9^{th} World Mathematics Team Championship 2018

Advanced Level Individual Round 1

English Version

Instruction: This round has 15 questions (**20 minutes**).
Each question is worth 2 points.
No point penalty for submitting wrong answer.
Blank answer will be assigned 0.5 point.

1. Find the value of $\sqrt{9}+\sqrt{\left(3-\sqrt{11}\right)^2}+\sqrt[3]{\left(2-\sqrt{11}\right)^3}$.

A) $8-2\sqrt{11}$ B) 4 C) 2 D) $2\sqrt{11}-2$ E) $2\sqrt{11}$

2. Find x if $2^{x-1}\times 5^{x-1}=0.1\times 10^{2x+5}$.

A) –5 B) –4 C) –3 D) 3 E) 5

3. If $\cos\alpha=-\frac{1}{4}$ and $\pi\le\alpha\le\frac{3\pi}{2}$, find $\sin 2\alpha$.

A) $\frac{\sqrt{15}}{4}$ B) $-\frac{\sqrt{15}}{4}$ C) $-\frac{\sqrt{15}}{8}$ D) $\frac{\sqrt{15}}{8}$ E) $\frac{\sqrt{15}}{16}$

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4. In how many ways one can choose a group of leader, deputy leader and two members from a group of 10 persons?

A) 2500 B) 2400 C) 5040 D) 2520 E) 210

5. One of the sides of a parallelogram equals 9 cm and the two altitudes are 6 cm and 10 cm. Find the length of the other side.

A) 5.4 cm B) 13 cm C) 8 cm D) 7 cm E) 15cm

6. In how many ways 3 Bulgarian girls and 3 Chinese girls can sit in a bench such that the three Chinese girls are not sitting together next to each other.

A) 144 B) 720 C) 576 D) 625 E) 216

7. Find $\left(3+\sqrt{5}\right)\left(\sqrt{10}-\sqrt{2}\right)\sqrt{3-\sqrt{5}}$.

A) $\sqrt{3}-\sqrt{5}$ B) $3\sqrt{2}$ C) 5 D) 8 E) 10

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8. The number of solutions of $\left|\begin{array}{l} x+xy+y=7 \\ x^2y+xy^2=12 \end{array}\right.$ is:

A) 1 B) 2 C) 4 D) 0 E) 3

9. The unit digit of the sum of all 3 digit numbers having only odd digits equals:

A) 1 B) 2 C) 3 D) 4 E) 5

10. For what values of *a* and *c* the range of the function $y=ax^2+c$ is the interval $\left[-\frac{1}{8},\infty\right)$?

A) $\left|\begin{array}{l} a=-\frac{1}{8} \\ c>0 \end{array}\right.$ B) $\left|\begin{array}{l} a>0 \\ c=-\frac{1}{8} \end{array}\right.$ C) $\left|\begin{array}{l} a<0 \\ c=-\frac{1}{8} \end{array}\right.$ D) $\left|\begin{array}{l} a=-\frac{1}{8} \\ c=0 \end{array}\right.$ E) $\left|\begin{array}{l} a=0 \\ c=-\frac{1}{8} \end{array}\right.$

11. A rectangle *ABCD* and a point *M* are such that $MA=2\sqrt{5}$, $MB=2$ and $MC=3$ find *MD*.

A) $\sqrt{5}$ B) 5 C) $2\sqrt{5}+1$ D) $2\sqrt{5}-1$ E) $2\sqrt{6}$

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12. If $\left(1+i\sqrt{3}\right)^{21}=a+bi$ for a and b real numbers find $\log_2 a^2$.

A) 42 B) 21 C) 48 D) 63 E) –48

13. The solution of $\left(x^2-5x+4\right)\sqrt{x-3}\le 0$ is:

A) [3,5] B) [3,4] C) [1,4] D) $[3,\infty)$ E) (3,4)

14. If $a=\log_{10}25$ find $\log_2 \sqrt[3]{625}$.

A) $\dfrac{4a}{3(2-a)}$ B) $\dfrac{4a}{3(a+2)}$ C) $\dfrac{3a}{2(3-2a)}$ D) $\dfrac{6a}{3a+4}$ E) $\dfrac{6a}{3a-4}$

15. Solve $\dfrac{x}{x^2+2}\sqrt{1+\dfrac{x^4+4}{4x^2}}=x+1$.

A) $-\dfrac{1}{2}$ B) $\dfrac{3}{2}$ C) $\dfrac{1}{2}$ D) $-\dfrac{3}{2}$ E) –2

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Advanced Level Individual Round 2

English Version

Instructions: This round has 8 questions (**40 minutes**).

Question numbers 1, 2, 3, 4, 5 and 6 are worth 4 points each.

Question numbers 7 and 8 are worth 8 points each.

No point penalty for submitting wrong answers.

1. For a positive integer n denote by $d(n)$ the number of all positive divisors of n. If $d(2n) = 28$ and $d(3n)=30$ find $d(6n)$.

2. Solve the equation $\left(\frac{1}{3}\right)^{\log_{\frac{1}{2}}\left(x^2-2x+5\right)} = 9-|x-1|$.

3. Point M is the midpoint of the side CD of a square $ABCD$. Point P on the diagonal AC is such that $\angle BPM = 90^\circ$. Find the ratio $\frac{PC}{PA}$.

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4. Compute $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\ldots\ldots}}}}$

5. The diagonals AC and BD of quadrilateral $ABCD$ are perpendicular.
If $AB = 60$ cm, $BC = 52$ cm and $CD = 25$ cm, find AD.

6. Find the number of 5-digit positive integers with sum of their digits equals to 22 and without 0 in their decimal representation.

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7. To prepare for WMTC a student solved n problems during 31 day period. It is known that:

- From day 1 to day 14 inclusive he solved in average 5 problems per day
- From day 6 to day 25 inclusive he solved in average 4 problems per day
- From day 12 to day 31 inclusive he solved in average 3 problems per day

Find the number of different values of n for which this is possible.

8. Find the number of ways 5 indistinguishable black balls and 5 indistinguishable white balls can be placed on a circle.

If an arrangement is obtained from another one by rotation, the two arrangements are considered to be the same.

Advanced Level Relay Round 1

R1-A 20 lines in the plane are such that no two are parallel and no three are concurrent. Find the number of regions the plane is divided by these lines.

R1-B T = TNYWR (The Number You Will Receive). Let a, b and c be positive integers such that $ab + bc + ca$ attains its maximum value when $a < b < c$ and $a + b + c = T$. Find b.

Advanced Level Relay Round 2

R2-A A five-digit number without digit 0 is called nice if it coincides with any of the numbers 12345 and 23456 in exactly two positions (for example 12457 is nice but 12456 is not). Determine the number of nice numbers.

R2-B T = TNYWR (The Number You Will Receive). Points C and D lie on a circle of center O and diameter 700 cm. If $CD = T$ cm find the distance from the point C to the line OD.

Advanced Level Relay Round 3

R3-A Consider the sequence $a_1 = 3$, $a_2 = 2$ and $\frac{2}{a_n} = \frac{1}{a_{n-1}} + \frac{1}{a_{n+1}}$ for $n \geq 2$. If $a_{2018} = \frac{p}{q}$ where p and q are relatively prime find q.

R3-B T = TNYWR (The Number You Will Receive). A word has length w and is composed by three letters only – a, b and c. The word is called *good* if it is possible to replace these letters by 1, 2 and 3 in some order such that the number obtained is divisible by 3. For how many values of w in the interval [T, 2018] all words of length w are good?

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Advanced Level Team Round

English Version

Instruction: This round has 14 questions (**40 minutes**).
Each question is worth 5 points.
No point penalty for submitting wrong answer.

1. Let a be the least integer for which the equation $x^3 + ax + 2a + 15 = 0$ has rational root. Find a^2.

2. Let a and b be real numbers such that the equation $\left|\left|x^2 - 1\right| - b\right| = a^2 + 1$ has exactly 5 distinct real roots. Find the least value of $2a + |b| + 49$.

3. If $a = \sqrt[3]{7} + \sqrt[3]{7 + 5\sqrt{2}}$ and $b = \sqrt[3]{7} - \sqrt[3]{7 - 5\sqrt{2}}$ find the value of $a^2 + b^2 - 2ab + 3$.

4. The diameter of a football (soccer) ball is $40\left(3 - \sqrt{6}\right)$ cm. Three football balls lay on the ground and each one touches the other two. Another football ball is put on the top of given balls such that it touches each of the three balls. Find the distance from the ground to the highest point of the given "pyramid" of four balls.

5. Let A be a set of positive integers such that for any two distinct x and y from A we have: $\left|\frac{1}{x} - \frac{1}{y}\right| \geq \frac{1}{25}$. Find the maximum possible number of elements of A.

6. All 10 digit numbers with distinct digits are written one after another in increasing order. Find the 1198-th digit in the sequence obtained.

7. Find the number of ordered pairs (a,b) of real numbers such that such that:

$$x^2 + ax + b \geq ax^2 + bx + 1 \geq bx^2 + x + a \qquad \text{for all real numbers } x.$$

8. For every nonempty subset A of the set $\{1, 2, 3,\ldots, 99, 105\}$ denote by $P(A)$ the product of elements of A. If S is the sum of all such products find the largest prime divisor of $S + 1$.

9. All ordered triples of positive integers (a, b, c) are written one after another by the following rules:

1. If $a_1b_1c_1 < a_2b_2c_2$ then (a_1, b_1, c_1) is before (a_2, b_2, c_2).
2. If $a_1b_1c_1 = a_2b_2c_2$ and $a_1 < a_2$ then (a_1, b_1, c_1) is before (a_2, b_2, c_2).
3. If $a_1b_1c_1 = a_2b_2c_2$ and $a_1 = a_2$ and $b_1 < b_2$ then (a_1, b_1, c_1) is before (a_2, b_2, c_2).

How many triples are there between (1,1,900) and (900,1,1) inclusive?

***T* = the answer of problem #8**

10. In a triangle ABC with $\angle ACB = T^\circ$ points P and Q on the side AB are such that $AP = BC$ and $BQ = AC$. If M, N and K are the midpoints of AB, CP and CQ respectively find $2\angle NMK$ in degrees.

***T* = the answer of problem #6**

11. Equilateral triangle of side length T is divided into equilateral triangles of side length 1 by lines parallel to its sides. Find the number of parallelograms bounded by the segments of the grid.

***T* = the answer of problem #7**

12. Find the greatest value of xy where x and y are integers such that

$$(T + 6)(x - y) + xy = 8.$$

***T* = the answer of problem #2**

13. A person climbs an escalator that is moving in the same direction. From the moment he steps on the escalator to the moment he reaches the end of the escalator he climbs T steps. When climbing two times faster from the moment he steps on the escalator to the moment he reaches the end of the escalator he climbs T+10 steps. Find the number of steps the escalator in rest has.

***S* = the answer of problem #3**

***T* = the answer of problem #5**

14. Two circles k_1 and k_2 of radii S and T, respectively, are externally tangent. If AB and CD are common tangents to k_1 and k_2 ($A, C \in k_1$ and $B, D \in k_2$) find $(AC + BD)^2$.

Advanced Level Team Round