend? *(See Example 8.)*



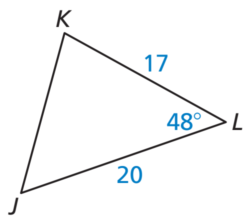
34. MODELING WITH MATHEMATICS A zip line is constructed across a valley, as shown in the diagram. What is the width w of the valley?



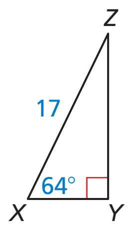
35. MODELING WITH MATHEMATICS You are on the observation deck of the Empire State Building looking at the Chrysler Building. When you turn 145° clockwise, you see the Statue of Liberty. You know that the Chrysler Building and the Empire State Building are about 0.6 mile apart and that the Chrysler Building and the Statue of Liberty are about 5.6 miles apart. Estimate the distance between the Empire State Building and the Statue of Liberty.

36. MODELING WITH MATHEMATICS The Leaning Tower of Pisa in Italy has a height of 183 feet and is 4° off vertical. Find the horizontal distance d that the top of the tower is off vertical.

37. MAKING AN ARGUMENT Your friend says that the Law of Sines can be used to find JK. Your cousin says that the Law of Cosines can be used to find JK. Who is correct? Explain your reasoning.



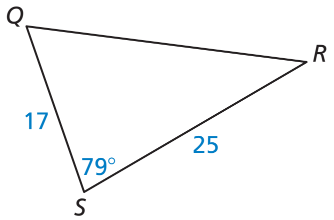
38. REASONING Use $\triangle XYZ$.



a. Can you use the Law of Sines to solve $\triangle XYZ$? Explain your reasoning.

b. Can you use another method to solve $\triangle XYZ$? Explain your reasoning.

39. MAKING AN ARGUMENT Your friend calculates the area of the triangle using the formula $A = \frac{1}{2}qr \sin S$ and says that the area is approximately 208.6 square units. Is your friend correct? Explain your reasoning.



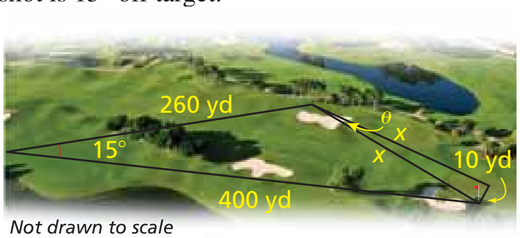
40. MULTIPLE REPRESENTATIONS You are fertilizing a triangular garden. One side of the garden is 62 feet long, and another side is 54 feet long. The angle opposite the 62-foot side is 58°.

a. Draw a diagram to represent this situation.

b. Use the Law of Sines to solve the triangle from part (a).

c. One bag of fertilizer covers an area of 200 square feet. How many bags of fertilizer will you need to cover the entire garden?

41. MODELING WITH MATHEMATICS A golfer hits a drive 260 yards on a hole that is 400 yards long. The shot is 15° off target.



Not drawn to scale

a. What is the distance x from the golfer's ball to the hole?

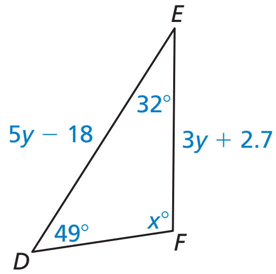
b. Assume the golfer is able to hit the ball precisely the distance found in part (a). What is the maximum angle θ (theta) by which the ball can be off target in order to land no more than 10 yards from the hole?

42. COMPARING METHODS A building is constructed on top of a cliff that is 300 meters high. A person standing on level ground below the cliff observes that the angle of elevation to the top of the building is 72° and the angle of elevation to the top of the cliff is 63°.

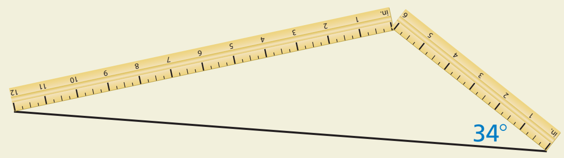
a. How far away is the person from the base of the cliff?

b. Describe two different methods you can use to find the height of the building. Use one of these methods to find the building's height.

43. MATHEMATICAL CONNECTIONS Find the values of x and y.



44. HOW DO YOU SEE IT? Would you use the Law of Sines or the Law of Cosines to solve the triangle?



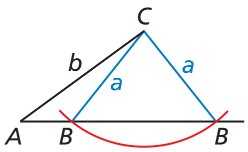
45. REWRITING A FORMULA Simplify the Law of Cosines for when the given angle is a right angle.

46. THOUGHT PROVOKING Consider any triangle with side lengths of a, b, and c. Calculate the value of s, which is half the perimeter of the triangle. What measurement of the triangle is represented by $\sqrt{s(s-a)(s-b)(s-c)}$?

47. ANALYZING RELATIONSHIPS The *ambiguous case* of the Law of Sines occurs when you are given the measure of one acute angle, the length of one adjacent side, and the length of the side opposite that angle, which is less than the length of the adjacent side. This results in two possible triangles. Using the given information, find two possible solutions for $\triangle ABC$. Draw a diagram for each triangle. (*Hint*: The inverse sine function gives only acute angle measures, so consider the acute angle and its supplement for $\angle B$.)

a. $A = 40°$, $a = 13$, $b = 16$

b. $A = 21°$, $a = 17$, $b = 32$

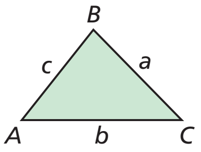


48. ABSTRACT REASONING Use the Law of Cosines to show that the measure of each angle of an equilateral triangle is 60°. Explain your reasoning.

49. CRITICAL THINKING An airplane flies 55° east of north from City A to City B, a distance of 470 miles. Another airplane flies 7° north of east from City A to City C, a distance of 890 miles. What is the distance between Cities B and C?

50. REWRITING A FORMULA Follow the steps to derive the formula for the area of a triangle,

Area $= \frac{1}{2}ab \sin C$.



a. Draw the altitude from vertex B to $\overline{AC}$. Label the altitude as h. Write a formula for the area of the triangle using h.

b. Write an equation for $\sin C$.

c. Use the results of parts (a) and (b) to write a formula for the area of a triangle that does not include h.

51. PROVING A THEOREM Follow the steps to use the formula for the area of a triangle to prove the Law of Sines (Theorem 9.9).

a. Use the derivation in Exercise 50 to explain how to derive the three related formulas for the area of a triangle.

Area $= \frac{1}{2}bc \sin A$,

Area $= \frac{1}{2}ac \sin B$,

Area $= \frac{1}{2}ab \sin C$

b. Why can you use the formulas in part (a) to write the following statement?

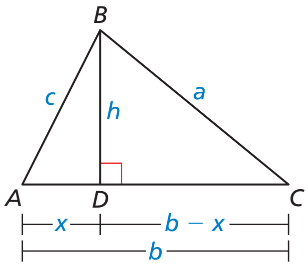
$$\frac{1}{2}bc \sin A = \frac{1}{2}ac \sin B = \frac{1}{2}ab \sin C$$

c. Show how to rewrite the statement in part (b) to prove the Law of Sines. Justify each step.

52. PROVING A THEOREM Use the given information to complete the two-column proof of the Law of Cosines (Theorem 9.10).

Given $\overline{BD}$ is an altitude of $\triangle ABC$.

Prove $a^2 = b^2 + c^2 - 2bc \cos A$



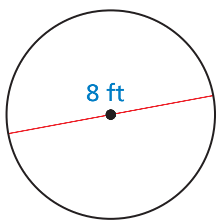
STATEMENTS	REASONS
1. $\overline{BD}$ is an altitude of $\triangle ABC$.	**1.** Given
2. $\triangle ADB$ and $\triangle CDB$ are right triangles.	**2.** ______________
3. $a^2 = (b - x)^2 + h^2$	**3.** ______________
4. ______________	**4.** Expand binomial.
5. $x^2 + h^2 = c^2$	**5.** ______________
6. ______________	**6.** Substitution Property of Equality
7. $\cos A = \frac{x}{c}$	**7.** ______________
8. $x = c \cos A$	**8.** ______________
9. $a^2 = b^2 + c^2 - 2bc \cos A$	**9.** ______________

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the radius and diameter of the circle. *(Skills Review Handbook)*

53.



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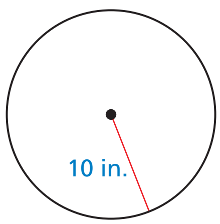

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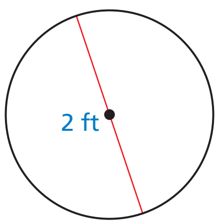

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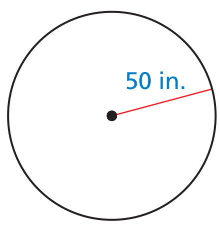


9.4–9.7 What Did You Learn?

Core Vocabulary

trigonometric ratio, *p. 492*
tangent, *p. 492*
angle of elevation, *p. 494*
sine, *p. 498*
cosine, *p. 498*
angle of depression, *p. 501*
inverse tangent, *p. 506*
inverse sine, *p. 506*
inverse cosine, *p. 506*
solve a right triangle, *p. 507*
Law of Sines, *p. 513*
Law of Cosines, *p. 515*

Core Concepts

Section 9.4

Tangent Ratio, *p. 492*

Section 9.5

Sine and Cosine Ratios, *p. 498*
Sine and Cosine of Complementary Angles, *p. 498*

Section 9.6

Inverse Trigonometric Ratios, *p. 506*
Solving a Right Triangle, *p. 507*

Section 9.7

Area of a Triangle, *p. 512*
Theorem 9.9 Law of Sines, *p. 513*
Theorem 9.10 Law of Cosines, *p. 515*

Mathematical Thinking

1. In Exercise 21 on page 496, your brother claims that you could determine how far the overhang should extend by dividing 8 by tan 70°. Justify his conclusion and explain why it works.

2. In Exercise 29 on page 503, explain the flaw in the argument that the kite is 18.4 feet high.

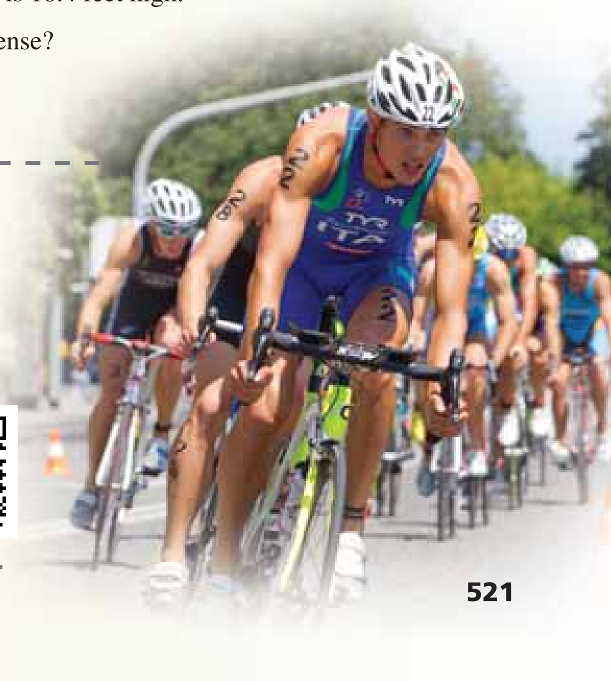
3. In Exercise 31 on page 510, for what values does the inverse sine make sense?

Performance Task

Triathlon

There is a big triathlon in town, and you are trying to take pictures of your friends at multiple locations during the event. How far would you need to walk to move between the photography locations?

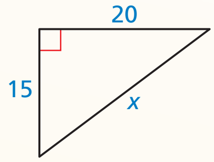
To explore the answers to this question and more, go to ***BigIdeasMath.com***.



9 Chapter Review

9.1 The Pythagorean Theorem (pp. 467–474)

Find the value of x. Then tell whether the side lengths form a Pythagorean triple.



$c^2 = a^2 + b^2$ — Pythagorean Theorem (Theorem 9.1)

$x^2 = 15^2 + 20^2$ — Substitute.

$x^2 = 225 + 400$ — Multiply.

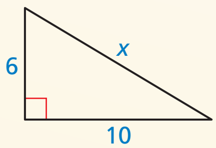
$x^2 = 625$ — Add.

$x = 25$ — Find the positive square root.

▶ The value of x is 25. Because the side lengths 15, 20, and 25 are integers that satisfy the equation $c^2 = a^2 + b^2$, they form a Pythagorean triple.

Find the value of x. Then tell whether the side lengths form a Pythagorean triple.

1.



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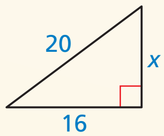

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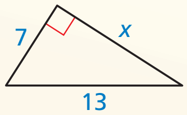


Verify that the segment lengths form a triangle. Is the triangle *acute*, *right*, or *obtuse*?

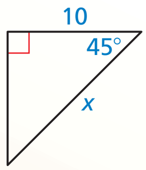
4. 6, 8, and 9

5. 10, $2\sqrt{2}$, and $6\sqrt{3}$

6. 13, 18, and $3\sqrt{55}$

9.2 Special Right Triangles (pp. 475–480)

Find the value of x. Write your answer in simplest form.



By the Triangle Sum Theorem (Theorem 5.1), the measure of the third angle must be 45°, so the triangle is a 45°-45°-90° triangle.

hypotenuse = leg • $\sqrt{2}$ — 45°-45°-90° Triangle Theorem (Theorem 9.4)

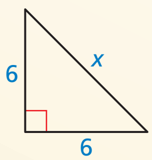
$x = 10 \cdot \sqrt{2}$ — Substitute.

$x = 10\sqrt{2}$ — Simplify.

▶ The value of x is $10\sqrt{2}$.

Find the value of x. Write your answer in simplest form.

7.



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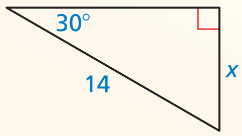
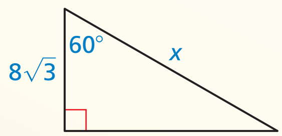
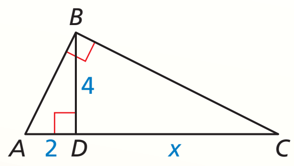


9.

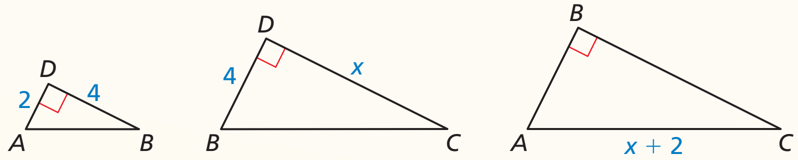


9.3 Similar Right Triangles (pp. 481–488)

Identify the similar triangles. Then find the value of x.



Sketch the three similar right triangles so that the corresponding angles and sides have the same orientation.



▶ $\triangle DBA \sim \triangle DCB \sim \triangle BCA$

By the Geometric Mean (Altitude) Theorem (Theorem 9.7), you know that 4 is the geometric mean of 2 and x.

$4^2 = 2 \cdot x$ Geometric Mean (Altitude) Theorem

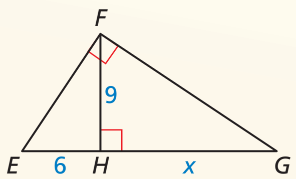
$16 = 2x$ Square 4.

$8 = x$ Divide each side by 2.

▶ The value of x is 8.

Identify the similar triangles. Then find the value of x.

10.



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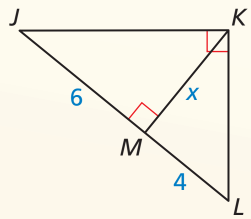
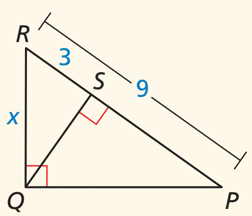


12.



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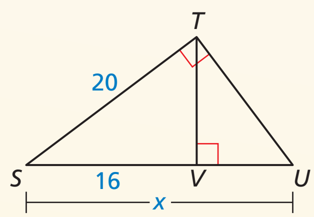


Find the geometric mean of the two numbers.

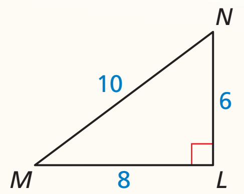
14. 9 and 25

15. 36 and 48

16. 12 and 42

9.4 The Tangent Ratio (pp. 491–496)

Find tan *M* and tan *N*. Write each answer as a fraction and as a decimal rounded to four places.

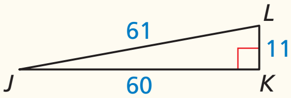


$$\tan M = \frac{\text{opp. } \angle M}{\text{adj. to } \angle M} = \frac{LN}{LM} = \frac{6}{8} = \frac{3}{4} = 0.7500$$

$$\tan N = \frac{\text{opp. } \angle N}{\text{adj. to } \angle N} = \frac{LM}{LN} = \frac{8}{6} = \frac{4}{3} \approx 1.3333$$

Find the tangents of the acute angles in the right triangle. Write each answer as a fraction and as a decimal rounded to four decimal places.

17.



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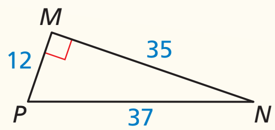

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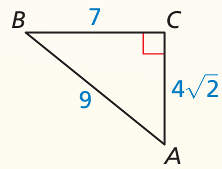
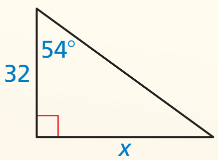


Find the value of *x*. Round your answer to the nearest tenth.

20.



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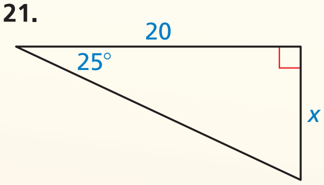
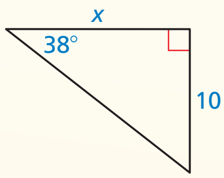
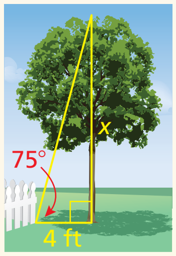


22.

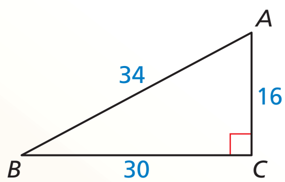


23. The angle between the bottom of a fence and the top of a tree is 75°. The tree is 4 feet from the fence. How tall is the tree? Round your answer to the nearest foot.



9.5 The Sine and Cosine Ratios (pp. 497–504)

Find sin *A*, sin *B*, cos *A*, and cos *B*. Write each answer as a fraction and as a decimal rounded to four places.



$$\sin A = \frac{\text{opp. } \angle A}{\text{hyp.}} = \frac{BC}{AB} = \frac{30}{34} = \frac{15}{17} \approx 0.8824$$

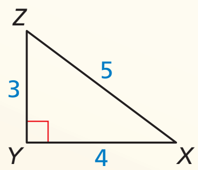
$$\sin B = \frac{\text{opp. } \angle B}{\text{hyp.}} = \frac{AC}{AB} = \frac{16}{34} = \frac{8}{17} \approx 0.4706$$

$$\cos A = \frac{\text{adj. to } \angle A}{\text{hyp.}} = \frac{AC}{AB} = \frac{16}{34} = \frac{8}{17} \approx 0.4706$$

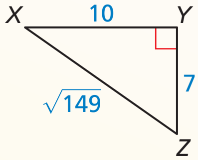
$$\cos B = \frac{\text{adj. to } \angle B}{\text{hyp.}} = \frac{BC}{AB} = \frac{30}{34} = \frac{15}{17} \approx 0.8824$$

Find sin X, sin Z, cos X, and cos Z. Write each answer as a fraction and as a decimal rounded to four decimal places.

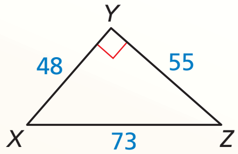
24.



25.

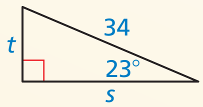


26.

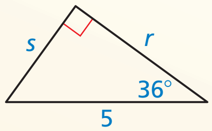


Find the value of each variable using sine and cosine. Round your answers to the nearest tenth.

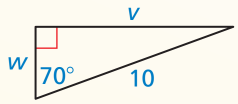
27.



28.



29.

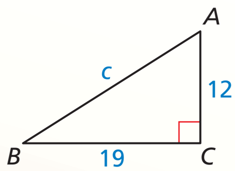


30. Write sin 72° in terms of cosine.

31. Write cos 29° in terms of sine.

9.6 Solving Right Triangles (pp. 505–510)

Solve the right triangle. Round decimal answers to the nearest tenth.



Step 1 Use the Pythagorean Theorem (Theorem 9.1) to find the length of the hypotenuse.

$c^2 = a^2 + b^2$ — Pythagorean Theorem

$c^2 = 19^2 + 12^2$ — Substitute.

$c^2 = 505$ — Simplify.

$c = \sqrt{505}$ — Find the positive square root.

$c \approx 22.5$ — Use a calculator.

Step 2 Find $m\angle B$.

$m\angle B = \tan^{-1}\frac{12}{19} \approx 32.3°$ — Use a calculator.

Step 3 Find $m\angle A$.

Because $\angle A$ and $\angle B$ are complements, you can write $m\angle A = 90° - m\angle B \approx 90° - 32.3° = 57.7°$.

▶ In $\triangle ABC$, $c \approx 22.5$, $m\angle B \approx 32.3°$, and $m\angle A \approx 57.7°$.

Let $\angle Q$ be an acute angle. Use a calculator to approximate the measure of $\angle Q$ to the nearest tenth of a degree.

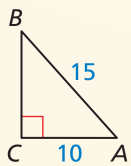
32. $\cos Q = 0.32$

33. $\sin Q = 0.91$

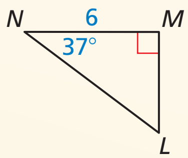
34. $\tan Q = 0.04$

Solve the right triangle. Round decimal answers to the nearest tenth.

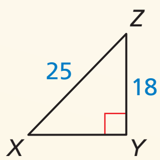
35.



36.



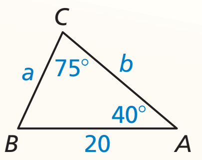
37.



9.7 Law of Sines and Law of Cosines *(pp. 511–520)*

Solve the triangle. Round decimal answers to the nearest tenth.

a.



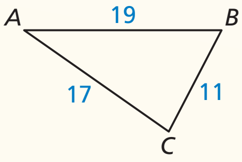
By the Triangle Sum Theorem (Theorem 5.1), $m\angle B = 180° - 40° - 75° = 65°$.

By the Law of Sines, you can write $\frac{a}{\sin 40°} = \frac{b}{\sin 65°} = \frac{20}{\sin 75°}$.

$\frac{a}{\sin 40°} = \frac{20}{\sin 75°}$	Write two equations, each with one variable.	$\frac{b}{\sin 65°} = \frac{20}{\sin 75°}$
$a = \frac{20 \sin 40°}{\sin 75°}$	Solve for each variable.	$b = \frac{20 \sin 65°}{\sin 75°}$
$a \approx 13.3$	Use a calculator.	$b \approx 18.8$

► In $\triangle ABC$, $m\angle B = 65°$, $a \approx 13.3$, and $b \approx 18.8$.

b.



First, find the angle opposite the longest side, $\overline{AB}$. Use the Law of Cosines to find $m\angle C$.

$$19^2 = 11^2 + 17^2 - 2(11)(17) \cos C$$ Law of Cosines

$$\frac{19^2 - 11^2 - 17^2}{-2(11)(17)} = \cos C$$ Solve for cos C.

$$m\angle C \approx 82.5°$$ Use a calculator.

Now, use the Law of Sines to find $m\angle A$.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$ Law of Sines

$$\frac{\sin A}{11} = \frac{\sin 82.5°}{19}$$ Substitute.

$$\sin A = \frac{11 \sin 82.5°}{19}$$ Multiply each side by 11.

$$m\angle A \approx 35.0°$$ Use a calculator.

By the Triangle Sum Theorem (Theorem 5.1), $m\angle B \approx 180° - 35.0° - 82.5° = 62.5°$.

► In $\triangle ABC$, $m\angle A \approx 35.0°$, $m\angle B \approx 62.5°$, and $m\angle C \approx 82.5°$.

Find the area of $\triangle ABC$ with the given side lengths and included angle.

38. $m\angle B = 124°, a = 9, c = 11$ **39.** $m\angle A = 68°, b = 13, c = 7$ **40.** $m\angle C = 79°, a = 25, b = 17$

Solve $\triangle ABC$. Round decimal answers to the nearest tenth.

41. $m\angle A = 112°, a = 9, b = 4$

42. $m\angle A = 28°, m\angle B = 64°, c = 55$

43. $m\angle C = 48°, b = 20, c = 28$

44. $m\angle B = 25°, a = 8, c = 3$

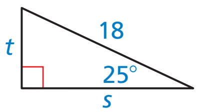
45. $m\angle B = 102°, m\angle C = 43°, b = 21$

46. $a = 10, b = 3, c = 12$

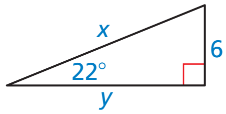
9 Chapter Test

Find the value of each variable. Round your answers to the nearest tenth.

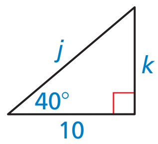
1.



2.



3.



Verify that the segment lengths form a triangle. Is the triangle *acute*, *right*, or *obtuse*?

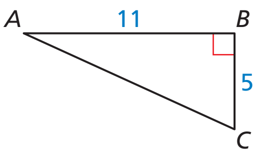
4. 16, 30, and 34

5. 4, $\sqrt{67}$, and 9

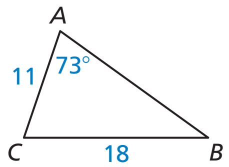
6. $\sqrt{5}$, 5, and 5.5

Solve $\triangle ABC$. Round decimal answers to the nearest tenth.

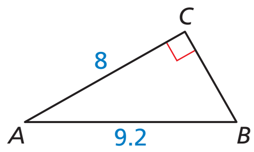
7.



8.



9.



10. $m\angle A = 103°$, $b = 12$, $c = 24$

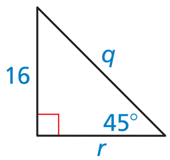
11. $m\angle A = 26°$, $m\angle C = 35°$, $b = 13$

12. $a = 38$, $b = 31$, $c = 35$

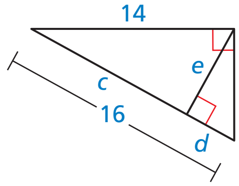
13. Write cos 53° in terms of sine.

Find the value of each variable. Write your answers in simplest form.

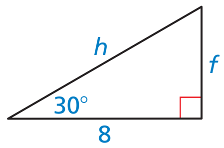
14.



15.



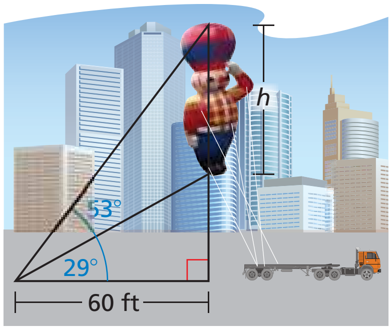
16.

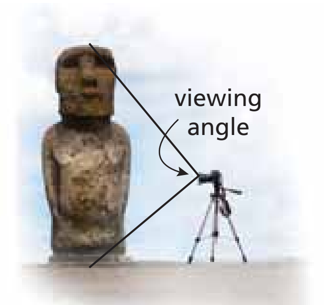


17. In $\triangle QRS$, $m\angle R = 57°$, $q = 9$, and $s = 5$. Find the area of $\triangle QRS$.

18. You are given the measures of both acute angles of a right triangle. Can you determine the side lengths? Explain.

19. You are at a parade looking up at a large balloon floating directly above the street. You are 60 feet from a point on the street directly beneath the balloon. To see the top of the balloon, you look up at an angle of 53°. To see the bottom of the balloon, you look up at an angle of 29°. Estimate the height h of the balloon.



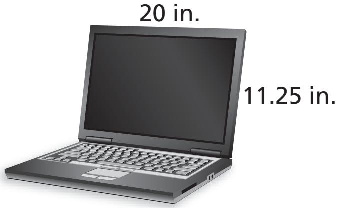


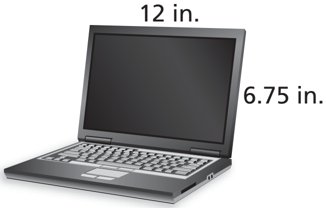
20. You want to take a picture of a statue on Easter Island, called a *moai*. The moai is about 13 feet tall. Your camera is on a tripod that is 5 feet tall. The vertical viewing angle of your camera is set at 90°. How far from the moai should you stand so that the entire height of the moai is perfectly framed in the photo?

9 Standards Assessment

1. The size of a laptop screen is measured by the length of its diagonal. You want to purchase a laptop with the largest screen possible. Which laptop should you buy? *(TEKS G.9.B)*

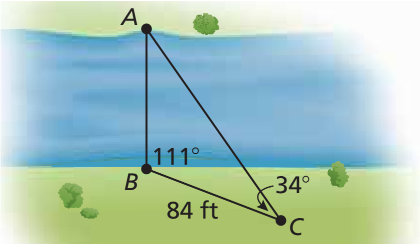
Ⓐ 

Ⓑ 

Ⓒ 

Ⓓ 

2. **GRIDDED ANSWER** A surveyor makes the measurements shown. What is the width of the river? Round to the nearest whole number. *(TEKS G.9.A)*



3. What are the coordinates of the vertices of the image of $\triangle QRS$ after the composition of transformations shown? *(TEKS G.3.B)*

Translation: $(x, y) \rightarrow (x + 2, y + 3)$

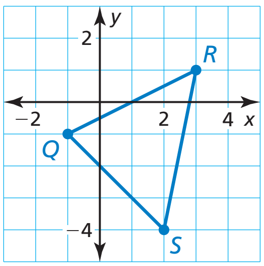
Rotation: 180° about the origin

Ⓕ $Q'(1, 2), R'(5, 4), S'(4, -1)$

Ⓖ $Q'(-1, -2), R'(-5, -4), S'(-4, 1)$

Ⓗ $Q'(3, -2), R'(-1, -4), S'(0, 1)$

Ⓙ $Q'(-2, 1), R'(-4, 5), S'(1, 4)$



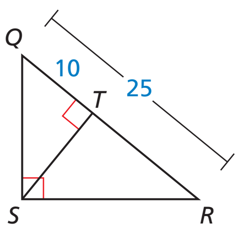
4. What is the length of $\overline{ST}$? *(TEKS G.8.B)*

Ⓐ $5\sqrt{6}$

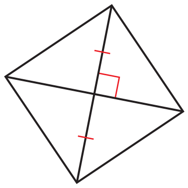
Ⓑ 15

Ⓒ $5\sqrt{10}$

Ⓓ $5\sqrt{15}$



5. Two properties are shown in the diagram. Which quadrilaterals have these two properties? *(TEKS G.6.E)*



I. kite
II. trapezoid
III. rhombus
IV. square

Ⓕ II and III only   Ⓖ III and IV only

Ⓗ I, II, and III only   Ⓙ I, III, and IV only

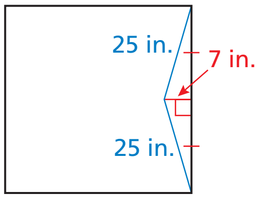
6. What is the approximate perimeter of the square? *(TEKS G.9.B)*

Ⓐ 96 in.

Ⓑ 144 in.

Ⓒ 192 in.

Ⓓ 200 in.



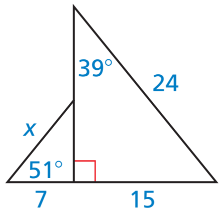
7. Are the triangles similar? If so, state the theorem you used and find the value of x. *(TEKS G.7.B)*

Ⓕ Yes, by the AA Similarity Theorem (Thm. 8.3); $x = 11.2$

Ⓖ Yes, by the SAS Similarity Theorem (Thm. 8.5); $x = 14.5$

Ⓗ Yes, by the AA Similarity Theorem (Thm. 8.3); $x = 51.4$

Ⓙ No, the triangles are not similar.



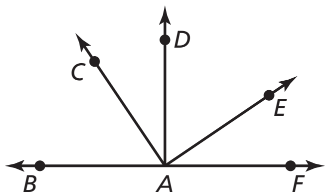
8. In the diagram, $\angle CAD$ and $\angle DAE$ are complements and $\overrightarrow{AB}$ and $\overrightarrow{AF}$ are opposite rays. What can be concluded about $\angle BAC$ and $\angle EAF$? *(TEKS G.6.A)*

Ⓐ $m\angle BAC = m\angle EAF$

Ⓑ $m\angle BAC + m\angle EAF = 45°$

Ⓒ $m\angle BAC + m\angle EAF = 90°$

Ⓓ $m\angle BAC + m\angle EAF = 180°$



9. What are the values of x and y? *(TEKS G.9.B)*

Ⓕ $x = 12\sqrt{3}, y = 24\sqrt{3}$ Ⓖ $x = 12, y = 24$

Ⓗ $x = 4\sqrt{3}, y = 8\sqrt{3}$ Ⓙ $x = 4, y = 8$