TRANSIENT STABILITY ANALYSIS
OF A MULTI MACHINE POWER SYSTEM

Huynh Chau Duy, Huynh Quang Minh and Ho Dac Loc

Faculty of Electrical and Electronics Engineering
HoChiMinh City University of Technology, VietNam

ABSTRACT

Transient stability analysis has recently become a major issue in the operation of power systems due to the increasing stress on power system networks. This problem requires evaluation of a power system's ability to withstand disturbances while maintaining the quality of service. Many different techniques have been proposed for transient stability analysis in power systems, specially for a multi-machine system. These methods include the time domain solutions, the extended equal area criteria, and the direct stability methods such as the transient energy function. However, the most methods must transform from a multi-machine system to an equivalent machine and infinite bus system [1]-[3], [5]-[6]. This paper introduces a method as an accurate algorithm to analyse transient stability for power system with an individual machine. It is as a tool to identify stable and unstable conditions of a power system after fault clearing with solving differential equations. This method is performed for an Northern VietNam power system.

1. INTRODUCTION

Multimachine equations can be written similar to the one-machine system connected to the infinite bus. In order to reduce the complexity of the transient stability analysis, similar simplifying assumptions are made as follows.

-Each synchronous machine is represented by a constant voltage source behind the direct axis transient reactance. This representation neglects the effect of saliency and assumes constant flux linkages.
-The governor's action are neglected and the input powers are assumed to remain constant during the entire period of simulation.
-Using the prefault bus voltages, all loads are converted to equivalent admittances to ground and are assumed to remain constant.
-Damping or asynchronous powers are ignored.
-The mechanical rotor angle of each machine coincides with the angle of the voltage behind the machine reactance.
-Machines belonging to the same station swing together and are said to be coherent. A group of coherent machines is represented by one equivalent machine.

2. MATHEMATICAL MODEL OF MULTIMACHINE TRANSIENT STABILITY ANALYSIS

The first step in the transient stability analysis is to solve the initial load flow and to determine the initial bus voltage magnitudes and phase angles. The machine currents prior to disturbance are calculated from, [5]

$$I_i = \frac{S_i^*}{V_i^*} = \frac{P_i - jQ_i}{V_i^*}, i = 1, 2, \dots, m \qquad (1)$$

Where
m is the number of generators
V_i is the terminal voltage of the ith generator
P_i and Q_i are the generator real and reactive powers.
All unknow values are determined from the initial power flow solution. The generator armature resistances are usually neglected and the voltages behind the transient reactances are then obtained, [5]

$$E_i^{'} = V_i + jX_d^{'}I_i \qquad (2)$$

Next, all load are converted to equivalent admittances by using the relation

$$y_{i0} = \frac{S_i^{*}}{|V_i|^2} = \frac{P_i - jQ_i}{|V_i|^2} \qquad (3)$$

To include voltages behind transient reactances, m buses are added to the n bus power system network. The equivalent network with all load converted to admittances is shown in Fig.1, [4]
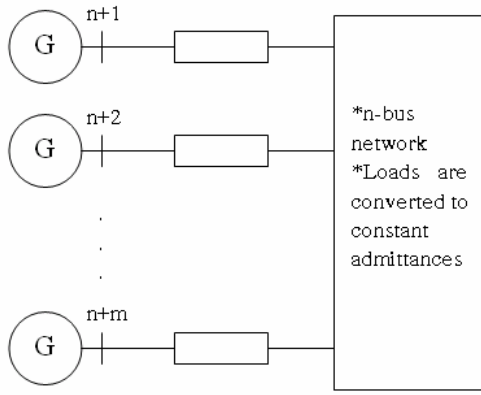

Fig.1 Power system representation for transient stability analysis

Nodes n+1, n+2, . . ., n+m are the internal machine buses, i.e., the buses behind the transient reactances. The node voltage equation with node 0 as reference for this network, is

$$
\begin{bmatrix} I_1 \\ I_2 \\ . \\ . \\ . \\ I_n \\ I_{n+1} \\ . \\ . \\ . \\ I_{n+m} \end{bmatrix} =
\begin{bmatrix} Y_{11} & \cdots & Y_{1n} & Y_{1(n+1)} & \cdots & Y_{1(n+m)} \\ Y_{21} & \cdots & Y_{2n} & Y_{2(n+1)} & \cdots & Y_{2(n+m)} \\ . & \cdots & . & . & \cdots & . \\ . & \cdots & . & . & \cdots & . \\ . & \cdots & . & . & \cdots & . \\ Y_{n1} & \cdots & Y_{nn} & Y_{n(n+1)} & \cdots & Y_{n(n+m)} \\ Y_{(n+1)1} & \cdots & Y_{(n+1)n} & Y_{(n+1)(n+1)} & \cdots & Y_{(n+1)(n+m)} \\ . & \cdots & . & . & \cdots & . \\ . & \cdots & . & . & \cdots & . \\ . & \cdots & . & . & \cdots & . \\ Y_{(n+m)1} & \cdots & Y_{(n+m)n} & Y_{(n+m)(n+1)} & \cdots & Y_{(n+m)(n+m)} \end{bmatrix}
\begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ . \\ V_n \\ E_{n+1}^{'} \\ . \\ . \\ . \\ E_{n+m}^{'} \end{bmatrix}
$$
$$(4)$$

Or

$$I_{bus} = Y_{bus}V_{bus} \qquad (5)$$

Where
I_{bus} is the vector of the injected bus currents
V_{bus} is the vector of bus voltages measured from the reference node.

The diagonal elements of the bus admittance matrix are the sum of admittances connected to it, and the off-diagonal elements are equal to the negative of the admittance between the nodes. The reference is that additional nodes are added to include the machine voltages behind transient reactances. Also, diagonal elements are modified to include the load admittances.
To simplify the analysis, all nodes other than the generator internal nodes are eliminated using Kron reduction formula [5]. To eliminate the load buses, the bus admittance matrix in (4) is partitioned such that the n buses to be removed are represented in the upper n rows. Since no current enters or leaves the load buses, currents in the n rows are zero. The generator currents are denoted by the vector I_m and the generator and load voltages are represented by the vector $E_m^{'}$ and V_n, respectively. Then, Equation (4), in terms of submatrices, becomes

$$
\begin{bmatrix} 0 \\ I_m \end{bmatrix} =
\begin{bmatrix} Y_{nn} & Y_{nm} \\ Y_{nm}^{t} & Y_{mm} \end{bmatrix}
\begin{bmatrix} V_n \\ E_m^{'} \end{bmatrix}
\qquad (6)
$$

The voltage vector V_n may be eliminated by substitution as follows.

$$0 = Y_{nn}V_n + Y_{nm}E_m^{'}, \qquad (7)$$
$$I_m = Y_{nm}^{t}V_n + Y_{mm}E_m^{'}. \qquad (8)$$

From (7),

$$V_n = -Y_{nn}^{-1}Y_{nm}E_m^{'} \qquad (9)$$

Now substituting into (8), we have

$$I_m = [Y_{mm} - Y_{nm}^{t}Y_{nn}^{-1}Y_{nm}]E_m^{'} = Y_{bus}^{red}E_m^{'} \qquad (10)$$

The reduced admittance matrix is

$$Y_{bus}^{red} = Y_{mm} - Y_{nm}^{t}Y_{nn}^{-1}Y_{nm} \qquad (11)$$

The reduced bus admittance matrix has the dimensions *(m x m)*, where m is the number of generators.

The electrical power output of each machine can now be expressed in terms of the machine's internal voltages

$$S^*_{ei} = E^{'*}_i I_i,$$

Or

$$P_{ei} = Re(E^{'*}_i I_i),\qquad(12)$$

Where

$$I_i = \sum_{j=1}^{m} E'_j Y_{ij}.\qquad(13)$$

Expressing voltages and admittances in polar form, i.e., $E'_i = \left|E'_i\right|\angle\delta_i$ and $Y_{ij} = \left|Y_{ij}\right|\angle\theta_{ij}$, and substituting for I_i in (12), result in

$$P_{ei} = \sum_{j=1}^{m} \left|E'_i\right|\left|E'_j\right|\left|Y_{ij}\right|\cos(\theta_{ij}-\delta_i+\delta_j)\qquad(14)$$

The above equation is the same as the power flow equation. Prior to disturbance, there is equilibrium between the mechanical power input and the electrical power output, and we have

$$P_{mi} = \sum_{j=1}^{m}\left|E'_i\right|\left|E'_j\right|\left|Y_{ij}\right|\cos(\theta_{ij}-\delta_i+\delta_j)\qquad(15)$$

The classical transient stability study is based on the application of a three-phase fault. A solid three-phase fault at bus k in the network results in $V_k = 0$. This is simulated by removing the kth row and column from the prefault bus admittance matrix. The new bus admittance matrix is reduced by eliminating all nodes except the internal generator nodes. The generator excitation voltages during the fault and postfault modes are assumed to remain constant. The electrical power of the ith generator in terms of the new reduced bus admittance matrices are obtained from (14). The swing equation with damping neglected, for machine i becomes

$$\frac{H_i}{\pi f_0}\frac{d^2\delta_i}{dt^2} = P_{mi} - \sum_{j=1}^{m}\left|E'_i\right|\left|E'_j\right|\left|Y_{ij}\right|\cos(\theta_{ij}-\delta_i+\delta_j)$$
$$(16)$$

Where
Y_{ij} are the elements of the faulted reduced bus admittance matrix
H_i is the inertia constant of machine i expressed on the common MVA base S_B.
If H_{Gi} is the inertia constant of machine i expressed on the machine rated MVA S_{Gi}, then H_i is given by

$$H_i = \frac{S_{G_i}}{S_{cb}} H_{Gi}\qquad(17)$$

Showing the electrical power of the ith generator by P_e^f and transforming (16) into state variable mode yields

$$\frac{d\delta_i}{dt} = \Delta\omega_i,\qquad(18)$$

$$\frac{d\Delta\omega_i}{dt} = \frac{\pi f_0}{H_i}(P_{mi} - P_{ei}^f).\qquad(19)$$

In transient stability analysis problem, we have two state equations for each generator. When the fault is cleared, which may involve the removal of the faulty line, the bus admittance matrix is recomputed to reflect the change in the networks. Next the postfault reduced bus admittance matrix is evaluated and the postfault electrical power of the ith generator shown by P^{pf}_i is readily determined from (14). Using the postfault power P^{pf}_i, the simulation is continued to determine the system stability, until the plots reveal a definite trend as to stability or instability. Usually the slack generator is selected as the reference machine are plotted. Usually, the solution is carried out for two swings to show that the second swing is not greater than the first one. If the angle differences do not increase, the system is stable. If any of the angle differences increase indefinitely, the system is unstable.
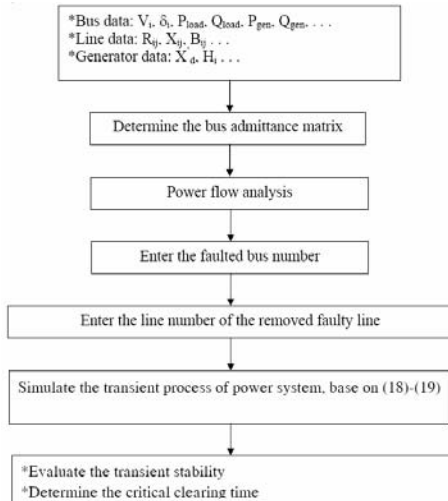
Fig.2 Flow chart of transient stability analysis
for a multimachine power system

3. SIMULATION

Based on the above procedure, a program which is written in Matlab programming language. It allows to analysis transient stability of a multimachine network subjected to a balance three-phase fault. The program prompts the user to enter the faulted bus number and the line numbers of the removed faulty line. The machine phase angles are tabulated and a plot of the swing curves is obtained. The simulation is repeated to find a critical stable state with the critical clearing time. The transient stability analysis problem is applied for the real case system, 220kV and 110 kV Northern VietNam power system with 119 buses, 18 generator units of 5 plants include HoaBinh hydro power plant, ThacBa hydro power plant, PhaLai thermal power plant, NinhBinh thermal power plant, UongBi thermal power plant, and 28 transformers.

Table 1 Analysis results summary for Northern
VietNam power system

| No.	Faulted bus number	Removed faulty line	Critical clearing time (second)	Clearing time (second)	Phase Angle Difference Characteristic	Analysis Result
1	2	2-18	0.07	0.060	Fig. 3	Stable
2	2	2-18	0.07	0.071	Fig. 4	Unstable
3	3	3-14	0.08	0.07	Fig. 5	Stable
4	3	3-14	0.08	0.09	Fig. 6	Unstable
5	12	12-24	0.33	0.3	Fig. 7	Stable
6	12	12-24	0.33	0.39	Fig. 8	Unstable
7	16	16-56	0.14	0.10	Fig. 9	Stable
8	16	16-56	0.14	0.18	Fig. 10	Unstable
9	19	19-78	0.17	0.15	Fig. 11	Stable
10	19	19-78	0.17	0.18	Fig. 12	Unstable
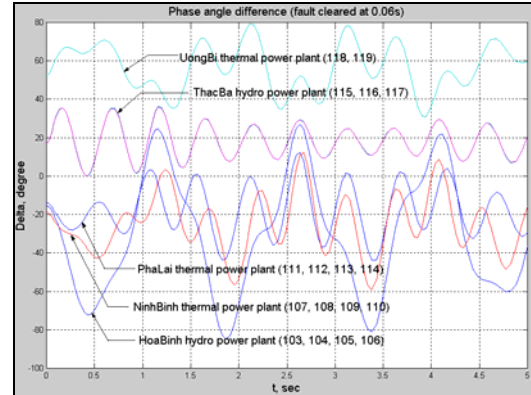
Fig.3 Three-phase fault on line 2-18
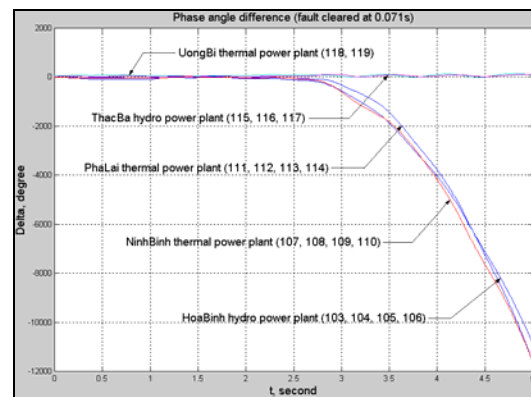(t_{CCT}=0.07s), the system is stable

Fig.4 Three-phase fault on line 2-18
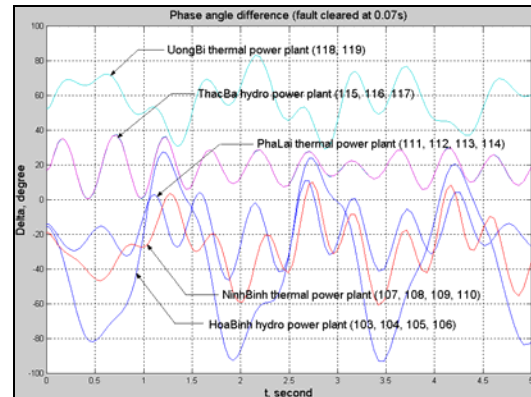(t_{CCT}=0.07s), the system is unstable

Fig.5 Three-phase fault on line 3-14
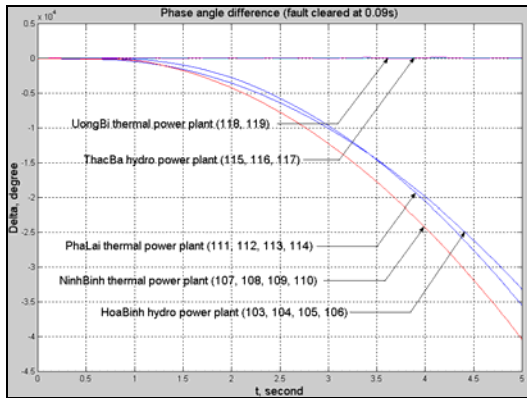(t_{CCT}=0.08s), the system is stable

Fig.6 Three-phase fault on line 3-14
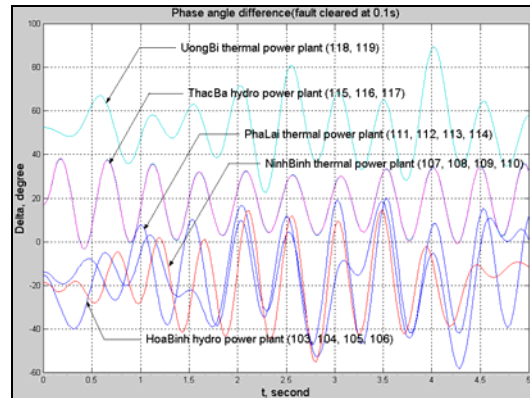(t_{CCT}=0.08s), the system is unstable

Fig.9 Three-phase fault on line 16-56
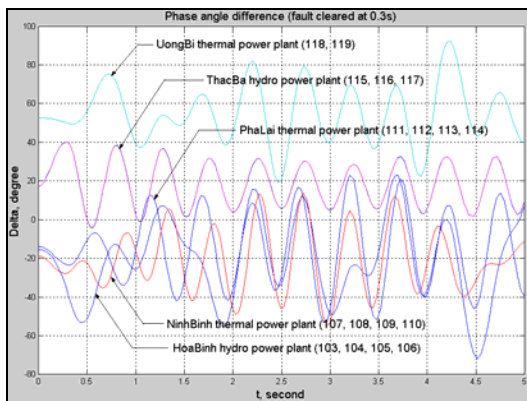(t_{CCT}=0.14s), the system is stable

Fig.7 Three-phase fault on line 12-24
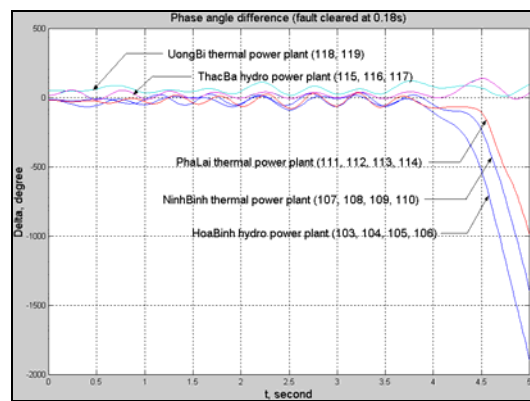(t_{CCT}=0.33s), the system is stable

Fig.10 Three-phase fault on line 16-56
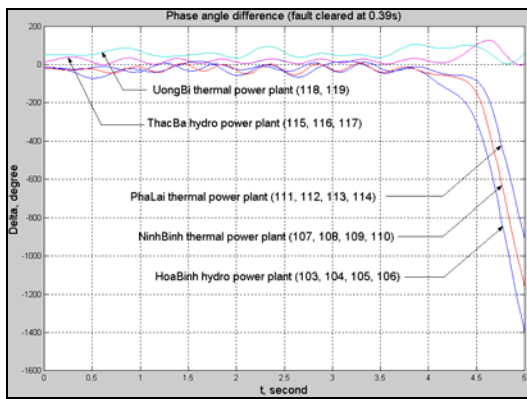(t_{CCT}=0.14s), the system is unstable

Fig.8 Three-phase fault on line 12-24
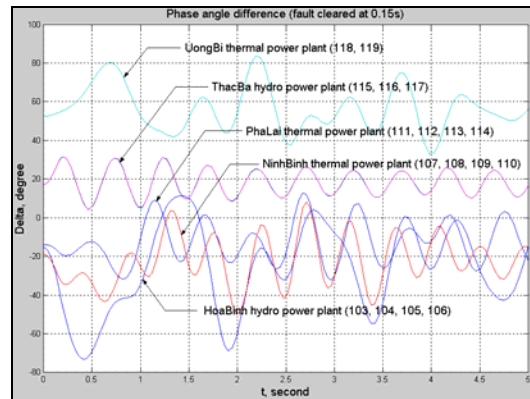(t_{CCT}=0.33s), the system is unstable

Fig.11 Three-phase fault on line 19-78
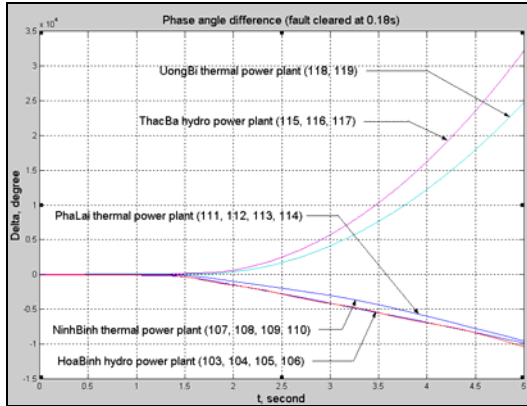(t_{CCT}=0.17s), the system is stable

Fig.12 Three-phase fault on line 19-78
(t_{CCT}=0.17s), the system is unstable

4. CONCLUSION

This analysis allows to assess that the system is stable, unstable and also allows to determine the critical clearing time of power system with three-phase faults. These results can be used effectively in planning or operation of power systems.

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