## Applications

1. a. 25 shirts would cost $\$ 70$. You could use a table by trying to find the cost $C$ for every value of $n$. Thus, the table would reflect values for $n=1,2,3, \ldots, 25$. You could use the graph by finding some coordinate pairs and then extending the line formed. You would go over to 25 on the $x$-axis (the axis representing the variable $n$ ), and then go up on the $y$-axis until you reach the line, and read the value of $C$.
b. The class could buy 30 shirts for $\$ 80$. You could use a table by trying to find the cost $C$ for every value of $n$. Thus, the table would reflect values for $n$ until you reached a value of $\$ 80$ for $C$. You could use the graph by finding some coordinate pairs and extending the line formed. You would go up to 80 on he $y$-axis (the axis representing the variable $C$ ), and then go over on the $x$-axis until you reach the line, and read the value of $n$.
c. Taleah is looking for the Cost $C$ of 15 T-shirts.
d. Keisha is looking to see if 30 T -shirts resulted in a cost of $\$ 80$.
2. a. Plan 1: the number of km walked Plan: the amount of money raised in dollars
Plan 3: the number of km walked
b. Plan 1: Solve for $x$. Divide 14 by 2, which would give you 7 kilometers walked; Plan 2: Solve the right side of the equation for y . Take 3.5 times 10 (35) and then add 10 , thus giving you $\$ 45$ (the amount raised); Plan 3: Solve for $x$. Take 100 minus 55 (45), and then divide that by 1.5 , which would give you 30 kilometers walked.
3. a. $y=45$
b. $x=22$
c. $x=3$
4. a. $y=-10$
b. $x=13$
c. You could use a table by trying to find the $y$ for every value of $x$. Thus, the table would reflect values of $x$ from 1 through at least 13. You could use the graph by finding the coordinate pairs. You would go over to 1 on the $x$-axis, and then go up or down on the $y$-axis until you reach the line, and read the value of $y$. You can do the same thing to find the $x$-value, either by looking at all values in a chart that give you either a particular $x$ - or $y$-value or by finding coordinate pairs on the graph. For example, you would go up to 50 on the $y$-axis and move along the $x$-axis until you reach the line. Then read the $x$-value of the coordinate pair, which is 13 .
5. 4 coins
6. 5 coins
7. 1 coin
8. coins
9. a.


There are 4 coins in each pouch; $x=4$.
b.


There are 7 coins in each pouch; $x=7$.
c.


There are 7 coins in each pouch $x=7$.
d.


There are 6 coins in each pouch; $x=6$.
e.


There are 2 coins in each pouch; $x=2$.
10. a. $y=5+0.50 x$. Here $x$ stands for the number of math questions Gilberto gets right, y stands for the total amount of money his grandfather gives him, 5 stands for the birthday money Gilberto gets even if he never answers a single question correctly, and 0.50 stands for the $\$ .50$ he gets for every question he gets right.
b. To buy a shirt that costs $\$ 25$, we need to solve the equation:

$$
\begin{aligned}
25 & =5+0.50 x \\
20 & =0.50 x \\
x & =40
\end{aligned}
$$

So, he will need to answer 40 questions correctly to buy the shirt.

Note: We anticipate that students may use other methods to solve this problem. It might be nice to discuss this exercise as a class to reinforce the idea that solving these kinds of problems algebraically yields the same answer as solving with a table or graph.
c. $y=5+0.50(12)$
$y=5+6$
$y=11$
So, he will make $\$ 11$.
Note: Again, students may use tables or graphs to solve this problem. Make sure
they understand that they can also solve this problem by substituting 12 for $x$.
Note: You can have students compare this to the equation for Alana's pledge plan. They are the same, but each represents a different context.
11. a. $x=21$; you can use the equation $3 x+15=78$ to represent the given information. Subtract 15 from each side of the equation to get $3 x=63$, and then divide each side by 3 .
b. $x=22$; you can use the equation $5 x-27=83$ to represent the given information. Add 27 to each side of the equation to get $5 x=110$, and then divide each side by 5 .
c. Answers will vary. Sample answer: If you add 6 to 9 times the mystery number, you get 87 . The mystery number is 9 .
12. a. $x=-3$
b. $x=-10$
c. $x=-\frac{3}{4}$
d. $x=10$
13. $C$
14. a. $x=5$
b. $x=\frac{25}{3}$
c. $x=-\frac{25}{3}$
d. $x=-5$
e. $x=5$

Note: Students may have various strategies for solving $3 x+5=20$, such as:

- Using fact families: $3 x+5=20$, so $3 x=20-5$.
if $3 x=15$, then $x=\frac{15}{3}$
- Using an "undoing" metaphor: Begin with $3 x+5=20$, subtract 5 from each side, and then divide by 3 on each side.
- Using properties of equality: $3 x+5=20,3 x+5-5=20-5$, and so on.
It would be a good idea to discuss whether each of these strategies will work effectively on ACE Exercise 12 also. Ask students to explain why the strategies will work or why they will not.

15. a. always equal: $-2(x-3)=-2 x+6$
b. never equal (the expressions differ by 12)
c. sometimes equal (only equal for $x=0$ )
d. never equal (the expressions differ by 9)
e. never equal (the expressions differ by 12)
f. always equal:

$$
2(3-x)=6-2 x=-2 x+6
$$

16. Equations $\mathrm{A} \& \mathrm{H}: x=2$

Equations B \& F: $x=6$
Equations C \& G: $x=-2$
Equations D \& K: $x=-6$
Equations E \& J: $x=2.5$
One strategy students might use to match the equations is to solve each equation for $x$. Another strategy is to simplify each equation in Group 1. For example, dividing each side of equation $E$ by 6 results in equation J. Subtracting 6 from both sides of equation $A$ yields equation H .
17. a. $x=2$
b. $x=-12$
c. $x=-6$
d. $x=-7$
18. a. $x=-2.5$
b. $x=4.5$
c. $x=-5.5$
d. $x=-2.5$
19. a. $x=1$
b. $x=2$
c. $x=1$
d. $x=1$
20. a. $y=-8$
b. $y=13$
c. $y=-2$
d. $y=8$
e. $y=4$
21. a. $x=\frac{4}{3}$
b. $x=-\frac{17}{3}$
C. $x=\frac{19}{3}$
d. $x=\frac{1}{6}$
22. You can look at either the $x$-axis or the $y$-axis to find the other coordinate pair by using the graph made by the line $y=4-3 x$.
23. a. Part (a) is about recognizing notation.

$$
\text { For example, }-1 x=-x \text { and } \frac{1}{4} x=\frac{x}{4}
$$

A \& D: $x=8 ; \mathrm{B} \& \mathrm{~F}: x=-8$; C\&E: $x=2$
b. G, J, M: $x=7$; H, K, L: $x=-5$
c. $\mathrm{N} \& \mathrm{P}: x=4.5 ; \mathrm{O} \& \mathrm{Q}: x=-8$; $R \& S: x=-3.5$
24. Student 1 subtracted 6 from the righthand side, instead of distributing the 6 throughout the parenthesis on the left-hand side (used subtraction to undo multiplication). Student 2 started to distribute the 6 throughout the parenthesis on the left-hand side, but only did it to the first term and forgot to do it to the second one (it should be 24 , as opposed to 4). The
correct solution is $x=-\frac{26}{3}$
25. Student 1: The first step is incorrect. Instead of subtracting 6, the student should have added 6. The correct answer is $64.5 \div 3.4 \approx 18.43$.
Student 2: The student's answer is wrong due to incorrect placement of the decimal point. The answer is approximately 18.4.
26. a. To solve $5 x+10=-20$, use the equation $5 x+10=y$. To use a table, scan down the table of $y$-values until you come to -20 . The corresponding $x$-value is the solution. To use a graph, graph the equation $5 x+10=y$ until you have a point whose $y$-coordinate is -20 . The corresponding $x$-coordinate $(x=-6)$ is the solution.
b. To solve this equation by graphing, graph the lines $y=4 x-9$ and $y=-7 x+13$. At the point where the graphs cross, the $x$-coordinate is the solution. To solve using a table, create a table for the left side and right side and look for an ordered pair ( $x, y$ ) that shows up on both tables. The x -coordinate of this ordered pair will be the solution.
27. a. $P=2$
b. $c=-30$
c. You could use a table by trying to find the $P$-value for every value of $c$. You could use the graph by finding the coordinates of points on the line. You would go over to con the $x$-axis (the axis representing the variable $c$ ), and then go up or down on the $y$-axis (representing the variable $P$ ) until you reach the line, and read the value of $P$. You can do the same thing to find the $c$-value, either by looking up all values in a chart that give you either a particular $c$ - or $P$-value or by finding coordinate pairs by using the graph.
28. a. i $m=79.75$
ii. $m=15.75$
iii. $m=25.99$
b. i. $d=12$
ii. $d=-4.921875$
iii. $d=26.328125$
29. a. Yes, Khong's method is correct. A factored expression is equivalent to the original expression, and dividing both sides of an equation by the same number results in another equivalent equation.
b. $40 x+20=120$

$$
\begin{aligned}
20(2 x+1) & =20(6) \\
2 x+1 & =6 \\
2 x & =5 \\
x=\frac{5}{2} & =2 \frac{1}{2}
\end{aligned}
$$

c. Khong is partially correct: his method does not work as well because all the numbers are prime. However, he could still factor out the GCF, which is 1 . He could also factor out another number but would get fractions in the new equation. For example, Khong could factor out seven to get
$7\left(x+\frac{3}{7}\right)=7\left(\frac{31}{7}\right)$.
d. Answers will vary. Sample answer:

$$
12 x+24=48
$$

$$
\begin{aligned}
12(x+2) & =12(4) \\
x+2 & =4 \\
x & =2
\end{aligned}
$$

30. a. About 71.3 T-shirts must be made and sold to break even. By setting the expressions for $E$ and $/$ equal to each other, you obtain $535+4.5 n=12 n$. Solving for $n$ gives $n=71.3$.
b. A loss, because the expenses, which are $535+4.5(50)=\$ 760$, exceed the income, which is $12(50)=\$ 600$.
c. There is a profit because when the income is $\$ 1,200$, the number of T-shirts is 100 . When the number of T-shirts is 100 , the expenses are $\$ 985$. Whenever the expenses are less than the income, there is a profit.
d. Answers will vary. Sample answer:

For the expenses graph, $(10,580)$ is on the graph. This means that the cost of making 10 T -shirts is $\$ 580$. We know this point will lie on the graph because $580=535+4.50(10)$.
$(12,144)$ is a point on the income graph. This means that if they sell 12
T-shirts, they will make $\$ 144$. We know this point will be on the graph because $144=12$ (12).
Note: You can also ask students to find when income is less than expenses of vice versa. They can use the graph to answer this question.
31. a. International Links would be cheaper unless the customer talks more than 625 minutes per month. Students may look at a graph or table of the relationships $C=50+0.10 m$ and $C=0.18 \mathrm{~m}$ to find this answer.
b. International Links is cheaper for $m<625$. World Connections is cheaper for $m>625$.
Students could represent the inequalities on a graph. In the graph below, the thicker part of each line below represents when each plan is the cheaper of the two. (See Figure 1.)
Students could also represent the inequalities on number lines.

International Links is Cheaper for $m<625$


World Connections is Cheaper for $m>625$.

32. a. Answers will vary. Students describe how many text messages they send and receive each month on average and calculate the better deal.
b. Possibly, depending on the number of texts that person sends and receives in a month. You can ask students to consider whether their recommendation would be the same for a friend or a parent.
c. The break-even point for the two plans is 400 texts. So, Walby Communications is cheaper for $t>400$, and Driftless is cheaper for $t<400$.
Students could represent the inequalities on a graph. In the graph below, the thicker part of each line below represents when each plan is the cheaper of the two.(See Figure 2.)

Figure 1


Figure 2


Students could also represent the inequalities on number lines.

Driftless Region is Cheaper for $t<400$.


Walby Communications is Cheaper for $\mathrm{t}>400$.


## Number of Text Messages

33. a. They must sell 75 roses to break even. By setting the expressions for cost and income equal to each other, you obtain $0.5 n+60=1.3 n$. Solving for $n$ gives $n=75$.
b. They do not make a profit, but have a loss of $\$ 20$, when they sell 50 roses. They make a profit of $\$ 20$ when they sell 100 roses.
They make a profit of $\$ 100$ when they sell 200 roses.
34. 


b. $(6,192)$ is the point of intersection. This tells us that the two cost plans are the same if Ruth uses them for 6 months.

## Connections

35. a. multiplication; -32
b. multiplication; -8
c. multiplication and then subtraction; -40
d. multiplication (twice) and then addition; 11
36. a. -4
b. -4
c. 4
d. 0
e. -1
f. -1
g. 1
37. a. No; in the first expression, only 5 is multiplied by 6 , but in the second expression, both 5 and 2 are multiplied by 6 .
b. No; they are opposites of each other.
c. Yes; they are equal because of the Commutative Property of Addition.
d. Yes; they are equal because of the Commutative Property of Multiplication.
e. No; they are opposites of each other.
f. Yes; both quantities have the same value: $\$ .50$.
g. No; 1.5 liters equals 1,500 milliliters, not 15 milliliters.
h. No; 2 out of 5 is 40 ,, not 50 ,.
38. a. $n=30+-3$ or $n=30-3,-3=n-30$ or $3=30-n$.
Answers will vary. Some students may think $n=30-3$ is easier, while other students may not.
b. $5=-36-n$ or $n=-36-5$. Answers will vary. Some students may think $n=-36-5$ is easier, while other students may not.

## Answers | Investigation 3

c. For part (a), add -3 to both sides;
$n=27$.
For part (b), subtract 5 from both sides;
$n=-41$.
Answers will vary. Some students may find fact families easier; others may find the properties of equality easier.
39. a. $A=x(5+4) ; A=5 x+4 x$
b. $A=1.5(7+x) ; A=10.5+1.5 x$
40. a. $x=\frac{3}{14}$
b. $x=\frac{14}{3}$
c. $x=\frac{1}{7}$
d. $x=\frac{10}{9}$
41. a. all numbers greater than -4
b.

c. all numbers greater than 3

d. $x \leq 3$; all numbers less than or equal to 3
e. $x>-3$

42. a. $n=4(60)-160=80$ chirps
b. $\quad t=\frac{150}{4}+40=77.5^{\circ} \mathrm{F}$
c. This would be when the number of chirps is 0 :
$t=\frac{0}{4}+40=40^{\circ} \mathrm{F}$.
d. The $y$-intercept gives the temperature when the number of chirps is 0 . The coefficient of $n$ means the ratio of the change in temperature to the change in number of chirps.

Note: An interesting question to raise is, "If the graph were extended to the left, what meaning would that part have?"

Cricket Chirps.


Number of Chirps
43. a. $T=0-\frac{1500}{150}=0-10=-10^{\circ} \mathrm{C}$
b. $26=t-\frac{300}{150}=t-2$; thus $t=28^{\circ} \mathrm{C}$
44. a. $180^{\circ}$
b. $360^{\circ}$
c. $720^{\circ}$
d. $1,440^{\circ}$
e. $3,240^{\circ}$
45. a. $60^{\circ}$
b. $90^{\circ}$
c. $120^{\circ}$
d. $144^{\circ}$
e. $162^{\circ}$
46. a. 5 sides
b. 8 sides
47. a. $x=4$, so the three side lengths are $6 \mathrm{~cm}, 8 \mathrm{~cm}$, and 10 cm .
b. $\quad x=\frac{24}{2 \pi} \approx 3.8 \mathrm{~cm}$
c. $\quad x=2$, so the side lengths are 2 cm and 10 cm .
d. The area of the right triangle in (a) is 24 cm 2 , and the area of the rectangle in (c) is $20 \mathrm{~cm}^{2}$.
48. a. $C=50+0.10 t$
b. $\quad 10.5$ hours $=630$ minutes, so

$$
C=50+0.10(630)=\$ 113
$$

c. $\quad 75=50+0.10 t$, so $t=250 \mathrm{~min}$
d. Part (c) should be an inequality. Many phone companies round up the number of minutes used, so the actual number of minutes may be less than what Andrea was charged.
49. a. $h=H-0.06 t$
b. Here, $h=160$ and $t=30$, so $160=H-0.06(30)$ which gives $H=161.8 \mathrm{~cm}$.
c. $6 \mathrm{ft} 6 \mathrm{in} .=198.12 \mathrm{~cm}$
$h=198.12-0.06(50)=195.12$
Therefore, he will be 195.12 cm tall, or about 6 feet 5 inches, at age 80.
d. In part (a), we assumed that a person's height will decrease by 0.06 cm each year. Because the person likely does not decrease by 0.06 cm every year, it could be rewritten as an inequality.
50. a. $h=61.412+2.317(46.2) \approx 168.5 \mathrm{~cm}$
b. $\quad h=81.688+2.392(50.1) \approx 201.5 \mathrm{~cm}$
c. femur: 39.1 cm
tibia: 31.4 cm
humerus: 27.7 cm
radius: 20.3 cm
d. femur: 50.9 cm
tibia: 42.4 cm
humerus: 36.8 cm
radius: 28.1 cm
e. The graphs will be straight lines going upward from left to right. The $x$-intercept tells the value for $x$ (femur, tibia, humerus, or radius length) when the height of the person is 0 , and the $y$-intercept tells the value for $y$ (the person's height) when the length of a bone is 0 . These values do not make sense in the context of the problem.

## Extensions

51. a. $1,500=150 A+40 C$
b. $1,500=150 A+40(10)$
$1,500-400=150 A+400-400$
$1,100=150 A$

$$
\frac{1100}{150}=\frac{150 A}{150}
$$

$$
7.33=A
$$

So, no more than 7 adults can get in.
c. $1,500=150(6)+40 C$
$1,500-900=900-900+40 C$
$600=40 C$
$\frac{600}{40}=\frac{40 C}{40}$
$15=C$
So, 15 children can get in.
52. a. $x=3$ ( $x$ equals an exact number, which is 3 , to make both sides of the expression equal.)
b. $\quad x=$ any number. For any value of $x$, the value of each side of the equation is the same-the two expressions are equivalent.
c. No solution. For any value of $x$, the value of the left side is 3 more than the value of the right side.
53. a. Frank's method works in general, except for multiplying and dividing by a negative number. This requires flipping the inequality sign, which is the issue in part (b).
b. The inequality $-2 x>4$ is equivalent to the inequality $2 x<-4$. One way to see this is to add $2 x-4$ to both sides of $-2 x>4$. Thus, we see that $-4>2 x$ or $x<-2$. That is, all numbers less than -2 are solutions of the original inequality. In general, multiplying or dividing both sides of an inequality by a negative number reverses the direction of the inequality.
54. a (See Figure 3.)
b. NY to SF: $d=270 t$, SF to NY: $d=330 t$
c. Plane Flying Time


Note: Notice that the graph has time on the horizontal axis and distance on the vertical axis. Students are accustomed to thinking of distance as depending on time. The equations in part (c) show this as well. However, this table is set up in reverse, in a way, as students are asked to find the time given the distance. Some students may think of time as dependent on distance and put time on the vertical axis. The related equations are $\frac{d}{270}=t$ and $\frac{d}{330}=t$, and the graph will show the distance as $x$ and the time as $y$.
d. Against the wind: $5,000 \div 270=18.52 \mathrm{~h}$; with the wind: $5,000 \div 330=15.15 h$

Figure 3

## Airplane Flight Times

| Distance (mi) | NYC to SF Time (h) | SF to NYC Time (h) |
| :--- | :--- | :--- |


| 0 | 0.00 | 0.00 |
| ---: | :---: | :---: |
| 200 | 0.74 | 0.61 |
| 400 | 1.48 | 1.21 |
| 600 | 2.22 | 1.82 |
| 800 | 2.96 | 2.42 |
| 1,000 | 3.70 | 3.03 |
| 1,200 | 4.44 | 3.64 |
| 1,400 | 5.19 | 4.24 |
| 1,600 | 5.93 | 4.85 |
| 1,800 | 6.67 | 5.45 |
| 2,000 | 7.41 | 6.06 |
| 2,200 | 8.15 | 6.67 |
| 2,400 | 8.89 | 7.27 |
| 2,600 | 9.63 | 7.88 |
| 2,800 | 10.37 | 8.48 |
| 3,000 | 11.11 | 9.09 |

55. All three methods are correct. The advantage in Jess's solution is getting rid of the fractions first by distributing. Another way to do this is what Terri did, multiplying the equation by 3 . Brian noticed that the expression in both sets of parentheses is the same, so he is correct that he can simplify the left side to $6 x-9$.
56. C is the correct answer. The expression on the right side will always be 6 greater than the expression on the left because the left side is adding 3 to $3 x$, whereas the right side is adding 9 to $3 x$.
57. C is the correct answer. This equation is an identity. $3(x+1)$ is equivalent to $3 x+3$.
58. a. $x>0$
b. any number greater than -3 and less than 3.

c. $|x| \geq 2$

d. any number greater than one, or between -1 and 0 .

e. All negative numbers will have negative sums, so those are excluded. Any number greater than 1 will work, but any number between 0 and 1 also works because $1 x$ will be greater than 1.

