# A DRIVERLESS ALTERNATIVE: FLEET SIZE AND COST REQUIREMENTS FOR A STATEWIDE AUTONOMOUS TAXI NETWORK IN NEW JERSEY 

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#### Abstract

Over the past six decades, as private automobiles have become more affordable and more universal among American families, cars' previously uncounted costs have come to the forefront of the modern transportation debate, with some activists calling for an end to cars. This paper identifies five transit criteria that a transportation system must satisfy if it hopes to dethrone the individually owned and operated car as king of the road. They are: 1) a solution to the congestion problem, 2) safety improvements over conventional manually operated cars, 3) a lesser impact on the environment, 4) economic feasibility, and 5) comfort and convenience to rival the automobile. Given recent advancements in the field of vehicle autonomy, a potential solution to the car's growing problems has presented itself: an autonomous taxi network (ATN). Drawing from the classic Personal Rapid Transit model as well as Mark Gorton's idea of Smart ParaTransit, two potential designs for an ATN are presented and compared to one another, and the viability of the ATN concept as a whole is explored using statewide transportation demand from the state of New Jersey. Using travel demand as generated by Talal Mufti in 2012, the Smart Para-Transit model emerges as the more economically viable implementation, requiring a fleet size between 1.6 and 2.8 million 6-passenger vehicles to meet the state's travel demand in its entirety, at a cost to consumers of $\$ 16.30$ to $\$ 23.50$ per person per day.


## INTRODUCTION

The 2013 paper Shared Autonomous Taxi Networks: An Analysis of Transportation Demand in NJ and a 21 st Century Solution for Congestion by Chris Brownell introduces five key transit criteria that an emerging transportation technology must satisfy if it hopes to challenge the personally-owned automobile as the preferred form of mobility in the United States. They are 1) The system must reduce congestion and decrease commuting times; 2) it must be safer than the conventional automobile; 3) it must have fewer negative environmental impacts than the conventional automobile; 4) it must be economically feasible; and 5) it must offer its passengers comfort and convenience to rival the automobile. An emergent next generation technology that potentially meets each of these requirements is the Autonomous Taxi Network (ATN). An ATN is defined by two key characteristics. First, it consists of fully autonomous vehicles - the taxis which drive passengers to their requested destinations. Second, the taxis are demand-responsive; they do not operate on a regular schedule such as a bus or a train, but are only deployed when a passenger indicates demand.

A full discussion of the means by which ATNs satisfy each of the five transit criteria, as well as a review of the emerging players in the autonomous vehicle space is included in [1]. This article focuses on the two distinct ATN models discussed therein: the Personal Rapid Transit (PRT) model and the Smart Para-Transit (SPT) model. Particular focus is given to the optimal fleet size and cost of operation for an Autonomous Taxi Network in the state of New Jersey, using the disaggregate state-wide travel demand model first implemented by Talal Mufti in [2].

## TWO MODELS FOR AN ATN

One key parameter that must be defined when designing an Autonomous Taxi Network is the method by which travelers are picked up and dropped off. In [3] Kornhauser et al. design a system that borrows its layout from the classic implementation of a Personal Rapid Transit, or PRT network. This model establishes stations - or taxi stands - across the state of New Jersey, in a grid, spaced 0.5 mile apart from one another. The PRT model assumes that passengers will walk to their closest station, which is at most 0.35 mile away. At a typical human walking speed of 3 mph , this corresponds to a seven minute walk at the absolute most, though the majority of passengers require five minutes or less to travel to their nearest station. Similarly, at the end of their trip, passengers disembark at a station and walk to their destination, again a maximum distance of 0.35 miles away. In the PRT model for an ATN, two riders are served by the same vehicle if their origin and destination "stations" are the same, and they arrive at the taxi stand within $t_{\text {max }}$ seconds of one another.

The second model discussed in this paper derives its set-up from a 2008 report by Mark Gorton [4] which introduces a transportation mode called Smart Para-Transit. Figure 1 shows an example of a Smart Para-Transit system condensing twelve individual trips from northern New Jersey to Manhattan into just two SPT trips. Gorton's system assumes that SPT vehicles are operated by human drivers, but they could easily be substituted by autonomous taxis in an ATN. The basic idea behind Gorton's SPT system is that individual riders request a trip to a given destination, at which point they are picked up by the SPT vehicle at a "central transit point." Along the way, the vehicle may stop at one or two other "central transit points" to pick up additional passengers. The drop-off works similarly to the pick-up, with the vehicle stopping at one or more locations.

The SPT model for an ATN allows the distance between nodes on the statewide transit grid to become larger, as the vehicle can move around within the origin pixel to pick up multiple
passengers before traveling to the destination pixel. In an SPT model ATN, "central transit points" can even be discarded, because the autonomous taxis can drive themselves to the passengers' doorsteps, and let these passengers off at the doorsteps of their destinations. In this way, the vehicle takes the place of the individual for intra-pixel travel. While the PRT model requires its users to walk up to 0.35 miles to the nearest station, the SPT model sees the vehicle providing mobility, gathering up passengers before a trip departs. Not only is this a benefit to the passenger, who exerts less energy to get to his taxi, it is also allows for a major increase in pixel size. In the PRT model, two people who live 0.6 mile from one another with a taxi stand directly in between walk for six minutes each, prior to boarding the vehicle. In the SPT model on the other hand, an autonomous taxi can pick up two passengers up to 2.5 miles apart in the same six minutes, assuming an average speed of 25 mph . In the SPT model, the distance between nodes in the transit grid increases from 0.5 to 1.5 miles, resulting in a maximum distance of 1.06 miles to the center of each pixel.


FIGURE 1 Representation of Mark Gorton's Smart Para-Transit System [4]

## DIVIDING NEW JERSEY INTO A PIXELATED TRANSIT GRID

For both the PRT model and the SPT model, the state of New Jersey and its surrounding areas must be broken up into a grid in which each pixel has a side length of $d_{s}$. In the case of the PRT model, $d_{s}$ is equal to 0.5 miles, and in the case of the SPT model $d_{s}$ is equal to 1.5 miles. Rearranging the formula for great circle distance found in equation (1), the X and Y coordinates for any point $P$ located at $\left(\operatorname{Lat}_{P}, \operatorname{Lon}_{P}\right)$ are calculated using Equations (2) and (3). In this analysis, the origin point $O$ is located at $(38.0,-76.0)$ or $\left(38^{\circ} \mathrm{N}, 76^{\circ} \mathrm{W}\right)$.

$$
\begin{gather*}
D=\sqrt{\{69.1 *(\operatorname{Lat}[b]-\operatorname{Lat}[a])\}^{2}+\{69.1 *(\operatorname{Lon}[b]-\operatorname{Lon}[a]) * \cos (\operatorname{Lat}[a] / 57.3)\}^{2}}  \tag{1}\\
X_{P}=\frac{69.1}{d_{s}} \cos \left(\frac{\operatorname{Lat}_{O}}{57.3}\right) \times\left(\operatorname{Lon}_{P}-\operatorname{Lon}_{O}\right) \tag{2}
\end{gather*}
$$

$$
\begin{equation*}
Y_{P}=\frac{69.1}{d_{s}} \times\left(L a t_{P}-L a t_{O}\right) \tag{3}
\end{equation*}
$$

The resulting grid that is formed via these equations can be seen in Figure 2, overlaid atop the Princeton, NJ area. The pixels in the figure have side length $d_{s}=0.5$, a representation of the PRT model ATN; the layout of the SPT model combines each three-pixel-by-three-pixel square from the PRT model into a single pixel. Once the state has been divided up into a grid of pixels, the trip files generated via Mufti's method [2] can be broken down into individual trips from an origin pixel ( $\mathrm{O} \_\mathrm{X}, \mathrm{O} \_Y$ ) to a destination pixel ( $\mathrm{D} \_\mathrm{X}, \mathrm{D} \_\mathrm{Y}$ ) at a given departure time $t_{\text {dep }}$. In the statewide trip data set generated for this paper, there are a total of $32,770,528$ trips taken by $9,054,849$ individuals, for an average of 3.62 trips per person in the simulated weekday. A statewide ordered trip file for the SPT model can be seen in Table 1, sorted by pixel of origin.


FIGURE 2 PRT Model Gridding Overlaid on the Princeton Area [3]
The trips listed on the left of Table 1 are those that originate at the westernmost point of the state $-(16,73)$ - a pixel that contains only one point of interest, Fort Mott State Park, and lies due south of Wilmington, DE. The park employs approximately 10 people, is visited by 80 patrons each day, according to Mufti's 2010 census-based employee and patronage data, and while only the first 19 trips originating at the park are shown in the table, a grand total of 65 trips in the ordered trip file originate from $(16,73)$, and by extension, from the park. While this number is lower than the expected 90 visits, it is a reasonable realization scenario for an average work day. The trips on the right of Figure 30 originate at the easternmost point in the analysis, (81, 139), which corresponds to Westchester County, NY and is one of the seven out-of-state locations in which New Jersey workers reside in Mufti's model. The final fifteen trips originating in Westchester County in the ordered trip file share a common destination and range in time from 6:57am until 9:00am, indicating a potential for ridesharing during the morning commute to $(75,138)$, a pixel that includes the towns of Rockleigh, NJ and Northvale, NJ, both in Bergen County.

TABLE 1 The First 19 and Last 19 Entries in the SPT Model Ordered Trip File


## CALCULATING FLEET SIZE AND TRAVEL COSTS

To compare the PRT model ATN with the SPT model ATN as they pertain to transit criterion four - economic feasibility - the required fleet size and travel costs for each system need to be determined. The first step in that process is to combine any individual person-trips that share the same origin pixel and destination pixel into one taxi trip, provided they depart within a given time window $t_{\max }$. This is done via equation (4), which determine $q_{x}$, the number of passengers present in taxi trip $x$. In the equation, $z_{x}$ corresponds to the row entry of the first person-trip in taxi trip x. Given a maximum vehicle occupancy of $q_{\max }$, taxi trips are filled by the first $q_{\max }$ passengers travelling from point A to point B within $t_{\max }$ seconds of the original departure time indicated in $z_{x}$, which is denoted $t_{z_{x}}$. If fewer than $q_{\max }$ passengers arrive within the given time period, $q_{x}$ is equal to the total number of person trips that originate within that time slot for the given origin-destination pair.

$$
\begin{equation*}
q_{x}=1+\sum_{i=1}^{q_{\max }-1} 1_{\left\{t_{z_{x+i}}-t_{z_{x}} \leq t_{\max }\right\}} \tag{4}
\end{equation*}
$$

The output of equation (4) can be seen in Table 2. The output rows have a very similar format to the rows in Table 1, and come from, again, the very beginning and very end of the ordered trip file. In Table 2, the $\mathrm{O}_{-} \mathrm{X}, \mathrm{O}_{-} \mathrm{Y}, \mathrm{D}_{-} \mathrm{X}, \mathrm{D}_{-} \mathrm{Y}$, and $t_{d e p}$ values no longer apply to person-trips, but to taxi trips, and for every taxi trip $x$, an occupancy $q_{x}$ has been added in the final column. The $q_{\max }$ value for this output is set at six passengers.

The data in Table 2 come from the SPT model, and comparing it to the data in Table 1, it is clear that the trips originating in the Fort Mott State Park pixel, at the left side of the table, do not offer as much opportunity for ridesharing as those originating in Westchester County, NY. The fifteen person-trips from $(81,139)$ in Westchester County to $(75,138)$ in Bergen County have been condensed into eight taxi trips, with vehicle occupancies ranging from one to three, whereas the Fort Mott pixel offers only two trips with the possibility for ridesharing.

TABLE 2 Ordered Taxi Trip File with Capacities

| Dep Time | O_X | O_Y | D_X | D_Y | Q_x |  | Dep Time | O_X | O_Y | D_X | D_Y | Q_x |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 66099 | 16 | 73 | 16 | 75 | 1 |  | 58277 | 81 | 139 | 75 | 135 | 1 |
| 61999 | 16 | 73 | 16 | 80 | 1 |  | 28391 | 81 | 139 | 75 | 136 | 1 |
| 62410 | 16 | 73 | 16 | 80 | 1 | $\ldots$ | 29123 | 81 | 139 | 75 | 136 | 2 |
| 66002 | 16 | 73 | 16 | 80 | 2 |  | 29676 | 81 | 139 | 75 | 136 | 4 |
| 44955 | 16 | 73 | 17 | 75 | 1 |  | 58036 | 81 | 139 | 75 | 137 | 2 |
| 62659 | 16 | 73 | 17 | 75 | 1 |  | 58884 | 81 | 139 | 75 | 137 | 1 |
| 44154 | 16 | 73 | 17 | 76 | 1 | $\cdots$ | 25024 | 81 | 139 | 75 | 138 | 1 |
| 65876 | 16 | 73 | 17 | 76 | 2 |  | 25695 | 81 | 139 | 75 | 138 | 2 |
| 45556 | 16 | 73 | 18 | 73 | 1 |  | 27298 | 81 | 139 | 75 | 138 | 1 |
| 66515 | 16 | 73 | 18 | 73 | 1 |  | 30241 | 81 | 139 | 75 | 138 | 3 |
| 44112 | 16 | 73 | 18 | 74 | 1 |  | 30578 | 81 | 139 | 75 | 138 | 1 |
| 45880 | 16 | 73 | 18 | 74 | 1 |  | 31399 | 81 | 139 | 75 | 138 | 3 |
| 52039 | 16 | 73 | 18 | 74 | 1 |  | 31751 | 81 | 139 | 75 | 138 | 2 |
| 65353 | 16 | 73 | 18 | 74 | 1 |  | 32124 | 81 | 139 | 75 | 138 | 2 |

In its entirety, the SPT output file shown in the table lists all the taxi trips needed to meet New Jersey's transportation demand as determined by Mufti's model. A total of 32,770,528 individual person-trips are reduced to a smaller number of taxi trips; the exact number depends on which model is selected, as well as the values of $q_{\max }$ and $t_{\max }$. The quantity of taxi trips required to meet the given demand in a number of different scenarios is discussed at length in the Results section. Whatever these parameters may be, the taxi trip output file shows the demand for vehicles over time at each node throughout the day, which becomes the basic input information for determining optimal fleet size and the cost of operation for the two Autonomous Taxi Network models.

The cost function employed in the Results section is linearly dependent upon the distance the vehicle travels between picking up passengers at supply node $m$ and dropping them off at demand node $n$. In addition, the analysis requires knowledge of both the departure time of each taxi trip and the arrival time at its destination, at which point the vehicle can be repurposed to serve another trip in the area. It is important to note that the pick-up and drop-off behavior of the two systems differs quite significantly. While a taxi trip in the PRT model ATN ends as soon as the vehicle reaches the centroid of its destination pixel, the SPT model autonomous taxi must serve a number of locations within both the origin and destination pixel to pick up and drop off its passengers. This behavior is modeled via the addition of the "Chauffeur Function," $f^{C h}\left(q_{x}\right)$ to the distance calculation (5) in the case of the SPT model; $f^{C h}\left(q_{x}\right)$ varies with $q_{x}$, the number of passengers in taxi trip $x$.

$$
\begin{gather*}
\operatorname{Dist}_{x}=C_{\text {circ }}\left(d_{s} \sqrt{\left(D_{-} Y_{x}-O_{-} Y_{x}\right)^{2}+\left(D_{-} X_{x}-O_{-} X_{x}\right)^{2}}\right)+2 C_{\text {circ }} f^{C h}\left(q_{x}\right)  \tag{5}\\
C_{\text {circ }}=\text { Circuity multiplier }(\text { set at } 1.2 \text { for this analysis }) \\
d_{s}=\text { Side length of each pixel } \\
f^{C h}=\text { The Chauffeur Function }
\end{gather*}
$$



FIGURE 3 Realizations of the Chauffeur Factor for Trip Occupancies of 1-6
The Chauffeur Function, shown for $q_{x}$ from one to six above, is a worst-case-scenario pick-up and drop-off approximation for the SPT model autonomous taxi network. If taxi trip $x$ has only one passenger, the taxi can pick up the passenger at her point of origin, drive her directly to her destination and then be free to relocate and serve another trip. Given the fact that this pick-up and drop-off can occur anywhere within the pixel, the value for $f^{C h}(1)$ is equal to 0 miles, and the distance between the centroids of the two pixels is multiplied by the circuity multiplier $C_{\text {circ }}$ to determine the total trip distance. However, when more than one passenger is present in an SPT model taxi trip, the vehicle cannot make only one pick-up and drop-off stop. In the worst case scenario for a three passenger trip, for example, the vehicle would have to travel $C_{\text {circ }} \times 2.24$ miles between picking up passenger one and passenger three, and another $C_{\text {circ }} \times$ 2.24 miles while dropping them off. The remaining worst case scenarios are plotted in Figure 3, using nine PRT model pixels to approximate one pixel in the SPT model grid. Of course, there is a possibility that in a three passenger trip, two or more passengers originate at the same location, but the Chauffeur Function is meant as a representation of the worst-case scenario. In practice, the distances travelled in the SPT model may be shorter than those calculated in equation (5). In the case of the PRT model, $f^{C h}\left(q_{x}\right)$ is equal to zero regardless of the value of $q_{x}$.

After the implementation of equation (5), trip time is calculated by multiplying the distance by the inverse of the average vehicle speed. In much of the analysis in the Results section, the average speed is assumed to be 30 miles per hour, though in some instances the inter-pixel driving is assigned a speed of 30 miles per hour while the "chauffeuring speed" - the distance added to the SPT model via the Chauffeur Function - is given a slower average value of 15 miles per hour to account for stops at passengers' destinations. In both cases however, the trips in the SPT model ATN are expected to have longer travel times than their counterparts in the PRT model ATN, due to the extra driving inherent in the SPT model's pick-up and drop-off scheme.

Given the calculation of taxi trip distances and arrival times, the total cost of the PRT system and SPT system can be compared. In each case the total cost - calculated in equation (6) - is a function of per-mile travel cost and fleet size. The total operational cost is approximated by the sum of the trip distances multiplied by a per-mile cost constant $c_{o p}$ and the total vehicle cost is the product of the fleet size and a per-vehicle cost constant $c_{v}$.

$$
\begin{equation*}
\text { Cost }_{\text {Model }} \approx c_{o p} \sum_{x=1}^{x_{\text {TOT }} \text { Dist }_{x}+c_{v} q_{\text {fleet }}} \tag{6}
\end{equation*}
$$

The required fleet size to meet all demand is denoted by $q_{\text {fleet }}$, and depends on which model is employed, as well as the vehicle occupancy $q_{\max }$ and the time delay $t_{\max }$. Fleet size is calculated by discretizing time into 48 thirty-minute segments, and determining how many vehicles are actively en route during each time segment. Whichever half-hour time period has the greatest number of active vehicles will be the period that determines fleet size. The Results section employs two different methods for determining fleet size. In the first, it is assumed that any vehicle that has finished a trip can be instantaneously repurposed to serve another trip anywhere in the state. In the second method, each vehicle is required to wait one hour between trips, giving it ample time to refuel or recharge and relocate to a nearby pixel where demand for a taxi trip has been indicated. The reality of the repositioning and refueling time is likely somewhere in between these two methods, which presents an opportunity for additional research.

## RESULTS

Both the PRT and SPT model describe a system that significantly outperforms the personally owned and operated car in transit criteria one through three, as discussed in [1]. In this section, the results of the enclosed equations and models are discussed, and the two models are compared to one another as they pertain to transit criteria four and five; additionally, they are compared to their true competitor, the current system of personally owned and operated automobiles.

Average Occupancy for Both Models Given $\left(\boldsymbol{q}_{\max }=\infty\right) ;\left(\boldsymbol{t}_{\max }=\mathbf{5}\right.$ min $)$
The first implementation of equation (4) performed in this analysis assumes a $t_{\max }$ equal to five minutes, and does not impose a maximum vehicle occupancy. The resulting average statewide taxi trip occupancy values are shown in Table 3 below. At first glance it is clear that the PRT model presents the opportunity for much less ridesharing than the SPT model. This result is to be expected, as each pixel in the SPT model has nine times the amount of land area of a single pixel in the PRT model. The trade-off between the two models is that while ridesharing is more prevalent in the SPT model, trips take a longer amount of time due to increased distance traveled within the origin and destination node, picking up and dropping off passengers.

TABLE 3 Statewide Occupancy for the First Implementation

| Statewide Occupancy for $\left(\boldsymbol{q}_{\max }=\infty\right) ;\left(\boldsymbol{t}_{\boldsymbol{\operatorname { m a x }}}=\mathbf{5} \min \right)$ |  |  |  |
| :---: | :---: | :---: | :---: |
| Model | Total Person-Trips | Total Taxi Trips | Average Occupancy |
| PRT | $32,770,528$ | $25,824,326$ | 1.269 |
| SPT | $32,770,528$ | $15,174,736$ | 2.160 |

Running a simulation without imposing a maximum vehicle occupancy allows for sanity checks regarding origin-destination pairs that experience very high volumes of travel within specific five-minute intervals throughout the day. In the SPT model, one such route takes place beginning at 7:50am, and lasts until 7:55am. The trip serves 1,896 passengers within this five minute span, and originates at pixel $(71,126)$, which corresponds to coordinates (40.73517, 74.04422), the Jersey City-Hoboken area. The destination that these passengers share is pixel ( 73,126 ) which corresponds to coordinates (40.73517, -73.98913 ), or Manhattan, New York. The purpose of this sanity check is to determine whether it is reasonable to expect that 1,896 people travel from the Jersey City-Hoboken area into Manhattan between 7:50am and 7:55am on an average business day. Indeed this origin-destination pair would be expected to serve a large number of people just before 8:00am on a business day, precisely when the commuters who live across the river from Manhattan travel to work in the city.

## Average Occupancy for Both Models Varying $\boldsymbol{q}_{\max }$ and $\boldsymbol{t}_{\text {max }}$

Having calculated the absolute best case average occupancy numbers for both models, it is evident that in practice neither model will achieve the occupancies shown in Table 3. To achieve these occupancies, the autonomous taxi network would need to include at least one vehicle with an occupancy of 1,896 to serve the trip from Hoboken to Manhattan at 7:50am. In reality, a vehicle size must be selected. Theoretically, an ATN could be served using vehicles as large as buses, with occupancies of 48 or more, and on the other end of the spectrum the taxis could be much smaller pods with only two seats. Average statewide occupancies for a PRT model ATN with $q_{\max }$ varying from two to forty-eight seats, and with $t_{\max }$ equal to five or seven minutes are plotted in Figure 4.


FIGURE 4 Statewide Occupancy for PRT Model Given Variable $\boldsymbol{q}_{\max }$

Focusing on the solid line in which $t_{\max }$ is equal to five minutes, the PRT model average occupancy is equal to the best case, $q_{\max }=\infty$ occupancy of 1.269 people per taxi trip when the vehicle occupancy is set to forty-eight passengers. For $q_{\max }$ between six and forty-eight, the average statewide taxi occupancy ranges from 1.255 to 1.269 , but with a maximum vehicle occupancy less than six there is a rapid drop-off in average trip occupancy to a value of 1.221 for a $q_{\max }$ of three, and 1.174 when $q_{\max }$ is equal to two.

The same calculations were performed for the SPT model resulting in occupancy numbers significantly higher than in the PRT model, plotted in Figure 5. The contour of the plot is very similar to the PRT model in Figure 4, and a major fall-off occurs when maximum occupancy is less than six.


FIGURE 5 Statewide Occupancy for SPT Model Given Variable $\boldsymbol{q}_{\max }$
Despite the clear benefits of using larger vehicles in both the PRT and SPT model ATNs, there are also significant costs associated with vehicles built for twelve people, for example, versus vehicles built for six. The remainder of this section focuses on two specific layouts of an ATN, one a PRT model and the other an SPT model, each with a $t_{\max }$ equal to five minutes, which is more convenient to passengers than a $t_{\max }$ of seven minutes, and a $q_{\max }$ equal to six passengers. The value of six has been selected because it is the beginning of the occupancy dropoff and because six-passenger autonomous vehicles could easily be mass-produced on current sedan platforms, and would be approximately the size of today's vehicles.

Further Analysis for SPT and PRT Model ATNs with ( $\boldsymbol{q}_{\max }=6$ ); $\left(\boldsymbol{t}_{\boldsymbol{m a x}}=5\right)$
Continuing with the analysis of two ATN models for a $q_{\max }$ equal to six and a $t_{\max }$ equal to five minutes, it is important to discuss the trade-off between ridesharing and total trip distance. As shown in the vehicle occupancy plots in Figures 4 and 5, the SPT model offers much more opportunity for ridesharing than the PRT model. This more efficient use of vehicle space does not come for free however. Because vehicles in the SPT model travel within their origin and destination pixels, the trip distances and trip times are longer than in the PRT model. In the table
below, the average trip distance for each model has been calculated using equation (5), and the average trip distance in the SPT model is 4.88 miles longer than the average in the PRT model. Despite this disparity however, the approximate total distance traveled by all vehicles over the entire day in the SPT model is still more than 80 million miles less than the total distance traveled in the PRT model.

TABLE 4 Comparison of the Two Models in Three Categories

| Average Distance per Trip and Approximate Total Distance for Both Models |  |  |  |
| :---: | :---: | :---: | :---: |
| Model | Total Taxi Trips | Average Trip Distance | Approximate Total Distance |
| PRT | $26,105,757$ | 16.29 miles | $425,263,000$ miles |
| SPT | $16,287,928$ | 21.17 miles | $344,815,000$ miles |

Because the total mileage of the PRT model is greater than that of the SPT model, and the number of trips needed to meet demand is drastically larger in the PRT model as well, the expectation is that the PRT model's cost will be much greater than the cost of an SPT model ATN. In the following three subsections, the required fleet size for both models is calculated in in two ways: 1) assuming instantaneous repositioning capability, and 2) assuming one hour of down-time between taxi trips.

## Calculating Fleet Size with Instantaneous Repositioning

The assumption that vehicles can instantaneously reposition themselves to meet any demand that arises within the network allows for a quick calculation of the best case scenario for fleet size. For each time segment, the total number of cars en route is calculated assuming an average speed of 30 miles per hour, and plotted in a graph such as the one in Figure 6, which corresponds to the PRT model. The main peak in the morning rush hour occurs between 7:30am and 8:00am, when there are $1,775,225$ vehicles on the road in the PRT model. Evening rush hour is much busier than morning rush hour however, as the majority of secondary trips to retail locations take place in the afternoon and evening, resulting in a daily maximum fleet size of $2,409,736$ vehicles on the road between $5: 00 \mathrm{pm}$ and $5: 30 \mathrm{pm}$.


FIGURE 6 Vehicles Required at 48 Time Steps in PRT Model Assuming Instantaneous Repositioning

In 2011, there were $7,609,467$ vehicles registered in the state of New Jersey, so a fleet size of $2,410,000$ would be a considerable reduction in the number of vehicles needed to meet New Jersey's transportation demand. The reduction in fleet size would further reduce congestion and the danger of accidents, and would furthermore be better for the roads. As expected, the fleet size required for an SPT model ATN is even smaller than that of a PRT model ATN. The busiest time of day in the SPT model is the time between $6: 00 \mathrm{pm}$ and $6: 30 \mathrm{pm}$, when there are $1,609,073$ vehicles actively moving passengers from their origins to their destinations.


FIGURE 7 Vehicles Required at 48 Time Steps in SPT Model Assuming Instantaneous Repositioning
While the instantaneous repositioning method is the most straightforward means to calculate approximate fleet size, its results are unattainable in practice. In an effort to determine a more realistic, if slightly over-cautious fleet size for both models, a system is implemented in the following section in which every taxi is required to take an hour break between trips to refuel and reposition itself.

## Calculating Fleet Size with One Hour Break between Trips

As expected, in the implementation that includes an extra hour of wait time between trips, the calculated fleet size is much higher than in the case of instantaneous repositioning. In Figure 8 for the PRT model, the busiest time period has changed from $5: 00 \mathrm{pm}-5: 30 \mathrm{pm}$ to $6: 00 \mathrm{pm}-$ 6:30pm. This can be easily accounted for: as the afternoon rush begins, each trip carries with it an hour of waiting. It is no coincidence that the new peak time is one hour after the peak time in Figure 6. As for the actual capacity calculation for the PRT model in Figure 8, there are $4,450,701$ cars on the road between 6:00pm and 6:30pm.


FIGURE 8 Vehicles Required at 48 Time Steps in PRT Model Assuming 1 Hour Repositioning Time
The results of the SPT model with one hour of repositioning and refueling time after every trip (Figure 9) are very similar to those shown in Figure 8. Like the PRT model, the SPT model experiences the busiest streets from $6: 00 \mathrm{pm}$ until $6: 30 \mathrm{pm}$, with $2,789,391$ vehicles on the road during that time.


FIGURE 9 Vehicles Required at 48 Time Steps in SPT Model Assuming 1 Hour Repositioning Time

Assuming that reality lies somewhere between instantaneous repositioning and hour-long required waiting time, the necessary fleet size for a PRT model ATN would be somewhere between $2,409,736$ vehicles and $4,450,701$ vehicles. For an SPT model ATN, the range would be between $1,609,073$ and $2,789,391$ vehicles. While the SPT model appears to be a more optimal set-up than the PRT model, it is worth noting that both models outperform the automobile in terms of limiting the vehicles on the road without sacrificing mobility. In the end, that is the goal of any alternative to the car: to afford the people of the future all the freedom and mobility we experience today, without any of the negative externalities including hassle, safety issues, environmental concerns, and time wasted behind the wheel.

## Cost Comparison of the Two Models

In this subsection, the PRT and SPT models are compared using both the upper and lower bound for fleet size, rather than attempting to pick a value in between the two bounds. The equation for total daily operation cost for each model is found in equation (7) below.

$$
\begin{equation*}
\text { Cost }_{\text {Model }} \approx c \sum_{x=1}^{x_{T O T}} \text { Dist }_{x}+c_{v} q_{\text {fleet }} \tag{7}
\end{equation*}
$$

The total cost estimates based on the results presented in this chapter are contained in Table 5. The value of c is set at $\$ 0.17$ per mile, which is the average cost of operating a personal vehicle according to research in [6]. For the value of $c_{v}$, the estimated cost of each vehicle, there is significant leeway as estimates for the cost of fully autonomous cars range anywhere from $\$ 100,000$ to $\$ 300,000$. Assuming that by the time an ATN is implemented, the equipment will have experienced a reduction in price, a cost per vehicle of $\$ 100,000$ has been selected and divided equally over a five year lifetime, coming to $\$ 54.76$ per car per day. The resulting daily costs for the PRT model range from $\$ 204$ million to $\$ 316$ million, or a per capita price of $\$ 22.69$ to $\$ 35.11$ for the entire system. The SPT model ranges from $\$ 147$ million to $\$ 211$ million total, or $\$ 16.30$ to $\$ 23.49$ per capita.

TABLE 5 Total Cost Estimations

| Estimating the Total Cost of the Two Models |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Model | $\boldsymbol{c}$ | Sum(Dist) | $\boldsymbol{c}_{\boldsymbol{v}}$ | $\boldsymbol{q}_{\boldsymbol{f l e e t}}$ | Cost |
| PRT Inst. | $\$ 0.17$ | $425,263,000$ miles | $\$ 54.76$ | $2,409,736$ | $\$ 204.2 \mathrm{M}$ |
| PRT 1 Hr. | $\$ 0.17$ | $425,263,000$ miles | $\$ 54.76$ | $4,450,701$ | $\$ 316.0 \mathrm{M}$ |
| SPT Inst. | $\$ 0.17$ | $344,815,000$ miles | $\$ 54.76$ | $1,609,073$ | $\$ 146.7 \mathrm{M}$ |
| SPT 1 Hr. | $\$ 0.17$ | $344,815,000$ miles | $\$ 54.76$ | $2,789,391$ | $\$ 211.4 \mathrm{M}$ |

While there is a slight overlap between the two cost windows, it is evident that the SPT model outperforms the PRT model in terms of transit criterion four. A preliminary comparison between the SPT model ATN and the personally owned and operated automobile shows that the cost of owning a car and driving for the average SPT trip distance of 21.17 miles 3.4 times per day would result in cost of operation of $\$ 12.24$ per person per day, using the cost-per-mile from [5]. When the cost of automobile ownership is included in the equation, the SPT model ATN cost of $\$ 16.30$ to $\$ 23.49$ per person per day will prove to be even more competitive with the conventional system of individually owned and operated cars, indicating that its adoption as a major form of transportation may not be as prohibitively expensive as previously thought.

## CONCLUSIONS

While this preliminary analysis has demonstrated the competitiveness of an SPT model ATN with personally owned vehicles, the key fifth transit criterion has been thus far omitted from this report. While less quantifiable than the other criteria on the list, comfort and convenience of a transportation alternative plays a major role in the psychology of the consumer - and the consumer will eventually dictate whether the concept of an ATN takes off in the coming decades.

By estimating the comfort and convenience factor as a weighted product of cost and trip time, both ATN models can be compared to the automobile in terms of transit criterion five. Regardless of the weighting method, the SPT model operates at a lower cost and, as discussed, removes the element of pedestrian travel that exists in the PRT model. In comparing the SPT model with the automobile, the data overwhelmingly suggest that autonomous taxis will use road space more efficiently and with a lower incident rate than automobiles, suggesting reduced trip times. In the event that the benefits of autonomous vehicles are fully realized as they pertain to travel time, the SPT model ATN will present a significant time savings as well as a comparable price, making it competitive or preferred to the automobile in each of the five criteria presented.

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