A Computer Algorithm for Sprinkler Hydraulic Calculations ©Ⓡ

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Introduction

In 1996, the National Fire Protection Association (NFPA) approved a new revision of Standard 13 – Standard for the Installation of Sprinkler Systems (NFPA-13). With this edition, the requirement of performing hydraulic calculations in lieu of pipe schedules for the design of sprinkler systems was made more stringent. Section 5-2.2.1 of NFPA-13 says,

> "… Pressure and flow requirements for Extra Hazard Occupancies shall be based on the hydraulic calculation methods of 5-2.3. The pipe schedule method shall be permitted only for new installations of 5000 ft^2 (465m^2) or less or for additions or modifications to existing pipe schedule systems sized according to the pipe schedules of Section 6-5…."

In this regard the standard provides only two exceptions. The first one allows the use of pipe schedules for Light and Ordinary Hazard systems with more than 5000ft^2 (465m^2) when the required flows on NFPA-13 table 5-2.2 is met with a minimum residual pressure of 50 psi (3.4 bar) at the highest elevation of sprinkler. The other exception deals with additions or modifications to existing Extra Hazard pipe schedule systems.

Beyond the constrains imposed by NFPA-13 and current city water supply in Puerto Rico, there is another reason for using hydraulic calculations rather than using pipe schedule systems. When a system is hydraulically designed, the system can be arranged as a grid or a loop. The benefit of this type of pipe arrangement is that pipe sizes can be reduced (when compared to pipe schedules) and made uniform. Besides, hydraulic calculations provide an insight into the working conditions of the sprinkler system and can help to tailor the system meet the available water supplies.

Hydraulic calculations of systems designed using pipe schedules (tree arrangement) can be done by hand. The labor is math intensive, but simplification tables for pipe and fitting head loss have been made available through various resources. For systems arranged in a grid or a loop, the computation becomes tedious and challenging. The use of computer algorithms is a welcome relief for these cases.

Many computer programs are available commercially for the design of sprinkler systems. The cost of such programs can be as high as $1,500. But this is an exorbitant cost for a task that can be implemented in a computer using the BASIC language. If an algorithm simple enough to be implemented would be available, almost everyone would agree that they would give it a try. The only problem is that many of the available algorithms need a little of magic in order to work for the design of sprinkler systems. Many program developers call this magic, coding secrets, and they concealed those secrets from any description on how the program works. Moreover, such secrets are not coding secrets at all, but physical interpretations or approximations on how the systems we want to design work. Should these "secrets" be made available, everyone could have a better understanding of how sprinkler systems work.

In the next section a simple method to solve pipe networks is explained. The method alone is not enough to solve a sprinkler design problem. A few "secrets" on how to use it for sprinkler systems will be discussed in later sections. Finally two examples will be presented to demonstrate the accuracy and versatility of this algorithm.

The Hardy Cross Method

From the available algorithms, the easiest one to implement in a computer is the Hardy Cross Method. In general the basic principles that govern the solution of any pipe network system are the following:

1. Conservation of mass at the nodes.
2. Uniqueness of pressure at a given point in the loop.

Hardy Cross assumption was that the conservation of mass at each node can be established initially without consideration of the uniqueness of pressure. Then the uniqueness of pressure can be used to calculate correction factors (*DQ*) in the flow rate for the different loops[1]. Since head loss can be defined in terms of flow rate, Hardy Cross establishes that the correction factor is equal to,

$$\Delta Q_i = -\frac{\sum \operatorname{sgn}(Q_j) h_j\left(\left|Q_j\right|\right)}{\sum \left.\frac{\partial h_j}{\partial Q}\right|_{\left|Q_j\right|}} \qquad \text{Equation 1}$$

In Equation 1, DQ_i is the correction factor for loop i, $h_j(|Qj|)$ is the head loss of element j evaluated at the absolute value of the flow rate Q_j, and $\left.\frac{\partial h_j}{\partial Q}\right|_{|Q_j|}$ is the derivative of the head loss with respect to flow rate evaluated at the absolute value of Q_j the flow rate of element j. The function sgn(Q_j) returns the sign value of the flow rate Q_j.

For example, Figure 1 presents a simple pipe network system. The system is composed of 5 Pipe Elements. Pipe Element 1 goes from Node A to node B, 2 goes from C to B, 3 goes from A to D, 4 goes from C to D and 5 goes from A to C. In this case flow is entering at nodes A and C and leaving from nodes B and D. Note how the flow rate is assumed to go in the direction of the arrows. This is very important, because the solution will be presented based on these assumed directions. A positive flow rate means the flow direction follows the arrow. A negative flow rate means the flow direction is against the arrow.

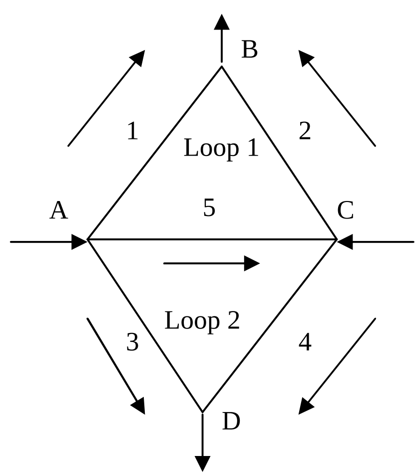

Figure 1 – Simple Pipe Network

There are two loops in the problem presented in Figure 1. Loop Number 1 is composed of Pipe Elements 1, 2 and 5. Loop 2 is composed of pipe elements 3, 5 and 4. When performing the summations in the correction factor numerator, a positive loop direction must be assumed. It can be either clockwise or counterclockwise. As long as all the loops use the same convention, the algorithm works. For this example let’s assume that loop sign convention is positive in the clockwise direction. The correction factor for elements that belong to Loop 1 would be,

$$\Delta Q_1 = \frac{\operatorname{sgn}(Q_1)h_1(|Q_1|) - \operatorname{sgn}(Q_2)h_2(|Q_2|) - \operatorname{sgn}(Q_5)h_5(|Q_5|)}{\left.\frac{\partial h_1}{\partial Q}\right|_{|Q_1|} + \left.\frac{\partial h_2}{\partial Q}\right|_{|Q_2|} + \left.\frac{\partial h_5}{\partial Q}\right|_{|Q_5|}}$$

Since it was established that the loop positive sign convention is clockwise and Pipe Elements 2 and 5 assumed flow rate is against this convention, a negative sign is used for h_2 and h_5. When applying this correction factor to each of the three elements in Loop 1, one must also follow the loop sign convention. The new calculated flow rates for the elements in Loop 1 are as follows,

$$Q_1^{new} = Q_1^{old} + \Delta Q_1$$
$$Q_2^{new} = Q_2^{old} - \Delta Q_1$$
$$Q_5^{new} = Q_5^{old} - \Delta Q_1$$

Note that a correction factor have to be calculated for Loop 2 and that all the elements that form part of Loop 2 have to be corrected accordingly. Because Pipe Element 5 forms part of Loop 1 and Loop 2, it will be corrected twice.

The process of correcting the flow rates using Equation 1 is continued until a certain established convergence criteria is achieved. The criteria can be relative or absolute. A relative criterion is more powerful than an absolute one, because it assures convergence based on the relative magnitude of the variable compared to its change. An absolute criterion needs to be carefully evaluated before convergence is declared. NFPA-13 establishes absolute criteria for the pressure to be less than 0.5 psi, which is reasonable for sprinkler systems. A more stringent criterion can be used based on the change of flow rates. It is easier to implement because the change in the flow rates is the correction factor calculated using Hardy Cross. If every pipe element flow rate is used to divide its correction factor, a relative criteria based on flow rate is achieved. This is a preferred method for convergence check.

Head Loss Equation for Pipe Elements

The head loss in a pipe can be calculated using numerous empirical formulas. Some are more accurate than other, some work for larger ranges of Reynolds Number than others.

NFPA-13 establishes that pipe friction losses shall be determined on the basis of the Hazen-Williams equation. The Hazen-Williams formula in USC units is as follows,

$$h = \frac{4.52 L Q^{1.85}}{C^{1.85} d^{4.87}} \qquad \text{Equation 2}$$

In Equation 2, h is the frictional resistance in pounds per square inch (psi), L is the pipe length in feet (ft.), Q is the flow rate in gallons per minute (GPM), d is the actual internal diameter of the pipe in inches (in.), and C is an empirical friction loss coefficient (dimensionless). C values depend on the type of pipe material. Table 1 list C values for different types of pipe materials.

Table 1 – Friction Loss Coefficient for Different Pipe Materials

Pipe material	C Value
Standard Underground	140
Steel Schedule 40	120
Thinwall	100-120
Copper Type M	150

The use of the Hazen-Williams equation in the Hardy-Cross formulation is a very easy implementation. Since the pipe diameter and length and the C-value do not change during the iterations, they are combined into a single K-factor as follows,

$$h = K Q^{1.85} \qquad \text{Equation 3}$$

$$K = \frac{4.52 L}{C^{1.85} d^{4.87}} \qquad \text{Equation 4}$$

The correction factor defined in Equation 1 based on the Hazen-Williams formula for a loop with pipe elements only would be as follows,

$$\Delta Q_i = -\frac{\sum \operatorname{sgn}(Q_j) K_j \left|Q_j\right|^{1.85}}{1.85 \sum K_j \left|Q_j\right|^{0.85}} \qquad \text{Equation 5}$$

Once again, one must be careful to use the loop sign convention when performing the summations in the numerator.

Head Loss Equation for Sprinkler Elements

In order to use The Hardy Cross method for hydraulic calculations of sprinkler systems, a head loss equation have to be used for the sprinklers. The head loss equation is also given in NFPA-13 as,

$$Q = K\sqrt{h} \qquad \text{Equation 6}$$

In this equation Q is the flow rate in gallons per minute (GPM), h is the head loss (psi) and K is head loss factor for a given sprinkler. A little of manipulation allows to put this equation into a more usable form for The Hardy Cross Method. Solving for h one obtains,

$$h = \frac{1}{K^2} Q^2 \qquad \text{Equation 7}$$

It is very helpful if this equation is re-arranged as,

$$h = K^{'} Q^2 \qquad \text{Equation 8}$$

$$K^{'} = \frac{1}{K^2} \qquad \text{Equation 9}$$

Notice that Equation 8 resembles Equation 3 for pipe elements, but instead of having an exponent of 1.85 it has 2. Let’s assume that Pipe Element 5 in Figure 1 is a sprinkler element. The correction factor for Loop 1 would become,

$$\Delta Q_1 = -\frac{\text{sgn}(Q_1)K_1|Q_1|^{1.85} - \text{sgn}(Q_2)K_2|Q_2|^{1.85} - \text{sgn}(Q_5)K_5^{'}|Q_5|^2}{1.85K_1|Q_1|^{0.85} + 1.85K_2|Q_2|^{0.85} + 2K_5^{'}|Q_5|^1}$$

This gives us a better understanding on how to calculate correction factors for loops with different types of elements. As we are going to see there is still one more element type that needs to be taken into account in order to fully implement The Hardy Cross method for sprinkler systems.

The Secrets of Hardy Cross for Sprinkler Systems

The Hardy Cross Method is a very useful method for evaluating flow and head loss distributions in pipe networks. Most examples where this method is shown assume that the inflows and outflows to the pipe network are known. But this is not the case for a sprinkler system. In a sprinkler system, the total flow rate is not known *a priori*. When one is designing a sprinkler system the only things that are known are,

1. Assumed pipe diameters
2. Pipe material and properties, mainly the C factor.
3. Pipe lengths
4. Required minimum flow rate at all sprinkler heads.

Item number 1 mentions that pipe diameters are assumed because although you may have an idea of the required pipe sizes, this item might require adjustment once the calculations are done for given pipe sizes. One of the many constrains while designing sprinkler systems is that the velocity on a given pipe element should be less than 20 feet per minute (FPM)[5]. The rationale behind this constrain is that above 20 FPM, the Hazen-William equation might yield inaccurate results of pipe friction. Another of the constrains is that the maximum pressure at a sprinkler head should be less than 60 psi[5]. Above this pressure, the water droplets that the sprinkler distributes are very small and can get vaporized by a fire before doing their job. For the moment these two constrains are going to be held to be checked once the calculations are performed. Item number 4 is calculated based on NFPA-13 depending on hazard type, most remote area and sprinkler flow rate density.

So how can we take into account the fact that we do not know the correct total flow rate and distribution among the sprinkler heads? If we look at the sprinkler system, one would see that all sprinkler heads are discharging at the same atmospheric pressure. This means that all sprinkler heads are discharging to a point that can be considered hydraulically the same. This point will be considered a ***pseudo node*** since it really does not exist in the pipe network. What this means in terms of Hardy Cross loop designation is illustrated in Figure 2.

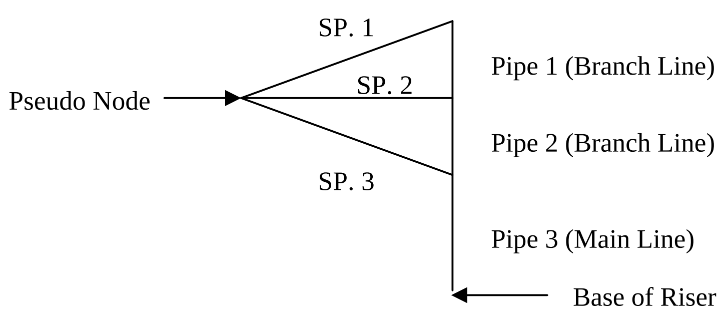

Figure 2 – The Pseudo Node

Figure 2 presents the last three sprinkler heads in a tree arrangement. In the Hardy Cross Method, each active sprinkler head will be considered an element on its own. All the exits of the sprinklers are connected to the same pseudo node. In the system illustrated by Figure 2, two loops can be formed. Loop 1 is formed by Sprinkler Head 1, Sprinkler Head 2, and Pipe 1, Loop 2 is formed by Sprinkler Head 2, Sprinkler Head 3 and Pipe 2. With the pseudo node and the Sprinkler Element, we have taken into account the problem of modeling sprinkler head behavior. Still this do not fully solves the problem of unknown inflows and outflows. One more element type must be considered in order to fully complete the implementation of Hardy Cross for sprinkler systems.

Because a sprinkler system in operation is an open system, the available pressure at the base of the riser will determine the total flow rate of water. One way to look at this problem is to assume a given pressure at the base of the riser. In reality, a sprinkler system uses all the available residual pressure at a given flow rate to move the water through all the pipes to the sprinkler heads. Because of this, the pressure we are assuming at the base of the riser is going to be equal to the head loss from the base of the riser to the pseudo node that we have already defined. Because of the uniqueness of pressure, this head loss is going to be the same independently of the path we take to calculate it. This is where the third element we haven't defined yet comes into play.

Let's say that we have an element connecting the base of the riser to the pseudo node we defined. This connection does not really exist, but it is an artificial way to stipulate a fixed pressure level between the base of the riser and the exit of all the sprinkler heads (pseudo node). This element carries no flow, but it has a constant head loss. The head loss is equal to the pressure at the base of the riser. This element is called a ***pseudo element***. A pseudo element can be used in the Hardy Cross to impose a fixed pressure level between two points [3]. In the flow correction equation for the loop, a pseudo element is taken into account by adding the head loss in the numerator, keeping in mind the loop sign convention. Since the head loss is constant, it's derivative is zero and thus, it does not shows up in the denominator.

There is still one more item that is of our concern. If we are assuming a fixed pressure level using the pseudo node, does this means that the minimum sprinkler flow rate will be achieved? No, the pseudo element in conjunction with the pseudo node only provides us with a way to convert a sprinkler system to a pipe network suitable for use of the Hardy Cross Method. For a given pressure level there is going to be a unique flow rate distribution among the sprinkler heads. This is warranted by the uniqueness of pressure. The only way (with what have been discussed so far) to implement this constrain is that after declaring convergence, the flow rates at all the heads have to be checked for compliance with minimum flow rate and pressure, and all pipe elements should be checked for maximum velocity.

There is still a way to make The Hardy Cross Method find the minimum pressure that is going to satisfy the minimum flow rate at all the sprinkler heads. The program should provide that on every iteration, the sprinkler head with the smallest flow rate be determined. If this sprinkler head flow rate is less than the required minimum flow rate, the fixed pressure level established by the pseudo element is raised by the amount needed to bring the sprinkler head flow rate to minimum compliance. If the opposite is occurring, i.e. the sprinkler head with the smallest flow rate is greater than the required minimum; the head loss of the pseudo element is decreased. The required change in the head loss can be calculated by the following equation.

$$\Delta h = K'\left(Q_{\min}^2 - Q^2\right) \qquad \text{Equation 10}$$

In this equation *Dh* is the required head loss change, *K'* is the *K*' factor of the sprinkler head with the smallest flow rate as defined on Equation 9, *Q* is the flow rate factor of the sprinkler head with the smallest flow rate, Q_{min} is the required minimum flow at all heads.

We have already completed all the requirements to fully implement sprinkler hydraulic calculations using the Hardy Cross into a computer program. This is a very powerful algorithm. It can be used for a tree, a loop or a grid system. The next section presents a simple tree system example. Afterwards we need to deal with other issues of the algorithm and of sprinkler systems.

Example 1- The Tree Arrangement

A tree system is by no means a hard problem to solve by hand. Pipe and fittings head loss tables make it even easier. But it can also be solved using the Hardy Cross Method as implemented in this article. In order to familiarize the reader with the workings of a problem using this algorithm, the next example problem is presented. This problem was taken from NFPA-13 Appendix A page 13-125. The example can dissipate any doubts you may have regarding the use of pseudo nodes and pseudo elements.

Figure 3 and 4 show the plan view and the elevation view of the example problem. In this problem, the most remote area consists of $1500ft^2$. Also, it was calculated by NFPA-13 to use 12 sprinklers in total with 4 sprinkler heads per line.

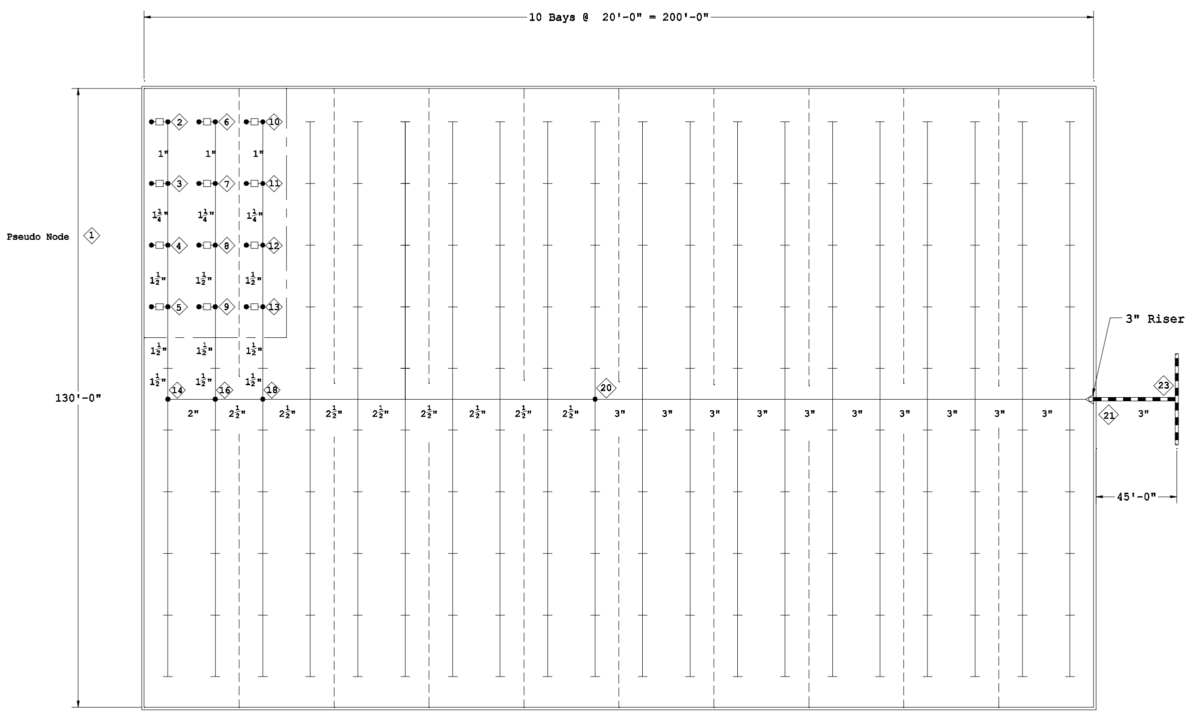

Figure 3 –Plan View (Example 1)

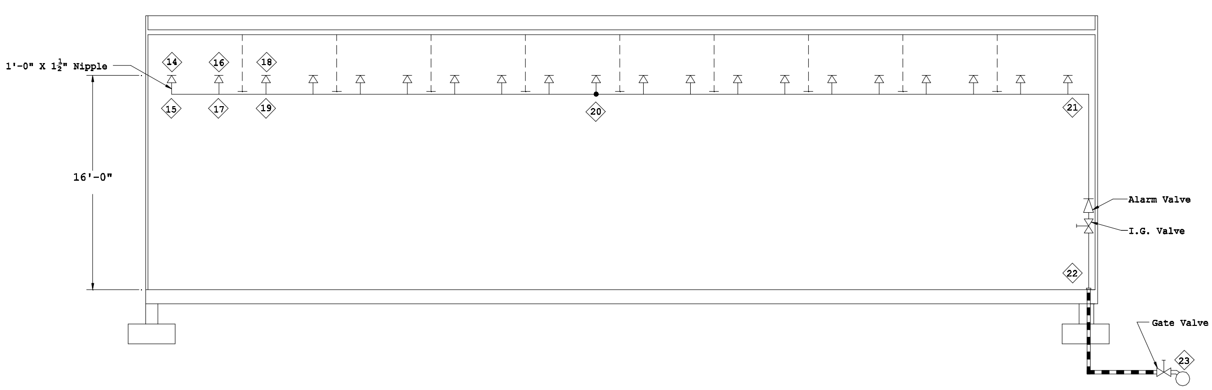

Figure 4 – Elevation View (Example 1)

Table 2. Pipe Element Data (Example 1)

Element	From Node	To Node	Estimated Flow Rate (GPM)	Equivalent Length (Ft.)	Diameter (Sch 40) (in.)	Pipe Friction Coefficient	Static Head Loss (Ft.)
1	2	3	19.5	13’	1”	120	
2	3	4	39.0	13’	1 ¼”	120	
3	4	5	58.5	13’	1 ½”	120	
4	5	14	78.0	19.5’	1 ½”	120	
5	6	7	19.5	13’	1”	120	
6	7	8	39.0	13’	1 ¼”	120	
7	8	9	58.5	13’	1 ½”	120	
8	9	16	78.0	19.5’	1 ½”	120	
9	10	11	19.5	13’	1”	120	
10	11	12	39.0	13’	1 ¼”	120	
11	12	13	58.5	13’	1 ½”	120	
12	13	18	78.0	19.5’	1 ½”	120	
13	14	15	78.0	17’	1 ½”	120	
14	15	17	78.0	10’	2”	120	
15	16	17	78.0	17’	1 ½”	120	
16	17	19	156	10’	2”	120	
17	18	19	78	17’	1 ½”	120	
18	19	20	234.0	70’	2 ½”	120	
19	20	21	234.0	110’	3”	120	
20	21	22	234.0	30’	3”	120	15’
21	22	23	234.0	82.2’	3” C.M.	150	

Table 3. Pseudo Element Data (Example 1)

Element	From Node	To Node	Head Loss (PSI)
1	23	1	60

Table 4. Sprinkler Element Data (Example 1)

Element	Node	Estimated Flow Rate (GPM)	K
1	2	19.5	5.65
2	3	19.5	5.65
3	4	19.5	5.65
4	5	19.5	5.65
5	6	19.5	5.65
6	7	19.5	5.65
7	8	19.5	5.65
8	9	19.5	5.65
9	10	19.5	5.65
10	11	19.5	5.65
11	12	19.5	5.65
12	13	19.5	5.65

Table 5. Loop Data (Example 1)

Loop	#1	#2	#3	#4	#5	#6	#7	#8
1	+PS1	-S12	-P12	-P17	-P18	-P19	-P20	-P21
2	-S8	-P8	-P15	-P16	+P17	+P12	S12	
3	-S4	-P4	-P13	-P14	+P15	+P8	S8	
4	+P1	+S1	-S2					
5	+P2	+S2	-S3					
6	+P3	+S3	-S4					
7	+P5	+S5	-S6					
8	+P6	+S6	-S7					
9	+P7	+S7	-S8					
10	+P9	+S9	-S10					
11	+P10	+S10	-S11					
12	+P11	+S11	-S12					

Legend: P – Pipe Element, S- Sprinkler Element, PS – Pseudo Element

Tables 2 provides the data for all the pipe elements, Table 3 provides the data for pseudo elements, Table 4 provides the data for sprinkler head elements and Table 5 provides the loop data. In regard to the loop data it is important that all the elements are present in at least one of the loops. When deciding how to make the loop that contains the pseudo element, any path can be used. The path selected was taken to be the one with the least elements present. This minimizes data input. One must be careful to use the same loop sign convention for all loops not containing the pseudo element. On Figure 5 a schematic representation of the sprinkler system is presented with the loops and sprinkler heads identified. Notice how Loop 1, the one that contains the pseudo element, follows the shortest path from the pseudo node to the base of the riser. It is important to notice that only for this loop, the positive sign convention has no effect as long as the correct positive or negative head loss in the pseudo element is used. If the path taken goes from the pseudo node to base of the riser in the pseudo element, it is negative (a negative head loss is equal to a pressure increase) because you are going from a node of less pressure to one with higher pressure. The opposite also holds true.

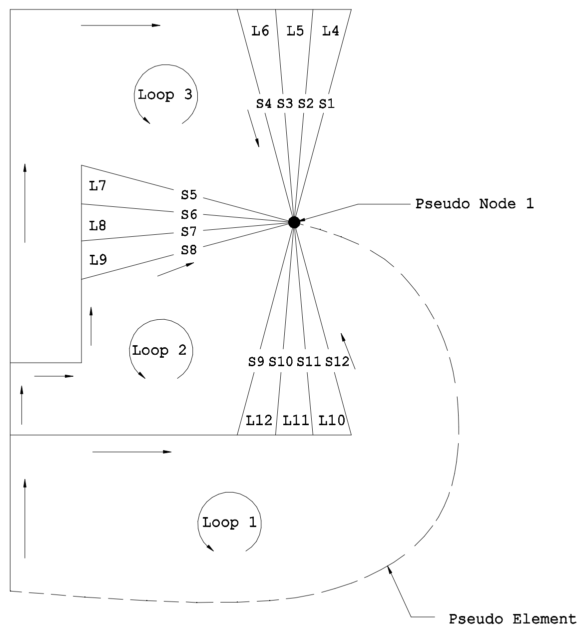

Figure 5 – Loops Diagram (Example 1)

List 1 presents the converged flow rate, head loss and velocity through all the elements. List 2 presents the pressures at the start and end of each element, the start being from where the flow is coming in, and the end where the flow is going. These results were obtained using a computer program that I developed based on the algorithm presented in this article. The program took 734 iterations in 9 seconds to find the solution on a Pentium 166MHz computer using Visual Basic 5.0 for Windows 95. Table 6 shows the hand computation presented in NFPA-13. As it can be seen, the results are almost identical. The hand calculation yielded a flow rate of 260.4GPM @66.3psi; the computer program results were 260.67GPM @66.47 psi. The small discrepancies are due to the rounding-off made in the hand computations. From these results it is obvious that the pseudo node and the pseudo element accomplished their task. They simulate the behavior of a sprinkler system when using The Hardy Cross Method.

For this problem, the NFPA-13 example also mentions that a city water supply with a static pressure of 90psi and 1000GPM @60psi of residual pressure is available. The results obtained with the hand calculation and the computer program is not what is going to happen

List 1 – Flow results using The Hardy Cross Method

Element	Element Type	Head Loss PSI	Flow Rate GPM	Velocity FPS
1	Pipe	1.6143	19.5000	7.2370
2	Pipe	1.6248	40.2794	8.6377
3	Pipe	1.7171	62.2714	9.8109
4	Pipe	4.6276	85.4762	13.4669
5	Pipe	1.6573	19.7792	7.3406
6	Pipe	1.6679	40.8534	8.7608
7	Pipe	1.7624	63.1550	9.9502
8	Pipe	4.7494	86.6843	13.6572
9	Pipe	1.7234	20.2014	7.4973
10	Pipe	1.7340	41.7215	8.9470
11	Pipe	1.8320	64.4913	10.1607
12	Pipe	4.9362	88.5111	13.9450
13	Pipe	4.0343	85.4762	13.4669
14	Pipe	0.7028	85.4762	8.1703
15	Pipe	4.1405	86.6843	13.6572
16	Pipe	1.0803	172.1604	11.5336
17	Pipe	4.3033	88.5111	13.9450
18	Pipe	16.2910	260.6715	17.4633
19	Pipe	8.8886	260.6715	11.3098
20	Pipe	8.9242	260.6715	11.3098
21	Pipe	5.0568	260.6715	11.9796
1	Pseudo	66.4734	0.0000	0.0000
1	Sprinkler	11.9117	19.5000	0.0000
2	Sprinkler	13.5260	20.7794	0.0000
3	Sprinkler	15.1507	21.9920	0.0000
4	Sprinkler	16.8678	23.2048	0.0000
5	Sprinkler	12.2552	19.7792	0.0000
6	Sprinkler	13.9125	21.0742	0.0000
7	Sprinkler	15.5804	22.3017	0.0000
8	Sprinkler	17.3428	23.5292	0.0000
9	Sprinkler	12.7840	20.2014	0.0000
10	Sprinkler	14.5074	21.5200	0.0000
11	Sprinkler	16.2414	22.7698	0.0000
12	Sprinkler	18.0734	24.0198	0.0000

List 2 – Pressure results using The Hardy Cross Method

Element	Element Type	Ps PSI	Pe PSI
1	Pipe	13.5260	11.9117
2	Pipe	15.1507	13.5260
3	Pipe	16.8678	15.1507
4	Pipe	21.4954	16.8678
5	Pipe	13.9125	12.2552
6	Pipe	15.5804	13.9125
7	Pipe	17.3428	15.5804
8	Pipe	22.0921	17.3428
9	Pipe	14.5074	12.7840
10	Pipe	16.2414	14.5074
11	Pipe	18.0734	16.2414
12	Pipe	23.0096	18.0734
13	Pipe	25.5298	21.4954
14	Pipe	26.2326	25.5298
15	Pipe	26.2326	22.0921
16	Pipe	27.3129	26.2326
17	Pipe	27.3129	23.0096
18	Pipe	43.6039	27.3129
19	Pipe	52.4925	43.6039
20	Pipe	61.4167	52.4925
21	Pipe	66.4734	61.4167
1	Pseudo	66.4734	0.0000
1	Sprinkler	11.9117	0.0000
2	Sprinkler	13.5260	0.0000
3	Sprinkler	15.1507	0.0000
4	Sprinkler	16.8678	0.0000
5	Sprinkler	12.2552	0.0000
6	Sprinkler	13.9125	0.0000
7	Sprinkler	15.5804	0.0000
8	Sprinkler	17.3428	0.0000
9	Sprinkler	12.7840	0.0000
10	Sprinkler	14.5074	0.0000
11	Sprinkler	16.2414	0.0000
12	Sprinkler	18.0734	0.0000

if the sprinkler system is connected to a city water supply. If the allowance for hose streams is not taken into consideration, more water is going to flow at a higher pressure. Actually, 304.03gpm @86.68psi is going to be the point of operation. This can be found graphically and using the hand calculation. To find this point, one has to draw the system curve for the sprinkler system and the city water supply curve. The intersection of both lines is the point of operation. The accuracy of a graphic solution is only as accurate as the definition of the drawn graph. A computational solution using the results of the hand calculation is also available. But because we need to check back to see if we exceed the pressures and velocities that are part of our constrains, it is a little bit lengthy approach. Once the point is found, one has to re-compute back to each sprinkler head. The Hardy Cross Method is suitable for finding this solution on its own without much hesitation. The results given above were calculated using an adapted pseudo element that simulates city water supply conditions. Compared to what can be obtained from the graph in Figure 6, it is quite close (approximately 300GPM@ 87psi). The developments of the city water supply element and that of a fire pump simulation element are beyond the scope of this article. Neverless, the development of advance elements is a continuation of the method established in this article.

There is still one more test that would convince us of the exceptional power of this algorithm. The next section deals with the solution to a grid problem.

Table 6 –Manual Calculations (Example 1)

STEP No.		Nozzle Ident. And Location	Flow In gpm	Pipe Size	Pipe Fittings and Devices	Equiv. Pipe Length	Friction Loss Psi Foot	Pressure Summary	Normal Pressure	Notes	Ref. Step
1	1	BL-1	q Q 19.5	1″		L 13.0 F T 13.0	C=120 0.124	Pt 11.9 Pe Pf 1.6	Pt Pv Pn	q=130x.15=19.5	
2	2		q 20.7 Q 40.2	1 ¼″		L 13.0 F T 13.0	C=120 0.125	Pt 13.5 Pe Pf 1.6	Pt Pv Pn	q=5.65(13.5)^.5	
2	3		q 22.0 Q 62.2	1 ½″		L 13.0 F T 13.0	C=120 0.132	Pt 15.1 Pe Pf 1.7	Pt Pv Pn	q=5.65(15.1)^.5	4
4	4	DN RN	q 23.2 Q 85.4	1 ½″	2T-16	L 20.5 F 16.0 T 36.5	C=120 0.237	Pt 16.8 Pe Pf 8.6	Pt Pv Pn	q=5.65(16.8)^.5	5
5		CM TO BL-2	q Q 85.4	2″		L 10.0 F T 10.0	C=120 0.070	Pt 25.4 Pe Pf 0.7	Pt Pv Pn	k=85.4/(25.4)^.5 k=16.95	
6		BL-2 CM TO BL-3	q 86.6 Q 172.0	2 ½″		L 10.0 F T 10.0	C=120 0.109	Pt 26.1 Pe Pf 1.1	Pt Pv Pn	q=16.95(26.1)^.5	6
7		BL-3 CM	q 88.4 Q 260.4	3″		L 70.0 F T 70.0	C=120 0.233	Pt 27.2 Pe Pf 16.3	Pt Pv Pn	q=16.95(27.2)^.5	
8		CM TO FIS	q Q 260.4	3″	E5 AV15 GV1	L 119.0 F T 140.0	C=120 0.081	Pt 43.5 Pe 6.5 Pf 11.3	Pt Pv Pn	Pe=15x0.433	8
9		THROUGH UNDER- GROUND TO CITY MAIN	q **Q 260.4**		E5 GV1 T15	L 50.0 F 32.0 T 82.2	C=150 TYPE 'M' 0.061	Pt 61.3 Pe Pf 5.0	Pt Pv Pn	COPPER 21x1.51=32	9
			q Q			L F T		Pt **66.3** Pe Pf	Pt Pv Pn		

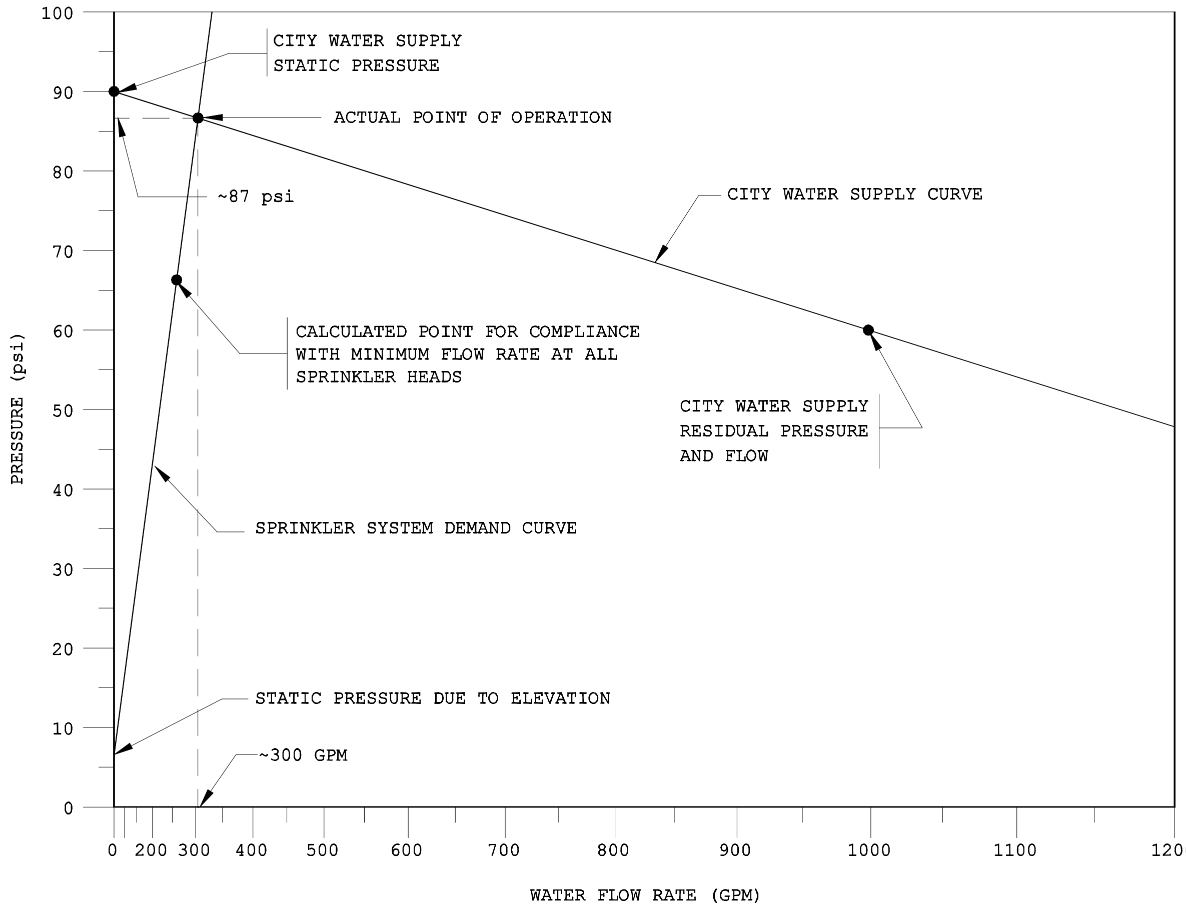

Figure 6 – Demand and Supply Graph

Example 2 – A Grid Arrangement

A grid arrangement provides a formidable way to distribute the water required by sprinkler systems. In a tree arrangement, a main line feeds the branches where the active sprinkler heads are located. Because of this, the main branch has to be capable of supplying all the water demanded by the system. In a grid arrangement the water is distributed to the active sprinkler heads using the branch lines that are not active. Unlike the tree arrangement, the branch lines are connected on both sides. This allows for a more efficient distribution of water and allows the use of smaller pipe sizes. The following example problem was taken from reference 2. Because the data input is more extensive, we are only going to show the results obtained. Figure 7 shows the solution obtained using the computer program that was developed for this article making use of this algorithm.

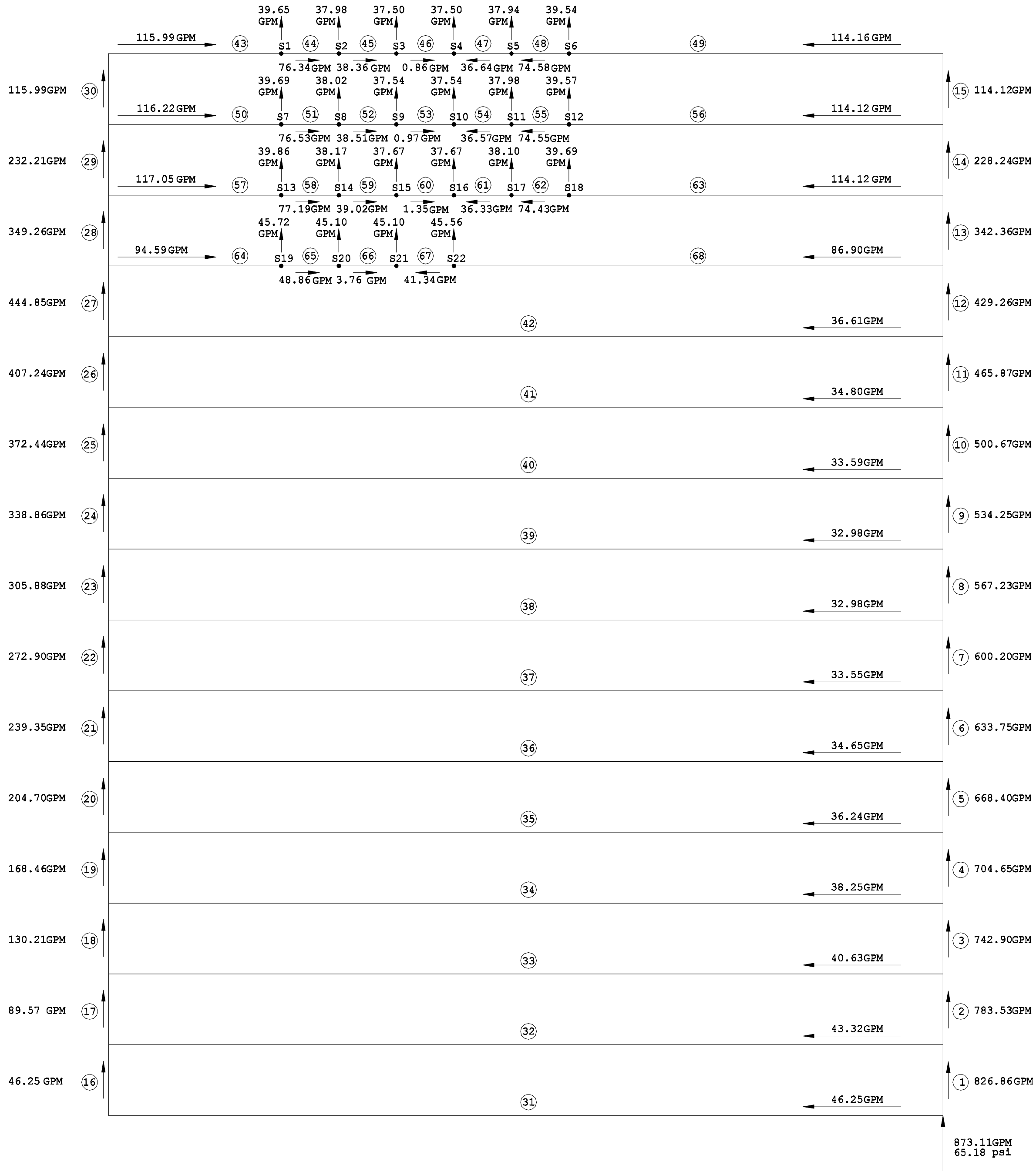

Figure 7 – Flow Distribution Solution (Example 2)

For this example, the branch lines were 1 ½", the supply side main was 4" and the tie-in main was 3". As it can be seen on Figure 7, the branch lines that are inactive are used to distribute the water to the far-end side of the branch lines that are active. The most hydraulically remote sprinkler heads are found in the last branch in the middle. NFPA-13 requires two other hydraulic computations where the remote zone is moved to the left and/or to the right one sprinkler head in other to establish that the most remote zone is being used for the calculations. Since the amount of active heads do not change for the required two other calculations, it is a simple matter of changing the lengths to the elements entering the remote area. In this case elements 43, 50, 57 and 64 on the left side and elements 49, 56, 63 and 68 on the right side lengths have to be changed accordingly. Using the algorithm shown in this article it is very simple to change the length data to accomplish this task. Mainly this will change the K factor of those pipe elements.

The solution found using the algorithm presented in this article showed that 873.11GPM @ 65.18psi are required to meet the minimum required flow rate at the active sprinkler heads (2224 iterations and 84 seconds). Compared to the solution offered in reference 4 (873.11 GPM @65.16psi), both solutions are very close. The differences can be due to the convergence criteria and tolerance used as well as the use of double precision (15 digits) in the program that was used for this article. As we have seen this algorithm can be used for any kind of sprinkler system configuration. There is still an aspect of the algorithm that we have not yet talked about. Next section explains another little secret.

Convergence of The Hardy Cross Algorithm

As with any computer algorithm, there are instances were the convergence rate is poor. Sometimes, the algorithm never converges to a solution. There is information that points out convergence problems with The Hardy Cross Method [1]. Whenever a loop contains elements with high pressure loss and elements with low pressure loss, convergence difficulties might arise. Neverless, by exercising carefulness in the selection of the loops and by using a few other techniques, such problems can be readily solved. One such technique is the use of relaxation factors. A relaxation factor reduces the possibility of the algorithm to diverge. A relaxation factor is implemented in The Hardy Cross Method by using only a fixed percentage of the flow correction in all the loops. In solving the problems I have presented in this article, a relaxation factor of 0.75 to 0.85 was used. Also, the correction to the pseudo elements in order to force compliance with minimum sprinkler head flow rate (Equation 10) has to use a relaxation factor. For the problems solved in this article a relaxation factor for the pseudo elements in the order of 0.1 to 0.2 was used. There are other methods available that show much better convergence rates. All of them are based on the same principles presented in this article. The use of a pseudo element and a pseudo node is the key to implementing any of the available methods for sprinkler systems.

Summary of the Hardy Cross Method

For those of you that are anxious to implement this algorithm in your computer, List 3 provides a cookbook recipe of the steps to the Hardy Cross Iteration Technique.

List 3 – The Hardy Cross Iteration Sequence

1. Calculate the K and K' of all elements. To include fitting pressure losses use the equivalent length method.
2. Assume a flow rate in all the elements. For sprinkler elements use the required minimum flow rate. Start working from the sprinklers toward the base of the riser.
3. Make sure that the assumed flow rate conforms to the conservation of mass principle at all nodes.
4. Calculate the correction factor using Equation 1 for all loops. Make sure you are following the loop sign convention established.
5. Apply the correction factor to all the element flow rates.
6. For each element divide the sum of the correction factors applied to the element by its flow rate.
7. Find which of those values is the biggest. This is your relative maximum flow rate error.
8. Use the absolute value of the error to compare against a tolerance.
9. Apply Equation 10 to the pseudo node and check the change in head at the pseudo node. Compared it to a preset tolerance.
10. If any of the above convergence check above fail, go to step 4.

Conclusions

As we have seen, the use of computer algorithms holds a promise for the solution of pipe network problems. The Hardy Cross Method is the simplest method to implement and is especially suitable for sprinkler systems. The results that can be obtained using this method are accurate. The use of the ***pseudo node*** and the ***pseudo element*** provides a correct simulation of a sprinkler system operation. The development of advance elements that allows the understanding of problems involving pumps, reservoirs and city water supply resources are possible with this method.

The secrets revealed in this article are not coding secrets, but interpretations of how to simulate physical reality using a mathematical model. The disclosure of these techniques allows engineers to understand how sprinkler systems work.

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