

A Conflict of Interests: The Case of Mark Black

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In this paper, the author describes the case of a teacher, Mark Black, as he struggles to adapt to the calls for the reform of teaching in California. Drawing on a set of interviews and observations that are part of a larger study (see Cohen, Ball, and Peterson in this volume), the author explores how Mark enacts the curriculum of Real Math, the textbook that his school district recently adopted. Through the lenses of his beliefs about the nature and structure of mathematical knowledge, his beliefs about how students best learn mathematics, and his beliefs about his role as a teacher, Mark transforms the innovative textbook into a more familiar, traditional elementary mathematics curriculum. The author discusses four real and perceived constraints that influence Mark's ability to enact the curricular policy proposed by the Framework and argues that teachers are themselves learners who need to be supported and nurtured as they try to change their practice.

I started in Arizona. I taught in Arizona in a real redneck type district and I can bring a group of kids in the first day and I can sit them down from the first minute and I can work them solid until the last minute of the last day. And I'll tell you, I've done it but it's no fun. But I can if I had to. I can sit them down, I can shut them up, and I can work them. (interview, 12/88)

Mark has been teaching fifth grade for 10 years. Initially certified in Arizona, he moved to California several years ago. Mark is an energetic and enthusiastic teacher, constantly moving around the room, speaking clearly, encouraging students to ask questions, patting kids on their backs. He is always asking questions, reminding students that it's okay to be confused. After all, what is his job, if it is not to help clear up their confusions?

Decisively in control of his class, Mark tolerates no disruptions and maintains a quiet and orderly classroom. Students seem to

have a good time: They smile often, are eager to answer his questions, and are willing to ask many of their own. Parents like having their children in his class for he has a reputation in the school for "straightening out" troubled kids. Moreover, his classes have scored high on the California Assessment Program (CAP) tests in the past. In fact, last year his class received the highest scores in the school, scoring even higher than the gifted class. In addition to their achievement, Mark is also concerned with students' self confidence since, as he put it, "you need confidence just to be successful in life." He believes kids acquire confidence through mastery of schoolwork and Mark works hard to help students master mathematics, the subject matter that was focal in our discussions.

The school district in which Mark works adopted *Real Math* (1987) as the textbook best suited to meet the goals of the Mathematics Curriculum *Framework* for California Public Schools (California State Department of Education, 1985). Mark himself had never

seen the *Framework* when we met in December; the closest he had come to it was a meeting with a representative from the textbook company in September. According to Mark, the representative, “tried to sell us on the textbook.” Mark is certain that the textbook represents the *Framework* authors’ intentions, although he is not sure about the specific goals of the *Framework*, having only heard about it in casual conversations with other teachers and school administrators. He’s almost certain that he agrees with its spirit which he interprets as making math “real” and “useful” to children. As he explained:

It’s something that I’ve said for years. People don’t sit around—somebody mentioned it the other day, except for accountants—people don’t sit around all day and do math problems. This is what they’re going to do in real life. They’re going to be out somewhere at a pizza party and they’re going to have left over pizza and they’re going to have to figure out how to divide it up or something along that line. You know what I mean? So, this is the sort of thing that can help them. To me, that’s why it’s important. To help them understand . . . math in a real setting type thing. (interview, 12/88)

Mark’s beliefs about the reactions to the *Framework*, his enactment of the curriculum, and his beliefs about teaching mathematics are the basis of this case. I observed Mark on three separate occasions, twice in December of 1988—4 months after he had started using the new textbook adopted by his school district—and then once more in April of 1989, 8 months into the academic year. On each occasion, Mark’s commitment to helping students learn mathematics was clear. In the first section of the case I describe in detail two instances of his teaching that reflect this concern. I then move on to an analysis of what Mark thinks about teaching mathematics for understanding and how he implemented the new curriculum. I close the case with a discussion of several conflicts inherent in Mark’s practice that influence his teaching and his implementation of the policy.

Inside Mark’s Classroom

December 1988: Teaching Long Division

Mark teaches in a large suburban school district in northern California in which students

attend school all year. His classroom is a rectangular room in a building that looks like a Quonset hut. Twenty-eight children were present when I visited, half of them were White, the other half a mixture of Black, Hispanic, Asian, Filipino, and Indian. Most students in his class come from middle- and lower middle-class backgrounds. Their seats were arranged in pairs, all facing the front of the room and the blackboard. Mark stood and spoke from a podium at the front of the room when the whole class was working together. At other times, he wandered throughout the room, checking students’ papers and working with students who were working at the blackboard.

Mark’s lesson came directly from the teacher’s manual of the textbook, *Real Math, Level 5* (pp. 106–107). Class began with a mental math exercise in which Mark read problems from the textbook, for example, 40 divided by 4, and students signaled thumbs up or thumbs down depending on whether or not there was a remainder. When students got stuck on the mental arithmetic, Mark had them work the problems out on scratch paper. During those times, he would walk around the room. As students completed problems, Mark would check them for correctness. If a student completed a problem correctly, sometimes Mark would assign him/her the role of student teacher—which meant that the student was free to walk around the room and confer with students who were having difficulties. Each time students worked at their desks, Mark and two or three of these “student teachers” would work with the students as they went through the problem in question. As it turned out, Mark used this system of “student teachers” frequently in his teaching of all subjects. Mark ended the mental math activity with “the problem of the day” which was 2 divided by 3. Most students seemed familiar with the way to solve this problem and no time was spent discussing the answer. Mark simply went through the solution steps at the board, frequently asking students to tell him what to do next.

Mark then moved to a “prophecy activity” in the text. Students read problems from the textbook aloud, and Mark talked them

through the solution. The problems required that students predict the solutions to problems, e.g., "If you start at 0 and add 2 each time, will you hit 20?" At this time, he stood at the front of the room with a clipboard on which he had a checklist. As students read the problems, Mark checked off that they had participated in a "speaking" activity. He explained to me later that he also checks them for reading and writing. According to Mark, he is responding to the call for integrating language skills throughout the elementary school curriculum by using this accounting system.

After students had gone through five problems of this sort, Mark moved on to the next page in the text and had another student read a word problem:

One day Laura did a lot of work at the library. She rode her bicycle home for lunch. She rode back to the library after lunch, and in the afternoon she rode home again.

"That's about 4 trips I made today between my house and the library," said Laura.

The odometer on her bicycle showed that she had ridden a total of 6.0 kilometers. "I'm going to figure out how far it is from my house to the library," she said. Laura did the problem this way:

$$\begin{array}{r} 1 \text{ R}2 \\ 4 \overline{) 6} \end{array}$$

Asking selected students to tell him how to solve the problem at each stage, Mark led the students through this problem, all the while focusing on the procedure for how to do division with decimals (lining up the columns, putting in the decimal, adding zeroes, bringing the zeroes down, subtracting, continuing his process until they reached zero). Clearly, students had learned a set of steps to go through in order to solve these problems, for example, set up the long division problem, put the decimal in the right place, etc., and the class discussion of the problem went immediately to those steps. Mark spent no time discussing the textual aspects of the problem as it was presented—the scenario, the characters, the problem. Rather, quickly he reduced the word problem to a mechanical division problem. Discourse in the class was

characterized by Mark asking pointed questions with right answers and students providing brief, one or two word responses. For example, in one part of the lesson, the class was working on the problem: $3 \div 8$. Mark had written on the board.

$$8 \overline{) 3.0}$$

Mark: Eight into 30, how many times Ashcon?

Ashcon: Three times.

Mark: Three times. Three times eight, Ashcon?

Ashcon: 24.

Mark: [Writing the work on the board.]

$$\begin{array}{r} .3 \\ 8 \overline{) 3.0} \\ \underline{24} \\ 60 \end{array}$$

Mark: Do you see what I did here? I subtracted; I added a new zero here. I mean, I subtracted; I got a six. I added a new zero; I brought it straight down. Eight into 60, Ashcon? Who knows [students start raising their hands]? Good. Heather, come on, I want you with us today. Ashcon.

$$\begin{array}{r} .37 \\ 8 \overline{) 3.0} \\ \underline{24} \\ 60 \\ \underline{56} \\ 4 \end{array}$$

Ashcon: Six.

Mark: Six, okay, I believe it would be seven times eight is [writes the problem on the board]. Now, if I subtract, I'm going to get a four; add my next 0; 8 into 40 goes 5 times; there it is. Okay, Ashcon? Questions? Okay, what? Do you understand that one, Ashcon? Did everybody understand it? Give me a yes signal if you understood it. Come on, everybody give me some sort of signal.

$$\begin{array}{r} .375 \\ 8 \overline{) 3.0} \\ \underline{24} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

In this instance, and in many others that I observed, Mark did most of the talking during mathematics class. When Ashcon proposes “six,” Mark barely pauses, sweeping on to “7,” the correct answer. During the final portion of the class, Mark had students work on the problems from the textbook. Students who felt comfortable with their skill worked at their desks, occasionally raising their hands with questions. Mark answered individual questions, prompted the next step in the procedure, pointed out mistakes. Most of the time, however, he spent at the chalkboard with students who were still uncomfortable or unsure of how to do the problems. About eight students came up, and worked problems on the board while he looked on. As students became more secure with the procedure, he sent them back to their seats, or they decided to go back on their own.

The conversations that Mark had with students during this portion of class reveal what his goal for the lesson was: mastery of the procedure for dividing with decimals. When students had difficulty, he would direct them to “put the decimal here,” “line the columns up properly,” or “bring the zero down.” When students were asked to “explain” their answers, it was sufficient and acceptable for students to explicate the steps taken toward the answer. For instance, Nefissa’s explanation is typical of what Mark wanted his students to be able to do:

Nefissa: [She has written the following on the board:]

$$\begin{array}{r} 1.2 \\ 5 \overline{) 6.0} \\ \underline{5} \\ 10 \\ \underline{10} \\ 00 \end{array}$$

Five goes into 6 once and one times five is five. And then 6 subtract five is one, so you put your decimal point in there, put your zero there. Bring down the zero. Five times, two times five is ten, so you put your ten there. (observation, 12/88)

All of the instruction that Mark provided while the students worked on problems was procedural. His talk was peppered with comments like, “put your decimal in the right

place,” “bring your zeroes down,” “what one little thing did you forget to do?” His comments to Melissa serve as an illustration:

Melissa, how are you doing? Are you sure? One thing you forgot to do is keep your numbers in a straight column. Right now you’ve got your 2 right above where the decimal needs to be but your decimal will go between here and here. See, because after you stop working with the five, you’re in the area where the decimals are, so then the numbers go behind the decimal. (observation, 12/88)

Mark also provided a lot of positive feedback when working with students at the board. He seemed to recognize that several students may have been there because they needed reassurance or attention more than help with the steps. Constantly and cheerfully exclaiming, “Perfect!” “Well done!” “Exactly!” Mark gives his students lots of praise and encouragement.

Mark’s focus on the standard procedure is illustrated in his reaction to the book’s use of an alternative strategy for notation in long division problems. When Sara noted that the book used a different procedure than the one Mark had taught them, he said to the class:

I don’t understand their procedure either. Let’s do it my way for now. Boys and girls, if you look at the examples up there, they do it a completely different way in this book. I’m sorry but I don’t know that way, so if you don’t know that way, do it as we’ve been doing it. Alright? I’m sorry, I don’t know that way so we can only do it the way I know. (observation, 12/88)

When I asked him about this comment in an interview after the observation, Mark explained:

I never saw that procedure before. This got really confusing. What they’re doing is they’re putting their little notation here after you subtract but they don’t show the procedure to get it and that’s going to lose a lot of these kids. First of all, there is something about this textbook that I’m not happy with—the fact that this is the first time in my life that I’ve ever seen this. No introduction, no explanation, not even in the teacher’s book. . . . I’m skipping this. . . . I’m forgetting that, and I’m going to do what I

understand because I can only teach what I understand. (interview, 12/88)

Mark is absolutely right, he can only teach what he understands. In my observations of his teaching in December, it appeared that Mark understood mathematics to be a set of procedures that students needed to master in order to solve exercises involving division, subtraction, multiplication, and addition: tools that could help students solve real word problems like measuring the size of a room or dividing up a pizza. While he was open to the possibility that there were alternative procedures that students could learn to solve those problems, he was aware of the limitations of his own knowledge of alternative procedures and concentrated on teaching students the methods he knew. He neither chose to help students generate their own algorithms nor explained how or why his procedures worked.

One exception to Mark's heavy emphasis on the procedural aspects of solving exercises occurred during my second observation of Mark in December. After my first observation, Mark and I spent several hours talking about the *Framework* and its emphasis on conceptual understanding. Thinking about what we had talked about the night before, Mark decided to spend a little time in class the next day showing the students what was happening when they divided using decimals. He explained to me, "I thought I'd do a little bit of what we were talking about last night." During the lesson, which consisted of solving more exercises involving decimals and division, Mark interrupted the routine and said (observation, 12/89):

Mark: Okay, I'm going to show you something now. Just pay attention for fun. Watch this. [Draws on the board.] We know that we can take 4 into 10 and we can get 2 and 5 tenths, right?

$$\begin{array}{r} 2.5 \\ 4 \overline{) 10.0} \\ \underline{8} \\ 20 \end{array}$$

Everybody understands because I already worked that. And now you're seeing that you can take four into ten and get 2 remainder two. Boys and girls watch this! Here I have a two [pointing to the numerator of the

fraction 2/4] and here I have a five [pointing to the decimal .5 in 2.5] but I can make this two turn into that five. Take this two here, everybody see where I got that two? Nothing up my sleeves. Take this four here, okay? [Writing on the board:]

$$\begin{array}{l} \frac{10}{4} = 2\frac{2}{4} \\ \text{and} \\ \frac{2}{4} = .5 \end{array}$$

This is now a fraction, which means, division.¹ You guys remember that? Whenever you have a fraction, it's actually a division problem: you're dividing the bottom number into the top number. Everybody with me on that so far? I've taken this and put it on top of that. Boys and girls, how many times does four go into 20?
Ss: 5.

Mark: Five, yes. Don't we have .5? How many get it? [About half the class raise their hands.] Good, mathematics *works!* There's no secrets, there's no tricks. [Quickly erases everything from the board.]

As an observer, I had questions concerning what students understood about what Mark was trying to do during these 5 minutes. Mark explained to me that he didn't have time to always "explain" the underlying rationale for why some mathematical procedures worked, but that he had wanted to show students—if only briefly—that division with decimals and fractions were related. When students asked him questions about his explanation, he repeated what had gone before, going through the same numerical manipulations. Several students continued to ask questions, unclear about the relationship between numerator 2 in $\frac{2}{4}$ ths and the decimal .5 in the 2.5, and he eventually said in frustration, "I only showed you that little thing *for fun*. We can't spend all this time talking about it."

Mark's teaching is familiar. His class looks like countless other mathematics classes: children learn how to manipulate numbers, solve problems, practice in class, do homework sets. Talk is teacher-centered; student participation consists of curt responses to simple, informational questions. He is a prototype of the "effective" teacher. Using an old script, Mark is acting out a part that has

been well articulated and clearly defined by process-product researchers of teaching. He asks his students dozens of questions, he smiles often, and he provides practice and preliminary explanations in which he models the strategy he is about to teach. He energetically walks around the room, patting students on their backs and providing extra help for those who express the most confusion. But *why* does Mark teach math this way. Is it because he sees mathematics as procedural? Alternatively, does he view the teaching and learning of mathematics in a way that shapes his teaching this way? Or is it because he has developed strategies for teaching familiar content that are habits hard to break, methods tried and true? A view of Mark teaching another topic, one that is new to the curriculum and to him, may help us begin to explore some of the reasons for Mark's pedagogical style and choices.

April 1989: Teaching Functions

When I returned to observe Mark in April, he was teaching inverse functions, a topic that was new to the fifth-grade curriculum in this school and one which Mark had never taught. Since we had last met, he had altered the seating arrangements of his students. Instead of pairs of desks facing the front of the room, the desks were arranged in three large circles of 11 children each. The lesson was a review of the work that they had been doing for the past 2 days, and Mark started the class by writing different functions on the board, asking students to generate the inverse of the function and then solve it. For example, the first 10 minutes of class went something like this (observation, 4/89):

Mark began class by writing the following on the board, occasionally glancing in the teacher's guide:

INVERSE FUNCTIONS

The inverse function does the opposite of whatever the function does.

Mark: Who can read that to me?

Girl: [Reading from the board] The inverse function does the opposite of whatever the function does.

Mark: Alright. Everybody remember this? What was the biggest problem you guys

had on last week's quiz? Remember on that paper? I mean the homework paper, not the quiz?

Boy: The arrows.

Mark: The arrows. Remember? Okay, so, let me give you a simple one here and let's begin.² [Writes the following on the board:]



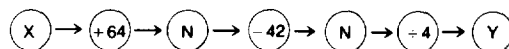
Mark: Watch your arrows, boys and girls! Okay, go ahead and do that. First of all, copy the function then give me the inverse. [Writes on the board:]

1. Copy the function
2. Give the inverse

[Mark then walked around the room while students worked the problem. When most were done, he went to the front of the room.] How many of you copied the function? I want you to have that practice. So you copy the function. Then you reverse the direction, don't you? It goes the opposite way. So if you are starting at y, you must subtract five to get your basic number in the middle and then the opposite is divide three.³ [Writes on the board:]



Alright, any questions? Any questions? I've got a dead group back there not paying any attention. Are there any questions? [continued silence] Okay. I know this is Monday and I know the weather's been warm, but that's okay. Are we ready?! Okay. [Writes on the board:]



The class then went through several more examples which followed the same pattern. Mark generated a function and wrote it on the board. He then gave students a couple of minutes to solve it (which consisted of copying the function and finding the inverse). After that, he wrote the correct answer on the board.

The remainder of the class continued in this pattern. Mark would present a problem; students would solve it. Mark would walk around the room checking students' work. He continued the "student teacher" system I had observed in the fall, asking students who

had finished their work correctly to help others who were having difficulties.

Like the lessons I observed in the fall, this one came from the textbook. Students were supposed to know that a function involved manipulating a number with a series of operations—addition, subtraction, multiplication, or division—to get a new number. For example, if you start with 5 and add 3, multiply by 2 and subtract 7, you end up with a new number, 9. Students were then supposed to learn that the “inverse” of this function involved coming up with a series of steps that would undo what had been done, that is, add 7 to 9, divide by 2, and subtract 3, to get the original 5. Mark’s goal was to have students be able to generate the inverse of any function that he put on the board. In the lesson, he emphasized the mechanics of the process—reversing the arrows, exchanging multiplication and division signs, and exchanging addition and subtraction signs. There was no discussion of why students might exchange multiplication and division signs, addition and subtraction signs. There was no discussion of function machines, what a “function” was or what an “inverse” was, and the focus of this exercise involved helping students learn to get the right answers, emphasizing the “hows” of generating the inverse without discussing the nature of functions, what was going on with all these numbers, why you would want to know the inverse of a particular function, or what the relationship between a function and its inverse is.

When I asked him later how he felt about the lesson, Mark said he thought this was important content because it was good preparation for pre-algebra since “functions are algebra.” More importantly, he thought that these problems gave students practice in the basics—addition, subtraction, multiplication, and division—since they had to use all of those operations to solve the problems. The lesson had the potential for communicating aspects of inverse functions that are important for students to know if they are to “understand” the nature of inverses—for example, *why* the arrows are there in the function, *why* you turn them around in the inverse. Representing functions with arrows helps communicate to the students the dy-

namic aspects of functions—how operations are done to variables, how numbers change. And while the inverses of simple linear functions are equivalent to reversing the operations in a linear fashion, not all inverses are so simply constructed. By defining inverse as “doing the opposite” or “reversing the arrows,” Mark oversimplifies the mathematical ideas at the heart of this lesson. “Inverse” for Mark’s students was a series of steps, not a mathematical idea. The first steps were, literally, put on the board:

1. Copy the function down
2. Write the inverse function

The second set of steps, just as important, was never written:

1. Go to Y, at the end of the function
2. Turn the arrow around
3. Exchange the sign in the circle for its opposite, e.g. substitute a $-$ for a $+$, a \div for an \times .
4. Turn the next arrow around
5. Exchange the sign in the circle for its opposite
6. Repeat until you reach the X

Math, as represented in this lesson, consisted of a set of steps that must be done in order. If students do all the steps, they will get the right answer. The lesson was bifocal: It provided many occasions for children to practice addition, multiplication, subtraction, and division and it gave students a new procedure, a procedure that would produce something called an inverse function. There was no evidence, though, that students had learned to think about or make sense of functions and their inverses.

Conflicts and Constraints

Teaching for Understanding: Levels of Knowing

From one perspective, Mark appears to be a good teacher. He asks his students many questions in a warm and enthusiastic way, checking their solutions, helping them through the steps of algorithms. He covers the content of the curriculum, making sure that he exposes students to all of the topics in the textbook. Students get a lot of drill and practice with addition, multiplication, subtraction, and division—operations that are considered, by some, the “basics” of elemen-

tary school mathematics. But if we switch lenses and look at Mark through the spectacles of the *Framework's* rhetoric, he looks different. Mark believes that mathematics is the mastery of algorithms. He teaches his students to acquire habits, like lining up numbers and following a set of steps, that allow them to manipulate and conquer algorithms. In one interview, I asked Mark to describe the teaching of procedures and rules. His description captures the essence of what I saw him do in his own classes:

[Procedures consist of steps . . .] step by step, by step, by step, you get a result. Rules, I guess are just similar that certain things *have* to occur in order to have a correct answer. So that's it. Step by step, and the correct steps would be rules and procedures. Correct steps to get the correct answers. You blow one of them, you make a mistake and naturally something's going to be wrong. That's why I send them back. I say, "No, you've made an error, go back and see if you can find it." (interview, 12/88)

Mark is committed to his students' mastery of "steps" through lots of practice. Yet consider the *Framework* authors' position on computation and algorithms in mathematics:

Those persons responsible for the mathematics program must assign primary importance to a student's understanding of fundamental concepts rather than to the student's ability to memorize algorithms or computational procedures. Too many students have come to view mathematics as a series of recipes to be memorized, with the goal of calculating the one right answer to each problem. The overall structure of mathematics and its relationship to the real world are not apparent to them. (California State Department of Education, 1985, p. 12)

Although Mark emphasizes the acquisition of algorithmic knowledge, he recognizes that there are other levels of understanding in mathematics. In our interviews, for example, he differentiated between the type of understanding that he aims for, which involves "setting" the algorithms in his students' minds so that they can successfully complete problems involving addition, subtraction, multiplication, and division, and the kind of understanding he doesn't have time to teach, which involves knowing "why" the algo-

rithms work, what a fraction is, or what multiplication means. This distinction surfaced first in a post-observation interview. When I asked him how he thought the lesson went, Mark explained:

I thought that, overall, the lesson went fine because they were doing division and they were comprehending. Now, did they understand deeper the meaning of fractions? I don't think so. Or decimals, I don't think so. But at least they understand the steps. [Interviewer: And what makes you think they did not understand the "deeper meaning"?] First of all, I haven't taught much about the deeper understanding because this book skims over it. I would wait 'til later in the year to be on this particular concept. And then, by then I would have developed decimals. They haven't developed it very well in this book. That's what I mean by they don't understand the deeper meaning. Probably if you asked them what's a decimal, most of them aren't going to be able to tell you that it's a part of a whole, or, even if they do mimic those words, what does that mean? They can't tell you. (interview, 12/88)

Mark's reaction to the text reflects a fundamental difference between the textbook authors' approach to teaching mathematics and his own. The textbook authors, for example, have structured the curriculum of *Real Math* to develop layers of understanding—beginning with intuitive concepts and slowly moving toward more explicit, sometimes algorithmic knowledge. The curriculum is also structured to interweave related ideas. For example, fractions and decimals are taught side by side instead of as separate and discrete topics within mathematics. They explain their approach to fractions in the teacher's manual:

Children have intuitive notions about fractions, because fractions are part of people's everyday language. "We're about halfway there"; "only about one third of these are good." In *Real Math* we use these intuitive understandings. In fifth grade, for example, the students estimate fractional lengths, areas, and so on to review fractional notation in a way that corresponds to their intuitive notions of fractions. We also do a lot with fractions of numbers, because that is a common use of fractions that the students

have encountered often outside mathematics class. Later on in the year, the students add and subtract fractions, including those with unlike denominators, using their intuitive notions to help them add, say, one quarter and one half. Then when we develop standard algorithms for adding and subtracting fractions, the students find that these procedures fit well with their understandings of the world and our language. (Willoughby, Bereiter, Hilton & Rubinshtein, 1987, p. xvi)

Rather than thinking of learning mathematics as the layering of understandings or the gradual development of understandings from intuitive to explicit, Mark has a building-block notion of mathematical understanding. Mathematical concepts rest upon a foundation of mathematical rules and procedures (intuition does not play a role in this conception). Students must first master procedures. They do so by learning about a series of topics, such as single digit subtraction and double digit subtraction, and gaining algorithmic mastery over each “type” of problem. After that foundation has been laid, teachers can explore more conceptual aspects of mathematics. In the best of all possible worlds, students would learn procedures and concepts because algorithmic knowledge alone, in Mark’s opinions, is rather useless:

If I put numbers on the paper and you can add them up and get a new number—so what? What can you do with it besides write them on the paper and do that. It’s like a child who can read out loud but can’t understand what they’re reading. It’s like, I’ve got a dog who can do certain tricks but she doesn’t know what she’s doing. So what? (interview, 12/88)

Mark believes that the ability to represent mathematics—with pictures, diagrams, models—is an example of a more advanced and sophisticated level of understanding in mathematics. So he believes that all children should first learn the mechanics, and then some students, if they have the ability and the disposition, may begin to develop the ability to represent those problems. He explained in one interview:

I wouldn’t count on any kids in my class [coming up with a pictorial representation]. You’ve got to understand one thing, there’s

no gifted kids in my class because they have been siphoned off. What they do in our district is they siphon off the highest talent, the gifted and put them in their own class. . . . So I wouldn’t count on anybody in my class coming up immediately with pictures unless we’ve had a lot of practice. (interview, 4/89)

For Mark, then, only the brightest students develop the cognitive ability to represent mathematical notions in pictorial forms. This belief too, seems in opposition with those that undergird the *Framework*. In that document, the authors argue that *all* students can and should develop “mathematical power and that *no* student should be limited to the computational aspects of the number strand” (California State Department of Education, 1985, p. 4). They also argue that the teacher’s eye must always be on the development of conceptual understandings, whereas Mark seems to believe that this is something a teacher should do only if there is enough time.

Conflict #1: The press of time, community, and tests. So why does Mark teach for rules and procedures if he recognizes that there are different levels of understanding, and he is clearly concerned about students “getting it?” Open about his choice, Mark named three causes: time, tests, and parental pressure. With limited time, for example, Mark believes that he can only work on the basic foundation—the rules and procedures. This is reflected in his reactions to the *Framework* authors’ claim that teaching for understanding is more important than teaching rules and procedures:

When do I have the time to teach? Because I barely got through what they would call here rules and formulas and procedures [in today’s class]. I didn’t have time to get into how to use it. Tomorrow I have another lesson to present. I agree with it—you’ve got to learn how to apply it, no doubt about it. No doubt about it. That’s what math is. But when?

Mark also mentioned parents as a source of pressure for covering the content:

These kids are going to be dragging their books home and one day a parent is going to look at it and say, “You’ve been in school 9

months and you're only on page 100? You've got 200 more pages. What's happened?" And they're going to be romping and stomping in here, saying to the principal, "This teacher is not going fast enough." And I'll tell you what. They can make your life very, very sticky, and I've had it happen where I taught very well and didn't go very fast and parents were screaming and squawking, "They're not going fast enough." You speed it up and then you know what you hear? You hear from the parents who have kids who are going too slow. One parent says you're going too slow; the next parent says you're going too fast. (interview, 12/88)

Finally, Mark remarked that the press to get high test scores on tests like the CAP also limited his ability to teach for deeper understandings. His problems with such tests were two-fold. First, he did not think the tests were designed to test the kind of material that was being presented in the *Framework* or the textbook, for example, the *Framework* authors' emphasis on conceptual understandings. Second, he believed that the test was one of the factors pushing him to cover content since students who did not know the "basic functions"—addition, subtraction, multiplication, and division—would perform poorly on the tests:

Teaching for understanding is what we are supposed to be doing. Now, I only have so many minutes of the day. I'm supposed to teach for understanding. Look at the last one. It's difficult to test, folks. That is the bottom line. It's funny they put it at the last one, because the bottom line here is that all they really want to know is how are these kids doing on the tests? They want me to teach in a way that they can't test, except that I'm held accountable to the test. It's a catch-22. [Rules and procedures are] easy to test. (interview, 12/88)

What is most paradoxical and troubling about Mark's talk is that there seems to be a real distinction in his mind between teaching and teaching for understanding, and even though he wishes that he had more time to teach the material so that his students would learn it, he is willing to simply "teach." According to his own self-reports, as well as the observations I made of his teaching, most of

Mark's teaching consists of showing students how to manage the procedures of mathematics. When he asks students whether they "understand" something, he is checking whether they have been paying attention or following his directions, not for the degree to which they have conceptually mastered the material. This is reflected in much of Mark's talk, both in and out of the classroom. Recall his comment to students, "Everybody understands, because I already worked that." He has taught something if he has told them about it and provided time to practice the steps. This belief has been clarified and reinforced by Mark's experiences in schools in which the press is to cover material and document performance, not to ensure understanding. Consider his remarks on the *Framework's* claim that teaching for understanding takes longer than teaching rules and procedures: "[Reading from the *Framework*] 'Teaching for understanding . . . takes longer to learn.' Hey, if I were spending the time to really get these kids to learn it, I might be several pages back" (interview, 12/88).

Mark made comments like this several times, in which he would explicitly state that his teaching did not involve making sure that students understood the material. He even stated that, given his limited resources and large class size, he didn't try to reach all of his students:

What with the testing, I know that the top ones are going to pass. What do I need to worry about them for? I've got 33 kids—what do I care? That's terrible to say. I care. But they're going to ace it no matter what. I get kids in there who get straight A's no matter what I do. They're going to get straight A's even if I didn't teach them. And I've got kids who are in there flunking, ok? I've got to bring them up; they need the help. And the middle ground are the ones who can do it, aren't really able to, and are going to make the most progress, and that's going to show. So I shoot for the middle ground. They're the ones to show me in a lesson who really got it. Well, there should be a core. There are those who are going to get it no matter what, there are those that will never get it, and there are those that you can move along. And as you move this thing

along, you bring them along as best as you can. (interview, 12/88)

Mark's concerns for parental pressure, students' performance on standardized tests, class size, and content coverage, combined with his beliefs about how children learn mathematics and their abilities to master some aspects of the subject, have put Mark in a position where he has chosen to teach only knowledge of procedures and skills because that is safer, more efficient, more manageable. Mark portrays himself as a teacher caught in a desperate tug of war: The state wants him to teach conceptual understanding but tests procedural knowledge; teaching for deeper understanding requires that compromises be made between breadth and depth—compromises that are often questioned by parents and the community. Mark's concerns about the press of time, parents, and tests are very real, and he is right in acknowledging the power they have over the choices that get made by teachers in schools. But Mark's talk also suggests that other factors are influencing his pedagogical decisions, a point that becomes clearer as we examine the ways in which Mark used the textbook.

"Following" the Book

I pretty much follow it step by step. That's the way I was always brought up in teaching. To me, texts are supposed to be sequenced . . . but math is generally, the way I understood it, sequenced so it kind of goes in stages. So, I kind of follow it step by step. However, I did skip a little here and there. When you get too long in one thing I move on. I move on to the next thing. (interview, 12/88)

As already noted, Mark had no exposure to the *Framework*, save our conversations about it in interviews. The mathematics *Framework*, for Mark, is but one of a series of curricular chimera introduced by the state to increase student achievement in California schools. Teachers, according to Mark, have had no input into these decisions but are nonetheless supposed to implement the *Framework* by using the textbooks adopted by their districts.⁴ Moreover, when the teachers were introduced to the textbook at the beginning of the school year, they were

told to follow it page by page. And according to Mark, that is just what he is doing.

But in conversations with Mark, it became clear that his claim to have "skipped a little here and there" is an understatement. For example, the text, which relies heavily on the use of manipulatives and games, is accompanied by a set of materials, materials that Mark has not "seen the need for yet":

They gave us a big box. But I haven't really had time to look at it. They have little game pieces and they have paper, fake money. I haven't even seen a need for that yet, to tell you the truth. I use the little [response] cubes. We use those quite a bit when the game is lined up with that. They have a whole series of games but it's hard to fit them in, that's the big thing. There's a whole box of materials; as I say, I haven't really looked at it. There's a practice book or a work book where you ditto off the pages. I use them to back things up.⁵ (interview, 12/88)

To avoid using the materials, Mark either had to skip lessons that have required them or had to translate lessons into ones that he could teach without the materials. This is especially problematic given Mark's claims that the textbook does not teach the conceptual aspects of the topics covered. What Mark fails to realize is that the conceptual territory is often covered through the use of manipulatives and story problems. Following the book, for Mark, has meant following the pages in order, but dropping lessons that don't fit with his sense of what students should be learning, adapting ones that require manipulatives so that they can be taught without those materials (perhaps turning them into something entirely different than the authors' intended lessons), and adding "backup" work that has included practice sheets that he has sent home for homework assignments—some of which have come from the workbook in the "box," others that he has from previous years of teaching. In addition, because he only spends about 30 minutes a day on mathematics (and often less), Mark has had to "streamline" lessons to save time. Through his adaptation of these materials, Mark may unwittingly be fulfilling his own prophecy: The students may not be developing deeper understand-

ings of the mathematical content presented in the textbook.

Mark's transformation of the curriculum is not a surprise. We know from research that teacher-proof materials are an illusion, and Mark exemplifies how a teacher's beliefs, knowledge, and concerns influence how curricular materials are used. But Mark's critical use of the textbook is not fueled by some malevolent wish to boobytrap the new mathematics *Framework*. He sincerely believes that he is following the text. And from his perspective, he is. Yet, in many ways, he is not. What is it about Mark, about his knowledge of teaching or of mathematics, about his instructional goals, about his dispositions that contributes to his translation of the curriculum?

Conflict #2: Competing conceptions of learning and teaching mathematics. For one, there is a clear dissonance between Mark's beliefs about how one learns the "basics" and how the textbook presents these basics. Mark believes that students should learn "the basic functions" in sequence: First you introduce addition—all types of addition, single digit and double digit—all the while providing a great deal of practice. Then you move on to subtraction, covering it completely and, again, providing plenty of practice. Once addition and subtraction have been mastered, you move on to multiplication and division, covering each separately and thoroughly. Related activities, for example, learning about decimals and working through applications of these basic functions, can be added on if you have time. However, the introduction of such activities should be held off until all students have mastered the procedures. And the procedures are best mastered if they are done separately, so as not to contaminate one another, since Mark believes that it is easier for students to master procedures if they concentrate on one at a time. Adding more "facts" to be memorized only confuses students in Mark's eyes, and he believes he should ease the learning of his students—reducing any potential sources of confusion or conflict.

The book that Mark uses is based on another set of assumptions. Rather than separating operations that are conceptually re-

lated, the book interweaves the teaching of addition and subtraction, multiplication and division; the text also starts with an emphasis on the intuitive before it gradually moves to a more explicated version of mathematical concepts. Representations, and the ability to generate and manipulate alternative representations of the subject matter, are central to the curriculum—not an add-on if there is time left after students master the procedures. Mark, while he applauds the "philosophy of the book," in his words, to "teach the meaning of these concepts," is troubled by what he considers a "pinball approach" to teaching. He likens the interweaving and spiralling of the curriculum to the painting of a house:

It's kind of like a coat of paint. You paint it one time and you let it dry. You paint it again and you let it dry. It might soak in and it might kind of chip off and things like that. So, I think they just figure a little smattering here and a little smattering next year. See, they're assuming these kids have had this since first grade or so, and they haven't. [The students] are not used to this pace. They're used to a pace where you do division until you basically have it and then you move on, you know. And they don't have it. And I really learned how to pace things where I could jump over addition and subtraction and just keep smattering that. I'd spend a long time. I usually spent the first two thirds of the year on multiplication, addition, subtraction and—well, addition and subtraction and then get beyond that, multiplication and division can take almost until the last quarter of the year and then you're into fractions and, you know, the other things like that. But, by then, most of your class is able to multiply and divide. (interview, 4/89)

Mark was very concerned about the fact that, in April, his students still didn't know how to multiply and divide large numbers:

Right now I have kids in my class that don't know how to multiply or divide yet, and the text isn't addressing it. Here's a page on multiplying, but it didn't just teach multiplication, it jammed decimals on top of it, too. They're getting confused by the decimals when they don't even know how to multiply well. (interview, 4/89)

In addition to the interweaving of topics, there are other features of the textbook of which Mark disapproves. For example, the textbook uses representations—symbolic, pictorial, and otherwise—throughout to help develop understanding. Mark, on the other hand, believes that the use of representations is an ability or skill that is developed after students master the procedures. He does not believe, for instance, that students should learn to represent mathematical ideas before they learn to manipulate the numbers involved. This is why he has dropped all aspects of lessons that deal with concrete objects or manipulatives. He seems unaware that the “box” does not contain supplementary materials to be used in spare time but, instead, contains essential tools for much of the teaching that the policy advocates and that is prescribed in *Real Math*. In some very real and fundamental ways, then, Mark’s view of mathematics teaching and learning conflicts with the one on which the textbook is based.

So here we see a teacher in conflict with the text: Teacher and text have fundamentally different assumptions about how mathematics is best learned and taught. Mark handles the frustration this clash produces in several ways: He skips parts of the text, he provides extra practice for students, and he peppers his teaching of the textbook with lessons that are taken from the *Scoring High* pamphlet provided by the district. Mark does not seem to recognize that his sporadic and inconsistent use of the text and its accompanying materials might be contributing to the difficulties his students are having with the material. This melange of activities and ideas is, in Mark’s eyes, “following the book,” for he does cover most of the lessons, dropping aspects that seem unimportant and adding practice and content that will ensure students’ success on traditional measures of performance.

Conflict #3: Knowledge of alternative pedagogical strategies. Recall Mark’s comment: “I can only teach what I understand.” Another factor that appears influential in Mark’s selective use of the textbook is his own lack of knowledge about how to teach mathematics in the ways suggested by the textbook. For example, although he ap-

plauded the use of manipulatives in mathematics teaching, Mark voiced concern over his own experience and knowledge of how to use such materials:

So I would work a lot with word problems and manipulatives, but I’m not well-trained in manipulatives and to be perfectly honest I don’t how; I don’t have any idea right off the top of my head how to make any manipulative for what we were doing in today’s lesson in division of fractions. (interview, 12/88)

On another occasion he remarked:

My teaching hasn’t been that much different [this year] except following a different text. As I was saying, my biggest hurdle to doing all these new methods—I call them new, but some of them are so old that I wasn’t a part of them when they were in before (they’re regenerating them because they are finding them to be valuable)—is my knowledge of what I’ve done all these years and I don’t know how to make the transition. And I don’t completely know all these methods in the math series. (interview, 12/88)

Mark and his students happily go through the motions, enacting the lessons laid out in the textbook. But their interpretation of those lessons is colored by what they know. For instance, Mark doesn’t know how to use manipulatives, nor does he know why one would use them in particular settings. Since he believes that representing mathematics is a higher order skill, one that follows the proficient use of algorithms and procedures, Mark chooses to skip over lessons that involve manipulatives, or drop the manipulatives from the lessons, in his race with the curriculum coverage clock.

Because they are simply going through the motions, Mark and his class bear little resemblance to the vision proposed by the *Framework*. Discourse in the class is highly constrained. While Mark invites students’ questions, only certain types of questions are allowed: questions about how to do the procedures. When students ask other types of questions, such as why something works, Mark responds by saying, “Remember what I taught you?” or “We don’t have time for that,” throwing the responsibility for answering the question back in the laps of the students. The only

inquiry that is encouraged is that which concerns the “steps” of a procedure. Students do not explore serious mathematical problems, they do not generate multiple solutions to problems, and they do not discuss and debate alternative interpretations and answers. The mastery of rules and procedures is the focus of Mark’s curriculum; no attention is paid to the underlying conceptual ideas. A good explanation is one that traces the steps of a procedure, not one that traces the student’s reasoning through a series of mathematical decisions.

This is not surprising. Mark has had no in-service training in the *Framework* or in the use of the textbook (with the exception of the beginning-of-the-year overview provided by the textbook company). Without the assistance of people who are willing to help teachers learn new ways of approaching mathematics, Mark is left to his own devices. Alone, he does the best he can: skips things he sees as irrelevant, alters assignments and activities to fit his understanding of mathematics and teaching mathematics, and interprets the textbook based on his own beliefs and orientations.

While Mark is sensitive to his own limitations, he is also cynical about the “expertise” of some of the individuals who are proposing these curricular changes. Mark resents “outsiders” who “have never been in classrooms” telling him how to teach:

I guess one thing that really is beginning to drive me up the wall in this business is the fact that every year somebody comes in and says, “Here it is folks, this is the best way to teach. This is it! This is the one that’s going to cure everything.” They’re like the old snake oil salesman. And yet *none* of them—well, I take that back because the last guy that came did—but most of them never say, “What do you do? What works? What doesn’t work? What do you need?” *None* of them! (interview, 12/88)

Given his wariness of outsiders, it is not surprising that Mark reacted to the textbook representative in the way that he did:

They take you through it and they show you a few sample lessons and they try to sell you that what they’ve sold the district is the best thing in the world. I don’t know; to tell you the truth usually I don’t listen to them. Because I don’t need someone telling me how

to work a textbook first of all. And second of all, she wasn’t making a lot of sense to me. Most of the people I talked to came out of there saying they would have done better to take the book home and read it. Another problem was, I can’t remember now if it was my vacation time or right in there, but I hadn’t even had time yet to work with the textbook. So I don’t even have time yet to know good or bad points that I’d like to ask about. I had nothing to go on, so really I got very little out of it. So what I did was, that night I went home and read the format, how it works, things like that. (interview, 12/88)

Mark’s lack of knowledge about alternative teaching strategies does not put him in conflict with the *Framework* as much as it constrains his ability to implement it in spirit. Mark wants to do the right thing, but it remains unclear what the right thing is. Mark read his textbook but the text has been unsuccessful in communicating to him the importance of thinking about mathematics, as well as its teaching and learning, in very different ways. Having a new textbook and a box of materials does not guarantee the appropriate use of them. Mark does not simply lack knowledge of *how* to teach with manipulatives or with cooperative groups, he also lacks knowledge of *why* a teacher might choose to use a particular strategy. Without such knowledge, he is left to interpret the materials in his own way, rejecting those that conflict with his own sense of what should be taught and experienced in a fifth-grade math class. While the authors of *Real Math* make an attempt at providing rationales for the choices they made, the authors do not recognize how powerful the lenses of traditional practice can be, nor how much they can influence what teachers read on and between the lines of their teacher’s manual.

Conflict #4: Competing calls for reform. Mark has reason to voice concerns; he works in a context in which there are multiple messages and reforms. Although the mathematics *Framework* has been implemented this year, Mark is aware of the impending implementation of similar frameworks in language arts, social studies, and science. Mark is poignantly aware of his own shortcomings as a teacher in these reformed classrooms:

And it's bad to say, but I'm finding my biggest hurdle right now to be integrating language, and now one of the big sweeps is literature, and I mentioned cooperative learning and now these new sweeps. I mean we've got four new sweeps coming at us right now. We've got, as I mentioned, integrating language, we're supposed to do that. We're supposed to do literature and, we're supposed to do cooperative learning and now this new math series. That's right now. That's on top of us right now. And to tell you the truth, I wasn't trained very well in any of those. (interview, 4/89)

Moreover, Mark knows that the CAP tests have not yet been altered to match the differences in content emphasis:

I guess another thing is all the stuff they pile on us to do. It's a lot of stress mainly because you know what they look at. They look at test results. And the tests are not written for any of this stuff—the tests are written for the old way. The tests are written for "Open the book, boys and girls. Do this activity, learn your nouns, verbs." Now they're saying, "No, don't teach nouns and verbs in isolation, teach writing of competency." But the test isn't for that and if they go down in the test they're going to come to me and say, "Mr. Black." and I'm going to say, "Wait a minute! You said to teach this, but the test is about that!" There is a lot of stress right now. If you were here long enough at this school, you would find a lot of deep seeded stress. (interview, 12/88)

Finally, Mark reminds us of the larger context in which all of these teachers work. Concerned about the learning of all school subjects, California has produced a series of frameworks that call for change in all content areas. Although those changes occur in cycles, with emphasis and resources being placed on one subject matter each year, teachers like Mark know that it takes more than one year with a new textbook to alter one's teaching.

Conclusion

The drums of reform echo loudly, and teachers like Mark hear the call to change their math teaching and their language arts teaching and their social studies teaching and their science teaching. And as he noted, the

situation is complicated by the fact that testing remains the same: Teachers are to teach new content, but student performance will still be measured with old measures until the new CAP tests are instated. Equally important is the fact that community evaluation is based on traditional conceptions of what and how things are taught in school. Parents expect teachers to teach their children as they themselves were taught. As he makes choices about what to do—what content areas to focus on, what teaching strategies to learn, how to prepare students for CAP tests and cover the new curriculum—Mark's world is a maelstrom of conflicting demands.

Mark needs help. Some of the reasons he needs help are those he himself noted: help in learning about new methods, help in finding time to teach for understanding, and resources for evaluating such understanding. But Mark also needs help for reasons he cannot see: While he speaks about different levels of mathematical understanding, Mark's own beliefs about what it takes to learn and know mathematics are in conflict with those that underline the *Framework*. He needs to learn to think about mathematics as a field of inquiry, not as a body of procedures. He needs to learn to think about the goals of learning mathematics as greater than the mastery of computational skills. And he probably needs to learn new things about the subject matter since his own knowledge of mathematics may be limited to the procedural aspects of the traditional curriculum.

Mark cannot make fundamental changes in his teaching without several kinds of support. First, he needs time and assistance in examining and evaluating his own assumptions about how children learn mathematics, comparing his own assumptions to those that undergird the *Framework*. Assumptions about what it means to know mathematics and how best the subject is taught have changed a great deal since Mark was taught to teach. He is a model teacher in the "effective teaching" paradigm, and there is much evidence that he has worked hard to learn to do that teaching well. But the changes encouraged by the *Framework* depend on another image of teaching, one that focuses on the student as well as on the teacher, on conceptual under-

standing as well as technical mastery. If Mark is to understand the nature of those changes, he needs a chance to examine the central differences between effective teaching and the teaching envisioned by the *Framework* authors. He cannot be led to believe that implementing the *Framework* involves adopting a few new activities and instructional strategies for at its heart the *Framework* assumes fundamentally different things about the nature of learning and knowing. Mark's conceptions of learning and teaching mathematics conflict with those inherent in the *Framework*. If we fail to acknowledge that such conceptions act as lenses through which teachers perceive and interpret curriculum, we see teaching like Mark's: an innovative curriculum edited to be familiar.

Second, Mark needs to think about the kinds of pedagogy best suited to facilitate the development of such understanding. This reform does not call for changing teaching across the board, no matter what. Rather, this reform is based on the belief that teaching methods should match educational goals and that teaching requires complex decision making about the use of a range of alternative pedagogical strategies. Mark needs to learn about the range of methods, including their respective strengths and weaknesses. His lack of knowledge about alternative methods constrains his ability to implement this reform.

Third, Mark needs practice and experience implementing strategies he has never used—gaining familiarity with new materials, adapting old strategies to meet new goals, crafting a version that draws on his strengths and minimizes his weaknesses. Learning to use new methods take time. As they become more familiar with methods, teachers acquire insight and understanding about each strategy—when and how it is most effective, how students react, what students need to know and be able to do in order to participate in the experience, what the nature of the teachers' and students' respective roles are in the activity. Such understanding is developed over time and best facilitated when teachers are given opportunities to practice, to make mistakes, to reflect on their experiences and those of their colleagues. Again, Mark's lack

of skill in the use of new methods restricts how much he can change his own practice.

Finally, Mark needs to work in a context that is sensitive to the complexity of teaching and the factors that influence classroom work. The tests are changing (cf. California State Department of Education, 1989). But the public, including parents, must be reeducated in their own conceptions of what and how students should be learning mathematics. If we want teachers to change their practices, we must provide safe, supportive environments that encourage those changes. Tests must be aligned with the goals of the curriculum, parents must be helped to see the benefits of the new curriculum, and administrators and teachers alike must understand that changes in practice are not easy, are often rocky, and always take time.

These types of support—room to examine beliefs and prior knowledge, new information, practice, and a safe and secure environment—are the kinds of support that we consistently urge teachers to provide students. The *Framework* authors acknowledge the complexity of learning to teach in this way when they state:

Teachers need the same opportunities to develop their understanding and their ability to apply their knowledge to new situations as students do, and such development does not occur in a 1-time 2-hour workshop on a single topic. Rather, well-planned, extended programs are needed in which teachers have the opportunity to see new techniques demonstrated in classrooms, try out new methods with their own students, and reflect on the changes in the curriculum. Further, teachers must receive coaching and support over a period of time to build their confidence and to see for themselves how content and methodology are related in their teaching. (California State Department of Education, 1985, p. 6)

Mark has yet to experience such support. What will happen to Mark? Will he continue to adapt the textbook to meet his more traditional vision of mathematics teaching? Will Mark encounter teachers or support staff who can begin to help him develop new skills like using manipulatives or coordinating cooperative learning? Or are Mark's beliefs about the nature of mathematics and how

children learn mathematics so ingrained in his pedagogical reasoning that he will be forever unable to implement this textbook—and perhaps the *Framework* authors' vision of mathematics teaching and learning—in ways that are more consistent with those documents?

This introduction to Mark and the changes he is making in his mathematics teaching ends, then, on an ironic note: The policy we are investigating calls for teaching mathematics for understanding, a kind of teaching that respects both the mind, dispositions, and interests of the learner as well as the difficulties inherent in learning anything in meaningful ways. Yet in its first year of implementation, Mark was not treated with a similar sense of respect for his needs as a learner. Instead of working with the *Framework*, a textbook became the messenger of the policy. Instead of being placed in settings where the policy could be explored, questions asked, alternative interpretations made, Mark heard through the grapevine that his teaching was supposed to change. Teachers, like their students, are learners who need to be taught in innovative, flexible ways. How the state of California and the school districts in which teachers like Mark work respond to the needs of the learners who comprise their teaching force will be a critical piece of the story we might someday be able to tell about the connections between this curriculum reform policy and its impact on classroom practice.

Notes

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¹What Mark is trying to help students see is the equivalence between $\frac{2}{4}$ and $.5$. He assumes that they do not understand that a remainder of $\frac{2}{4}$ ths is another way of expressing the decimal $.5$ and that 10 and $\frac{2}{4} = 10.5$.

²This representation is of the function $3x + 5 = y$. The textbook authors introduce the students to the notion of function by having them create a "function machine." The function machine allows you to put some number into it and get others out. The rules that govern what happens inside the machine are represented by addition, subtraction, multiplication or division symbols. After students have mastered the visual representation of a function in the form of this machine, the textbook authors use the representation of function sentences that consists of circles and arrows. The arrows indicate that a number is being placed in a machine; the circles represent the machine and its special operation. This function, then, has two machines associated with it. First, students are to replace the variable x with a number, say, 3 . Three is then placed in the first machine in which it is multiplied by 3 . This new number 9 , is represented as N in the number sentence. That number is then placed in the second machine in which 5 is added to it. The product of these operations is the answer, in this case, 14 . Students in Mark's class have already learned to substitute numbers in function sentences like these. What they are reviewing in this lesson is the construction, and validation through testing, of inverses of functions.

³The arrows in this inverse function, while reversed, serve the same purpose: They point the student in the direction of the function machine in which to place the number. In constructing the inverse of functions, students must reverse the arrows and decide what function machines would reverse the work done in the original ones. In the case of the function under discussion here, the original machine added 5 to the number. The inverse of that machine would then subtract 5 . To use the same example, if $Y = 14$, the first step of the inverse function involves placing the 14 into the first function machine in which 5 is subtracted, leaving the student with 9 or N . The next arrow directs the student to place 9 (or N) into the second function machine which has been designed to undo what its counterpart did in the original function, that is, divide by 3 . Students divide 9 by 3 , obtaining the original used in the first function.

⁴Mark is mistaken about the participation of teachers in the development of the state's policy. Teachers are an integral part of the state's policy making in all areas, and they hold positions on all essential committees: curriculum, textbook, and testing. Moreover, within Mark's school district

teachers participated in the review of textbooks and the subsequent adoption of *Real Math* by the district.

⁵An important element in Mark's theory of what it takes to learn mathematics is the role of practice, or what Mark refer to as "backup." Throughout our interviews, Mark constantly made reference to the textbook's lack of practice problems. Without practice, the knowledge of procedures does not "set" in students' mind and, moreover, the teacher lacks feedback about how much students understand.

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