Applications

1. An even number minus an even number will be even. Students may use examples, tiles, the idea of "groups of two," or the inverse relationship between addition and subtraction.
   - Using an example: 16 – 4 is 12.
   - Using tiles: For example, if you take away one rectangle with a height of 2 from another rectangle with a height of 2, you will still have a rectangle with a height of 2.
   - Using groups of 2: If you have an even number of objects, you can bundle the number of objects into groups of 2. If you take away some bundles of 2 from a group of bundles of 2, you are still left with bundles of 2.
   - Using the inverse relationship between addition and subtraction: Students may know that if $a + b = c$, then $a = c - b$. In this way, the question is asking "If c and b are even, is a even or odd?" In the equation $a + b = c$, if the values of b and c are even, then the value of a must also be even, because even + even = even.
2. An odd number minus an odd number is even. If you have a rectangle with one extra square and you take away a rectangle with one extra square, you have taken away the extra square, and you are left with a rectangle with a height of 2.
3. An even number minus an odd number is odd. If you have a rectangle with a height of 2 and you subtract a rectangle with one extra square, you have broken up a pair of squares on the original rectangle and are left with another rectangle with an extra square.
4. An odd number minus an even number is odd. If you have a rectangle with one extra square and you subtract a rectangle with a height of 2, you are left with a rectangle with an extra square.
5. Evens have ones digits of 0, 2, 4, 6, or 8, and they are divisible by 2. Odds have ones digits of 1, 3, 5, 7, or 9, and they are not divisible by 2.
6. A sum is even if all of the addends are even, or if there is an even number of odd addends. Otherwise, the number is odd.
7. $4 \times (3 + 6)$ and $(4 \times 3) + (4 \times 6)$, total area 36
8. $(4 + 2) \times 7$ and $(4 \times 7) + (2 \times 7)$, total area 42
9. $5 \times (3 + 6 + 2)$ and $(5 \times 3) + (5 \times 6) + (5 \times 2)$, total area 55

For Exercises 10–12, the area of the largest rectangle is the sum of the areas of the two smaller rectangles. To find the dimensions of each rectangle, first find a common factor of each pair of numbers. Each Exercise has multiple possible dimensions.

10. (See Figure 1.)

Figure 1

Dimensions of 39 Square Unit Rectangles and Partitions

	Small	Medium	Large
Possible Rectangle 1	3 × 4	3 × 9	3 × 13
Possible Rectangle 2	1 × 12	1 × 27	1 × 39

11. (See Figure 2.)

12. (See Figure 3.)

13. $3 \times (4 + 6) = 3 \times 10$ or
$(3 \times 4) + (3 \times 6) = 12 + 18$

14. $3 \times (5 + 1 + 3) = 3 \times 9$ or
$(3 \times 5) + (3 \times 1) + (3 \times 3) = 15 + 3 + 9$
(See Figure 4.)

Figure 2

Dimensions of 49 Square Unit Rectangles and Partitions

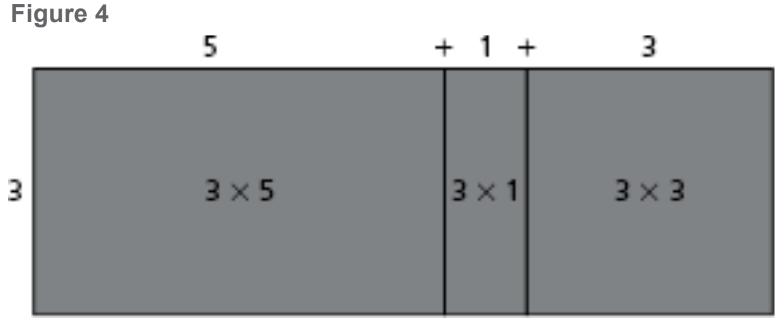
	Small	Medium	Large
Possible Rectangle 1	7 × 3	7 × 4	7 × 7
Possible Rectangle 2	1 × 21	1 × 28	1 × 49

Figure 3

Dimensions of 48 Square Unit Rectangles and Partitions

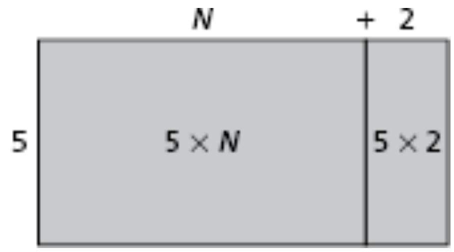
	Small	Medium	Large
Possible Rectangle 1	1 × 18	1 × 30	1 × 48
Possible Rectangle 2	2 × 9	2 × 15	2 × 24
Possible Rectangle 3	3 × 6	3 × 10	3 × 16
Possible Rectangle 4	6 × 3	6 × 5	6 × 8

Figure 4



15. $N \times (2 + 6) = 8N$ or
$(N \times 2) + (N \times 6) = 2N + 6N$
(See Figure 5.)

16. $5 \times (N + 2)$ or $(5 \times N) + (5 \times 2)$, $5N + 10$



17. $9 \times (30 + 4) = (9 \times 30) + (9 \times 4)$
$= 270 + 36 = 306$
(See Figure 6.)

Figure 5

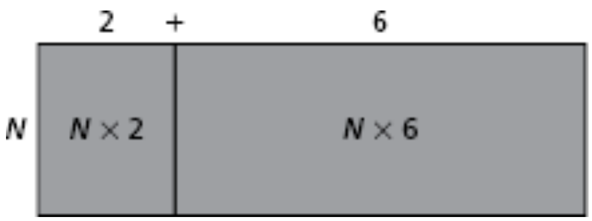
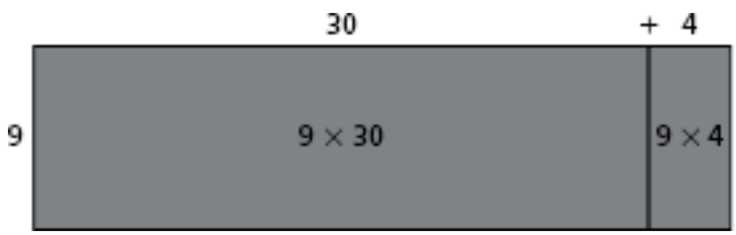
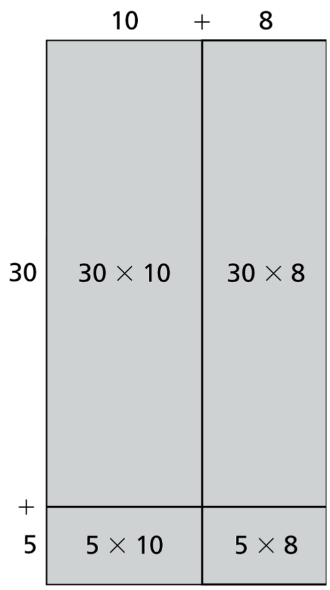


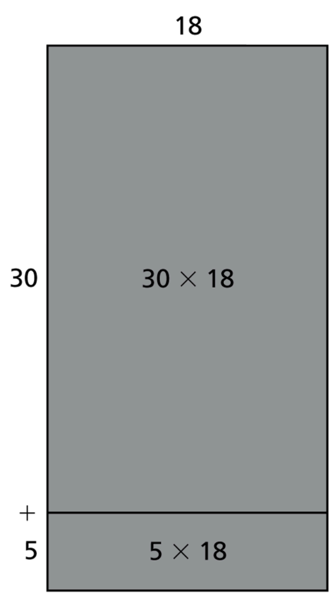
Figure 6



18. $35 \times 18 = (30 + 5) \times (10 + 8)$
$= (30 \times 10) + (5 \times 10) + (30 \times 8) + (5 \times 8)$
$= 300 + 50 + 240 + 40$
$= 630$



$(30 + 5) \times 18 = (30 \times 18) + (5 \times 18)$
$= 540 + 90$
$= 630$



Note: Some students may write $35 \times (20 - 2) = 700 - 70 = 630$, although this is not connected to the typical multiplication algorithm. The arithmetic may be easier with these numbers.

19. a. Answers will vary. Possible answer: $30 + 30$

b. Answers will vary. Possible answer: 6×10

c. Answers will vary. Possible answer:
$6 \times 10 = 6 \times (5 + 5)$
$= (5 \times 6) + (5 \times 6)$
$= 30 + 30$

20. a. $90 = 20 + 70 = 10(2 + 7)$

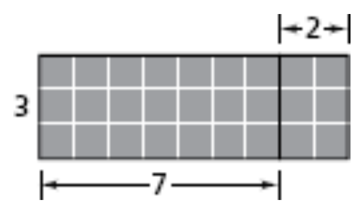
b. $90 = 36 + 54 = 9(4 + 6)$

21. a. The black number in the lower-right-hand square is the sum of the red numbers in the right-most column and also the sum of the red numbers in the bottom row.

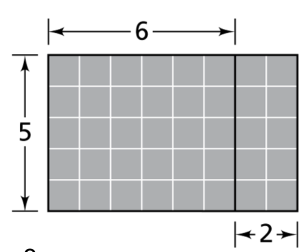
b. The same relationship will hold for any four numbers in the border squares. The black number is always the sum of the red numbers in the right-hand column and the red numbers in the bottom row.

c. Shalala is correct. The sum of the bottom row is $6(2 + 8) + 3(2 + 8)$. The sum of the last column is $2(6 + 3) + 8(6 + 3)$. From the first expression, you can factor $(2 + 8)$ to get $(2 + 8)(6 + 3)$. If you factor $(6 + 3)$ from the second expression, you get $(6 + 3)(2 + 8)$. By the Commutative Property of Multiplication, these two products are equal.

22.



23.



24. $m = 3$

25. $m = 10$

26. $m = 1$

27. $m = 6$

28. $(3 + 4) \times 2$

29. $12 \div 6 \times 2$

30. $11 \times 2 + 1$

31. $3^2 + 3^2$

32. $2 + 5 \times 3 = 17$

33. $2 \times 5 + 3 = 13$

34. $2 \times 5 \times 3 = 30$

35. $2 \times 5 - 3 = 7$

36. Answers will vary. $3 + 2 + 4(1) = 9$ or $3 + (2 + 4)(1) = 9$ or $(3 + 2 + 4)(1) = 9$.

37. $3 + (2)(4 + 1) = 13$

38. $(3)(2 + 4 + 1) = 21$

39. $3 + (2)(4) + 1 = 12$

40. $3 + 2 + 4 + 1 = 10$

41. a. $4 + 3(6 + 1) = 25$, which is a multiple of 5.

 b. $4(3) + 6(1) = 18$, which is a factor of 36.

42. a. Answers will vary. Possible answer:
$$\begin{aligned} 21 &= 3 \times 7 \\ &= 3 \times (5 + 2) \\ &= 3 \times 5 + 3 \times 2 \\ &= 15 + 6 \end{aligned}$$

 b. Answers will vary. Possible answer:
$$\begin{aligned} 24 &= 2 \times 12 \\ &= 2 \times (10 + 2) \\ &= 2 \times 10 + 2 \times 2 \\ &= 20 + 4 \end{aligned}$$

 c. Answers will vary. Possible answer:
$$\begin{aligned} 55 &= 5 \times 11 \\ &= 5 \times (10 + 1) \\ &= 5 \times 10 + 5 \times 1 \\ &= 50 + 5 \end{aligned}$$

 d. Answers will vary. Possible answer:
$$\begin{aligned} 48 &= 2 \times 24 \\ &= 2 \times (20 + 4) \\ &= 2 \times 20 + 2 \times 4 \\ &= 40 + 8 \end{aligned}$$

43. The student interpreted exponents as multiplying the two numbers. 3^2 is not 6, and 3^3 is not 9. The correct answer is 9.

44. The student performed multiplication before exponentiation; 2×3^2 is not 6^2, but 18. The correct answer is 26.

45. The student added before he subtracted; $18 - 6 + 6$ is not $18 - 12$, but $12 + 6$. The correct answer is 18.

46. The student multiplied before he divided; $24 \div 6 \times 4$ is not $24 \div 24$, but 4×4. The correct answer is 16.

47. Any number will work. Explanations will vary. Sample:

 Step 1: Choose 7.

 Step 2: $7 + 15 = 22$

 Step 3: $(7 + 15) \times 2 = 44$

 Step 4: $(7 + 15) \times 2 - 30 = 14$

 $14 = 2 \times 7$, which is double the original number.

 Alternatively,
$$\begin{aligned} (n + 15) \times 2 - 30 &= n \times 2 + 30 - 30 \\ &= n \times 2 \end{aligned}$$
 which is double the original number.

48. Choose N. Then,
$$\begin{aligned} ((N \times 2 + 6) - 3) &= (N \times 2 \div 2) + (6 \div 2) - 3 \\ &= N + 3 - 3 \\ &= N \end{aligned}$$

49. This can be solved algebraically. An area model works as well.

Let N = the area of a rectangle.

N

$= N$

Double it.

N	N

$= 2N$

Add 6.
(See Figure 7.)

Divide by 2.
(See Figure 8.)

Subtract 3.

$\frac{1}{2}N$	$\frac{1}{2}N$

$= N$

50. 6

51. 2

In Exercises 52–57, each case could be explained by the Distributive Property and knowledge of place value.

52. True. 432 = 400 + 32. So
$50 \times 432 = 50(400 + 32)$ and
$50(400 + 32) = 50 \times 400 + 50 \times 32$.

53. True. $50 \times 368 = 50(400 - 32)$
$= 50 \times 400 - 50 \times 32$

54. False. If the equation involved subtraction instead of addition, then it would be true.
$50 \times 800 = (50 \times 1{,}000) - (50 \times 200)$, since
$800 = 1{,}000 - 200$.

55. False. $90 \times 70 = (90 \times 30) + (90 \times 40)$;
$90 \times 30 > 70 \times 20$ and
$90 \times 40 > 50 \times 20$, so
$(90 \times 30) + (90 \times 40) > (70 \times 20) + (50 \times 20)$.
Alternatively,
$90 \times 70 = 9 \times 7 \times 100 = 9 \times 7 \times 5 \times 20$.
$9 \times 7 \times 5 \neq 70 + 50$, though, because
$70 + 50$ is even and $9 \times 7 \times 5$ is odd.

56. False. 50 is not multiplied by the sum (400 + 32). It is added to the product of 400 and 32.

57. True. $6 \times 17 = 6(20 - 3) = 6(20) - 6(3)$.
Each expression is 102.

58. Yes; each expression has a value of 12.

59. **a.** Mrs. Lee is correct. Because you do multiplication before subtraction, Mrs. Lee's expression will calculate the area of the yard and swing set first, then take the difference of those areas to find the remaining area.

b. Mr. Lee is correct. Because you operate in parenthesis first, Mr. Lee's expression will calculate the difference in length first to find the length of the lawn, then multiply by the width to find the area.

Figure 7

N	N	6

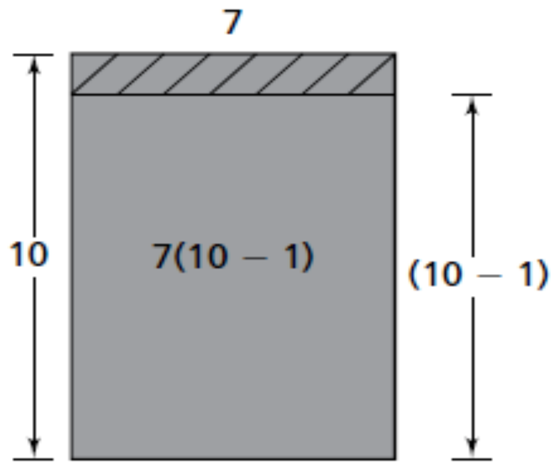
$= 2N + 6$

Figure 8

$\frac{1}{2}N$	$\frac{1}{2}N$	3

$= N + 3$

60.



The expression in expanded form is
7 × 10 – 7 × 1.
If we simplify within parentheses first, we find the expression is equal to 63:
7(10 – 1) = 7 × 9 = 63.
If we distribute the 7, we find that the expression is still equal to 63:
7(10 – 1) = 7 × 10 – 7 × 1 = 70 – 7 = 63.

61. There are 36 • 12 = 432 trading cards and 36 • 2 = 72 stickers.

62. 30(12 – 3) = 30 • 9 = 270, or $270. Alternatively, students might find the cost for all students 30 • 12 = 360 and then subtract the total discount 30 • 3 = 90 for a total of 270, or $270.

63. Tuesday's high temperature is 3 degrees colder than Sunday's high temperature. Students could use a variable, n, to represent Sunday's temperature. Then Tuesday's temperature can be represented by $n + 5 - 8$, which simplifies to $n - 3$, so Tuesday's high temperature is 3 degrees colder than n, Sunday's temperature.

Another method is to choose a few examples to see the relationship. Suppose Sunday's high temperature is 60 degrees. Then Monday's high temperature is 65 degrees, and Tuesday's high temperature is 57 degrees. For any starting amount (Sunday's high temperature), Tuesday's high temperature will be 3 degrees colder.

64. Elijah collected $264, $192 for the school and $72 for his homeroom. Students might calculate the two parts first: 24 • 8 = 192 (school) and 24 • 3 = 72 (homeroom). Solving it this way uses the Distributive Property, because 24(11) = 24(8 + 3) = 24(8) + 24(3).

65. $360. One way to solve this is to multiply 15(6)(4) = 360. Another number sentence is 15(1 + 3 + 2)4 = 360. In the second equation, students could distribute either the 15 or the 4 to each addend inside the parentheses.

Connections

66. A

67. 3,500

68. 1,750

69. 100

70. 6,000

71. 938

72. 3,200

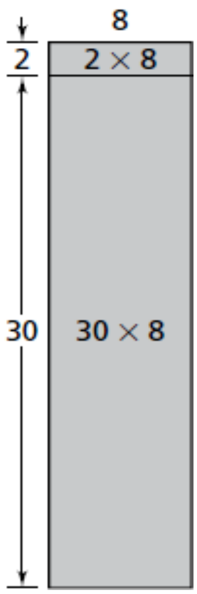
73. 100

74. 900

75. 4 × 5 = 4 × (3 + 2)
= (4 × 3) + (4 × 2)

76. **a.** 32 × 12 is the area of a rectangle with dimensions 32 and 12. The sum of the areas of the bottom two rectangles in Jim's figure is 32 × 2 = (30 + 2) × 2 = 30 × 2 + 2 × 2 = 60 + 4 = 64, the first partial product in the example. The sum of the areas of the upper two rectangles is 32 × 10 = (30 + 2) × 10 = 30 × 10 + 2 × 10 = 300 + 20 = 320, the second partial product. By adding these areas together, we get the final product, 384.

b. $32 \times 8 = (2 + 30) \times 8 = (2 \times 8) + (30 \times 8)$
$= 16 + 240 = 256$



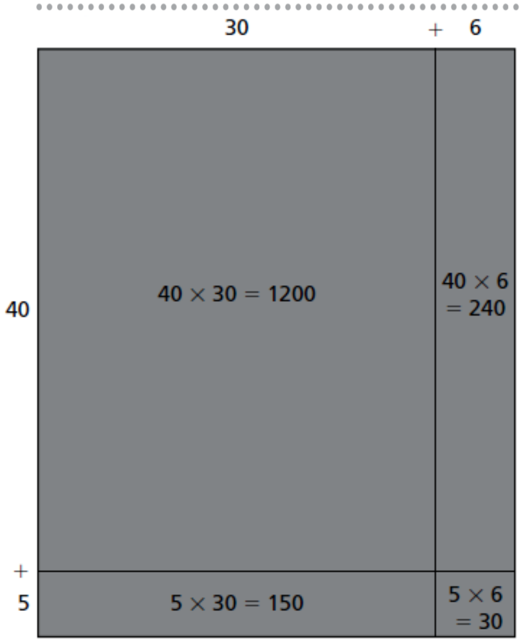
c. Jim's rectangle:
(See Figure 9.)
1,200 + 240 + 150 + 30 = 1,620

Basilio's method:
$45 \times 36 = 45(30 + 6) = 1{,}350 + 270 = 1{,}620$

77. n must be 2 less than a multiple of 5.
$3(n + 2)$ is a multiple of 5, and 3 is not a multiple of 5, so $(n + 2)$ must be a multiple of 5. Therefore, n must be 2 less than a multiple of 5. (i.e., n must be 3, 8, 13, etc.)

78. n = 0, 2, or 6. The factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24. If a number r is a factor of another number q, then the prime factorization of r is a subset of the prime factorization of q.
$24 = 2 \times 2 \times 2 \times 3$
$3(n + 2) = 3 \times 1$ or 3×2 or $3 \times 2 \times 2$ or $3 \times 2 \times 2 \times 2$.
So $n + 2 = 1$ or 2 or 2×2 or $2 \times 2 \times 2$.
$(n + 2)$ cannot equal 1, because then n would be negative, not a whole number.
If $n + 2 = 2$, then $n = 0$.
If $n + 2 = 2 \times 2$, then $n = 2$.
If $n + 2 = 2 \times 2 \times 2$, then $n = 6$.

Figure 9



79. n = 1. The factors of 20 are 1, 2, 4, 5, 10, and 20.

n cannot be 0 because 4(0) + 6 = 6, which is not a factor of 20.
n could be 1, because 4(1) + 6 = 10, which is a factor of 20.
n cannot be 2 because 4(2) + 6 = 14, which is not a factor of 20.
n cannot be 3 because 4(3) + 6 = 18, which is not a factor of 20.
n cannot be 4 because 4(4) + 6 = 22, which is greater than 20.
n cannot be any whole number greater than 4 because any whole number greater than 4 will also result in a number greater than 20.
Therefore, n = 1.
The reasoning for Exercise 78 may also be used for Exercise 79, but it results in more instances where N would not be a whole number, and therefore such a method may feel less efficient.

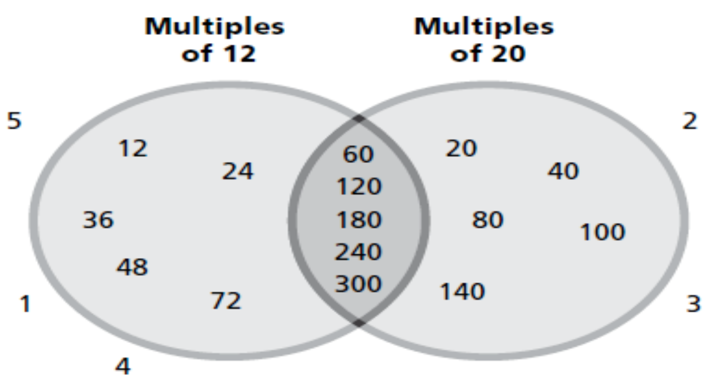
Extensions

80. a. Possible answer: Multiples of 12 that are not multiples of 20 include 12, 24, 36, 48, and 72. Multiples of 12 AND 20 include 60, 120, 180, 240, and 300. Multiples of 20 that are not multiples of 12 include 20, 40, 80, 100, and 140. Multiples of neither include 1, 2, 3, 4, and 5.
(See Figure 10.)

b. The multiples in the intersection are all divisible by 60.

81. Either two of the numbers are odd or two are even. If the first number is odd, you will have two odd numbers in the group. If the first number is even, you will have two even numbers.

Figure 10



82. a. Yes, Mirari is correct. Since every third number on the number line is divisible by 3, any three consecutive whole numbers must include one that is divisible by 3.

b. No, Gia's statement is not true. For example, 12 + 13 + 14 = 39, which is not divisible by 6 because it is not even. The sum of three consecutive numbers is divisible by 6 only if the first number is odd.

Let n be any whole number. Then $2n + 1$ is odd. The sum of the three consecutive numbers beginning with an odd number is $(2n + 1) + (2n + 2) + (2n + 3) = 6n + 6$, which is divisible by 6. If the consecutive numbers began with an even number, then the sum could be represented by $2n + (2n + 1) + (2n + 2) = 6n + 3$, which has a remainder of 3 when divided by 6.

c. Yes, Kim is correct. Any 3 consecutive whole numbers will include a multiple of 3 and a multiple of 2. The product of any group of numbers in which two of the numbers have 2 and 3 as factors must be a multiple of 6 (i.e., the product must be divisible by 6).

d. The product of any four consecutive numbers is divisible by 2, 3, 4, 6, 8, 12, and 24. Of the four consecutive numbers, one must be a multiple of 3 and one must be a multiple of 4. In addition to the multiple of 4, there will be one other even number. Since the product has 4 and another even number as its factors, the product will have at least three 2's in its prime factorization. Therefore, the product is divisible by $2 \times 2 \times 2 = 8$. Since 8 and 3 are relatively prime, 3×8, or 24, is also a factor of the product. Therefore, all factors of 24 are also factors of the product.

83. 5

84. 6

85. a. Stage 5: 1 + 3 + 5 + 7 + 9 = 25

Stage 6: 1 + 3 + 5 + 7 + 9 + 11 = 36

Stage 7:
1 + 3 + 5 + 7 + 9 + 11 + 13 = 49

Stage 8:
1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = 64

b. $1 + 3 + 5 + \ldots + 39 = 20^2 = 400$

c. Stage 24; 47. The sum will be 576 in row 24, because $576 = 24^2$. The last number in this row is 47 because 47 is the twenty-fourth odd number. Notice that to find the nth odd number, you can multiply n by 2 and subtract 1. (This famous pattern is the sum of the consecutive odd numbers: The sum in each row is the square of the number of numbers in the row.)

86. a. 4 = 2 + 2, 6 = 3 + 3, 8 = 3 + 5, 10 = 3 + 7 or 5 + 5, 12 = 5 + 7, 14 = 7 + 7 or 3 + 11.

b. Possible answers: 100 = 3 + 97, 100 = 11 + 89, 100 = 17 + 83, 100 = 29 + 71, 100 = 41 + 59, 100 = 47 + 53.

c. No, because there are no even prime numbers other than 2. If you added the number 2 to an odd prime, the sum would be odd.

87. a. In general, Boris is not correct, but he is correct for 996. The proper factors of 996 are 1, 2, 3, 4, 6, 12, 83, 166, 249, 332, and 498. The sum of the factors is 1,356, which is greater than 996.

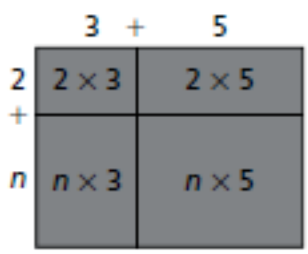
The proper factors of 975 are 1, 3, 5, 13, 15, 25, 39, 65, 75, 195, and 325. The sum of these is 761, which is less than 975, so 975 is not an abundant number, even though it has the same number of factors as 996.

b. No, there are no square numbers in the list. Square numbers have an odd number of factors, but the numbers in the list all have an even number of factors. Also, students may use their calculators to find that the square root of the least number, 975, is about 31.22. 31 × 31 = 961, which is less than the least number on the list, and 32 × 32 = 1,024, which is greater than the greatest number on the list.

88. a. Yes;

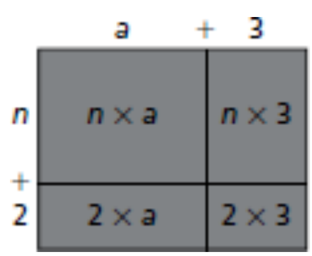
$$\begin{aligned} 36 \times 15 &= (30 + 6) \times 15 \\ &= (30 \times 15) + (6 \times 15) \\ &= 30 \times (10 + 5) + 6 \times (10 + 5) \\ &= 30 \times 10 + 30 \times 5 \\ &\quad + 6 \times 10 + 6 \times 5 \\ &= 300 + 150 + 60 + 30 \\ &= 540 \end{aligned}$$

b.



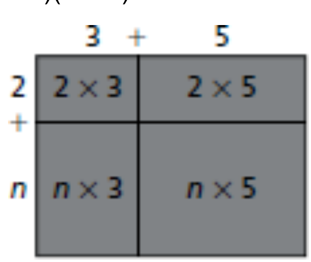
$$\begin{aligned} (2 + n)(3 + 5) &= (2 + n)(3) + (2 + n)(5) \\ &= 2(3) + n(3) + 2(5) + n(5) \\ &= 6 + 3n + 10 + 5n \\ &= 16 + 8n \end{aligned}$$

c.



$(n + 2)(a + 3) = an + 2a + 3n + 6$

d.



$$\begin{aligned} (a + b)(c + d) &= a \times c + a \times d \\ &\quad + b \times c + b \times d \\ &= ac + ad + bc + bd \end{aligned}$$

89. Let N and R be any whole numbers. Then $2N$ and $2R$ are even numbers. $2N + 1$ and $2R + 1$ are odd numbers.

a. $2N + 2R = 2(N + R)$. Since it has 2 as a factor, $2(N + R)$ is an even number. The sum of two even numbers is even.

b. $(2N + 1) + (2R + 1) = 2N + 2R + 2$

$= 2(N + R + 1)$

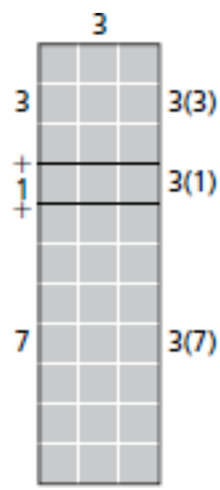
The sum of two odd numbers is even.

c. $2N + (2R + 1) = 2N + 2R + 1$

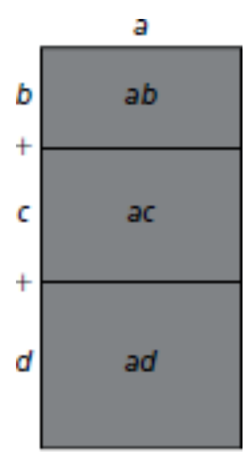
$= 2(N + R) + 1$

The number $2(N + R)$ is even. An even number plus 1 is odd, so $2(N + R) + 1$ is odd.

90. a.



b.



91. Answers will vary. Possible answers are $1 + 2 + 3 + 4 = 10$, $(1)(2)(3)(4) = 24$, $4 \div 2 + 3 - 1 = 4$, and $1^2 \times 3^4 = 81$.