AC 2009-2284: A COMPILATION OF EXAMPLES FOR USING EXCELL IN SOLVING HEAT TRANSFER PROBLEMS

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A Compilation of Examples for Using Excel for Solving Heat Transfer Problems

Abstract

Excel spreadsheet available on most desktop or laptop computers can serve as an effective and inexpensive computational tool in a heat transfer course. This paper focuses on the application of "Solver" and "Goal Seek" functions of Excel in solving those heat transfer problems requiring iteration solution process. It provides a collection of examples demonstrating the application of Excel in solving heat transfer problems. Some of the examples have been previously presented at various conferences including regional meetings, but not all can be easily accessed. The paper is augmented with additional example to expand the range heat transfer problem areas previously presented. Therefore, one aim of the paper is to provide a choice for selection of examples for integration into a heat transfer course. Some of the examples provided in this paper can be easily integrated into an introductory undergraduate heat transfer course. Those examples employing higher level mathematical functions or numerical schemes can be used in an advanced undergraduate or an introductory graduate level heat transfer course. The procedures and examples presented in this paper were well received by undergraduate and graduate students enrolled in an introductory graduate level heat transfer course.

Introduction

In an introductory undergraduate heat transfer course the coverage of topics includes introductions to basic modes of heat transfer, solutions of steady state and transient conduction problems, free and forced convection, and an exposure to radiation heat transfer. Analytical solutions are typically limited to one-dimensional steady-state heat conduction problems, one-dimensional transient conduction problem subject to simplest form of boundary condition, and evaluation of radiation view factors for objects displaying simple geometries. Solutions to heat convection problems are based on the empirical formulas provided in the textbooks. To demonstrate the application of heat transfer concepts, the course coverage typically includes one-dimensional heat conduction in fins of uniform cross-sectional area and the analysis of parallel or counter flow heat exchangers. Many of the more complex analytic solutions to heat transfer problems given in the textbooks[1-15] are in forms of graphs or charts. A few examples include graphs for fin efficiencies, transient temperature distribution charts for heat transfer in slabs, cylinders, or spheres (Heisler Charts), heat exchanger correction factors, NTU-effectiveness charts, and radiation shape (view) factor charts. Many mechanical engineering programs also offer a more advanced general heat transfer course to serve advanced undergraduate or entry level graduate students. The duel level course provides a more in-dept coverage of the topics included in an undergraduate heat transfer course. Introductions to condensation and boiling heat transfer processes may also be included in dual level course coverage. Integration of computational tool in a heat transfer course is an effective way to aid students in solving more complex problems, especially those requiring an iterative trial and error approach.

Prior to the introduction of personal computers (PCs) in the early 1980's, complex computer codes were needed for numerical solution of heat transfer problems. Access to mainframe

computers and proficiency in such programming languages as FORTRAN and PASCAL were necessary for solving complex heat transfer problems. As the personal computers became more available and affordable, and as the operating systems became more user friendly, their applications were gradually integrated into introductory heat transfer courses. Simple programming languages such as BASIC were used for solving simple heat transfer problems in late 1980s or early 1990s. During this period, all mechanical engineering programs required a course in one of the structured computer programming languages. However, in more recent years, many degree programs no longer require a course in one of the structured programming languages. The trend is now shifted toward using software packages to solve problems numerically.

Currently, many publishing companies provide computer software with heat transfer textbooks[10-15]. The most commonly used software packages accompanying heat transfer text books are Interactive Heat Transfer (IHT)[16] and Engineering Equation Solver (EES)[17]. These programs are general purpose, non-linear equation solvers with built-in property functions. They are capable of exploring and graphing the effects of change in variables on the solution to a given problem. The most significant advantage of these software programs is that no prior knowledge of programming language is necessary in their applications. Other software packages are also available in the market that could be employed for solving heat transfer problems. Most of these software packages are extremely useful tools for heat transfer analysis and design in an undergraduate or an introductory graduate level heat transfer courses. These include Microsoft Excel spreadsheet, Mathcad, MATLAB, and Maple. All these software programs can be used to solve open-ended problems or parametric studies of heat transfer problems. Excel, which is available on almost all desktop or laptop computers, is an example. Recently, we compared these software packages for cost and ease of application for integration into a heat transfer course[18].

This paper focuses on the application of Microsoft Excel in solving heat transfer problems. It will provide several examples demonstrating the use of "Solver" and "Goal Seek" tools of Excel in solving problems requiring iterative processes. The examples include solutions to heat transfer problems involving: i) one-dimensional conduction in fins, ii) one-dimensional transient conduction, iii) transient conduction in a semi-infinite region, iv) two-dimensional conduction using finite difference formulation, v) laminar flow over an isothermal flat plate, and iv) heat exchanger analysis. Some of examples given in this paper for problems involving heat conduction in fins, heat exchangers, and solution of boundary layer problems were presented previously by the author and others in various regional and national conferences. Since not all the papers previously presented might be easily accessible to all readers, some of the examples are repeated in this paper. However, the paper is augmented by examples covering three additional heat transfer areas as listed above. Therefore, this paper provides a wider range of choice of examples for integration into a heat transfer course.

Excel Spreadsheet

It has been shown[19-26] that Excel is an effective computational tool for solving heat transfer problems. Functions included in this software include 39 engineering functions, as well as various math and trigonometry functions. Among the engineering functions are Bessel

functions, error functions, and other functions appearing in heat transfer equations. To use functions in the Excel worksheet, the insert button on the Excel menu bar is clicked. Then selecting function among a list of options, a dialogue box appears on the screen, as shown on Fig. 1. One can search for the desired function by typing a description of the function (financial, engineering, etc.) in the search box or using the "select category" box by scrolling through options for the desired function.

For problems requiring iterative calculations, the "Goal Seek" or "Solver" tools can be employed. By using the tool menu and selecting the solver option a dialog box appears, as shown in Fig 1. By selecting the target cell and fixing the desired value for that cell, values in the selected cells automatically change to correspond to the solution given for the target cell. This will be demonstrated later in several examples.

The following sections demonstrate how "Goal Seek" or "Solver" functions of Excel can be used as a tool to solve heat transfer problems requiring trial and error processes. Solutions to several example problems requiring trial and error iterative processes are presented to demonstrate the effectiveness of Goal Seek and Solver functions of Excel.

Examples of Application of Excel

A. One-Dimensional Heat Conduction in Fins

The coverage of the analytical solution of conduction in fins in undergraduate heat transfer textbooks is usually limited to fins of uniform cross-sectional area. For more complex fin configurations, only efficiency charts are provided in most heat transfer textbooks[1-15]. Analysis for fins of variable cross-sectional areas or annular fins results in more complex differential equations. The solutions for temperature distribution involve complex functions such as Bessel functions. The analyses for these types of fins are not typically fully covered in an introductory heat transfer course. Instead the results are shown in the form of fin efficiency charts.

The fin efficiency is defined as

$$\eta_f = \frac{q_{act}}{q_{max}} = \frac{q_{act}}{hA(T_o - T_\infty)} \tag{1}$$

where, T_o and T_∞ are the base of the fin and the ambient temperatures, respectively, h is the heat transfer coefficient, A is the fin surface area, q_{act} denotes the actual heat transfer, q_{max} represents the maximum theoretical heat transfer by assuming that the entire fine is at the base temperature.

Fin efficiency charts approximate the rate of heat transfer, but do not provide any information on the temperature distribution in fins. Microsoft Excel, can be a useful tools in solving heat conduction problems for a variety of fin configurations. Several modern textbooks[9-12] provide expressions for the efficiency of most common fin shapes. Somerton, *et.al.*[19] and Karimi[20] have demonstrated the use of Excel spreadsheet in solving one dimensional heat conduction problems in fins. The followings are two examples[20].
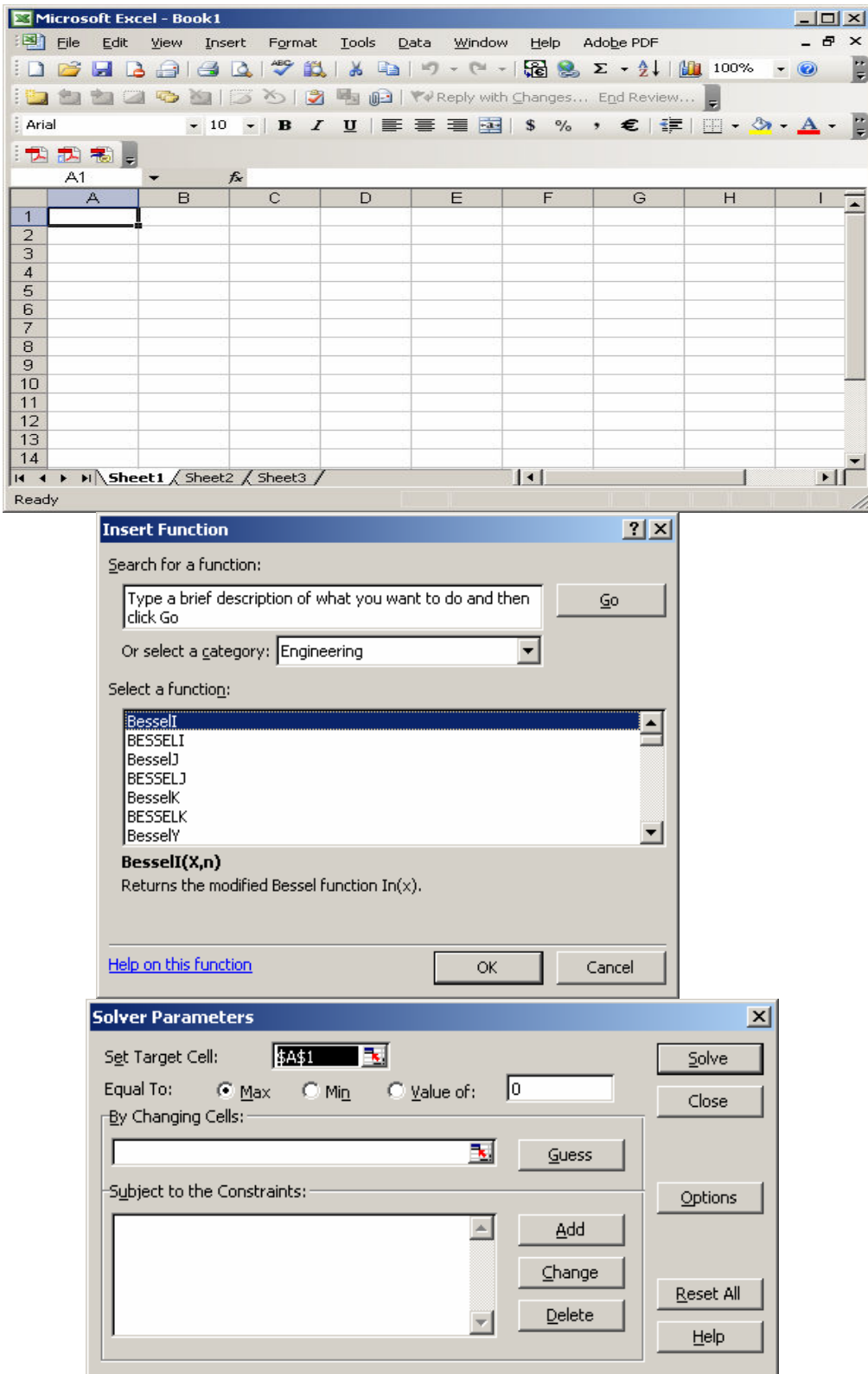
Fig. 1. Excel worksheet, function selection menu, and solver dialogue box

Example 1

A straight fin of triangular profile (axial section) 0.1 m in length, 0.02 m thick at the base, and 0.2 m in depth is used to extend the surface of a wall at 200°C. The wall and the fin are made of mild steel (k = 54 W/m.°C). Air at 10 °C (h = 200 W/m²·°C) flows over the surface of the fin. Evaluate the temperature at 0.05 m from the base and at the tip of the fin. Determine the rate of heat removal from the fin and the fin efficiency.
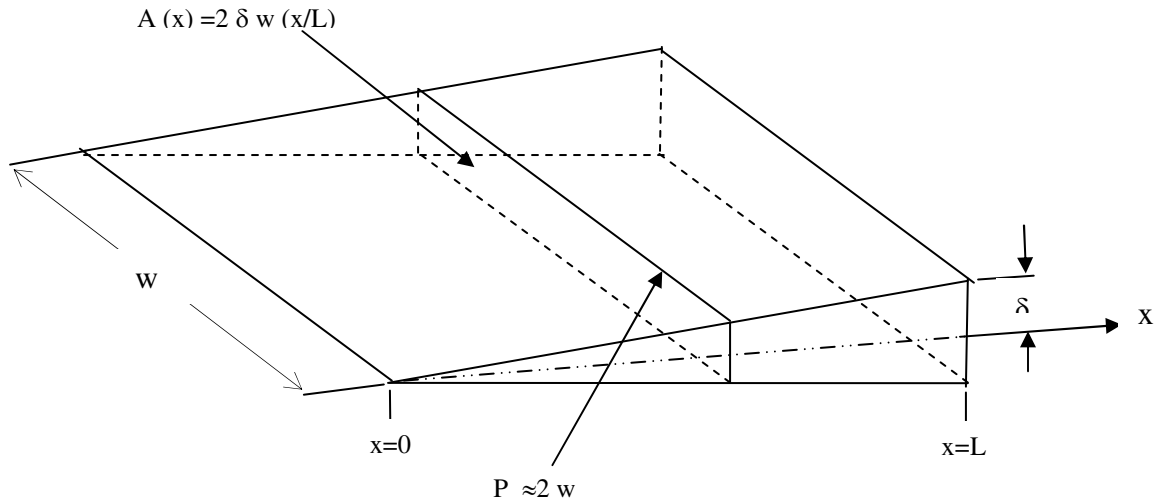

Fig. 2. Sketch of triangular fin in Example 1

Solution

An analytical solution to this problem[7] gives the following expression for the dimensionless temperature distribution

$$\theta = \frac{T - T_\infty}{T_L - T_\infty} = \frac{I_o\left(2\sqrt{hLx/k\delta}\right)}{I_o\left(2\sqrt{hL^2/k\delta}\right)} = \frac{I_o\left(m\sqrt{Lx}\right)}{I_o(mL)} \tag{2}$$

where, L is the length of the fin, x is the distance from the tip of the fin, δ is one half of the thickness at the base, $m = 2\sqrt{h/k\delta}$, and I_o is the modified Bessel function of the first kind of order zero.

The rate of heat removal can be calculated by evaluating heat transfer at the base of the fin, where x=L.

$$q = -kA\frac{dT}{dx}\bigg|_{x=L} \tag{3}$$

Thus the rate of heat transfer at the base can be expressed by

$$q = -2w\sqrt{hk\delta}\,(T_o - T_\infty)\frac{I_1\left(2\sqrt{hL^2/k\delta}\right)}{I_o\left(2\sqrt{hL^2/k\delta}\right)} \tag{4}$$

where, w represents the width of the fin. The rate of heat removal from the base is equal to –q. Therefore, the fin efficiency can be determined by the following relation

$$\eta_f = \frac{-q}{2h\sqrt{L^2 + \delta^2}\left[w(T_o - T_\infty)\right]} \tag{5}$$

The formulation of solution in Excel for this problem is shown in Fig. 3. The data given in the problem statement are first entered into the cells of the worksheet. Using these data, the formulas for the evaluation of m, mL, $m\sqrt{xL}$, I_o(mL), I_1(mL), I_o(mL), $I_o(m\sqrt{xL})$, θ, T, q, and η are entered into appropriate cells of the worksheet. To enter formulas an "=" sign is first entered into the cell followed with the terms needed for the evaluation of the formula. The basic mathematic operators used are +, -, * (multiplication), /, and ^ (power). The calculated results are presented in Fig. 4. By pressing CTRL + ` (grave accent) one can switch between the worksheet displaying formulas and their resulting values.

The worksheet shown in Fig. 4 can be expanded to evaluate the temperature profile in the fin and plot the results. To achieve this, the values for x ranging between 0 and 0.1 are entered in column A (cells A16 through A-26), as shown in Fig. 5. Then the cells B16 through E16 are highlighted and copied into lower rows by clicking on the bottom boundary corner of cell E16 and dragging it all the way to cell E26. By this copying action the values of $m\sqrt{xL}$, $I_o(m\sqrt{xL})$, θ, and T are automatically calculated for each value of x listed in column A. To plot T as a function of x, the cells A15 through A26 and E15 through E26 were first highlighted by pressing the Ctrl and the mouse appropriate key (usually left key) while moving the cursor over the indicated cells. Then by clicking the chart wizard icon on the menu bar of the worksheet, a menu appears offering several standard options for plotting data. The x-y (scatter) option was selected and the four steps of chart wizard were preformed by providing the necessary information in each step and pressing the next button. Finally the Finish button was pressed to show the results in the worksheet.

It should be noted that the derivation of equations for temperature profile and heat transfer are based on the assumption of one-dimensional heat conduction in the axial direction of the fin. For this assumption to be valid, the Biot number, Bi, must satisfy the following condition

$$Bi = \frac{hL_{ch}}{k} = \frac{h(A/P)}{k} \prec 0.1 \tag{6}$$

where, L_{ch} is a characteristic length, A is the cross sectional area, and P is the perimeter of the fin. For fins of circular cross sectional area, L_{ch} can be represented by the radius, R.
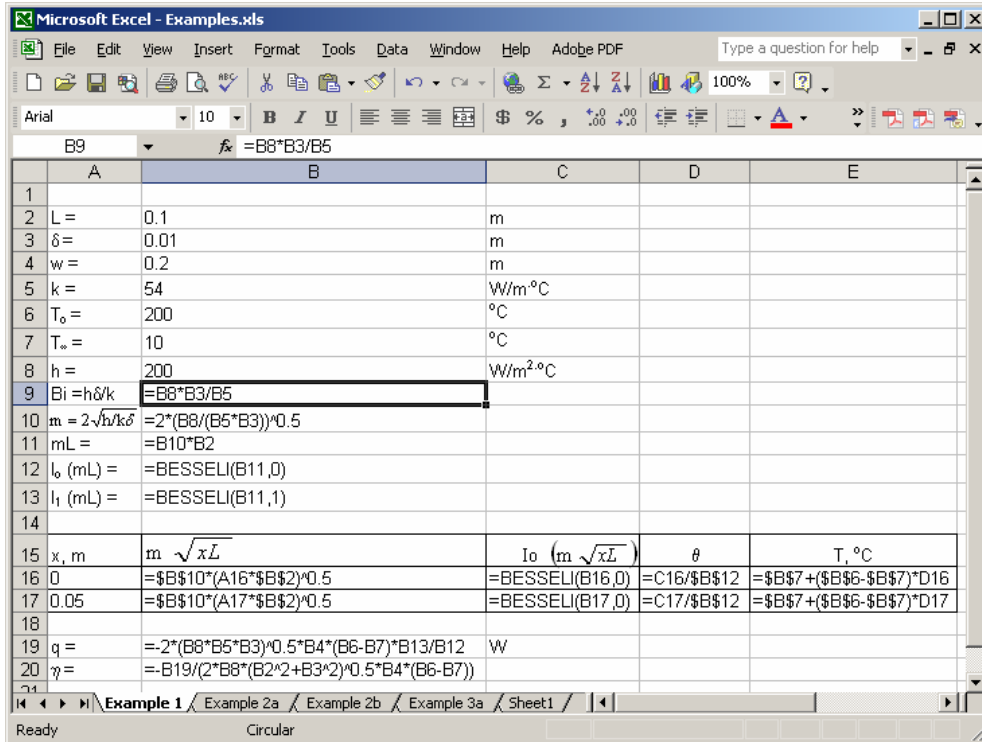
Microsoft Excel - Examples.xls

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B9 f_x =B8*B3/B5

	A	B	C	D	E
1					
2	L =	0.1	m		
3	δ =	0.01	m		
4	w =	0.2	m		
5	k =	54	W/m·°C		
6	T_o =	200	°C		
7	T_∞ =	10	°C		
8	h =	200	W/m²·°C		
9	Bi =hδ/k	=B8*B3/B5			
10	m = 2$\sqrt{h/k\delta}$	=2*(B8/(B5*B3))^0.5			
11	mL =	=B10*B2			
12	I_o (mL) =	=BESSELI(B11,0)			
13	I_1 (mL) =	=BESSELI(B11,1)			
14					
15	x , m	m $\sqrt{xL}$	Io $\left(m\sqrt{xL}\right)$	θ	T, °C
16	0	=$B$10*(A16*$B$2)^0.5	=BESSELI(B16,0)	=C16/$B$12	=$B$7+($B$6-$B$7)*D16
17	0.05	=$B$10*(A17*$B$2)^0.5	=BESSELI(B17,0)	=C17/$B$12	=$B$7+($B$6-$B$7)*D17
18					
19	q =	=-2*(B8*B5*B3)^0.5*B4*(B6-B7)*B13/B12	W		
20	η =	=-B19/(2*B8*(B2^2+B3^2)^0.5*B4*(B6-B7))			
21					

Example 1 / Example 2a / Example 2b / Example 3a / Sheet1 /

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Fig. 3. Excel formulation of the solution for problem in Example 1

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E22 f_x

	A	B	C	D	E	F
1						
2	L =	0.1	m			
3	δ =	0.01	m			
4	w =	0.2	m			
5	k =	54	W/m·°C			
6	T_o =	200	°C			
7	T_∞ =	10	°C			
8	h =	200	W/m²·°C			
9	Bi =hδ/k	0.03704				
10	m = 2$\sqrt{h/k\delta}$	38.49				
11	mL =	3.8490				
12	I_o (mL) =	9.9248				
13	I_1 (mL) =	8.51004				
14						
15	x , m	m$\sqrt{xL}$	Idm$\sqrt{xL}$	θ	T, °C	
16	0	0	1	0.101	29.144	
17	0.05	2.7217	3.9076	0.394	84.807	
18						
19	q =	-677.23	W			
20	η =	0.4433				
21						

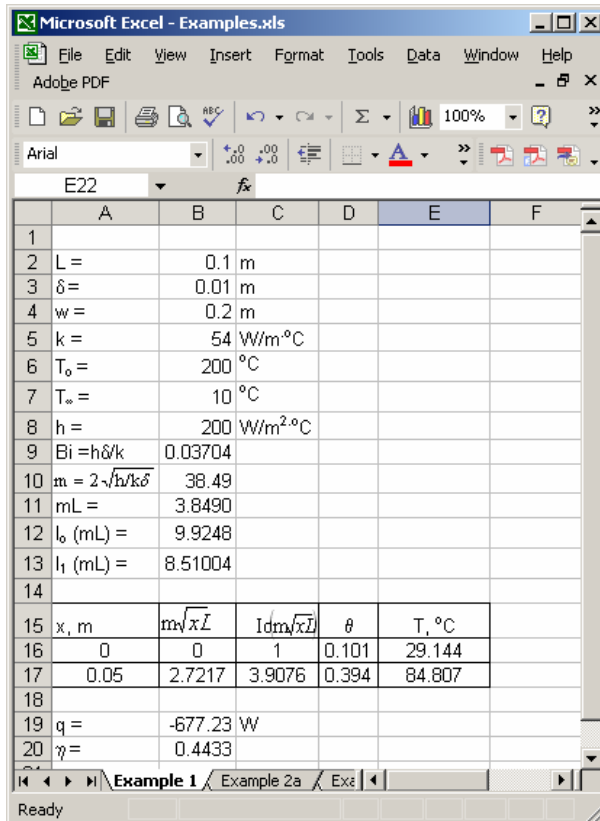Example 1 / Example 2a / Exa

Ready
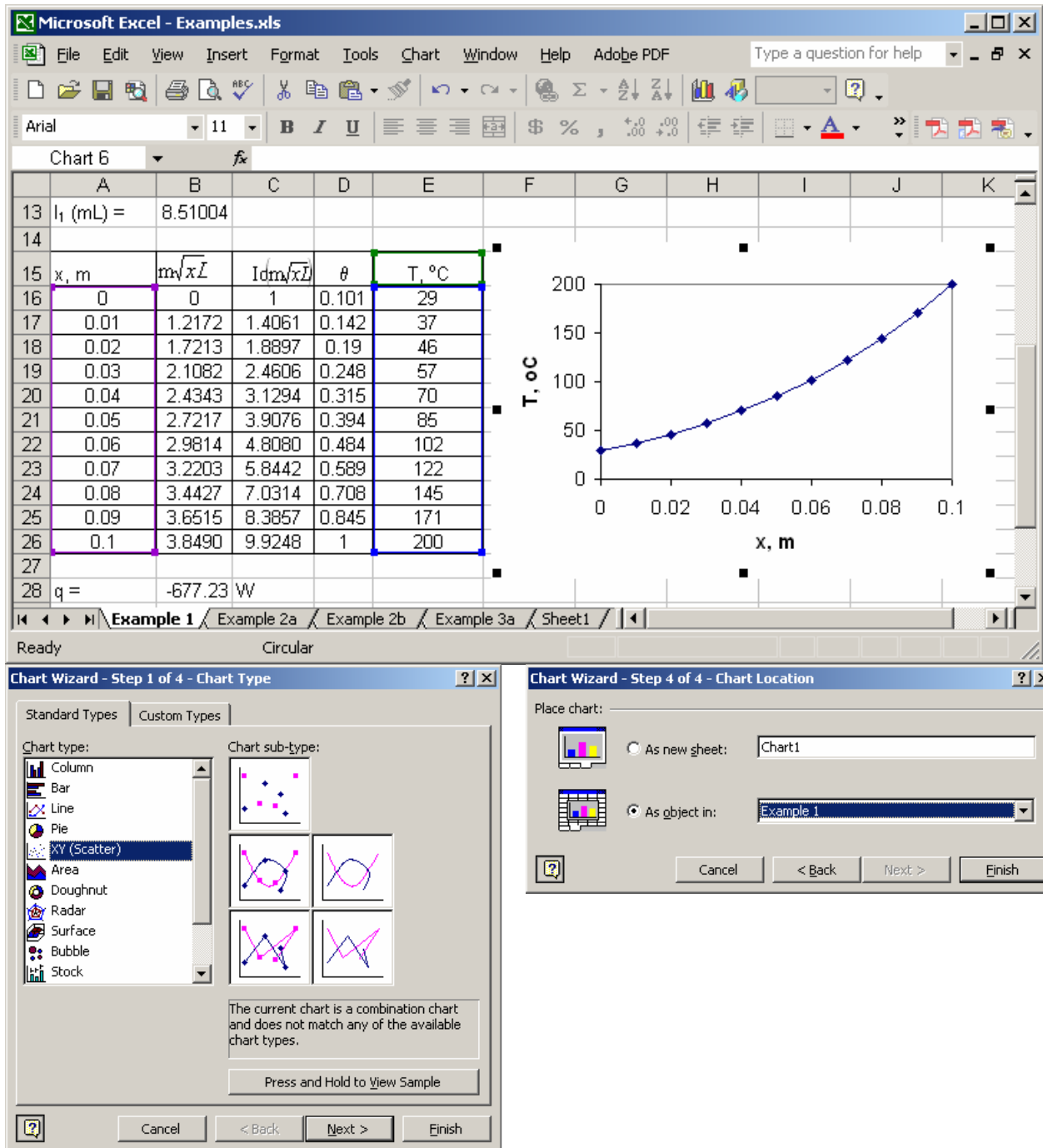
Fig. 4. Solution to Example problem 1

Fig. 5. Procedure for the evaluation and plotting of the temperature profile in Example 1

Example 2

A fin of triangular profile (axial section) 0.1 m in length, 0.02 m thick at the base, 0.2 m in depth is used to extend the surface of a wall at 200°C. The wall and the fin are made of mild steel (k = 54 W/m·°C). Air at 10°C (h = 200 W/m²·°C) flows over the surface of the fin. Evaluate the distance from the base where the temperature is 175°C.
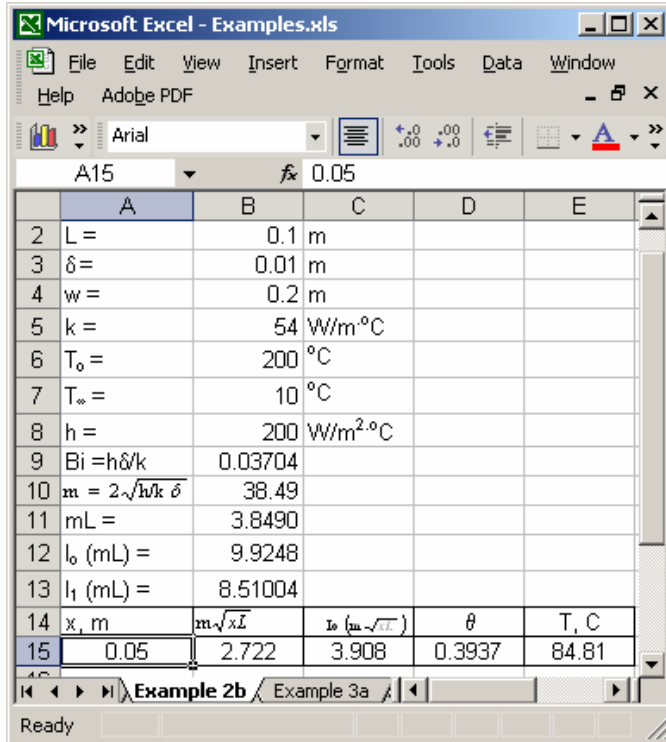
Solution

The solution to this problem is based on the same equations used in the previous example. However, in this case the distance, x, cannot explicitly be determined, since it is a part of the argument for Bessel function in Eq. 2. A trial and error procedure is required to solve this problem.

An Excel spreadsheet can be used to solve this problem. One method is to use the same solution used in example 1, but in this case the values of x in the spreadsheet can be changed to achieve the desired temperature. The result of this procedure is shown in Fig. 6.

A simpler way to solve the problem is to take the advantage of "Goal Seek" tool in Excel. The procedure and the final solution are shown in Fig.7. Figure 7-a shows the value of the temperature at an arbitrary position in the fin. By using the tool menu and selecting the Goal Seek option a dialog box appears, as shown in Fig 7. The target cell (temperature in this case, cell E16) then is selected and its value is set to a desired value for that cell (175). The cell that its value must be changed is identified (cell A16). After clicking on the Solve button, the value in the selected cell A16 (x) automatically changes to a value that yields the desired temperature of 175 $^\circ$C in the target cell (E16). The solution is presented in Fig. 7-b.

	A	B	C	D	E	F
2	L =	0.1	m			
3	δ =	0.01	m			
4	w =	0.2	m			
5	k =	54	W/m·$^\circ$C			
6	T_o =	200	$^\circ$C			
7	T_∞ =	10	$^\circ$C			
8	h =	200	W/m$^{2.\circ}$C			
9	Bi =hδ/k	0.03704				
10	m = $2\sqrt{h/k\delta}$	38.49				
11	mL =	3.8490				
12	I_o (mL) =	9.9248				
13	I_1 (mL) =	8.51004				
14	x, m	$m\sqrt{xL}$	$I_o (m \sqrt{xL})$	θ	T, C	
15	0	0	1	0.1008	29.14	
16	0.05	2.722	3.908	0.3937	84.81	
17	0.06	2.981	4.808	0.4844	102.04	
18	0.07	3.220	5.844	0.5889	121.88	
19	0.08	3.443	7.031	0.7085	144.61	
20	0.09	3.651	8.386	0.8449	170.54	
21	0.091	3.672	8.531	0.8596	173.32	
22	0.092	3.692	8.678	0.8744	176.13	
23	0.0915	3.682	8.604	0.8670	174.72	
24	0.09155	3.683	8.612	0.8677	174.86	
25	0.09156	3.683	8.613	0.8678	174.89	
26	0.09157	3.683	8.615	0.8680	174.92	
27	0.09158	3.683	8.616	0.8681	174.95	
28	0.091598	3.684	8.619	0.8684	175.00	

Fig. 6 Solution of Example 2 by a trial and error procedure

(a) Initial guess
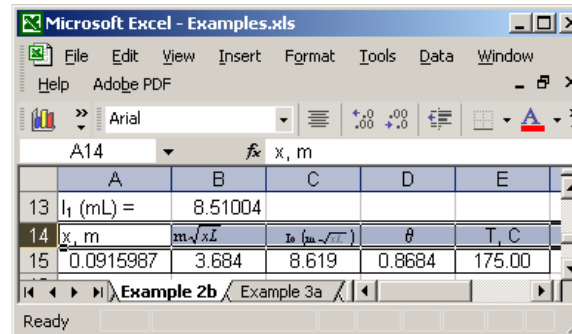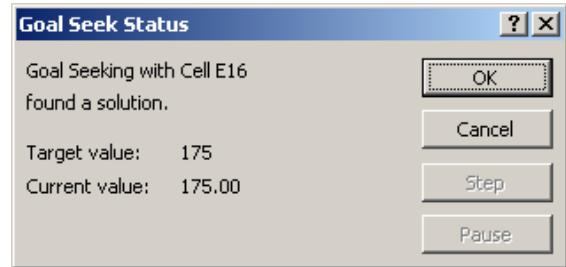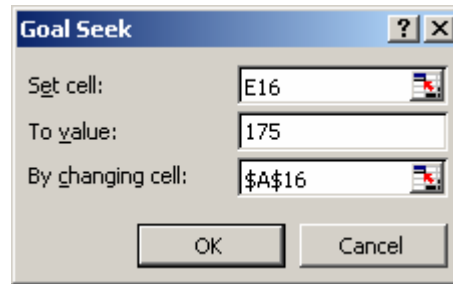
(b) Final solution

Fig 7   Procedure of using the Goal Seek tool to find x where T =175°C.

B. Transient One Dimensional Heat Conduction

Consider a wall of thickness L, initially at a uniform temperature of T_i. One surface of the wall (at x= 0) is insulated and the other surface temperature (at x =L) is suddenly lowered to T_∞. The analytical solution for the temperature profile of this conduction problem is presented in most heat transfer textbooks and the dimensionless temperature profile can be expressed as[27]

$$\theta = \frac{T - T_\infty}{T_i - T_\infty} = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)} \cos\left(\lambda_n \frac{x}{L}\right) Exp\left(-\lambda_n^2 . Fo\right) \tag{7}$$

where $\lambda_n = \frac{(2n-1)\pi}{2}$ ($n = 1,2,3,........$) and $Fo = \frac{\alpha t}{L^2}$

When Fo > 0.2, only a few terms of the series solution in Eq. (7) is necessary for the evaluation of the temperature profile[9]. Excel can be used to show this behavior. Figure 8 shows the evaluation of Θ at x/L = 0.5 when Fo = 0.2, using Excel. It shows that the fourth term in series solution improves the accuracy of Θ only by 0.001 (or 0.1 %). For Fourier numbers grater than 0.2, even less number of terms are necessary for the evaluation Θ. However, for Fo < 0.2, more terms are necessary for the series in Eq. (7) to converge. For example, Fig. 9 shows that for Fo = 0.01 and x/L = 0.99, over 90 terms are necessary for the series to fully converge.

Excel is a useful tool for evaluating transient temperature distribution in a wall from Eq. (7). When a problem requires the evaluation of position, x, or time, t, from Eq. (7), a trial and error procedure is necessary which is very tedious. The Solver function of Excel can be employed to simplify the trial and error iteration process. The following example demonstrates the use of Solver function of Excel in solving one-dimensional transient conduction problems when an iteration process is required.
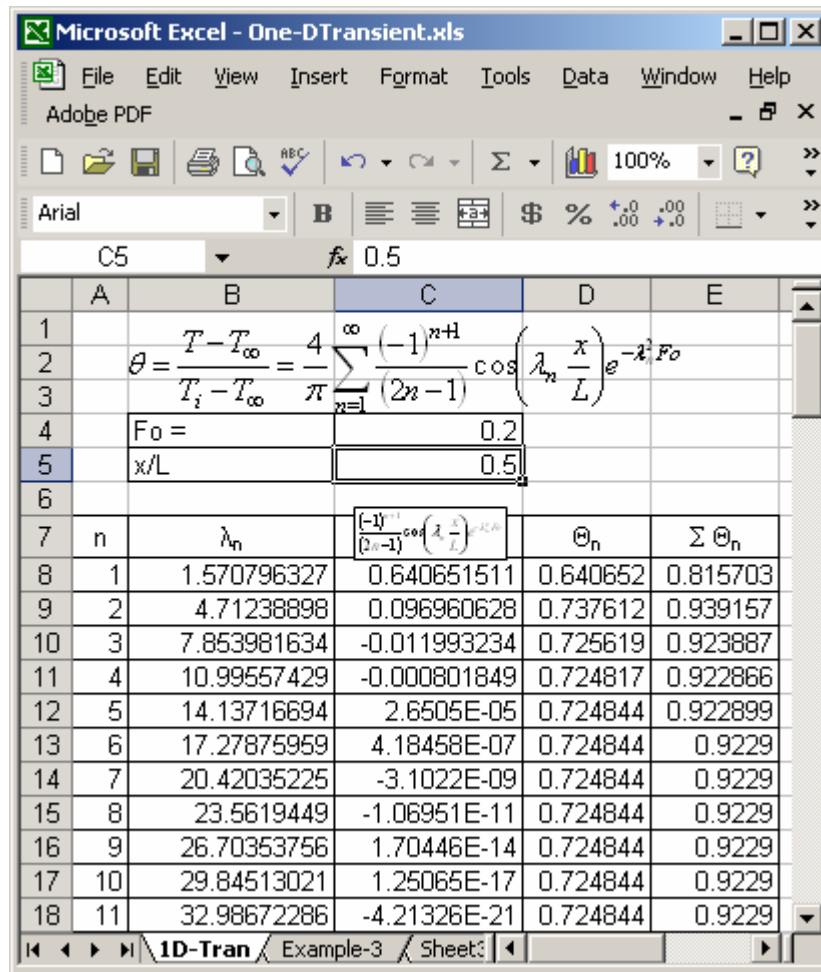

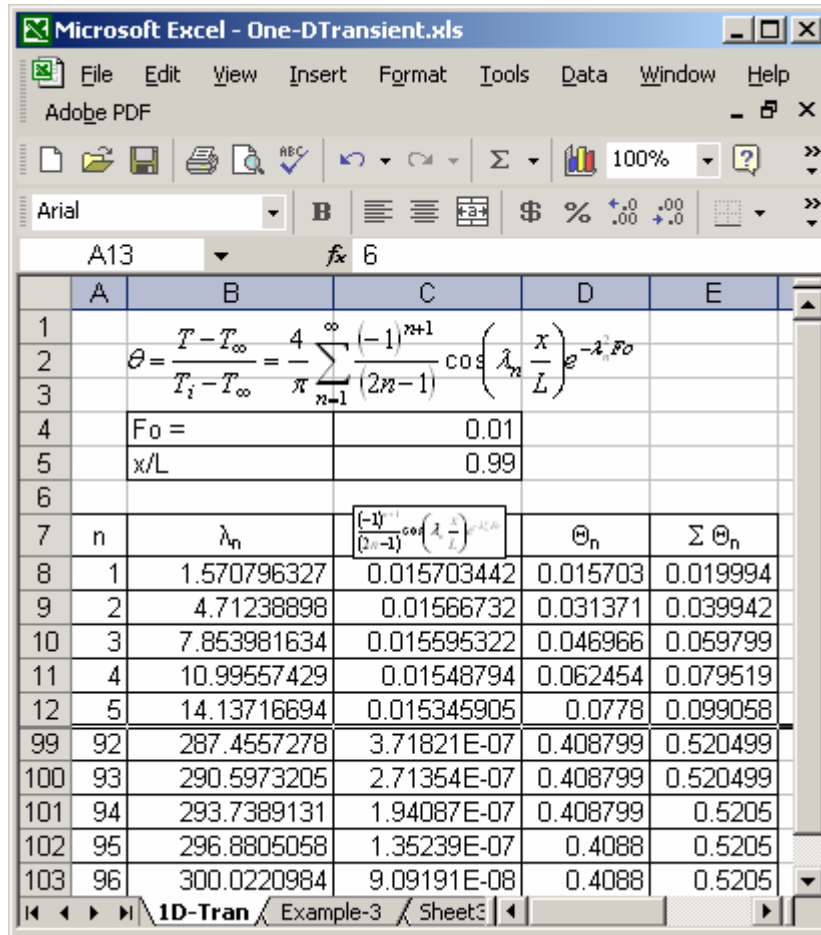Fig. 8 Evaluation of dimensionless temperature using Excel for a 1-D transient conduction problem, Fo = 0.2

The spreadsheet shows the following:

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Menu: File Edit View Insert Format Tools Data Window Help
Adobe PDF

Toolbar: Arial, 100%

Cell reference: A13, fx 6

Row 1–3 (cell B):

$$\theta = \frac{T - T_\infty}{T_i - T_\infty} = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(2n-1)}\cos\left(\lambda_n \frac{x}{L}\right)e^{-\lambda_n^2 Fo}$$

	A	B	C	D	E
4		Fo =		0.01	
5		x/L		0.99	
6					
7	n	λ_n	$\frac{(-1)^{n+1}}{(2n-1)}\cos\left(\lambda_n\frac{x}{L}\right)e^{-\lambda_n^2 Fo}$	Θ_n	$\Sigma\,\Theta_n$
8	1	1.570796327	0.015703442	0.015703	0.019994
9	2	4.71238898	0.01566732	0.031371	0.039942
10	3	7.853981634	0.015595322	0.046966	0.059799
11	4	10.99557429	0.01548794	0.062454	0.079519
12	5	14.13716694	0.015345905	0.0778	0.099058
99	92	287.4557278	3.71821E-07	0.408799	0.520499
100	93	290.5973205	2.71354E-07	0.408799	0.520499
101	94	293.7389131	1.94087E-07	0.408799	0.5205
102	95	296.8805058	1.35239E-07	0.4088	0.5205
103	96	300.0220984	9.09191E-08	0.4088	0.5205

Sheet tabs: 1D-Tran / Example-3 / Sheet3

Fig. 9 Evaluation of dimensionless temperature for a1-D transient conduction problem, Fo = 0.01, x/L = 0.99

Example 3

A stainless steel plate has a thickness of 6 cm and is initially at 400 $^{\circ}$C. Both surfaces of the plate are suddenly lowered to 60 $^{\circ}$C. Calculate the time required for the temperature at 0.5 cm below the surface to reache 350 $^{\circ}$C. The thermal diffusivity of plate is α = 4.4x10^{-6} m/s^2.
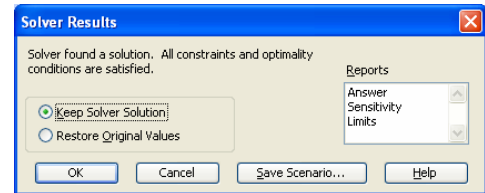
Solution

The solution to this problem can be obtained from Eq. (7). In this problem the center of the wall represents the insulated wall at x= 0, T_∞ represents the temperature at x = L when t > 0. Since the problem requires the evaluation of time, t, a trial and error process is necessary to solve this problem. The procedure, using Excel spreadsheet, and the final solution are shown in Fig.10. In this procedure, as shown in Fig. 10-a, an assumed time of t =180 seconds was used as a first guess and was entered into cell C7. Other parameters given in the problem statement were also entered into the appropriate cells of the spreadsheet. The value of x (measured from the center of the plate) was set to 0.025 meter in cell C8. Cells C10 and C11 were formulated for the evaluation of Fourier number and x/L, respectively. The values of n were entered in cells A14 through A115. For each n, the corresponding formulas for λ_n, the function inside the Σ sign of Eq. (7), and Θ_n were entered in columns C through E (rows 14 though 102), respectively. The value of T (x, t) was evaluated in cell F115.
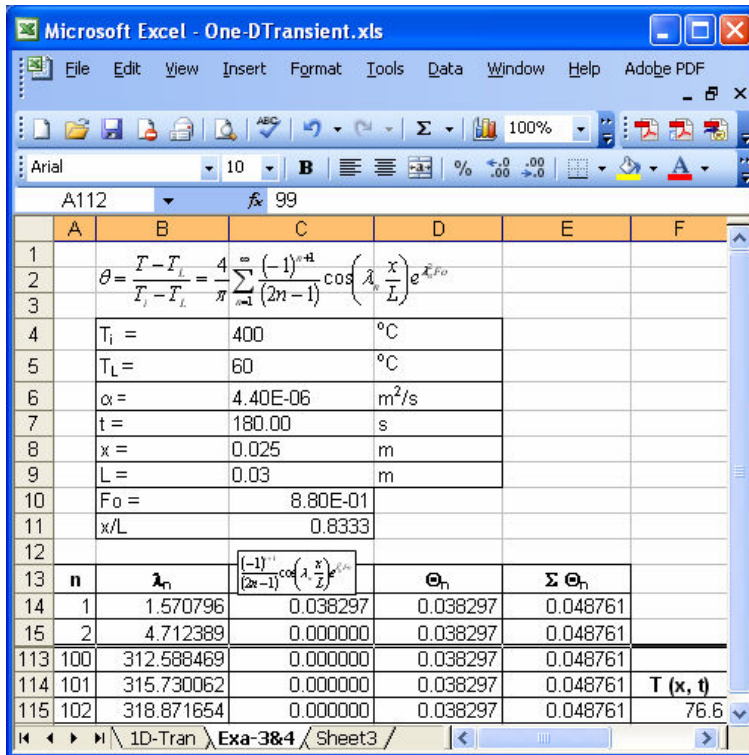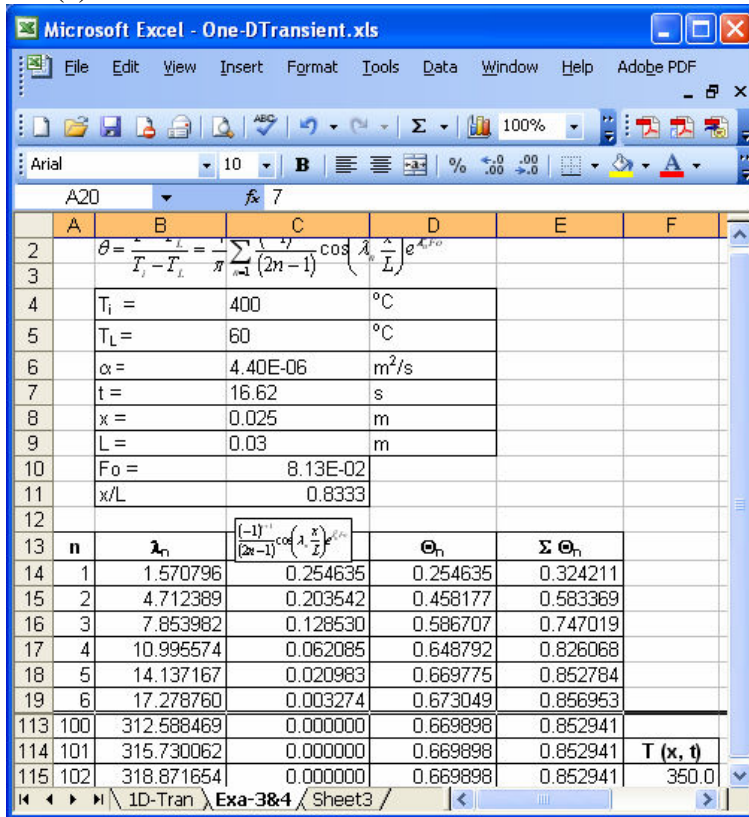
(a)- Initial solution based on assumed value of time
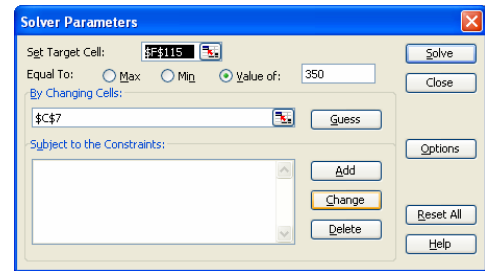
(b) – Solver dialoug box

(c) –Solver confirmation of results

(d)- final solution, showing t = 16.62, s

Fig. 10. Solution procedure and the results for Example problem 3

Based on the assumed value of t = 180 seconds, Fig 10-a shows that the temperature at 0.5 cm below the surface is 76.6 °C. Note that since the calculated value of Fo in cell C10 is larger than 0.2 (0.88), only the first term in Eq. (7) is needed for the series to converge. The calculated temperature in cell F115is different from the desired value of T = 350 °C. Therefore, the time must be changed in an iterative process until the desired temperature is achieved. Solver can be utilized for the iteration process.

By using the tool menu and selecting the Solver option, a dialog box appears, as shown in Fig 10-b. The target cell (temperature in this case, cell F115) is selected, its value is set to the desired value for that cell (350), and C7 is identified as the cell that its value (time) needs to be changed during the iteration process. When the Solve button is clicked, the value in cell C7 (t) automatically changes to a value that yields the desired temperature of 350 °C in the target cell (F115). The final solution presented in Fig.10-d shows that at 0.5 cm below the surface the temperature reaches 350 °C after 16. 62 seconds (cell C7). Note that for the final solution, several terms in the series are necessary for Eq. (7) to converge.

Analytical solutions for one-dimensional transient conduction problems subjected to convective heat transfer at boundaries result in infinite series expressions similar to Eq. (7). However, in these equations the infinite series solutions contain eigenvalues which are not periodic. For example the transient temperature distribution in an infinitely long cylinder subjected for the case involving convective heat transfer at the surface may be expressed as[7]

$$\theta = \frac{T - T_\infty}{T_i - T_\infty} = \sum_{n=1}^{\infty} \left(\frac{2 j_1(\lambda_n)}{\lambda_n \left[J_o^2(\lambda_n) + J_1^2(\lambda_n) \right]} \right) J_o\left(\lambda_n \frac{r}{R} \right) Exp\left(-\lambda_n^2 Fo \right) \qquad (8)$$

where, the eigenvalues λ_n are evaluated from the following relationship

$$\lambda_n J_1(\lambda_n) = \frac{hR}{k} J_0(\lambda_n) \qquad (9)$$

Since the solution of Eq. (9) for the determination of eigenvalues does not yield periodic behavior, the evaluation of the arguments r/R or Fo from equation (8) is complex. Recently, Dent, *et.al*[22] described a procedure for using Excel to evaluate the temperature from infinite series equations given for one-dimensional transient heat conduction problems. Although the procedure described in this paper for using Excel for solving transient heat conduction problems, the procedure requires employing Visual Basic Application to create macros functions and subroutines. This requires some programming knowledge. Therefore no examples are included from these papers.

C. Transient Heat Conduction in a Semi-infinite Slab.

The following is an example of a transient heat conduction problem in a semi-infinite region, which its solution requires a trial and error process. The solution shows how Solver or Goal Seek functions of Excel can be employed effectively to solve the trial and error problem.

Example 4

A semi-infinite concrete slab (k = 0.8 W/m.^{o}C) having a uniform temperature of 55 ^{o}C is suddenly exposed to an air stream at 10 ^{o}C. The average heat transfer coefficient on the surface is 15 W/m^2.^{o}C. Determine the distance below the surface of the slab where the temperature reaches 45 ^{o}C after 20 minutes. Thermal diffusivity of concrete is $\alpha = 5.31 \times 10^{-7}$ m/s^2.

Solution

An analytical solution for transient temperature distribution in a semi-infinite slab is expressed as[7]

$$\theta = \frac{T - T_\infty}{T_i - T_\infty} = erf\frac{\zeta}{2} + \exp(\beta\zeta + \beta^2)\left[erfc\left(\frac{\zeta}{2} + \beta\right)\right] \tag{10}$$

where,

$$\beta\zeta = \frac{hx}{k} \tag{11}$$

$$\beta^2 = \frac{h^2 \alpha t}{k^2} \tag{12}$$

erf denotes error function and, erfc is the complimentary error function.

Since the location x is a part of the arguments for erf and erfc, it can not be found explicitly from Eq. (10). Therefore, the solution requires a trail and error procedure. Figure 11 shows the result of the trial and error process, using an Excel spreadsheet. The formulas for the parameters of Eq. (10) wer entered into cells B9 through F9. An initial value was assumed for x (0.002, m) and entered into cell A-9. The initial guess for x resulted in a value of 39.4 ^{o}C for the temperature (cell F9). Cells B9 through F9 were copied into the following rows and the value of x was changed in each row until column F produced a temperature close to 45 ^{o}C. Figure 11 shows that the location where T = 45 ^{o}C is somewhere between x = 0.014 m (T= 44.8 ^{o}C) and x = 0.016 m (T = 45.6 ^{o}C).

Solver tool of Excel can be used to speed up the iteration process. Again the formulas for the parameters in Eq. (10) were entered into cells B-9 through F-9. The solution process is presented in Fig. 12. An initial value was assumed for x (0.001, m) and entered into cell A-9, which resulted in a corresponding temperature of 38.9 ^{o}C (Fig 12-a). Using the tool menu on the spreadsheet, Solver was selected. A dialog box appeared (Fig. 12-b) for setting the Solver parameters. In this box the target cell was set to F9 and its value was set to 45. Cell A9 was identified for the parameter that its value had to change by the Solver. Then the Solve button was clicked which produced the final result as shown in Fig. 12-c. The solution shows that at x = 0.0144 m below the surface, the temperature is 45 ^{o}C.

D. Finite Difference Solution of Two Dimensional Heat Conduction Problem

For a steady-state, two-dimensional heat conduction in a system having uniform properties and no heat generation, the general heat conduction equation reduces to

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{13}$$

Microsoft Excel - semi-infinite slab.xls

G16

$$\theta = \frac{T - T_\infty}{T_i - T_\infty} = erf\frac{\zeta}{2} + \exp\left(\beta\zeta + \beta^2\right)\left[erfc\left(\frac{\zeta}{2} + \beta\right)\right]$$

$$\beta\zeta = \frac{hx}{k} \qquad\qquad \beta^2 = \frac{h^2\,\alpha t}{k^2} \qquad\qquad \zeta := \frac{x}{\sqrt{\alpha\cdot t}}$$

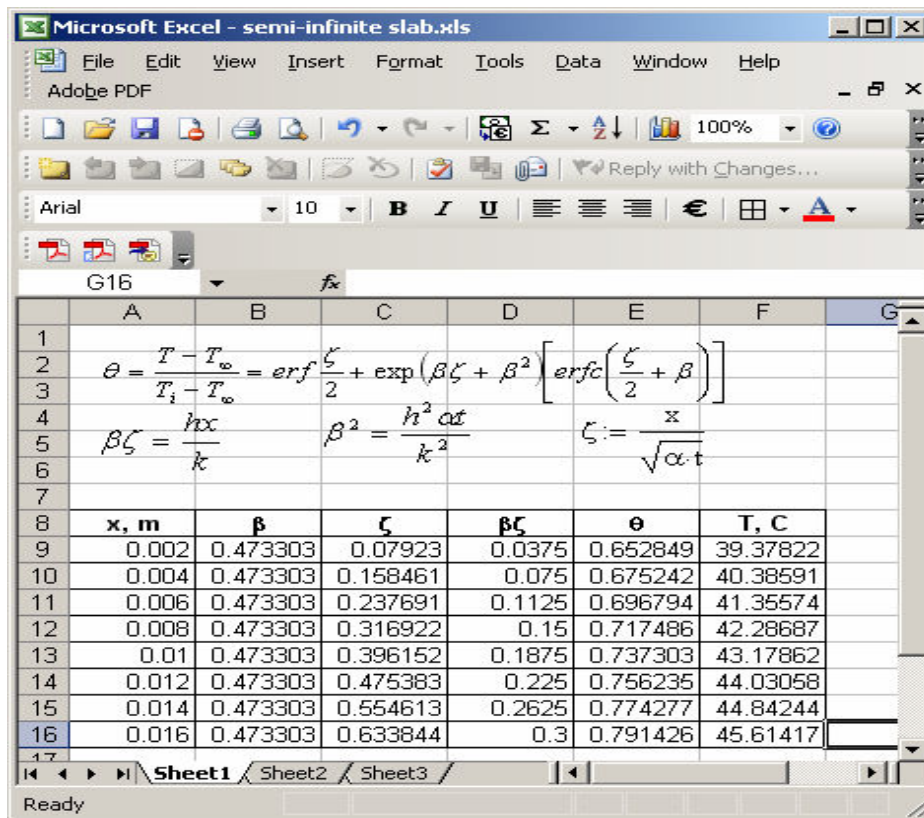| x, m	β	ζ	βζ	θ	T, C
0.002	0.473303	0.07923	0.0375	0.652849	39.37822
0.004	0.473303	0.158461	0.075	0.675242	40.38591
0.006	0.473303	0.237691	0.1125	0.696794	41.35574
0.008	0.473303	0.316922	0.15	0.717486	42.28687
0.01	0.473303	0.396152	0.1875	0.737303	43.17862
0.012	0.473303	0.475383	0.225	0.756235	44.03058
0.014	0.473303	0.554613	0.2625	0.774277	44.84244
0.016	0.473303	0.633844	0.3	0.791426	45.61417

Sheet1 / Sheet2 / Sheet3

Ready

Fig. 11   Solution of example 4 using a trial and error process

To find an approximation of temperature distribution, subject to specified boundary conditions, the system is typically divided into a network of nodal points and a discretization scheme is used to develop finite difference equations for each nodal point using Eq. (13) and the boundary conditions. As a result the heat conduction equation reduces to a system of algebraic equations which can be solved using a matrix inversion scheme, Gauss-Seidel iteration method, or other iteration procedures. The following example demonstrate the use of Solver function of Excel in steps required to solve a heat conduction problem requiring Gauss-Seidel iteration process.
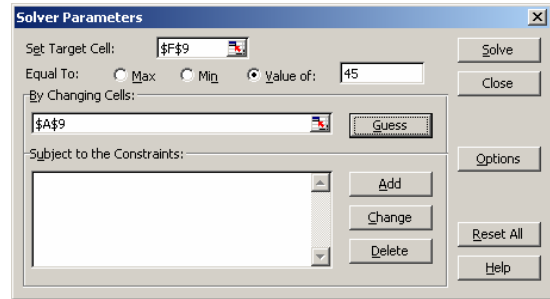
Example 5

In a solid section illustrated in the adjacent figure, the left surface is insulated. The right and bottom surfaces are maintained at 100 $^{\circ}$C and 150 $^{\circ}$C, respectively. The top surface is exposed to a convective environment at T_∞ = 20 $^{\circ}$C. The heat transfer coefficient at the top surface is h = 30 W/(m^2.$^{\circ}$C) and the thermal conductivity of the solid k = 5.0 W/(m.$^{\circ}$C). There is no heat generation in the solid. The solid is divided into a nodal network where $\Delta x = \Delta y = 10$ cm. Find the temperatures at nodal points 1 through 9.
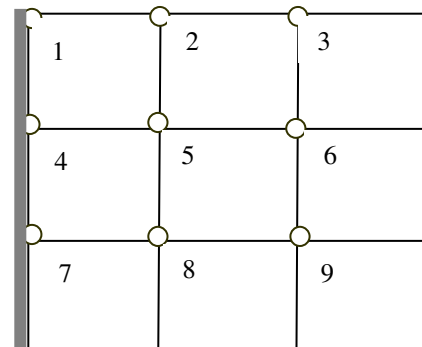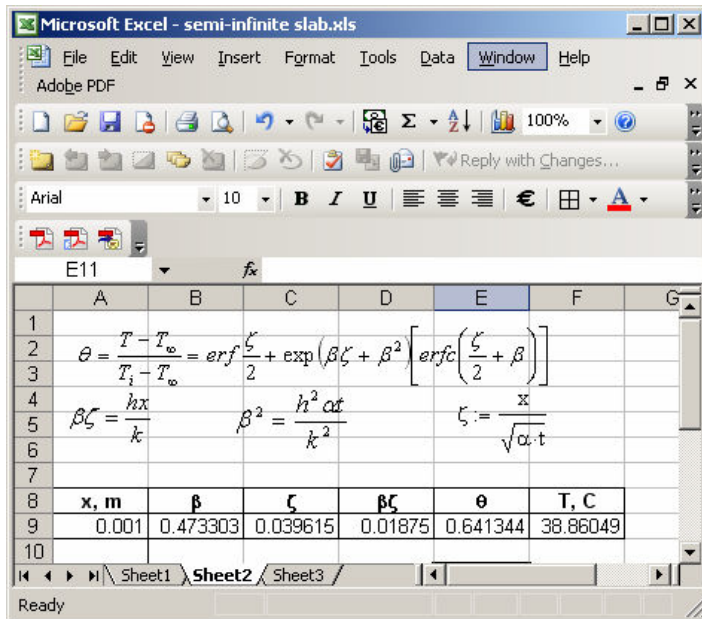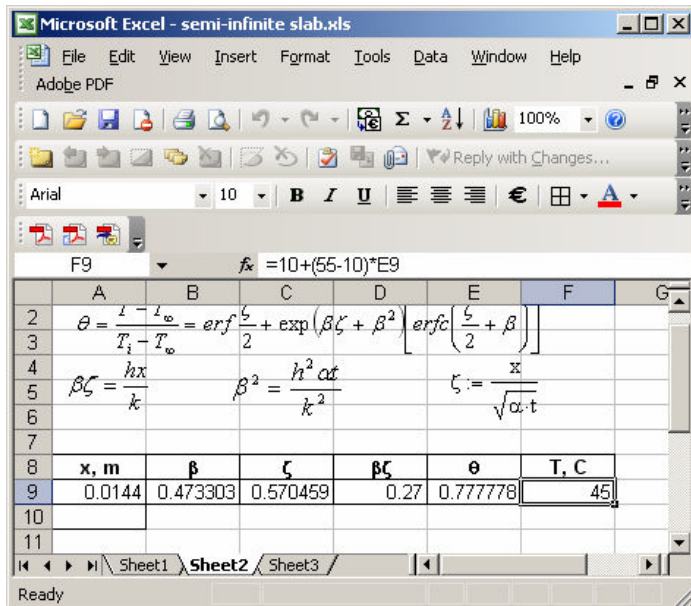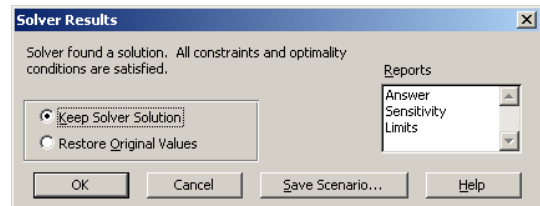
(a)

(b)

(c)

(d)

Fig. 12   Solution of Example 4, using Solver

Solution

Conducting an energy balance around each nodal point results in the following finite difference equations:

$$T_1 = \frac{T_2 + T_4 + BiT_\infty}{(2 + Bi)} \tag{14}$$

$$T_2 = \frac{T_1 + T_3 + 2T_2 + 2BiT_\infty}{2(2 + Bi)} \tag{15}$$

$$T_3 = \frac{T_2 + 2T_6 + T_R + 2BiT_\infty}{2(2 + Bi)} \tag{16}$$

$$T_4 = \frac{T_1 + 2T_5 + T_7}{4} \tag{17}$$

$$T_5 = \frac{T_2 + T_4 + T_6 + T_8}{4} \tag{18}$$

$$T_6 = \frac{T_3 + T_5 + T_9 + T_R}{4} \tag{19}$$

$$T_7 = \frac{T_4 + T_B + 2T_8}{4} \tag{20}$$

$$T_8 = \frac{T_5 + T_7 + T_9 + T_B}{4} \tag{21}$$

$$T_9 = \frac{T_6 + T_8 +_B + T_R}{4} \tag{22}$$

where, Bi = h Δx/k

Gauss-Seidel iteration procedure can be employed to solve for temperatures at each nodal point. As a first step in this process, some values are assumed for T_1 through T_9. Then in the following steps Eqs. (14) through (22) are used to calculate new values for the temperatures at each nodal point. At each steps of the calculation, the most recent calculated values of the temperatures are used in the right hand side of Eqs. (14) through (22) and the new values of the temperatures at each nodal point is compared with the previous value to check if there is any significant changes. When all values of $|T_{i,\,new} - T_{i,\,old}|$ are less than a sufficiently small number, δ, the calculation process is seized. In most cases the procedure converges to final values of temperatures. However, there are situation that at each step process the values of $|T_{i,new} - T_{i,old}|$ might get larger. In these situations the new calculate value can be relaxed by adjusting the new values by the weighted values from the new and the previous iteration steps[28].

Figure 13 illustrates the Gauss-Seidel iteration process for solving this example problem. As initial guesses, all temperatures were set equal to zero. Then the formulas from Eqs (14) through (22) were entered into cells B11 through J11. The formulation used the last calculated values in the right hand side of equations. The formulas for the calculation of $|T_{i,new} - T_{i,old}|$ were also entered in cells K11 through S11 and cell T11 was formulated to identify the largest value of $|T_{i,old}| = \Delta T_{max} = \delta$. The formulas in row 11 were copied into the following rows. The iteration process was stopped when $\delta < 0.01$. Figure 13 shows that this condition was met on the 24th step. Cells B34 though J34 give the resulting values of temperatures at the nine nodal points.

Example 6
In a solid section illustrated in the figure for example 5, the left surface is insulated. The right and bottom surfaces are maintained at 100 ^{o}C and 150 ^{o}C, respectively. The top surface is

exposed to a convective environment where the heat transfer coefficient is h = 30 W/(m^2.°C). The thermal conductivity of the solid k = 5.0 W/(m.°C). There is no heat generation in the solid. The solid is divided into a nodal network where $\Delta x = \Delta y$= 10 cm. It is required to maintain T$_6$ at 90 °C. What should be the temperature of the convective fluid (T$_\infty$) to meet this requirement?

Fig. 13 Solution of Example 5 using Gauss-Seidel iteration technique

Solution

Since the value of T$_\infty$ which is necessary for the evaluation of T$_1$, T$_2$, and T$_3$, from Eqs. (14), (15), and (16), respectively, the solution to this example problem requires a second iteration scheme in addition to Gauss-Seidel iteration procedure. The Solver tool of Excel can be utilized to solve this problem. Figure 14 demonstrates the solution technique.

Microsoft Excel - Finite Diff-Exa.xls

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G33 fx =(D33+F33+$B$1+J32)/4

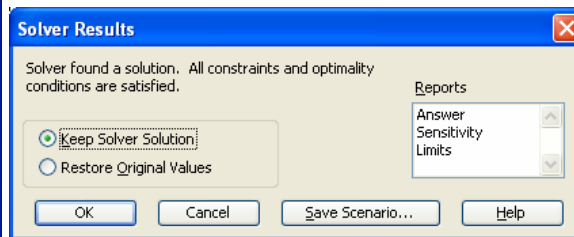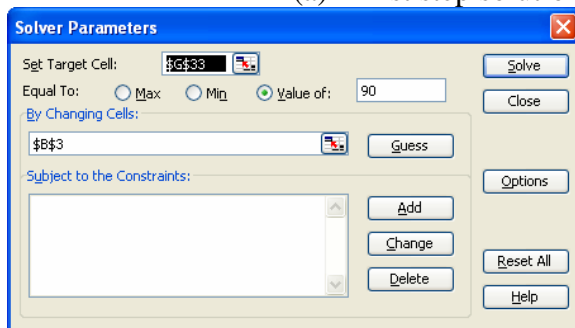|	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
1	T_R =	100	°C																	
2	T_B =	150	°C																	
3	T_∞ =	20	°C																	
4	Δx=	Δy=	0.1	m																
5	h =	40	W/(m²·°C)																	
6	k =	5	W/(m·°C)																	
7	Bi =	hΔx/k=		0.80									ΔT_v=	$T_{i,new}$ -$T_{i,old}$						
8	Step	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	ΔT_1	ΔT_2	ΔT_3	ΔT_4	ΔT_5	ΔT_6	ΔT_7	ΔT_8	ΔT_9	ΔT_{max}
9	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00										
10	1	5.71	6.73	24.77	1.43	2.04	31.70	37.86	47.47	82.29	5.71	6.73	24.77	1.43	2.04	31.70	37.86	47.47	82.29	82.29
11	2	8.63	12.41	37.11	12.64	26.06	61.37	64.40	80.69	98.01	2.92	5.67	12.34	11.21	24.02	29.66	26.54	33.21	15.72	33.21
30	21	59.78	61.03	67.92	90.37	91.03	93.66	119.63	119.09	115.69	0.01	0.01	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.01
31	22	59.78	61.03	67.92	90.37	91.04	93.66	119.64	119.09	115.69	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.01
32	23	59.79	61.03	67.92	90.37	91.04	93.66	119.64	119.09	115.69	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Sheet1 / Sheet2 / Sheet3 /

(a) – first step solution based on initial guess for T_∞

Solver Parameters

Set Target Cell: $G$33
Equal To: ○ Max ○ Min ⊙ Value of: 90
By Changing Cells:
$B$3   Guess
Subject to the Constraints:
Add
Change
Delete

Solve
Close
Options
Reset All
Help

Solver Results

Solver found a solution. All constraints and optimality conditions are satisfied.

⊙ Keep Solver Solution
○ Restore Original Values

Reports
Answer
Sensitivity
Limits

OK   Cancel   Save Scenario...   Help

(b) – Solver dialogue box and confirmation of solution

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G32 fx =(D32+F32+$B$1+J31)/4

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
1	T_R =	100	°C																	
2	T_B =	150	°C																	
3	T_∞ =	3.82	°C																	
4	Δx=	Δy=	0.1	m																
5	h =	40	W/(m²·°C)																	
6	k =	5	W/(m·°C)																	
7	Bi =	hΔx/k=		0.80									ΔT_v=	$T_{i,new}$ -$T_{i,old}$						
8	Step	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	ΔT_1	ΔT_2	ΔT_3	ΔT_4	ΔT_5	ΔT_6	ΔT_7	ΔT_8	ΔT_9	ΔT_{max}
9	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00										
10	1	1.09	1.29	19.18	0.27	0.39	29.89	37.57	46.99	81.72	1.09	1.29	19.18	0.27	0.39	29.89	37.57	46.99	81.72	81.72
11	2	1.65	4.95	30.51	10.00	22.96	58.80	63.49	79.54	97.08	0.56	3.66	11.33	9.73	22.57	28.90	25.93	32.55	15.36	32.55
30	21	49.54	51.26	60.24	84.40	85.58	90.00	116.93	116.67	114.17	0.01	0.01	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.01
31	22	49.54	51.26	60.24	84.41	85.58	90.00	116.94	116.67	114.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
32	23	49.54	51.26	60.24	84.41	85.59	90.00	116.94	116.67	114.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Sheet1 / Sheet2 / Sheet3 /

(c) – final solution

Fig. 14 Solution of Example problem 6 utilizing Gauss-Seidel iteration method and Solver

To solve this problem, a value was assumed for T_∞ (20 °C) and the procedure described for example problem 5 was used to evaluate all nodal point temperatures, including T_6. Figure 14-a shows that $T_6 = 96.56$ °C based on the assumed value of T_∞. Therefore, the value of T_∞ needs to be changed in a trial and error process in order to achieve the desired value for T_6. The Solver tool of Excel was used to speed up the iteration process. Using the tool menu on the spreadsheet, Solver was selected. The target cell was set to G32 in the dialogue box and its value was set to 90, as shown in Fig. 14-b. Cell B3 was identified as the parameter that its value had to be changed by the Solver. Then the Solve button was clicked which produced the final result as shown in Fig. 14-c. The solution shows that T_∞ must be 3.82 °C in order to maintain T_6 at 90 °C.

There have been other attempts in the recent years to use Excel to solving heat conduction problems via finite difference scheme. Sarker and Ketkar[21] have described the use of Excel in solving one-dimensional transient heat conduction problems. In this work the general heat diffusion equation in a cylindrical coordinate system was simplified by assuming no internal heat generation and ignoring heat transfer in the axial and angular directions. The resulting equation was transformed into finite difference equations and the resulting matrix for the system of equations was solved using Excel. Baughn [23] developed a unified numerical technique for solving multi-dimensional steady state and transient conduction problems using Excel spreadsheet. The numerical scheme uses Gauss-Seidel iteration process for steady state problems and explicit method for transient problems. The solution method in both papers includes the use of macros and subroutines, therefore, these examples are not included.

E. Similarity Solution for Laminar Flow over Isothermal Flat Plate

Assuming steady incompressible laminar flow with constant fluid properties, the continuity, momentum, and energy equations, respectively, are given by the following relations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{23}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \tag{24}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{25}$$

Using the Blasius method, the continuity and momentum equations reduce to a single ordinary differential equation.

$$2\frac{d^3 f}{d\eta^3} + f\frac{d^2 f}{d\eta^2} = 2f''' + f \cdot f'' = 0 \tag{26}$$

subject to the following boundary conditions

$$f\big|_{\eta=0} = \frac{df}{d\eta}\bigg|_{\eta=0} = f'\big|_{\eta=0} = 0 \qquad \text{and} \qquad \frac{df}{d\eta}\bigg|_{\eta\to\infty} = f'\big|_{\eta\to\infty} = 1 \qquad (27)$$

where

$$\eta \equiv y\sqrt{u_\infty/vx} \qquad (28)$$

It can be shown[11] that the velocity components u and v can be express as

$$u = u_\infty \frac{df}{d\eta} \qquad (29)$$

$$v = \frac{1}{2}\sqrt{\frac{vu_\infty}{x}}\left(\eta\frac{df}{d\eta} - f\right) = \frac{1}{2}\sqrt{\frac{vu_\infty}{x}}(\eta.f' - f) \qquad (30)$$

Therefore, the velocity components in the hydrodynamic boundary layer can be obtained from the solution of Eq. (26).

Defining the following expression for the dimensionless temperature

$$\theta = \frac{T - T_w}{T_\infty - T_w} \qquad (31)$$

Similar to the momentum equation, the energy equation, Eq. (25), can also be transformed into an ordinary differential equation, as presented below

$$\frac{d^2\theta}{d\eta^2} + \frac{Pr}{2}f\frac{d\theta}{d\eta} = \theta'' + \frac{Pr}{2}f.\theta' = 0 \qquad (32)$$

Where, Pr is the Prandtl number. The applicable boundary conditions for Eq. (32) are

$$\theta\big|_{\eta=0} = 0 \qquad \text{and} \qquad = \theta\big|_{\eta\to\infty} = 1 \qquad (33)$$

The standard procedure to find the velocity profile in the hydrodynamic boundary layer is to obtain a solution to Eq. (26) using a numerical scheme. The results of the solution of Eq. (26) are used in Eq. (32) to determine the temperature profile in the thermal boundary layer. The solution of equation (26) is necessary for the evaluation of shear stress and skin friction of the flat plate. The solution of Eq. (32) is needed to derive an expression for the heat transfer coefficient and Nusselt number. The results from the solution of Eq. (26) are presented in forms of table or graphs in many undergraduate heat transfer textbooks. Very few textbooks show the temperature profile resulting from the solution of Eq. (32).

Example 7
Use a numerical scheme to solve Eqs (13) and (19) to evaluate velocity profile in the hydrodynamic boundary layer and temperature profile in the thermal boundary layer, respectively. Use the results to develop relationships for local friction factor and Nusselt number.

Solution

The standard procedure for obtaining a numerical solution to Eq. (26) typically uses a Runga Kutta procedure for solving initial valued differential equations[29]. However, since the value of f" at η =0 is unknown at the initial step of the calculation process, a shooting method is usually employed to solve the differential equation. The solution of the Eq. (26) in Excel through this procedure requires some programming skill and the use of Visual Basic for Applications (VBA).

In two separate works Fakheri[24] and Naraghi[25] demonstrated the use of Excel in solving the boundary layer problem without the employment of VBA. The following is a summary of procedure used by Naraghi[25] who used the Solver tool of Excel to obtain solutions to Equations (26) and (32).

Using a forward finite difference method, the first, second, and third derivatives of function f the following relations were obtained

$$f_{i+1} = f_i + f_i'(\Delta\eta) \tag{34}$$

$$f_{i+1}' = f_i' + f_i''(\Delta\eta) \tag{35}$$

$$f_{i+1}'' = f_i'' + f_i(\Delta\eta) \tag{36}$$

From Eq. (26)

$$f_i''' = -\frac{f_i \cdot f_i''}{2} \tag{37}$$

A procedure was developed to solve Eq. (26) in an Excel worksheet using Eqs. (34) through (37). Figure 15 shows the process and the results of the procedure. As shown in Fig. 15-a, η was set to zero (0) in cell A3 and the boundary conditions from Eq. (27), f =f' = 0, were entered into cells B3 and C3. From Eq. (37), f'''=0 at η = 0. Therefore, cell E3 was also set to zero. The value of f" at η = 0 is unknown and it must be determined by trial and error. Therefore, an arbitrary value of 0.8 was selected and entered into cell D3. To obtain accurate results, the increment for Δη was set to 0.01 and entered into cell G1. The value of η was increased by an increment of Δη in cells A4 through A803 where η = 8. Equations (34) through (37) were entered in cells B4 through E4, respectively. These cells were highlighted and their contents (formulas) were copied into the following rows, through row 803. The third boundary condition requires that f' approaches 1.0 as ηbecomes very large. This condition is not satisfied by the cell C803 in Fig. 15-a. Therefore, the value of f" in cell D3 must be changed until the value of f' in cell C803 approaches 1.

To speed up this trial and error process the Solver tool of Excel was employed. The tool menu was used and Solver tool was selected. A dialog box appeared for entering the parameters for the Solver tool. As shown in Fig 15-b, the target cell was set to C803, the target value was set to 1, and D3 was selected for the cell which its value had to change. After clicking on the solver

button, the values of the variables in the worksheet change to the final results. The results are presented in Fig. 15-c.

(a)- fist step solution

(b) – Solver dialogue box
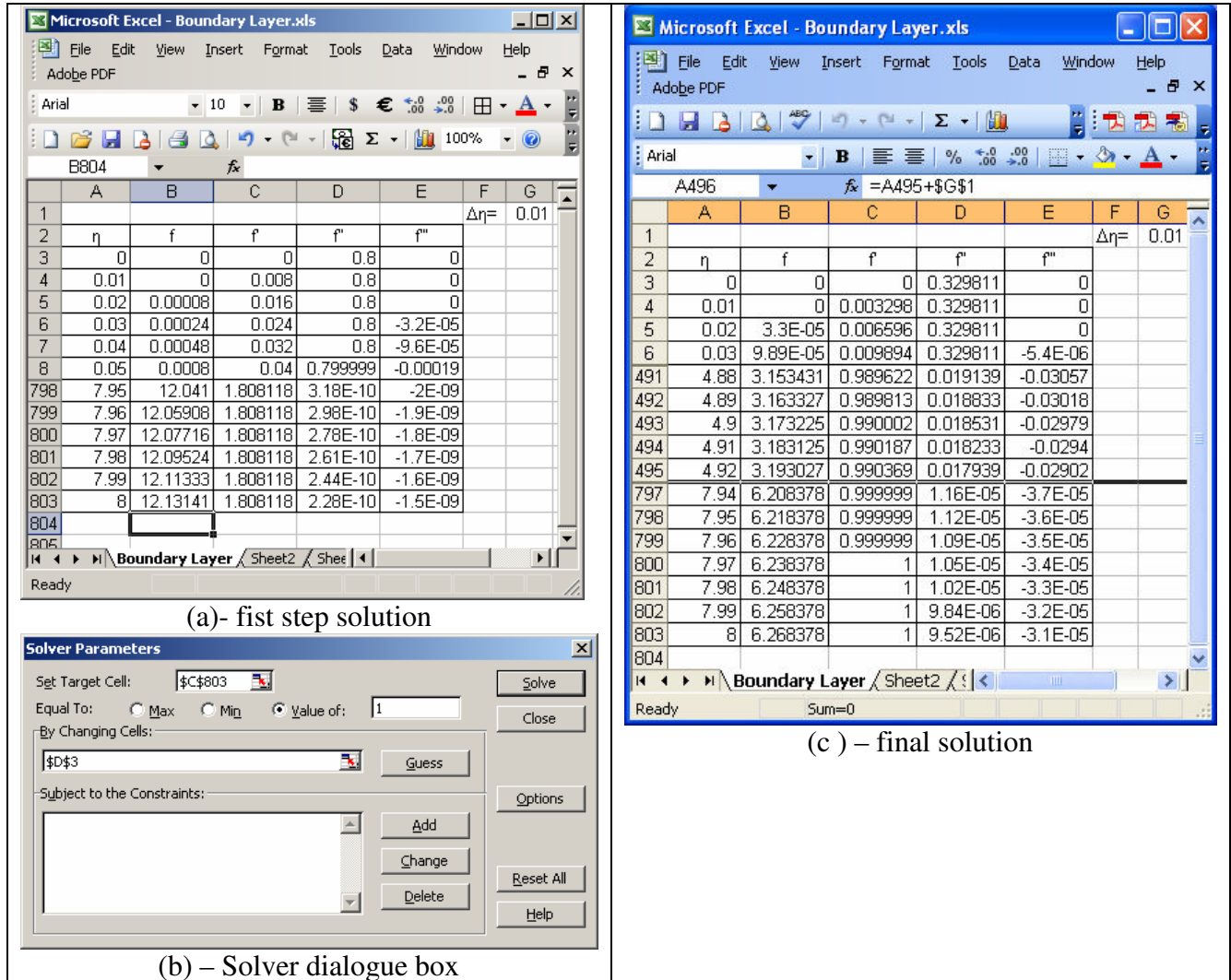
(c) – final solution

Fig. 15   Solution of hydrodynamic boundary layer equation, Eq. (26)

A comparison of data in Fig. 15-c with the accepted value in the literature indicates that the results are highly accurate. The boundary layer thickness is defined as a location away from the surface of the plate where $u = 0.99\ u_\infty$.   Therefore, Eq. (29) indicates that at the edge of boundary layer thickness, f' must be equal to 0.99.   Fig 15-c shows that , f'=0.99.   This compares very well with the established value of $\eta = 4.92$ (a relative error of 0.4%). The resulting value for f'' at $\eta = 0$ is used for the evaluation of shear stress at the wall $\tau_{w,x}$ and the local friction coefficient, $C_{f,x}$. Fig. 15-c shows a value of 0.3298 for f'' at $\eta = 0$ which compares well with the published value of 0.332 (0.7% relative error). Therefore, based on this results the following relations can be expressed for $\tau_{w,x}$ and $C_{f,x}$.

$$\tau_{w,x} = 0.3298\mu u u_\infty \sqrt{u_\infty/\nu x} \tag{38}$$

$$C_{f,x} = \frac{\tau_{w,x}}{\rho u_\infty^2/2} = \frac{0.6596}{\mathrm{Re}_x^{1/2}} \tag{39}$$

A similar procedure was used to solve Eq. (32) for the thermal boundary layer for various Pr numbers. Naraghi[25] used this procedure to solve Eq. (32) in Excel for values of Pr number ranging between 0.6 and 100. Again a forward finite difference method was used the express the first and second derivatives of function θ and resulting equations were rearranged into the following forms

$$\theta_{i+1} = \theta_i + \theta_i'(\Delta\eta) \tag{40}$$

$$\theta_{i+1}' = \theta_i' + \theta_i''(\Delta\eta) \tag{41}$$

From Eq. (32)

$$\theta_i'' = -\frac{\mathrm{Pr}\, f_i.\theta_i'}{2} \tag{42}$$

The Excel worksheet that resulted in the solution of hydrodynamic boundary layer equation was expanded to include columns for θ, θ', and θ'', as shown in Fig. 16. The boundary condition θ ($\eta = 0$) = 0 was entered into cell F3. From Eq. (37), $\theta'=0$ at $\eta = 0$. Therefore, cell H3 was also set to zero. The value of θ' at $\eta = 0$ is unknown and must be determined by trial and error. Again, an arbitrary value of 0.8 was selected and entered into cell G3. A specific value for Pr number was entered into cell I1 (2.0 in this case). Equations (40) through (42) were entered in cells F4 through H4, respectively. These cells were highlighted and their contents (formulas) were copied into the following rows, through row 803. The second boundary condition in Eq. (33) requires that θ should approach 1.0 as η becomes very large. This condition is not satisfied by the cell C803's value in Fig. 16. Therefore, the value of θ in cell G3 must be changed until the value of θ' in cell C803 approaches 1.0. Again the Solver tool of Excel was employed to satisfy the second boundary condition. Again the target cell was set to F803, the target value was set to 1.0, and G3 was selected for the cell which its value to be changed. After clicking on the solver button, the values of the variables in the worksheet change to the final results. The results are presented in Fig. 17.

The thermal boundary layer thickness is defined as a location away from the surface of the plate where $\theta = 0.99$. Figure 17 shows that this condition is met where $\eta = 3.79$ for Pr = 2.0.

The local Nusselt number can be expressed as a function of $\theta'(\eta = 0)$

$$Nu_x = \frac{hx}{k} = \mathrm{Re}_x^{1/2}\,\theta'(0) \tag{43}$$
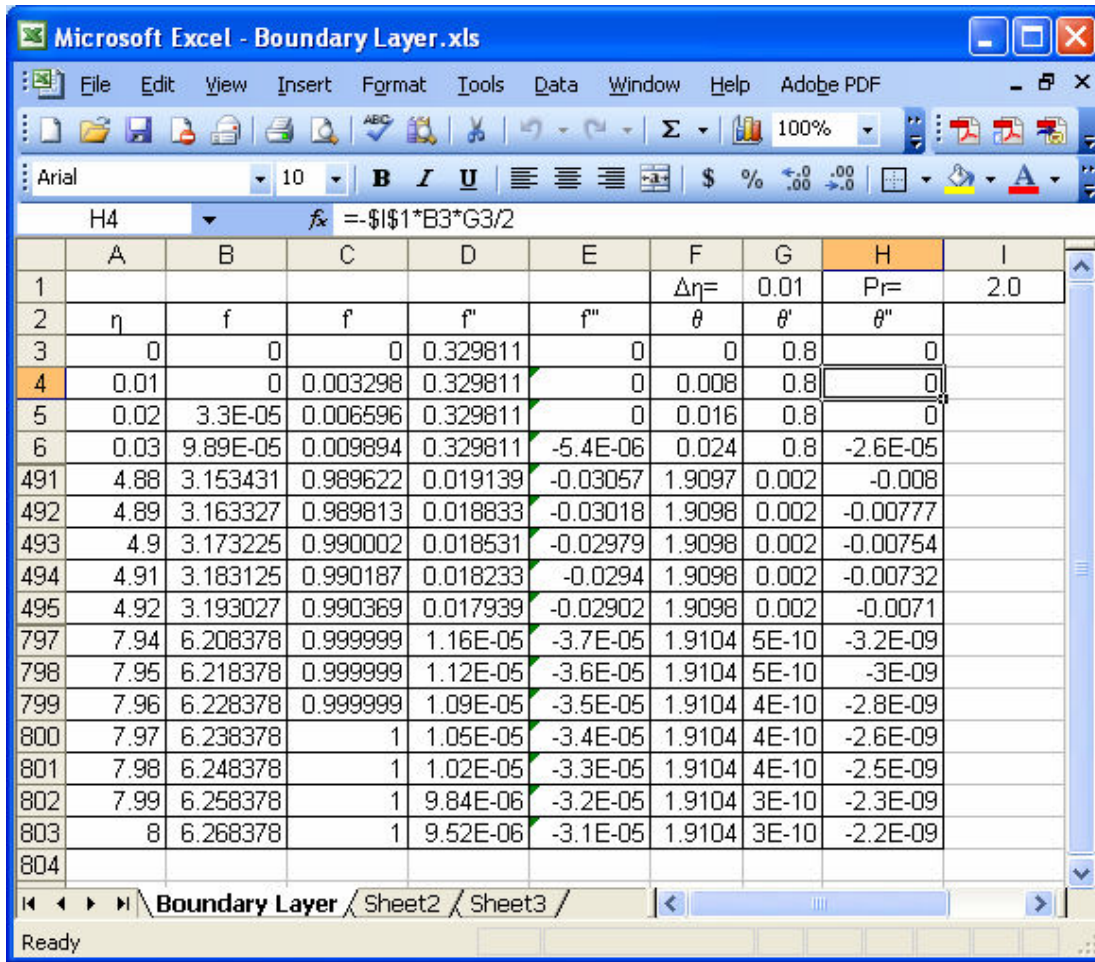
Fig. 16   Initial attempt to solve the thermal boundary layer equation, Pr =2.0

The procedure described was repeated for pr numbers ranging between 0.6 and 100. For each value of Pr, the corresponding value of θ'(0) were entered into an Excel worksheet and the results were plotted on a graph. The equation for trend-line resulted in the following correlation (R^2 =0.9999)

$$\theta'(0) = 0.3313\,\mathrm{Pr}^{0.3357} \tag{44}$$

Combining Eqs (43) and (44) yields the following relationship

$$Nu_x = \frac{hx}{k} = 0.3313\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{0.3357} \tag{45}$$

Equation (45) compares well with the established relationship for the local Nu_x.

$$Nu_x = \frac{hx}{k} = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3} \tag{46}$$

Microsoft Excel - Boundary Layer.xls

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A386   fx =A385+$G$1

	A	B	C	D	E	F	G	H	I
1						Δη=	0.01	Pr=	2.0
2	η	f	f'	f''	f'''	θ	θ'	θ''	
3	0	0	0	0.329811	0	0	0.419	0	
4	0.01	0	0.003298	0.329811	0	0.0042	0.419	0	
5	0.02	3.3E-05	0.006596	0.329811	0	0.0084	0.419	0	
6	0.03	9.89E-05	0.009894	0.329811	-5.4E-06	0.0126	0.419	-1.4E-05	
7	0.04	0.000198	0.013192	0.329811	-1.6E-05	0.0168	0.419	-4.1E-05	
380	3.77	2.077052	0.938403	0.083243	-0.08696	0.9895	0.026	-0.05483	
381	3.78	2.086436	0.939235	0.082373	-0.08645	0.9897	0.025	-0.05392	
382	3.79	2.095828	0.940059	0.081509	-0.08593	0.99	0.025	-0.05302	
383	3.8	2.105229	0.940874	0.080649	-0.08541	0.9902	0.024	-0.05213	
384	3.81	2.114637	0.941681	0.079795	-0.08489	0.9905	0.024	-0.05124	
385	3.82	2.124054	0.942479	0.078946	-0.08437	0.9907	0.023	-0.05037	
495	4.92	3.193027	0.990369	0.017939	-0.02902	0.9997	0.001	-0.00372	
797	7.94	6.208378	0.999999	1.16E-05	-3.7E-05	1	3E-10	-1.7E-09	
798	7.95	6.218378	0.999999	1.12E-05	-3.6E-05	1	2E-10	-1.6E-09	
799	7.96	6.228378	0.999999	1.09E-05	-3.5E-05	1	2E-10	-1.5E-09	
800	7.97	6.238378	1	1.05E-05	-3.4E-05	1	2E-10	-1.4E-09	
801	7.98	6.248378	1	1.02E-05	-3.3E-05	1	2E-10	-1.3E-09	
802	7.99	6.258378	1	9.84E-06	-3.2E-05	1	2E-10	-1.2E-09	
803	8	6.268378	1	9.52E-06	-3.1E-05	1	2E-10	-1.1E-09	
804									

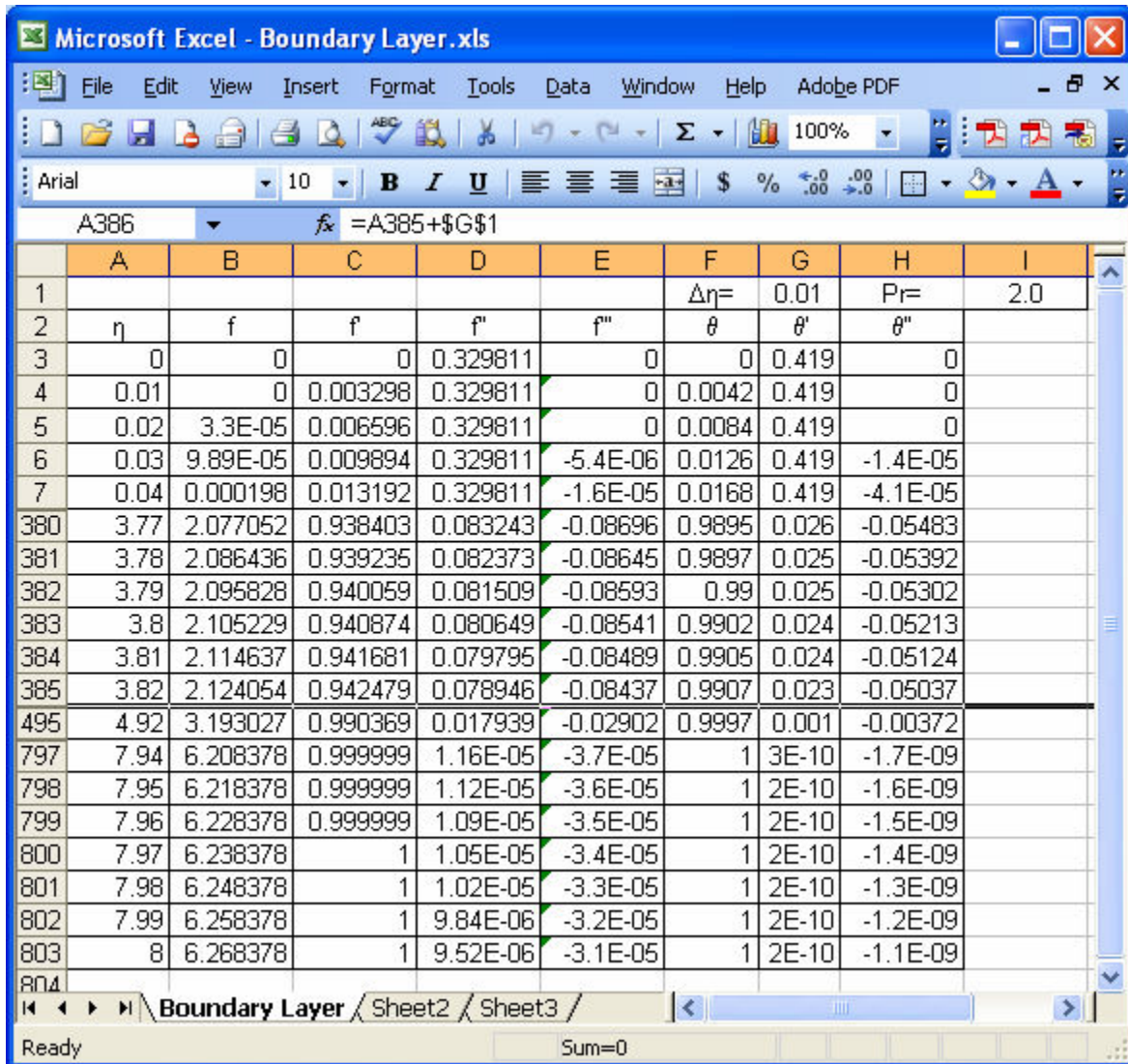|◄ ◄ ► ►|\ Boundary Layer / Sheet2 / Sheet3 /

Ready   Sum=0

Fig. 17   Results of the solution for the thermal boundary layer equation, Pr =2.0

F. Heat Exchanger Analysis

In an undergraduate heat transfer course students are introduced to two different types of heat exchangers analysis: Logarithmic Mean Temperature Difference (LMTD) and Effectiveness-NTU methods.

In a heat exchanger the flow heat capacity rate is defined as

$$C = \dot{m}c_p \tag{47}$$

where, $\dot{m}$ denotes the mass flow rate and c_p represents the specific heat of a given fluid stream. The rate of heat transfer from or to each fluid stream can be calculated from the following relations

$$q = C_h \ (T_{h,i} - T_{h,o}) = \ C_c \ (T_{c,o} - T_{c,i}) \qquad (48)$$

where, q is the rate of heat transfer, T denotes temperature; subscripts c and h identify cold and fluids, respectively; and subscripts i and o represent inlet and outlet conditions respectively.

The heat transfer rate, based on heat transfer concepts, is expressed as

$$q = UA_s \ (F) \ LMTD \qquad (49)$$

where, U is the overall heat transfer coefficient, A_s is the surface area separating the two fluid streams, LMTD is the logarithmic mean temperature difference between the two fluid streams, and F is an appropriate correction factor which value depends on the type of heat exchanger and flow conditions.

In general, LMTD can be expressed as

$$LMTD = \frac{\Delta T_a - \Delta T_b}{\ln(\Delta T_a / \Delta T_b)} \qquad (50)$$

where, ΔT_a and ΔT_b are the temperature differences between the two fluid streams at the terminal points of the heat exchangers, as shown in Fig. 18.
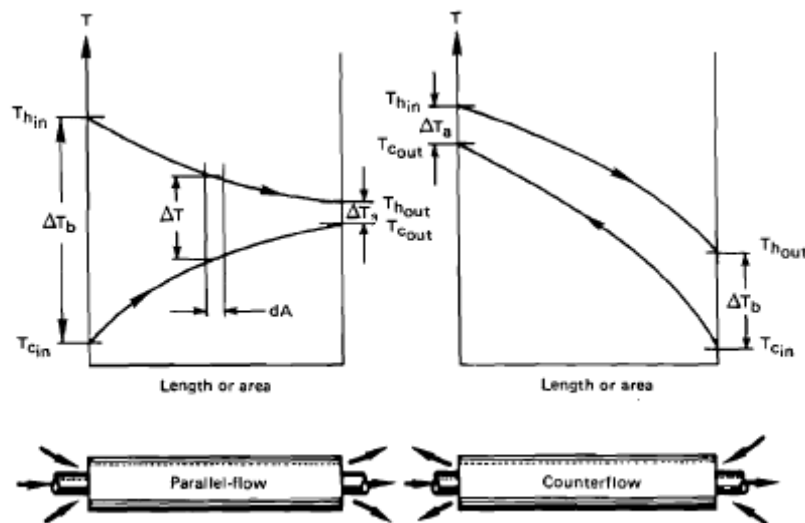

Fig. 18   Temperature profile of hot and cold fluids in parallel-flow and counter flow heat exchanges[7]

The terminal point temperature differences shown for the counter-flow heat exchangers in Fig. 18 are used in Eq. (50) for the evaluation of LMTD of fluid streams in any other types of heat exchangers such as shell-and-tube or cross flow heat exchangers.

For parallel-flow and counter-flow heat exchangers, the correction factor in Eq. (49) has a value of F=1. For other types of heat exchangers, specific charts or equations are used for the correction factor, F. For example, Fig. 19 is a correction factor chart for a one shell-pass, even number tube-pass heat exchanger.
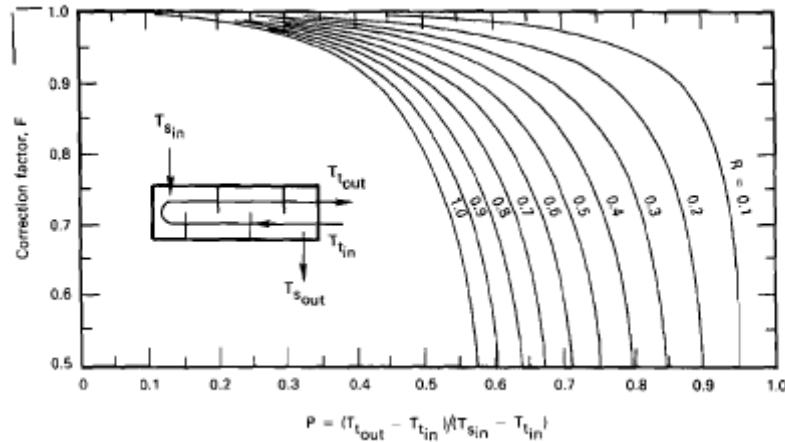

Fig. 19. LMTD correction factor, F, for a one shell-pass, even number of tube-passes heat exchanger.[7]

As shown in Fig. 19, the correction factor, F, is a function of two parameters P and R. The first parameter is defined as

$$P = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{c,i}}$$
(51)

Since the denominator in Eq. (51) represents the maximum temperature difference between the two fluid streams, the value of P is always less than one. The second parameter, R, is defined as

$$R = \frac{T_{h,i} - T_{h,o}}{T_{c,o} - T_{c,i}} = \frac{C_c}{C_h}$$
(52)

Depending on the flow heat capacity ratios (or temperature changes for the hot and cold fluids), the value of R could be less than one or greater than one. If the value of R in Eq. (52) ends up be greater than one, then R should be replaced by 1/R and P replaced by PR, since Fig. 19 displays curves only for R values that are less than or equal to one. In other words,

$$F = F(P,R) = F(RP, 1/R)$$
(53)

LMTD method is useful for sizing heat exchangers. That is when the inlet and outlet temperatures of the fluid streams are known or could be calculated directly from Eq. (49), LMTD and correction factor, F, can be easily evaluated and used in Eq. (50) to calculate the surface area or the overall heat transfer coefficient of heat exchangers. However, when two of the terminal temperatures of a heat exchanger are unknown and must be evaluated, LMTD can not be evaluated explicitly from Eq. (50). Hence, in these situations an iterative procedure is required for the evaluation of LMTD and the correction factor. In these cases the effectiveness-NTU method is employed for the analysis of heat exchangers.

The effectiveness of a heat exchanger is defined as

$$\varepsilon = \frac{\text{actual heat transfer}}{\text{maximum possible heat transfer}}$$

or

$$\varepsilon = \frac{C_h\left(T_{h,i} - T_{h,o}\right)}{C_{\min}\left(T_{h,i} - T_{c,i}\right)} = \frac{C_c\left(T_{c,o} - T_{c,i}\right)}{C_{\min}\left(T_{h,i} - T_{c,i}\right)} \tag{54}$$

where, the $C_{\min}$ is the smaller of the C_h and C_c. Combining Eqs (48) and (54) it follows that

$$q = \varepsilon C_{\min}\left(T_{h,i} - T_{c,i}\right) \tag{55}$$

The number of transfer units is defined as

$$NTU = \frac{UA}{C_{\min}} \tag{56}$$

The capacitance ratio is defined as

$$C_R = \frac{C_{\min}}{C_{\max}} \tag{57}$$

The derivation of heat exchanger effectiveness equation for parallel-flow is given in most heat transfer textbooks. For a parallel flow heat exchanger, the effectiveness is expressed as

$$\varepsilon = \frac{1 - \exp\left[-NTU\left(1 + C_R\right)\right]}{1 + C_R} \tag{58}$$

In this form the effectiveness is explicitly expressed as a function of NTU and C_R. Alternatively, NTU could be expressed as a function of ε and C_R. For a parallel-flow heat exchanger, NTU is given as

$$NTU = \frac{\ln[1 - \varepsilon(1 + C_R)]}{1 + C_R} \tag{59}$$

Heat transfer textbooks provide effectiveness charts for several types of heat exchangers. For example, Fig. 20 displays the effectiveness chart for a single pass cross-flow heat exchanger, one fluid unmixed.
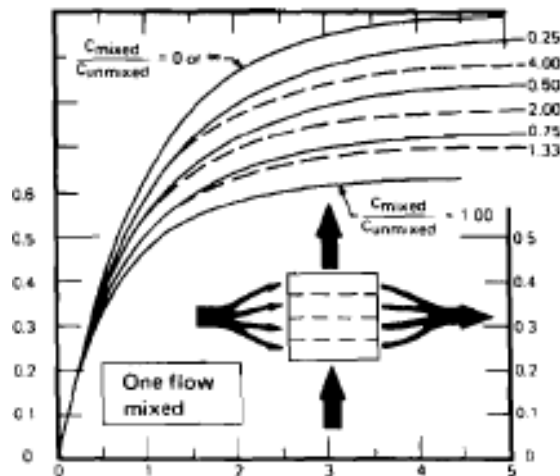

Fig. 20 Effectiveness chart for a one-pass cross-flow heat exchanger, one fluid unmixed[7]

More recent heat transfer textbooks also provide equations for $\varepsilon = \varepsilon(NTU, C_R)$ or $NTU = NTU(\varepsilon, C_R)$ for several types of heat exchanger.

There are two inherent problems with using charts in thermal analysis of heat exchanger systems. First, the accuracy of solutions is highly dependent on how precise one can read the charts, but also in problems requiring several stages of iteration, the process could become extremely tedious. Use of equations for correction factors or the effectiveness in a numerical scheme increases the accuracy and eases the task of solving problems involving repeated calculations. Microsoft Excel is one of the tools that can be used in solving heat exchanger problems. Recently, Karimi[26] descried methods of application of Excel in heat exchanger analysis and provided a few examples. Two examples are included

Example 8
Water at 15 °C with a mass flow rate of 8 kg/s is available to cool hot oil from 90 °C to 30 °C. The oil mass flow rate is 4 kg/s. A shell-and-tube heat exchanger with one-shell pass and four-tube-passes is proposed for this process. Using uniform c_p values of 2.5 kJ/(kg °C) and 4.2 kJ/(kg °C) for oil and water, respectively, and assuming an overall heat transfer coefficient of 250 W/(m^2.°C) for the heat exchanger
a) determine the surface area of the heat exchanger
b) plot the heat exchanger surface area as a function of water mass flow rate, when the mass flow rates vary between 6 and 30 kg/s.

Solution

This problem can be easily solved, using the LMTD method and the correction chart in Fig. 19. Using the specific heat and mass flow rate data given in the problem statement, Eq. (47) yields the following results:

C_c = 33.6 kW/°C

C_h = 10 kW/°C

The results are substituted into Eq. (48) to obtain $T_{c, o}$

q = $C_h \Delta T_h$ = 10 kW/°C (90-30) °C = 600 kW

q = 600 kW = C_c ($T_{c, o} - T_{c,i}$) = 33.6 kW/°C ($T_{c, o}$ -15) °C.   This gives

$T_{c, o}$ = 32.86 °C

Then,

ΔT_a = 90 − 32.86 = 57.14 °C

ΔT_b = 30 − 15 = 15 °C

Substituting ΔT_a and ΔT_b into Eq. (50), yields

LMTD = 31.51 °C

In order to evaluate the correction factor, the terminal temperatures of the heat exchanger are substituted into Eqs. (51) and (52) to find the values of P and R.

$$P = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{c,i}} = 0.2381$$

$$R = \frac{T_{h,i} - T_{h,o}}{T_{c,o} - T_{c,i}} = 3.36$$

However, since R> 1, in order to use Fig. 19 to evaluate the correction factor, P needs to be replaced with PR, and R replaced with 1/R

PR = (0.2381) (3.36) = 0.8

1/R = 1/3.36 = 0.2978

Then from Fig.19, the correction factor is approximated as

F = 0.74

Substituting the known values into Eq. (49) the heat exchanger area is calculated

$q = UA_s (F) LMTD$

600 kW = 0.250 kW/(m^2.°C) A_s (0.74) (31.51 °C)

A_s = 102.9 m^2

The same procedure can be used to solve part (b) of this example, by varying the mass flow rate of water. However, it is clear that the manual solution of part (b) will consume a great deal of time without adding much to the learning process. Employing Excel will ease and speed up the calculation process.

In order to use Excel to solve part (b) of this example, the correction factor chart, Fig. 19, must be replaced by an appropriate equation. For a one-shell-pass and even number of tube passes, the equation for the correction factor is given as[8]

$$F = \frac{\sqrt{1 + R^2}}{1 - R} \ln\left(\frac{1 - RP}{1 - P}\right) \left[\ln\left(\frac{2 - P\left(1 + R - \sqrt{1 + R^2}\right)}{2 - P\left(1 + R + \sqrt{1 + R^2}\right)}\right)\right]^{-1} \tag{60}$$

where, R and P are defined by Eqs. (51) and (52), respectively.

Equation (60), along with equations for C_c, LMTD, P, R, were used in an Excel spreadsheet to determine the surface area of the heat exchanger by varying the cooling water mass flow rates. Table 1 represents the results of the heat exchanger area calculations. Excel was used to plot the heat exchanger area as a function of water mass flow rate, as shown Fig. 21.

Table 1. Excel spreadsheet calculation of heat exchanger area for example 8.

m_w, kg/s	C_c kW/°C	C_h, kW/°C	ΔT_b, °C	$T_{c,o}$, °C	ΔT_a, °C	LMTD °C	P	R	F	A, m^2
6	25.2	10	15	38.81	51.19	29.48	0.3175	2.52	0.500	162.81
7	29.4	10	15	35.41	54.59	30.65	0.2721	2.94	0.675	115.99
8	33.6	10	15	32.86	57.14	31.51	0.2381	3.36	0.750	101.51
9	37.8	10	15	30.87	59.13	32.17	0.2116	3.78	0.796	93.73
10	42.0	10	15	29.29	60.71	32.70	0.1905	4.2	0.827	88.76
11	46.2	10	15	27.99	62.01	33.12	0.1732	4.62	0.850	85.28
12	50.4	10	15	26.90	63.10	33.48	0.1587	5.04	0.867	82.69
13	54.6	10	15	25.99	64.01	33.78	0.1465	5.46	0.881	80.68
14	58.8	10	15	25.20	64.80	34.03	0.1361	5.88	0.892	79.08
15	63.0	10	15	24.52	65.48	34.25	0.1270	6.3	0.901	77.77
16	67.2	10	15	23.93	66.07	34.45	0.1190	6.72	0.909	76.67
17	71.4	10	15	23.40	66.60	34.61	0.1120	7.14	0.915	75.74
18	75.6	10	15	22.94	67.06	34.76	0.1058	7.56	0.921	74.95
19	79.8	10	15	22.52	67.48	34.90	0.1003	7.98	0.926	74.26
20	84.0	10	15	22.14	67.86	35.02	0.0952	8.4	0.931	73.65
21	88.2	10	15	21.80	68.20	35.13	0.0907	8.82	0.934	73.11
22	92.4	10	15	21.49	68.51	35.23	0.0866	9.24	0.938	72.64
23	96.6	10	15	21.21	68.79	35.32	0.0828	9.66	0.941	72.21
24	100.8	10	15	20.95	69.05	35.40	0.0794	10.08	0.944	71.83
25	105.0	10	15	20.71	69.29	35.48	0.0762	10.5	0.946	71.48
26	109.2	10	15	20.49	69.51	35.55	0.0733	10.92	0.949	71.16
27	113.4	10	15	20.29	69.71	35.61	0.0705	11.34	0.951	70.87
28	117.6	10	15	20.10	69.90	35.67	0.0680	11.76	0.953	70.60
29	121.8	10	15	19.93	70.07	35.73	0.0657	12.18	0.955	70.36
30	126.0	10	15	19.76	70.24	35.78	0.0635	12.6	0.956	70.13

Example 9
Consider a cross-flow heat exchanger containing a tube bank that consists of a square array of 100 thin-walled tubes (10x10), each 2.5 cm in diameter and 5 meter long. The tubes are aligned with a transverse pitch of 5 cm. Water is used in this heat exchanger to cool hot air from 800 K, to 500 K. Water makes a single pass through each tube entering at 12 °C. Hot air enters the heat exchanger with a velocity of 5.0 m/s in a cross flow over tubes with a mass flow rate of 2.25 kg/s. Determine the water mass flow rate and the exit temperature.
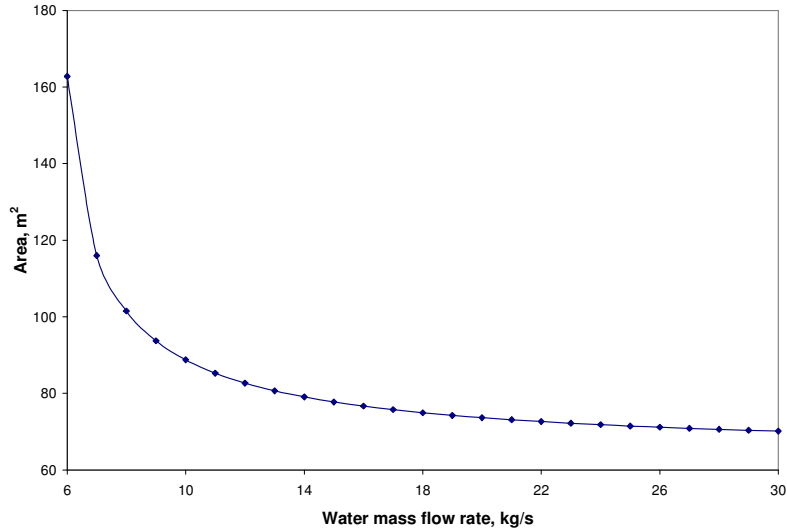
Fig. 21   Example 8, Variation of heat exchanger area with the mass flow rate of cooling water

Solution

The following property values are given for air at an average temperature of 650 K: $c_{p,a} = 1063$ J/kg.K, $\mu_a = 322.5 \times 10^{-7}$ N.s/m, $\nu_a = 60.21 \times 10^{-6}$ m^2/s, $k_a = 0.0497$ W/m.K, and $Pr_a = 0.69$. Assuming an average temperature of 340 K for water, the following property values are obtained $c_{p,w} = 4188$ J/kg.K, $\mu_w = 420 \times 10^{-6}$ N.s/m, $\nu_w = 5.35 \times 10^{-7}$ m^2/s, $k_w = 0.660$ W/m.K, and $Pr_w = 2.66$

For the external flow over an aligned tube bundle, as shown in Fig. 22, the maximum velocity is given by

$$u_{max} = \frac{S_T}{S_T - D} u_{\infty} \tag{61}$$

where u_{∞} is the free stream velocity, D denotes the tube diameter, S_T represents the transverse pitch. Using the data given in the problem statement
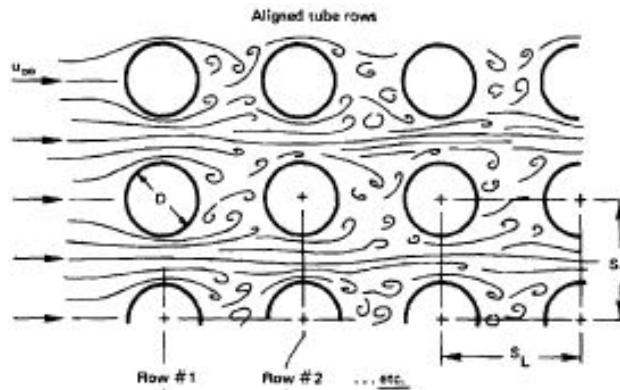 $u_{max} = 10$ m/s

Fig. 22. External flow over an aligned tube bundle[7]

The Reynolds number based on the maximum flow velocity is expressed as

$$\mathrm{Re}_{D,max} = \frac{u_{max} D}{\nu} \tag{62}$$

where, m is the kinematic viscosity. Using the kinematic viscosity value given for air
$\mathrm{Re}_{D,max} = 4152.1$

For air flow across a tube bundles consisting of 10 or more rows, Grimsion[30] gave the following correlation for the average Nusselt number

$$Nu_{D,max} = \frac{h_o D}{k} = C \, \mathrm{Re}_{D,max}^m \qquad \begin{bmatrix} n_L \geq 10 \\ 2000 < \mathrm{Re}_{D,max} < 40{,}000 \\ \mathrm{Pr} \approx 0.7 \end{bmatrix} \tag{63}$$

where n_L represents the number of rows, and h_o is the external heat transfer coefficient. The values of C and m depend on the ratios of S_T/D and S_L/D. For the case when $S_T/D = S_L/D = 2$ (this example), the values of C and m are given as[30] 0.229 and 0.632, respectively. Based on these values, Eq. (51) reduces to

$$Nu_{D,max} = 0.229 \, \mathrm{Re}_{D,max}^{0.632} \tag{64}$$

Substituting the value of $\mathrm{Re}_{D,max} = 4152.1$, into Eq. (64)

$\mathrm{Nu}_{D,max} = 44.318$
Then,
$h_o = k \, \mathrm{Nu}_{D,max}/D = 88.11 \ \mathrm{W/m^2.K}$

For the internal flow, the Reynolds number is defined as

$$\mathrm{Re}_D = \frac{u \, D}{\nu} = \frac{4(\dot{m}_w)_i}{\pi D \mu_w} \tag{65}$$

where $(\dot{m}_w)_i$ represents the mass flow rate in each tube. If Re_D indicates a fully developed laminar flow, the Nusselt number, assuming constant surface temperature, is given as

$$Nu_D = \frac{h_i \, D}{k} = 3.66 \tag{66}$$

where, h_i represents the heat transfer coefficient inside the tubes. When Re_D indicates a fully developed turbulent flow, the Nusselt number, can be approximated by Dittus-Boelter equation[31]

$$Nu_D = \frac{h_i \, D}{k} = 0.023 \, \mathrm{Re}_D^{0.8} \, \mathrm{Pr}^{0.4} \tag{67}$$

Ignoring the thermal resistance of the tube wall, the overall heat transfer coefficient is expressed as

$$U = \left[\frac{1}{h_i} + \frac{1}{h_o} \right]^{-1}$$ (68)

The surface area of the heat exchanger is evaluated, using the following relationship

$$A = N \pi DL = 100 \pi (0.025 \text{ m}) (4 \text{ m}) = 31.42 \text{ m}^2$$

NTU was defined in Eq. (56) as $NTU = \dfrac{UA}{C_{min}}$.

Using the specific heat and mass flow rate data, C_h is calculated from Eq. (47)

$$C_h = C_a = 2364.8 \text{ W/ K}$$

The rate of heat transfer is calculated from Eq. (48)

$$q = C_h \Delta T_h = 2364.8 \text{ W/ K} (800-500) \text{ K} = 709,425 \text{ W}$$

At this point there exist too many unknowns to solve the heat exchanger problem directly either by the LMTD method or the effectiveness method. For the LMTD, the exit temperature of water is unknown and cannot be calculated directly. For the effectiveness method NTU, C_R, and ε cannot be calculated directly without the knowledge of the mass flow rate of water. Therefore, an iterative procedure is required to solve this heat exchanger problem.

We will employ the effectiveness-NTU method in the iterative procedure described below. Some steps in the procedure depend on which fluid is assumed to represent the C_{min}. When $C_c = C_w$ is chosen as the C_{min}, the steps operation is presented in [*brackets and italic*]. If the one assumption does not converge to an answer, then the other assumption can be implemented in the iteration process.

1. Assume $C_h = C_a$ represents C_{min} [*Assume $C_c = C_w$ represents C_{min}*]
2. Assume a value for $C_R = C_{min}/C_{max}$
3. Evaluate $C_c = C_w = C_{max} = C_{min}/C_R = C_h/C_R$ [*evaluate $C_c = C_w = C_{min} = C_{max}C_R = C_hC_R$*]
4. Calculate the total mass flow rate of water, $\dot{m}_w = C_w/c_{p,w}$; $(\dot{m}_w)_i = \dot{m}_w/100$
5. Use the calculated value of $(\dot{m}_w)_i$ in Eq. (65) to evaluate Re_D
6. If the flow is laminar use Eq. (66) to evaluate Nu_D. Otherwise use Eq. (67)
7. Calculate the internal heat transfer coefficient from the results in step 6
8. Evaluate the overall heat transfer coefficient from Eq. (68)
9. Evaluate NTU from Eq. (56)
10. Substitute the values of NTU and C_R in an appropriate effectiveness equation. For a cross-flow heat exchanger when fluid representing C_{min} is mixed and fluid representing C_{max} is unmixed, the effectiveness, ε, is expressed as

$$\varepsilon = 1 - \exp\left\{ -C_R^{-1} \left\{ \left[1 - \exp\left(-C_R NTU \right) \right] \right\} \right\}$$ (69)

[*or a cross-flow heat exchanger when fluid representing C_{max} is mixed and fluid representing C_{min} is unmixed, the effectiveness, ε, is expressed as*

$$\varepsilon = \left(\frac{1}{C_R}\right)\left(1 - \exp\{-C_R[1 - \exp(-NTU)]\}\right) \qquad] \qquad (70)$$

11. Use the value of C_c, evaluated at step 3, is used in Eq. (54), to calculate $(T_{c,o} - T_{c,i})$

$$\varepsilon = \frac{C_c(T_{c,o} - T_{c,i})}{C_{min}(T_{h,i} - T_{c,i})} = \frac{(T_{c,o} - T_{c,i})}{C_R(T_{h,i} - T_{c,i})}, \text{ or } (T_{c,o} - T_{c,i}) = \varepsilon C_R(T_{h,i} - T_{c,i})$$

[*Use the value of C_c, evaluated at step 3, in Eq. (54), to calculate $(T_{c,\,o} - T_{c\,i})$*)

$$\varepsilon = \frac{C_c(T_{c,o} - T_{c,i})}{C_{min}(T_{h,i} - T_{c,i})} = \frac{(T_{c,o} - T_{c,i})}{(T_{h,i} - T_{c,i})}, \text{ or } (T_{c,o} - T_{c,i}) = \varepsilon(T_{h,i} - T_{c,i})]$$

12. Calculate $(T_{h,\,i} - T_{h,\,o})$ from Eq. (51)

$$q = C_h(T_{h,i} - T_{h,o}) = C_c(T_{c,o} - T_{c,i}), \text{ or } (T_{h,i} - T_{h,o}) = (C_c/C_h)(T_{c,o} - T_{c,i}) = (T_{h,i} - T_{h,o})/C_R$$

[*Calculate $(T_{h,\,i} - T_{h,\,o})$ from Eq. (51)*

$$q = C_h(T_{h,i} - T_{h,o}) = C_c(T_{c,o} - T_{c,i}), \text{ or } (T_{h,i} - T_{h,o}) = (C_c/C_h)(T_{c,o} - T_{c,i}) = C_R(T_{h,i} - T_{h,o})]$$

13. If the value of $(T_{h,i} - T_{h,o})_{cal}$ evaluated in step 12 is the same (or approximately the same) as the actual value of $(T_{h,i} - T_{h,o})_{act}$ (determined from the values given in the problem statement), stop the process and use the last values of C_c and $(T_{c,o} - T_{c,i})$ to evaluate the mass flow rate and exit temperature for water. Otherwise, assume a new value for C_R, go to step 3 and repeat the iteration process.

Excel was employed to implement the procedure described above. In this process it was assumed that $C_h = C_a$ represents C_{min}. Table 2 shows the results of the iteration process. It shows that when $C_R = 0.26105$, the calculated value of ΔT_h converges to the actual value of $\Delta T_h = 300$. Then water exit temperature can be calculated from

$$T_{w,\,o} = T_{w,\,i} + \Delta T_c = 12 + 78.31 = 90.31 \text{ }^{\circ}C$$

Table 2 shows that the water mass flow rate in each tube is 0.0216 kg/s or the total mass flow rate of water is 2.16 kg/s.

The Goal Seek or the Solver tools of Excel can be employed to speed up the iteration process for solving the example problem 9. Fig. 23-a shows the assumed (C_R) and calculated values for each step of the procedure described earlier for the iteration process. It shows that for an assumed value of $C_R = 0.4$ the procedure calculates $(\Delta T_h)_{calc} = 278.66 \text{ }^{\circ}C$, which is different from the actual value of $\Delta T_h = 300 \text{ }^{\circ}C$ given in the problem statement.

By using the tool menu on the Excel worksheet and selecting the Solver option a menu appears as shown in Figure 23-b. In this menu we can set the target cell (K22) equal to a value of 300 $[(\Delta T_h)_{calc} = 300 \text{ }^{\circ}C]$. We also identify the cell (A22, C_R) that its value needs to be changed during the iteration process. By clicking on the Solve button, the Solver will search for a value of C_R that results in a value of $(\Delta T_h)_{calc} = 300 \text{ }^{\circ}C$. The values of all other cells will be changed to correspond to the final value of C_R. The final results obtained from Excel's Solver tool for example 9 is shown in Fig. 23-c.

Table 2   Iteration process results from Excel spreadsheet for example 9

step 2	step 3	Step 4	step 5	step 6	Step 7	step 8	step 9	step10	step 11	step 12
C_R	$C_c = C_{max}$	$(m_w)_i$	Re_D	Nu_D	h_i	U	NTU	ε	ΔT_c	$(\Delta T_h)_{calc}$
0.1	23648	0.0565	6847.0	39.82	1051.2	81.29	1.0800	0.6407	33.00	329.98
0.2	11824	0.0282	3423.5	22.87	603.8	76.89	1.0214	0.6030	62.11	310.55
0.3	7883	0.0188	2282.3	16.53	436.5	73.31	0.9739	0.5702	88.10	293.67
0.21	11261	0.0269	3260.5	21.99	580.6	76.50	1.0163	0.5995	64.84	308.76
0.22	10749	0.0257	3112.3	21.19	559.4	76.12	1.0112	0.5961	67.54	307.00
0.23	10282	0.0245	2977.0	20.45	539.9	75.74	1.0063	0.5927	70.21	305.26
0.24	9853	0.0235	2852.9	19.77	521.8	75.38	1.0014	0.5894	72.85	303.54
0.25	9459	0.0226	2738.8	19.13	505.0	75.02	0.9966	0.5861	75.46	301.85
0.26	9095	0.0217	2633.5	18.54	489.4	74.66	0.9919	0.5829	78.05	300.17
0.27	8758	0.0209	2535.9	17.99	474.9	74.32	0.9873	0.5796	80.60	298.52
0.261	9060	0.0216	2623.4	18.48	487.9	74.63	0.9915	0.5825	78.30	300.01
0.262	9026	0.0216	2613.4	18.43	486.5	74.59	0.9910	0.5822	78.56	299.84
0.2611	9057	0.0216	2622.4	18.48	487.8	74.63	0.9914	0.5825	78.33	299.99
0.26105	9059	0.0216	2622.9	18.48	487.9	74.63	0.9914	0.5825	78.31	300.00

Integration into Heat transfer Course

Examples included in this paper were integrated into an advanced heat transfer course open to advanced undergraduate students and entry level graduate students. The integration covered two semesters when the course was offered in the last three years. The enrollment included approximately 10 % undergraduate and 90% master level students. All students had previously taken an undergraduate introductory heat transfer course. Is solving problems that required iteration process, student were given a choice of using Excel or any other software available them. Even though students had easy access to such software programs as IHT or MATLAB, approximately 70% of students chose Excel to complete their assignments. The author has not taught an introductory undergraduate heat transfer course in the last two years, but he plans to integrate the examples into the undergraduate course when the next opportunity of teaching the course. However, the author has successfully integrated the use of Solver or Goal Seek tools of Excel in the undergraduate thermodynamic courses for solving problems requiring trail and error processes.

Summary

The application of Excel spreadsheet in solving a variety of heat transfer problems was demonstrated through several examples. It was shown that Excel is a useful computational tool when the solution to problems requires (a) varying one of the parameters, (b) plotting the results of calculations, and (c) an iteration process. Excel is accessible to all students and, typically is available on most desktop and laptop computers.

Microsoft Excel - HX-Examples.xls

	A	B	C	D	E	F	G	H	I	J	K
1		Air				Water					
2	m_a =	2.25	kg/s		m_w =	?	kg/s				
3	$c_{p,a}$ =	1051	J/kg.K		$c_{p,w}$ =	4188	J/kg.K		L=	4	m
4	μ_a =	3.23E-05	N.s/m		μ_w =	4.20E-04	N.s/m		A =	31.42	m^2
5	ν_a =	6.02E-05	m^2/s,		ν_w =	5.35E-07	m^2/s,				
6	k_a =	0.0497	W/m.K		k_w =	0.66	W/m.K				
7	Pr_a =	0.69			Pr_w =	2.66					
8	$T_{h,i}$ =	800	K		$T_{c,i}$ =	285	K				
9	$T_{h,o}$ =	500	K		$*T_{c,o}$ =	?	K				
10	u_∞ =	5	m/s								
11	C_h =	2364.75	W/K		$*C_c$ =	?	W/K				
12	q =	709425	W				W				
13	D =	0.025	m		D =	0.025	m				
14	S_T =	0.05	m				m				
15	u_{max} =	10.0	m/s				m/s				
16	$Re_{D,max}$ =	4152.1									
17	$Nu_{D,max}$ =	44.318									
18	h_o =	88.105	W/m².K								
19											
20	step 2	step 3	Step 4	step 5	step 6	Step 7	step 8	step 9	step10	step 11	step 12
21	C_R	C_c =C_{max}	$(m_w)_i$	Re_D	Nu_D	h_i	U	NTU	ε	ΔT_c	$(\Delta T_h)_{calc}$
22	0.4	5912	0.0141	1711.7	13.14	346.8	70.25	0.9333	0.5411	111.47	278.66

Example 1 / Example 2 / Example 2a / Example 3 / Example 3a

(a) – First step of iteration process

Solver Parameters

Set Target Cell: $K$22

Equal To: ○ Max ○ Min ● Value of: 300

By Changing Cells:

$A$22

Subject to the Constraints:

Solve | Close | Guess | Options | Add | Change | Delete | Reset All | Help

(b) – Solver dialogue box

Microsoft Excel - HX-Examples.xls

K22 fx =J22/A22

	A	B	C	D	E	F	G	H	I	J	K
17	$Nu_{D,max}$ =	44.318									
18	h_o =	88.105	W/m².K								
19											
20	step 2	step 3	Step 4	step 5	step 6	Step 7	step 8	step 9	step10	step 11	step 12
21	C_R	C_c =C_{max}	$(m_w)_i$	Re_D	Nu_D	h_i	U	NTU	ε	ΔT_c	$(\Delta T_h)_{calc}$
22	0.261045	9059	0.0216	2622.9	18.48	487.9	74.63	0.9914	0.5825	78.31	300.00
23											
24											

Example 1 / Example 2 / Example 2a / Example 3 / Example 3a

(c) -Final solution

Fig. 23. First step in using Solver for the iteration process in Example problem 9

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Nomenclature

A = surface area or cross-sectional area, m^2
Bi= Biot number
C= Fluid capacitance, W/K
C_R= capacitance ratio, C_{min}/C_{max}
c_p specific heat, J/kg.K
D diameter, m
F= correction factor
Fo= Fourier number
h = heat transfer coefficient, $W/m^2 \cdot K$
$I_o(x)$, $I_1(x)$ = modified Bessel function of the first kind of order zero, order one
$J_o(x)$ or $J_1(x)$ =Bessel function of the first kind of order zero or order one
$K_o(x)$, $K_1(x)$ = modified Bessel function of the first kind of order zero, order one
k = thermal conductivity, $W/m \cdot K$
L= length, m
$\dot{m}$ = mass flow rate, kg/s
LMTD= logarithmic mean temperature difference, K
NTU= number of transfer units
Nu Nusselt number
P= correction factor parameter or perimeter
Pr = Prandtl number
q = heat transfer rate, W
R = correction factor parameter or radius
Re = Reynolds number
S pitch
T = temperature, ^{o}C or K
U = overall heat transfer coefficient, $W/m^2 \cdot K$

Greek letters

α = thermal diffusivity, m^2/s
Δ difference
ε heat exchanger effectiveness
θ = dimensionless temperature parameter, a ratio of temperature differences
η_f = fin efficiency
μ = viscosity, N.s/m
ν = kinematic viscosity, m^2/s
τ = shear stress

Subscripts

Act= actual
c= cold fluid stream
ch= characteristic
h = hot fluid stream
i = inlet condition
L linear
min= minimum value
max maximum value
o = outlet condition or location at x=0
T transverse
∞ = ambient condition
f = fin