Frequency-Domain Finite Element Methods for Electromagnetic Field Simulation: Fundamentals, State of the Art, and Applications to EMI/EMC Analysis

Andreas C. Cangellaris
Center for Electronic Packaging Research, ECE Department
University of Arizona, Tucson, AZ 85721, U.S.A.

Abstract—**This paper provides a critical review of frequecy-domain finite element methods and their applications to the modeling of electromagnetic interactions in complex electronic components and systems. Emphasis is placed on latest advances in finite element grid generation practices, element interpolation function selection, and robust, highly absorbing numerical grid truncation techniques for modeling electromagnetic interactions in unbounded domains. These advances have helped enhance the robustness and accuracy of the method. Finally, the advantages of domain decomposition techniques for the modeling of complex geometries are examined. Such domain decomposition techniques are expected to play an important role in the continuing effort to extend the applications of frequency-domain finite methods beyond the subcomponent-level to component and system modeling for electromagnetic interference and electromagnetic compatibility analysis and design.**

I. Introduction

There are two attributes of the method of finite elements that have prompted the rapid growth of its application to the modeling of electromagnetic interactions in electronic systems. One of them is its superior modeling versatility where structures of arbitrary shape and composition can be modelled as precisely as the desirable model complexity and available computer resources dictate. The second, is common to all differential equation-based numerical methods, and has to do with the fact that the matrix resulting from the discretization of the governing equations is very sparse, which implies savings in computer memory for its storage as well as in CPU time for its inversion. Clearly, these two attributes come at the expense of an increase in the degrees of freedom used in the numerical approximation of the problem since now, contrary to integral equation methods, the entire space surrounding all sources of electromagnetic fields needs be incorporated in the numerical model. Nevertheless, because of the sparsity of the resulting matrix and the simplicity with which complex geometries can be modeled, this increase in the degrees of freedom of the approximation is an acceptable penalty.

Over the past ten years, a significant volume of literature has been generated on the application of the finite element method to a variety of electromagnetic scattering and radiation problems. The book by J.M. Jin [1] serves both as a tutorial on the procedures for the application of the finite element method to the approximation and solution of electromagnetic boundary value problems, and as a rather thorough survey of the classes of problems that have been tackled. Considering the power of the aforementioned attributes, one would have expected that the method of finite elements would have gained in popularity among EMC/EMI engineers and would have established itself as the method of choice in the analysis and prediction of EMI and the design of electromagnetically compatible systems. Nevertheless, a literature review indicates that this is not the case. As an example we mention that in a special issue of the *IEEE Transactions on Electromagnetic Compatibility*, dedicated to computational methods for EMI/EMC analysis, very few papers on finite elements appeared, and the applications presented where limited to rather simple problems of low complexity [2]-[3].

Before one attempts to search for drawbacks in the method of finite elements that have prevented its proliferation as an EMI/EMC prediction tool, one has to keep in mind that application of electromagnetic CAD for component and system EMI/EMC is actually still at its infancy. The reason for this is that the complexity of an integrated electronic component, subsystem or system is such that accurate modeling of source, coupling mechanism, and receiver of electromagnetic noise is almost prohibitive using a single numerical method for solving Maxwell's equations, irrespective of the type of the method used. More specifically, considering the tremendous variation in feature size from chip, to package, to board, to cables, to shields, it becomes clear that the number of elements required for the discretization of such a system for finite element analysis of electromagnetic interactions is out of the reach of today's most powerful supercomputers.

In view of the above and recognizing that an electromagnetic analysis tool will be useful as a CAD tool only if simulation times are in the order of minutes or at most a few hours, this paper examines the latest advances in the method of finite elements that are expected to help the method establish itself as a reliable candidate for EMI/EMC problem solving either at the component level or in conjunction with reduced-order electromagnetic

0-7803-3207-5/96/$5.00

models of subsystems. As a matter of fact, it is this area where the method of finite elements can have an important impact. Indeed, current practices of EMI/EMC analysis concentrate on rather simplistic, individual source-to-victim models, which often suffer from their inability to capture the impact of surrounding conducting, dielectric, and magnetic material topology on the electromagnetic interaction. The finite element method allows for the development of a more precise model that will lead to higher accuracy in noise prediction and thus facilitate the design of electromagnetically compatible electronic modules.

Finally, the potential of domain decomposition methods for reducing the complexity of the original problem will be examined. The basic idea behind such methods is the partitioning of the domain of interest into smaller ones and the development of the solution in a piecewise manner, one subdomain at a time, using different types of both numerical and analytic techniques. The inherent parallelism of such approaches combined with the smaller size of the subdomains makes them extremely well-suited for massively-parallel computation.

II. Mathematical Framework for Finite Element Analysis

The focus of this paper is on the numerical approximations of Maxwell's equations with time-harmonic field variation. Therefore, the following discussion pertains to linear sources and materials. However, time-domain finite methods that can handle transient electromagnetic interactions in the presence of nonlinear sources and nonlinear media are possible and are currently the topic of vigorous research within the computational electromagnetics community. As a matter of fact, the finite-element formulation in [2] is such that both transient and time-harmonic electromagnetic simulations can be effected within a single mathematical framework.

In order to review the basic steps involved in the finite element approximation of electromagnetic boundary-value problems, let us consider the double-curl equation for the electric field, $\mathbf{E}$, which, in a source-free, isotropic and linear medium with position-dependent magnetic and electric properties has the form

$$\nabla \times \left(\frac{1}{j\omega\mu} \nabla \times \mathbf{E} \right) + j\omega\hat{\epsilon}\mathbf{E} = 0. \tag{1}$$

The time dependence $\exp(j\omega t)$ is assumed ($j = \sqrt{-1}$), and the complex permittivity, $\hat{\epsilon} = \epsilon - j\sigma/\omega$, is used to account for any conduction and/or dielectric losses in the medium.

For the purposes of finite element solutions, a weak form of (1) is required. For node based finite element expansions the unknown vector field is approximated in terms of scalar basis functions, ϕ_i,

$$\mathbf{E} = \sum_i \mathbf{E_i} \phi_i, \tag{2}$$

where $\mathbf{E_i}$ denotes the unknown vector field value at node i. The relevant weak form, in the spirit of Galerkin's approximation, is

$$\left\langle \left(\frac{1}{j\omega\mu} \nabla \times \mathbf{E} \right) \times \nabla\phi_i \right\rangle + \left\langle j\omega\hat{\epsilon}\mathbf{E}\phi_i \right\rangle = - \oint \frac{1}{j\omega\mu} \hat{\mathbf{n}} \times (\nabla \times \mathbf{E}) \phi_i ds \tag{3}$$

where $\langle\ \rangle$ and $\oint$ indicate integration over the domain of interest and its boundary, respectively, while $\hat{\mathbf{n}}$ is the outward unit normal on the boundary. For edge element expansions, vector basis functions, $\mathbf{N_i}$, are used for the expansion of the field,

$$\mathbf{E} = \sum_i E_i \mathbf{N_i}, \tag{4}$$

where E_i are the unknown coefficients in the expansion. The relevant weak form is

$$\left\langle \left(\frac{1}{j\omega\mu} \nabla \times \mathbf{E} \right) \cdot (\nabla \times \mathbf{N_i}) \right\rangle + \left\langle j\omega\hat{\epsilon}\mathbf{E} \cdot \mathbf{N_i} \right\rangle = - \oint \frac{1}{j\omega\mu} \hat{\mathbf{n}} \times (\nabla \times \mathbf{E}) \cdot \mathbf{N_i} ds \tag{5}$$

For two-dimensional problems, a scalar version of (3) is readily obtained. More specifically, for a transverse magnetic to z (TM_z) polarization, the fields, $\mathbf{E} = \hat{\mathbf{z}}E$, $\mathbf{H} = \hat{\mathbf{x}}H_x + \hat{\mathbf{y}}H_y$, are independent of z and (3) reduces to

$$\left\langle \left(\frac{1}{j\omega\mu} \nabla_{\mathbf{xy}} E \right) \cdot \nabla_{\mathbf{xy}} \phi_i \right\rangle + \left\langle j\omega\hat{\epsilon}E\phi_i \right\rangle = \oint \frac{1}{j\omega\mu} (\hat{\mathbf{n}} \cdot \nabla E) \phi_i dl \tag{6}$$

where $\nabla_{\mathbf{xy}} = \hat{\mathbf{x}}\partial/\partial x + \hat{\mathbf{y}}\partial/\partial y$. For transverse electric to z (TE_z) polarization of the two-dimensional fields, where $\mathbf{H} = \hat{\mathbf{z}}H$, $\mathbf{E} = \hat{\mathbf{x}}E_x + \hat{\mathbf{y}}E_y$, the weak form is easily found from (6) by duality.

For static problems ($\omega = 0$), a scalar potential, Φ, is often introduced, and the electric or magnetic fields are obtained in terms of the gradient of the potential. For such cases, it is a weak statement of Laplace's equation for the scalar potential that is being derived.

The most well-known attribute of Galerkin's method, where the solution is sought in a finite-dimensional subspace of the class of admissible functions for the problem of interest using the same set of functions as trial and test functions, is the symmetry of the resulting stiffness matrix given a symmetric weak formulation. However, another important merit of Galerkin's method is that, if a symmetric weak formulation is used, Galerkin's approximate

solution exactly conserves energy in the electromagnetic field despite the fact that it satisfies the vector Helmholtz equation only approximately over the domain of interest. This is easily shown starting from (5) and using the complex conjugate of the field, $\mathbf{E}^*$, as the test function and Faraday's law to introduce the magnetic field in the boundary integral

$$\left\langle \left(\frac{1}{j\omega\mu} \nabla \times \mathbf{E} \right) \cdot (\nabla \times \mathbf{E}^*) \right\rangle + \left\langle j\omega\hat{\epsilon}\mathbf{E} \cdot \mathbf{E}^* \right\rangle = \oint (\hat{\mathbf{n}} \times \mathbf{H}) \cdot \mathbf{E}^* ds \qquad (7)$$

The complex conjugation of (7), use of Faraday's law to simplify the integrand in the first term on the left, and use of the constitutive relations $\mathbf{B} = \mu\mathbf{H}$, $\mathbf{D} = \hat{\epsilon}\mathbf{E}$, result in the following equation

$$\langle -j\omega\mathbf{B} \cdot \mathbf{H}^* \rangle + \langle j\omega\mathbf{D}^* \cdot \mathbf{E} \rangle = \oint (\mathbf{E} \times \mathbf{H}^*) \cdot \hat{\mathbf{n}} ds \qquad (8)$$

Clearly, the resulting expression is Poynting's theorem for time-harmonic fields. Thus energy conservation in the electromagnetic field is satisfied exactly by the approximate solution.

The importance of the aforementioned result is rather significant. Considering the various potential sources of error in the development of a numerical solution to a boundary value problem, it is definitely advantageous to be able to work with a weak statement that is consistent with the correct physics of the field we are attempting to calculate. For the electromagnetic field problems of interest, the calculated field quantities will be acceptable only if they satisfy both energy conservation and electric charge conservation. As a matter of fact, the latter has been found to be extremely important in the finite element solution of three-dimensional vector electromagnetic problems. In later sections, it is pointed out that modifications to the weak statement in (3) and careful selection of the vector basis functions in (5) are needed to prevent the contamination of the numerical solution from spurious fields caused by the lack of enforcement of charge conservation in the original weak statement.

In order to illustrate the development of the numerical approximation of the electromagnetic boundary value problem, let us consider the weak statement in (5). Substitution of (4) into (5) and testing with each and every one of the vector basis functions $\mathbf{N_j}$ results in a linear system of simultaneous equations

$$\sum_{i=1}^{M} A_{ij} E_i = f_j, \quad j = 1, 2, \ldots, M \qquad (9)$$

where the elements, A_{ij}, of the so-called stiffness matrix are given by

$$A_{ij} = \left\langle \left(\frac{1}{j\omega\mu} \nabla \times \mathbf{N_i} \right) \cdot \left(\nabla \times \mathbf{N_j} \right) \right\rangle + \left\langle j\omega\hat{\epsilon}\mathbf{N_i} \cdot \mathbf{N_j} \right\rangle \qquad (10)$$

while the elements of the forcing vector are

$$f_j = -\oint \frac{1}{j\omega\mu} \hat{\mathbf{n}} \times (\nabla \times \mathbf{E}) \cdot \mathbf{N_j} ds \qquad (11)$$

M is the number of degrees of freedom in the approximation. One of the important attributes of the method of finite elements is that that the basis functions used have local support, i.e. they are non-zero only over a set of adjacent elements. This is what leads to the sparsity of the resulting system since most of the elements A_{ij} are zero. The forcing vector is formed by contributions of the surface integrals over the domain boundaries. From the uniqueness theorem, the tangential component of the magnetic field, $-(1/j\omega\mu)\hat{\mathbf{n}} \times (\nabla \times \mathbf{E})$, on the domain boundaries is all that is needed for a unique solution of Maxwell's equations inside the domain. This tangential magnetic field on the domain boundary is used to account for all sources exterior to the domain of interest. The surface term in (5) is used also for enforcing tangential magnetic field continuity conditions at material interfaces. As far as tangential electric field boundary conditions are concerned, they are taken into account in the construction of the basis functions. This is discussed in more detail in Section 4.

III. Grid Generation

Numerical grid generation is probably the most critical step in a finite element analysis of electromagnetic wave interactions. During the early stages of the application of the finite element method to modeling of electromagnetic interactions, the emphasis was on mathematical model and weak statement formulations and their subsequent use in the analysis of propagation, radiation and scattering problems in conjunction with rather simple geometries. Consequently, the important issue of automatic generation of finite element grids appropriate for electromagnetic propagation and scattering problems received rather limited attention. Apparently, the assumption was that grid generation practices used in low-frequency electromagnetic field modeling and/or other areas of engineeering in which the method of finite elements was already being used, could be adopted without significant alterations. This turned out to be a rather false assumption, simply because the specific physics of the phenomenon that is being analyzed needs be taken into account before a discrete model is built for its quantification. The famous *sampling theorem* of modern communication theory for the sampling of a band-limited signal serves as a simple, yet powerful example.

Extensive research over the past few years has led to the understanding and quantification of the impact of the discrete approximations of wave equations on the accuracy and quality of the simulated wave solutions [5]-[8]. The findings from such analysis are now being used as

guidelines and/or constraints in the development of procedures for automatic grid generation. The following is a list of undebatable results that are used to guide the construction of a finite element grid for the modeling of electromagnetic wave propagation, radiation and scattering problems.

First and foremost, the grid must provide for adequate wavelength resolution in order to minimize numerical dispersion (i.e., artificial, discretization induced dependence of phase velocity on electrical grid size) and thus control phase error. The number of degrees of freedom per wavelength, required to achieve a fixed accuracy, is dependent on the degree of the interpolating polynomials used in the constyruction of the basis functions, the electrical size of the domain (i.e., the size of the domain in wavelengths), and the boundary conditions used for grid truncation. Furthermore, problem geometry and material properties impact the way the finite element grid is constructed. Electromagnetic fields exhibit singular behavior at material wedges and corners (especially metallic), which cannot be resolved properly by the polynomial interpolating functions used for the finite element approximation. Thus, grid refinement or use of special elements are required in the vicinity of such geometric features to improve solution accuracy. In addition, abrupt changes in the electric permittivity and the magnetic permeability of materials need be accompanied by appropriate grid size changes in order to both prevent the occurrence of (nonphysical) numerical reflections and maintain the conditioning of the finite element matrix. Finally, the grid generation process should be such that element degeneracy (i.e. triangles degenerating toward line segments, quadrilaterals approaching triangles, or tetrahedrons degenerating to quadrilaterals) never occurs. Such elements lead to ill-conditioning and may eventually render the finite element solution useless.

While it is often tempting to adopt the philosophy that the finer the grid the better the quality of the solution, one needs to remember that an excessively fine mesh (unless needed) wastes computational resources and thus it should be avoided. Consequently, it is important that the grid generation process is such that adaptive mesh refinement is possible. In other words, for the purposes of computation efficiency, the analysis begins with the construction of an initial mesh the density of which has been decided on the basis a specific set of constraints perinent to the material properties, the characteristics of the sources and anticipated spatial variation of the electromagnetic fields. A simulation is attempted using this mesh. Next, an error estimator is used to examine the quality of the solution over the entire grid and thus identify areas where mesh refinement is needed to improve solution accuracy. Such mesh refinement is then effected using appropriate mesh refinement schemes that maintain element quality and preserve mesh conformity to material boundaries. A new solution is then attempted with the refined grid, and the process is repeated until the desirable solution accuracy has been achieved.

While the manipulation of hexahedral meshes, i.e., meshes that use the so-called brick elements, is much simpler than that of a tetrahedral mesh, tetrahedral meshes are the ones that have been adopted for automatic grid generation. (In two dimensions, brick elements reduce to quadrilaterals while tetrahedra reduce to triangles.) The reason for this is that triangles and tetrahedra are simplices in two and three dimensions, respectively. Complete polynomial expansion functions are defined easily on simplex elements. Furthermore, an arbitrary domain, no matter how complicated it might be, can always be discretized automatically into a set of simplex elements using Delaunay tesselation [9]. As a matter of fact, grid generation software based on the Delaunay algorithm and its variants is now commercially available and is being used for finite element grid generation with fairly satisfactory results. Consequently, most of the recent advances on automatic mesh generation and mesh refinement are based on Delaunay triangulation.

With regards to automatic mesh generation, new procedures have been developed for preventing the occurence of degenerate (sliver) elements, for the redistribution of existing nodes and/or the efficient incorporation of new nodes for improving the quality of the tetrahedra (i.e. making them as equilateral as possible), and for preserving prescribed material boundaries during the process of initial mesh generation as well as during subsequent mesh refinement [10],[11].

As far as mesh refinement is concerned, a variety of procedures are currently available. At this point, it is important to point out that in addition to the so-called *h refinement*, where new nodes are introduced in the grid while the order of the polynomial approximation remains the same, solution accuracy can be improved by increasing the order of the polynomial interpolation while keeping the number of nodes the same. This so called *p refinement* tends to be less popular than *h refinement*, mainly due to the desire of maintaining low interpolation function complexity within regions of high material complexity. However, one should keep in mind that *h refinement* cannot be pushed to arbitrarily small element sizes. The reason for this is that as the mesh is progressively refined the eigenvalue spectrum of the stiffness matrix widens; hence the system becomes progressively more ill-conditioned [12]. This progressive ill-conditioning of the stiffness matrix as the mesh is refined results in an increase in the round-off error which counteracts the decrease in approximation error. At some point, approximation error and round-off error become equal, and any further mesh refinement will result in deterioration in the accuracy of the solution. As pointed out in [12], the round-off error does not depend strongly on the degree of the interpolating polynomials.

It depends mainly on element size, h, and, for the second-order problems of interest in electromagnetic modeling, the dependence is of the form $O(h^{-2})$. Consequently, the way to maintain accuracy and postpone the deleterious effect of round-off is to increase the order of the interpolating polynomials (*p refinement*). The numerical experiments in [13] support this conjecture in a very illustrative manner.

We conclude this section by pointing out that before adaptive mesh refinement can be effected, a reliable *a posteriori* error estimate to be used as a refinement criterion is required. A variety of error estimators have been proposed over the past few years [14]-[17]. As expected, they depend on the physical attributes of the fields as well as the choice of functions and interpolating elements used in the finite element approximation.

IV. The Choice of Elements

Most of the original applications of the finite element method to vectorial electromagnetic field modeling in three dimensions were based on the so-called nodal elements, using field representations of the form shown in (2) and weak statements such as the one in (3). In other words, the degrees of freedom were defined to be the three components of the unknown fiield quantity at the element nodes. It was soon found that such approximations were plagued by the occurrence of non-physical, spurious modes [18]. These spurious solutions manifested themselves as modes with nonzero divergence, and were caused by the inability of the aforementioned choice of interpolation and weak statement to enforce Gauss' law for divergence-free solutions. Consequently, a variety of approaches were proposed for eliminating these spurious modes within the context of nodal elements [19]-[21].

A different approach to eliminating spurious modes that has gained popularity over the past few years is the use of the so-called edge elements [22]. Edge elements use vector basis functions of the form shown in (4). They have the unique property that the degrees of freedom associated with these elements are the circulations of the vector field along the edges of the mesh. The important implication of this fact is that edge elements impose the continuity of only the tangential components of the unknown electric (or magnetic) field across element boundaries, which is obviously consistent with the physical constraints for electromagnetic field solution uniqueness inside a domain. Additional advantages of edge elements are: a) Boundary conditions at material interfaces are automatically enforced through the natural boundary condition in the weak statement (see the surface integral term in (5)). b) Dirichlet boundary conditions are easily imposed along element edges. This second property becomes extremely useful when dealing with conducting boundaries, especially those that exhibit wedges and corners.

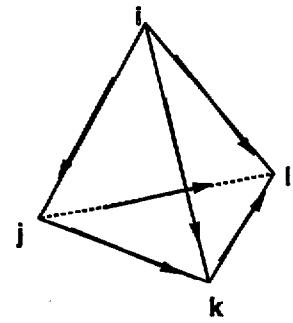

Fig. 1. Whitney 1-form edge element.

The most popular version of these edge elements is the so-called Whitney 1-form. It was long before the method of finite elements was becoming a popular tool in boundary value problem solving that Whitney described a family of polynomial forms on a simplicial mesh with special properties that made them attractive for electromagnetic field representations [23]. These polynomials are of, at most, degree one on tetrahedra. Any two p-forms are said to conform on a surface if they take the same values at any given set of p vectors tangent to the surface. Finally, p-forms are uniquely determined by integrals on p-simplices. Let us consider, for example, the popular Whitney 1-forms (Fig. 1). They are associated with mesh edges. Each edge in the tetrahedral mesh contributes an independent basis function. In other words, the degrees of freedom of the approximation are associated with the element edges; this is the reason they are called "edge elements." For an edge $e = \{i, j\}$ connecting vertices i and j the basis function is given by

$$\mathbf{N_e} = \mathbf{W_e} = \zeta_i \nabla \zeta_j - \zeta_j \nabla \zeta_i \tag{12}$$

where ζ_i $(i = 1, 2, 3, 4)$ are the bary-centric (or simplex) coordinates of the tetrahedron. It is easily shown that the circulation of $\mathbf{W_e}$ is 1 along edge e and 0 along all other edges. With the unknown field quantity interpolated over a tetrahedron as

$$\mathbf{E} = \sum_{e=1}^{6} E_e \mathbf{W_e} \tag{13}$$

the aforementioned property implies that the degrees of freedom of the approximation are the circulations of the field along the edges of the tetrahedron. Two easily obtained properties are the continuity of tangential components across facets (element interfaces) and the zero divergence of the basis functions. In view of this last property, it is clear that by using edge elements the requirement for divergence-free numerical solutions is built in the basis functions.

Despite their simplicity and their suitability for electromagnetic field representation, the Whitney 1-forms have

the drawback that, being first-order polynomials, the interpolation error in the approximated field is only first order. Consequently, a very fine wavelength resolution is required for controling numerical dispersion, which translates to large number of unknowns. To circumvent this difficulty, higher-order vector finite elements have been proposed by several authors, starting with the pioneering work of Nedelec [24]. These elements are called tangential vector finite elements, and continue to be a topic of extensive research [25]-[28]. However, it should be mentioned that the number of degrees of freedom associated with these high-order elements on tetrahedra increases rapidly with the order of the element, and this is an issue that needs be weighted properly when trading-off computation complexity and computation efficiency with solution accuracy.

However, as mentioned earlier, use of higher-order elements reduces the dimensionality of the approximation, improves the conditioning of the finite element system, and leads to increased accuracy in the numerical differentiation of the finite element solution for post-processing purposes. Since higher-order tangential vector elements are much more expensive than nodal elements, hybrid approaches have been proposed as an alternative. The basic idea is to use nodal elements over those regions that involve either homogeneous media or media with smooth variations in their electromagnetic properties. Edge elements, on the other hand, are used at material interfaces as well as in the vicinity of conducting wedges, corners and, in general, points where several material interfaces meet and thus the definition of a vector normal is not unique. Mur has shown that the use of such combinations of edge and nodal elements in electric or magnetic field-based finite element formulations yields optimum computational results [29]. In such mixed-element formulations, it is important that the divergence-free character of the fields and the continuity of the flux across interelement boundaries are imposed explicitly in a weighted sense. This is necessary for the elimination of spurious modes, and helps increase solution accuracy and improve convergence [30]. Finally, Boyse and Seidel have used such a combination of edge and nodal elements in conjunction with the finite element approximation of a scalar and vector potential formulation of Maxwell's equations [31].

V. Grid Truncation for Unbounded Problems

An important class of problems in EMI/EMC analysis deals with noise radiated out of a component or system or radiated noise interacting with a system with part of it getting scattered and part coupled to the system components. In order to model such electromagnetic radiation and scattering problems using finite methods, the domain of computation needs to be truncated by a (non-physical) mathematical boundary that encloses all radiation sources and/or scatterers. On this mathematical boundary an appropriate operator has to be used to effect the reflectionless transmission of radiation out of the domain. Such an operator can be effected using an integral equation statement of Huygen's principle. A variety of such *global* grid truncation operators have been proposed [1]. Such global operators have the undesirable property that their numerical approximation results in a fully populated submatrix which, unless managed properly, penalizes the sparsity of the matrix resulting from the finite element approximation inside the domain.

To overcome this difficulty, significant effort was devoted over the past few years on the development of local, differential equation-based operators for grid truncation [32]. Such local operators, often called absorbing boundary conditions (ABCs), are approximate and thus their performance is inferior to the one achieved by the global operators. However, they help retain the sparsity of the stiffness matrix.

Despite the successful development and application of such local operators for the truncation of finite element grids with either nodal or edge elements, the reflection levels resulting from their applications have not been as small as needed for applications relevant to EMI/EMC problems. For example, it is well-known that common-mode currents resulting from imbalances in interconnects and improperly designed current return paths are significant contributors to radiated emissions from boards, despite their very small amplitudes. If the absorbing boundary condition causes an appreciable level of reflection, the spurious reflected fields may alter the common-mode currents in the interconnects in the system and thus lead to totally erroneous calculations of radiated emission levels. Clearly, the availability of an almost reflectionless grid truncation methodology that maintains the sparsity of the stiffness matrix is highly desirable for EMI/EMC finite element modeling.

It was only three years ago that a breakthrough was achieved by Berenger in this area of numerical grid truncation [33]. His aproach was based on the numerical construction of anisotropic absorbing media with the property that the interface between such a medium and a homogeneous space is (theoretically) reflectionless for all frequencies and all angles of incidence (except at grazing). These absorbers were called *perfectly matched layers* PMLs. Even though Berenger's implementation of such PMLs involved a peculiar splitting if the field components so that the PMLs could be applied in conjunction with transient electromagnetic simulations using the finite difference time domain method, several researchers explored and continue to explore alternative implementations of PMLs that are more suitable for finite element applications [34]-[37]. As clearly illustrated in [36], such PMLs may be effected using a properly constructed anisotropic medium. To illustrate the properties of such

a medium, consider the case of a planar interface parallel to the $x-y$ plane in a cartesian coordinate system. Let the medium on the left be a homogeneous, isotropic medium with constant permittivity ϵ and constant permeability μ. To effect a reflectionless interface the permittivity and permeability tensors of the medium on the right need be defined as follows, $[\epsilon]/\epsilon = \text{diag}\{a, a, a^{-1}\}$, and $[\mu]/\mu = \text{diag}\{a, a, a^{-1}\}$. Furthermore, to effect absorption of the transmitted wave as it propagates inside the PML, we choose $a = 1 - js_z$, $s_z > 0$.

As mentioned earlier the construction of optimum PMLs is currently the subject of intensive research. It turns out that the numerical implementation of PMLs is not reflectionless. The origin of these reflections is the numerical discretization of the fields as well as the fact that the thickness of the PML has to be truncated for numerical purposes. From a variety of studies performed by several researchers on the effectiveness of the anisotropic PML in conjunction with finite element grid truncation in the frequency domain, the current state of the art could be summarized as follows: a) The effectiveness of the PML is strongly dependent on the values assigned to its material properties, its thickness and its distance from the scatterer. Typically PML thicknesses of $0.1\lambda - 0.2\lambda$ and distances from the scatterer of 0.2λ are expected to lead to very good PML performance. b) The possibility exists for optimizing the performance of the PML for a given thickness by allowing its material properties to vary smoothly, in a way similar to that used in conjunction with FDTD simulations of transient waves. c) While a choice of the form $a = 1 - js$, $s > 0$, will facilitate the absorption of propagating waves, the absorption of waves exhibiting either only evanescent or both evanescent and propagating behavior needs to be assisted by allowing the real part of a to assume values greater than 1 [38]-[40]. d) The material properties of the PML tend to slow down the convergence of the iterative solvers used for the solution of the finite element system.

To summarize, the use of anisotropic absorbers for finite element grid truncation is expected to impact significantly our ability to simulate time-harmonic electromagnetic interactions in unbounded regions using the method of finite elements. However, there remains a lot of work to be done for this grid truncation methodology to reach its full potential.

VI. Domain Decomposition

Domain decomposition is a rigorous approach for the solution of linear boundary value problems in complicated domains. As a matter of fact, it is routinely applied for the development of analytic solutions to boundary value problems in physics and engineering when geometries with multiple homogeneous regions connecting along separable boundaries are present. What we consider here is an extension of the aforementioned concept to complicated, not-necessarily separable domains.

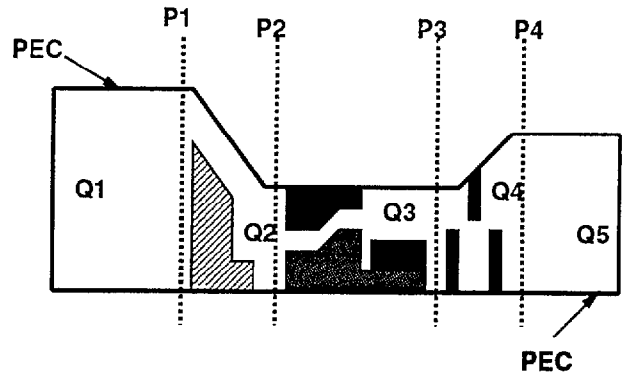

Fig. 2. A longitudinal cross section of a complicated structure. Partition planes P_1 through P_4 are used to decompose the structure in smaller subdomains.

The domain decomposition approach presented here is significantly different from other techniques of the same name [41],[42]. It is probably most closely related to the "connection scheme" of [43]; however, it is more versatile and more general in the type and complexity of structures it can model. Our presentation will concentrate on the motivation behind such an approach and the fundamental steps for its implementation. For a more concrete mathematical discussion references [44]-[45] should be consulted.

The rationale for the use of domain decomposition is best explained with the aid of an example structure. Fig. 2 illustrates a longitudinal cross section of a rather complicated shielded environment. Let us assume that the objective is the numerical modeling of noise interactions within this structure. While the development of a numerical grid for the entire structure could be attempted, it is rather obvious that the material complexity in the central part of the structure will require a much finer grid than the rather homogeneous end regions to maintain a desirable numerical solution accuracy. Furthermore, if the structure length is several wavelengths at the frequencies of interest, the size of the resulting finite element system could become prohibitively large for even the largest supercomputers available.

A domain decomposition approach to the solution of this problem proceeds as follows. First, the mathematical partitions P_1 through P_4 are used to decompose the structure into five smaller domains Q_1 through Q_5. Clearly, the choice of the partitions of Fig. 2 was influenced by the variability in material complexity. The power of the domain decomposition approach lies on the linearity of the governing equations and the uniqueness theorem for the solution to Maxwell's equations which is conveniently expressed by the weak statements (3) and (5). For example, from (5) it is clear that $\mathbf{E}$ is uniquely specified inside a domain if the tangential magnetic field is defined every-

where over the domain boundary. On the basis of this observation, the domain decomposition approach introduces appropriate expansions for the tangential magnetic fields on the partitions. These expansions are in terms of properly selected known, linearly independent expansion functions, and the unknown coefficients in these expansions become the fundamental degrees of freedom of the problem. For each expansion function on each partition a solution for the electric field inside the two subdomains adjacent to the specific partition is generated. The set of interior solutions thus generated within each subdomain due to all the expansion function excitations on the enclosing partitions constitute a basis that can be used for the representation of the unknown field inside the subdomain. In other words, using the principle of superposition, the unknown field inside each subdomain is represented as the sum of the generated interior solutions, each one weighted by the coefficient in the expansion for the tangential magnetic field on the partitions that is associated with the specific excitation function. Finally, the solution for the unknown coefficients in the tangential magnetic field expansions on the partitions is effected by enforcing the continuity of the tangential electric field across the partitions and solving the resulting linear system.

Simply expressed, domain decomposition is a technique for shifting the unknowns of a large and/or complicated boundary value problem from the continuum of points within the total structure to coefficients on properly selected partitions. Its chief strength is the independence of the subdomains. The first consequence of this independence is computational flexibility. Because each subdomain is modeled independently, different subdomains may be analyzed with different techniques. Analytic solutions may be used for separable subdomains, while finite methods or integral equation techniques may be used for subdomains of high complexity. The second consequence of the independence of subdomains is modularity. Many subdomains may be present in the structure; however, only some may be unique. Numerical solutions need be calculated only for the unique subdomains. The third major consequence of the independence of subdomains is parallelism. More specifically, two degrees of parallelism are present in the generation of the subdomain interior solutions. The solutions for different subdomains are independent as well as the solutions for different excitation functions for a given subdomain. Given enough computation resources, all these solutions may be generated in parallel.

A number of additional benefits come from modeling smaller domains. Numerical grid generation and adaptive refinement is easier in smaller domains. Partitioning allows the separation of disparate mesh regions without the need for transition areas between fine meshes and coarse meshes. Accuracy of the solution can be improved also. Indeed, domain decomposition results in smaller matrices and well-conditioned matrices due to better uniformity of the numerical grid. Consequently, round-off error is reduced and solution accuracy is improved.

VII. Discussion and Conclusions

Electromagnetic modeling for EMI/EMC analysis and design of components and systems is one of the most challenging areas for numerical electromagnetic simulation specialists. Topological complexity, material variability variability, large variability in feature sizes and broadband frequency analysis, are the main reasons for the aforementioned complexity of EMI/EMC simulations. All of these factors contribute to a rapid growth in the number of degrees of freedom needed for the numerical approximation of the problem, as one attempts to increase the sophistication and accuracy of the computer model. Very soon computer resource requirements exceed availability or, the turn-around time for numerical simulation becomes prohibitively long and thus inappropriate for use as a design aid. Inevitably, simplifications need to be made, and a reduced model is developed with significant effort placed on its ability to encompass the most important contributions to the process under investigation.

So far, applications of frequency-domain finite element methods have followed such a path. Evaluation of the effectiveness of conducting shields with slots or seems have benefited from finite element modeling [3],[46]. More recently, commercially available finite element software is being used for multiconductor interconnect transmission-line parameter extraction, extraction of lumped capacitive/inductive equivalents for electrical modeling of interconnect and package discontinuities, as well as electronic package inductance and capacitance modeling. Such models are subsequently used for crosstalk and switching noise prediction in packaged electronic systems. The modeling of electronic circuits, described in terms of their voltages and currents, in conjunction with vectorial field solvers based on finite elements has been demonstrated also [3],[47]. Other recent applications include frequency-dependent, scattering parameter characterization of package and interconnect discontinuities, investigation of the impact of the heatsink over a chip package on radiated emissions from the package, as well as the prediction of resonances inside a shielded box for a given layout of the various components.

The above list is expected to grow substantially as computing technology continues to advance rapidly, providing us with higher computation speeds, larger memory resources, parallelism and distributed computing. Continuing research in advancing the state-of-the-art in automatic grid generation and refinement, improving the performance and robustness of reflectionless grid truncation, and automating the application of domain decomposition approaches, will help enhance the power of the finite ele-

ment method to solve realistic EMI/EMC problems.

REFERENCES

[1] J.M. Jin, *The Finite Element Method in Electromagnetics*, John Wiley & Sons, Inc., New York, 1993.

[2] R. Laroussi, G.I. Costache, "Finite-element method applied to EMC problems," *IEEE Trans. Electromagnetic Compatibility*, vol. 35, no. 2, pp. 178-184, May 1993.

[3] J.R. Brauer and B.S. Brown, "Mixed-dimensional finite element models of electromagnetic coupling and shielding," *IEEE Trans. Electromagnetic Compatibility*, vol. 35, no. 2, pp. 235-241, May 1993.

[4] D.S. Dixon, M. Obara, and N. Schade, "Finite element analysis (FEA) as an EMC prediction tool," *IEEE Trans. Electromagnetic Compatibility*, vol. 35, no. 2, pp. 241-248, May 1993.

[5] A. Bayliss, C.I. Goldstein, and E. Turkel, "On accuracy conditions for the numerical computation of waves," *Journal of Computational Physics*, vol. 59, pp. 396-404, 1985.

[6] R. Lee, and A.C. Cangellaris, "A study of discretization error in the finite element approximation of wave solutions,', *IEEE Transactions on Antennas and Propagation*, vol. 40, pp. 542-549, 1992.

[7] W.R. Scott, "Errors due to spatial discretization and numerical precision in the finite-element method," *IEEE Transactions on Antennas and Propagation*, vol. 42, pp. 1565-1570, 1994.

[8] G.S. Warren, and W.R. Scott, "An investigation of numerical dispersion in the vector finite element method using quadrilateral elements," *IEEE Transactions on Antennas and Propagation*, vol. 42, pp. 1502-1508, 1994.

[9] J.C. Cavendish, D.A. field, and W.H. Frey, "An approach to automatic three-dimensional finite element generation," *Int. J. Numer. Methods Eng.*, vol. 21, pp. 329-347, 1985.

[10] N.A. Golias and T.D. Tsiboukis, "An approach to refining three-dimensional tetrahedral meshes based on Delaunay transformations," *Int. J. Numer. Methods Eng.*, vol. 37, pp. 793-812, 1994.

[11] J.-F. Lee and R. Dyczij-Edlinger, "Automatic mesh generation using a modified Delaunay tessellation," submitted to *IEEE Antennas and Propagation Magazine*.

[12] G. Strang and G.J. Fix, *An Analysis of the Finite Element Method*, Prentice-Hall, Englewood Cliffs, N.J., 1973.

[13] W.R. Scott, "Errors due to spatial discretization and numerical precision in the finite-element method," *IEEE Trans. Antennas Propagat.*, vol. 42, pp. 1565-1570, 1994.

[14] I. Babuska, O.C. Zienkiewicz, J. Gago, E.R. de Oliveira, Eds., *Accuracy Estimates and Adaptive Refinement in Finite Element Computation*, John Wiley & Sons, Inc., New York, 1986.

[15] P. Fernandes, P. Girdinio, and G. Molinari, "Techniques for h-refinement in adaptive meshing algorithms," *COMPEL*, vol. 13, supplement A, pp. 329-334, 1994.

[16] J.F. Lee, D.K. Sun, and Z.J. Cendes, "Full-wave analysis of dielectric waveguides using tangential vector finite elements," *IEEE Trans. Microwave Theory Tech.*, vol. 39, no. 8, pp. 1262-1271, 1991.

[17] N.A. Golias and T.D. Tsiboukis, "Consitutive inconsistency: Rigorous solution of Maxwell equations based on a dual approach," *IEEE Trans. on Magnetics*, vol. 30, no. 5, pp. 3586-3589, Sept. 1994.

[18] J.P. Webb, "The finite-element method for finding modes of dielectric loaded cavities," *IEEE Trans. Microwave Theory Tech.*, vol. 33, no. 7, pp. 635-639, 1985.

[19] A.J. Kobelansky and J.P. Webb, "Eliminating spurious modes in finite-element waveguide problems by using divergence-free fields," *Electron. Lett.*, vol. 22, pp. 569-570, 1986.

[20] K.D. Paulsen and D.R. Lynch, "Elimination of vector parasites in finite element Maxwell solutions," *IEEE Trans. Microwave Theory Tech.*, vol. 39, no. 3, pp. 395-404, 1991.

[21] W.E. Boyse, D.R. Lynch, K.D. Paulsen, and G.N. Minerbo, "Nodal based finite element modeling of Maxwell's equations," *IEEE Trans. Antennas Propagat.*, vol. 40, no. 6, pp. 642-651, 1992.

[22] A. Bossavit, "Simplicial finite elements for scattering problems in electromagnetism," *Computer Methods in Applied Mechanics and Engineering*, vol. 76, pp. 1262-1271, 1989

[23] H. Whitney, *Geometric Integration Theory*, Princeton U.P., 1957.

[24] J.C. Nedelec, "Mixed finite elements in R^3," *Numerical Mathematics*, vol. 35, pp. 315-341, 1980.

[25] J.C. Nedelec, "A new family of mixed finite elements in R^3," *Numerical Mathematics*, vol. 50, pp. 57-81. 1986.

[26] S.V. Polstyanko and J.F. Lee, "H_1(curl) tangential vector finite element method for modeling anisotropic optical fibers," *IEEE Journal of Lightwave technology*, in press.

[27] I. Bardi et al, "Higher order edge elements in electromagnetic field modeling," *Proceedings of 1995 International AP-S Symposium/URSI Meeting*, Newport Beach, CA, June 1995.

[28] T.D. Tsiboukis, *private communication*, Department of Electrical Engineering, Aristotle University of Thessaloniki, Thessaloniki, 54006, Greece.

[29] G. Mur, "Optimum choice of finite elements for computing three-dimensional electromagnetic fields in inhomogeneous media," *IEEE Trans. on Magnetics*, vol. 24, pp. 330-333, 1988.

[30] G. Mur, "Finite-element modeling of three-dimensional time-harmonic electromagnetic fields in inhomogeneous media," *Radio Science*, vol. 26, pp. 275-280, 1991.

[31] W.E. Boyse and A.A. Seidl, "A hybrid finite element method for 3-D scattering using nodal and edge elements," *IEEE Trans. Antennas Propagat.*, vol. 42, no. 1, pp. 1436-1442, 1994.

[32] D. Givoli, "Non-reflecting boundary conditions," *J. Comput. Physics*, vol. 94, pp. 1-29, 1991.

[33] J.P. Berenger, "A perfectly matched layer for the absorption of electromagnetic waves," *J. Comput. Physics*, vol. 114, pp. 185-200, 1994.

[34] W.C. Chew and W.H. Weedon, "A 3-D perfectly matched medium from modified Maxwell's equations with stretched coordinates," *Microwave and Optical Technology Lett.*, vol. 7, no. 13, pp. 599-604, 1994.

[35] C.M. Rappaport, "Perfectly matched absorbing boundary conditions based on anisotropic lossy mapping in space," *IEEE Microwave and Guided Wave Lett.*, vol. 5, no. 3, pp. 90-92, 1995.

[36] Z.S. Sacks, D.M. Kingsland, R. Lee, and J.-F. Lee, "A perfectly matched anisotropic absorber for use as an absorbing boundary condition," *IEEE Trans. Antennas Propagat.*, vol. 43 , no. 12, pp. 1460-1463, 1995.

[37] L. Zhao and A.C. Cangellaris, "A general approach for the development of unsplit field time domain implementations of perfectly matched layers for FD-TD grid truncation," *IEEE Microwave and Guided Wave Lett.*, (accepted for publication), June 1996.

[38] M.A. Gribbons, W.P. Pinello, and A.C. Cangellaris, "A stretched coordinate technique for numerical absorption of evanescent and propagating waves in planar waveguiding structures," *IEEE Trans. Microwave Theory Tech.*, vol. 43, no. 12, pp. 2883-2889. 1995.

[39] Z. Wu and J. Fang. "Numerical implementation and performance of perfectly matched layer boundary condition for waveguiding structures," *IEEE Trans. Microwave Theory Tech.*, vol. 43, no. 12, pp. 2676-2683, 1995.

[40] J.-F. Lee, ECE Department, Worcester, and R. Lee, EE Department, Ohio State University, *private communication.*

[41] G.L. Hennigan, S. Castillo, and E. Hensel, "Using domain decomposition to solve symmetric, positive-definite systems on the hypercube computer," *Int. J. Num. Meth. Eng.*, vol. 33, pp. 1941-1954, 1992.

[42] G.F. Carey, "Parallel subdomain and element by element techniques," in *Parallel Supercomputing: Methods, Algorithms,*

and Applications, G.F. Carey, ed., John Wiley & Sons, Inc., Chicester, UK, 1989.

[43] T.-M. Wang and H. Ling, "Electromagnetic scattering from three-dimensional cavities via a connection scheme," *IEEE Trans. Antennas Propagat.*, vol. 39, no. 10, pp. 1505-1513, 1991.

[44] C.T. Spring and A.C. Cangellaris, "Application of domain decomposition methods for the electromagnetic anaysis of large linear two-dimensional structures," *J. Electromagnetic Waves Applic.*, vol. 9, no. 1/2, pp. 175-192, 1995.

[45] C.T. Spring, "Domain decomposition in the rigorous electromagnetic analysis of large structures," Ph.D. Dissertation, ECE Department, University of Arizona, Tucson, AZ, 1995.

[46] S.D. Gedney and R. Mittra, "Electromagnetic transmission through inhomogeneously filled slots in a thick conducting plane - Arbitrary Incidence," *IEEE Trans. Electromagnetic Compatibility*, vol. 34, no. 4, pp. 404-415, 1992.

[47] J.R. Brauer, B.E. MacNeal, L.A. Larkin, and V.D. Overbye, "New method of modeling electronic circuits coupled with 3D electromagnetic finite element models," *IEEE Trans. Magnetics*, vol. 27, pp. 4085-4088, 1991.