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A Framework Analysis

TIMSS Advanced 2015 AND Advanced Placement Calculus & Physics

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Executive Summary

While the results of the TIMSS Advanced 2015 assessments are not yet known, in recent years U.S. students have underperformed relative to their international counterparts on international assessments such as TIMSS and PISA. While international assessment data are frequently used to compare performance cross-culturally, the analysis of these data often lacks adequate contextualization for country-by-country comparisons. To ensure maximum utility of the assessments, an examination of each participating country's curriculum content and standards in relation to those on the international assessment is critical. Only then can one make cross-cultural comparisons effectively.

The current study investigates the alignment between AP Physics and AP Calculus frameworks as compared to the TIMSS Advanced Physics and Mathematics frameworks. Due to the rigor of AP course work and the performance of AP students versus non-AP students on college course work, we expect AP student performance on TIMSS Advanced may exceed the performance of the general population. Moreover, meaningful comparisons of country achievement rankings are not possible without a study that considers alignment, and thus, there may also be additional compelling findings from the future performance report that can make a potentially significant contribution to the literature, and to policy discussions.

The methodology applied in this alignment study is modeled after Norman Webb (Webb, 1997). External alignment and curriculum specialists in mathematics and physics were engaged to evaluate the alignment of AP Physics 1: Algebra-Based and AP Physics 2: Algebra-Based Curriculum framework and AP Calculus framework to the TIMSS Advanced Frameworks according to three criteria: (1) Categorical concurrence, (2) Depth of knowledge, (3) Balance of representation. The mathematics and science groups approached the alignment criteria commensurate with their respective frameworks.

For categorical concurrence in physics, degree of alignment between the AP learning objectives and each of the TIMSS Advanced topics were rated as:

- Complete alignment — The learning objective aligns with the full description of the TIMSS topic.
- Partial alignment — The learning objective aligns to a part of the TIMSS topic description.
- Extended alignment — The learning objective aligns to all or part of the TIMSS topic description but also includes knowledge and/or uses of that knowledge that extend beyond what is described in the TIMSS topic.
- Not covered — The learning objective does not align with any of the 23 TIMSS topics.

Of the 250 AP Physics 1 and 2 learning objectives, 180 (72%) are covered within one or more of the 23 TIMSS Advanced Physics topics, while 70 (28%) of the learning objectives are not covered in the TIMSS Advanced Physics Framework. After aligning the AP learning objectives to the TIMSS topic, a holistic judgment was made as to whether the collection of AP learning objectives represents partial, complete, or extended alignment of the TIMSS topic as a whole. None of the 23 TIMSS Advanced Physics topics had "no alignment" with the AP Physics 1 and 2 curriculum frameworks, as each physics topic was represented in at least one AP learning objective, two (9%) had partial alignment, 14 (61%) had complete alignment, and seven (30%) had extended alignment.

The depth of knowledge alignment for physics was measured by considering the three cognitive domains (knowing, applying, and reasoning) outlined in the TIMSS Advanced Physics Framework and the science practices outlined in the AP Physics learning objectives. The least aligned cognitive domain was knowing, containing only 11.6% of the total AP learning objectives. Applying was the domain with the strongest alignment with the AP learning objectives, with reasoning also being strongly aligned.

For balance of representation in physics, the percentages of AP Physics 1 and 2 items aligned with each of the seven TIMSS Advanced Physics topic areas were determined. Important differences were found between the TIMSS Advanced Physics assessment and the AP Physics Exams in the proportional emphasis on certain topic areas. Most notably, AP students who have completed only AP Physics 1 and then take the TIMSS Advanced Physics assessment will encounter numerous items in Magnetism and Electromagnetic Induction and Atomic and Nuclear Physics (comprising 34% of the TIMSS assessment) that are not included in AP Physics 1.

For categorical concurrence in calculus, degree of alignment methodology between the AP content topics and each of the TIMSS Advanced topics followed the same strategy used by the physics team, defined as follows:

- Complete alignment — The content topic aligns with the full description of the TIMSS topic.
- Partial alignment — The content topic aligns to a part of the TIMSS topic description.
- Extended alignment — The content topic aligns to all or part of the TIMSS topic description but also includes knowledge and/or uses of that knowledge that extends beyond what is described in the TIMSS topic.
- Not covered — The content topic does not align with any of the 22 TIMSS topics.

Of the 46 AP Calculus AB content topics, all are at least partially covered or extend beyond one or more of the 22 TIMSS Advanced Mathematics topics, and 44 (96%) of the AP Calculus AB content topics are completely covered. Of the 67 AP Calculus BC content topics, all are at least partially covered or extend beyond one or more of the 22 TIMSS Advanced Mathematics topics, and 56 (84%) of the AP Calculus BC content topics are completely covered. When the AP Calculus AB framework was compared to the 22 TIMSS Advanced Mathematics topics, three TIMSS topics (14%) had no alignment, seven (32%) had partial alignment, 10 (45%) had complete alignment, and two (9%) had extended alignment. The comparison of the AP Calculus BC framework to the TIMSS Advanced Mathematics topics resulted in two TIMSS topics (9%) had no alignment, five (23%) had partial alignment, nine (41%) had complete alignment, and six (27%) had extended alignment.

To assess depth of knowledge in calculus, each of the 80 AP Calculus AB and BC items was assigned a TIMSS Advanced cognitive domain. AP Calculus content topics were categorized according to the TIMSS topics separately for AP Calculus AB and BC to assess balance of representation across frameworks. Of the AP Calculus AB content topics, 4.5% did not match any of the TIMSS topics. When focusing on the balance of representation for the three calculus topics, the derivatives representation (43.2%) is almost double compared to the representation of limits and integrals (20.5%). The AP Calculus BC balance of representation ranges from 1.5% to 28.4% across seven of the eight topics, with no representation for trigonometry. Compared to the AP Calculus AB content topics, a much greater proportion of AP Calculus BC content topics (19.4%) did not match any of the TIMSS topics. Similar to the AP Calculus AB, when focusing on just the three calculus topics the derivatives representation is almost double (28.4%) compared to the representation of limits and integrals (14.9%).

With achievement scores being used to support reform efforts, policymakers must be aware of the degree to which international assessments and curricula are aligned. This study will allow for a comparison of TIMSS results as it relates to AP Calculus and AP Physics.

Introduction

The results from the latest Trends in International Mathematics and Science Study (TIMSS) 2011 showed that U.S. fourth- and eighth-grade students underperformed their peers academically, particularly those in Asian countries (Martin, Mullis, Foy, & Arora, 2012; Mullis, Martin, Foy, & Stanco, 2012). Results from TIMSS Advanced 1995, the last administration of TIMSS Advanced in which the U.S. participated prior to 2015, showed advanced U.S. 12th-grade students performing among the bottom two countries (Mullis et al., 1998).

Some media channels and education policymakers have called for education reform based on the scores from international assessments. However, we must first challenge the alignment of international assessments to our curriculum and standards, as well as explore whether possible solutions may already exist. For example, research has shown that AP students outperform their non-AP peers in later college course work and exams (Patterson & Ewing, 2013; Patterson, Packman, & Kobrin, 2011). Moreover, the AP Program offers a variety of courses for advanced high school students to engage in college-level course work. Possibly, the rigor of AP courses elicit higher performance on the TIMSS Advanced assessment, as we will explore in the future performance report using 2015 TIMSS Advanced data.

International assessment data often lack adequate contextualization for analysis and comparison of performance cross-nationally. One of the reasons is that frameworks of international assessments are not written to be aligned to any single country's standards. Thus, it is paramount to examine country curriculum content before making country-by-country comparisons on performance data, and certainly before making meaningful inferences or policy decisions.

This report will determine to what degree the AP Physics 1 and 2 and AP Calculus AB and BC frameworks are aligned with the TIMSS Advanced Physics and Mathematics frameworks. This will enable an exploration of any differences in content coverage and levels of complexity will set the stage for a future performance report that will evaluate AP student performance on TIMSS Advanced. The current report will expose meaningful differences in frameworks that may speak to performance outcomes. In 2015, the U.S. as well as several cooperating countries will participate in TIMSS Advanced, providing the optimal opportunity to recognize AP and non-AP students within the TIMSS Advanced sample and investigate the differences in performance. The field trial determined that the established sampling plan ensured adequate AP Physics 1 and 2, and AP Calculus representation, as well as non-AP students to which they can be compared for the final country-to-country performance report.

Methodology

Examination of the current literature on alignment and linking studies guided the determination of methodology chosen for this study. A study undertaken by Neidorf, Binkley, Gattis, and Nohara (2006), with the support of NCES (National Center for Education Statistics), sought to compare the content of NAEP (National Assessment of Educational Progress), TIMSS, and PISA (Programme for International Student Assessment) assessments by taking a detailed look at their respective frameworks. The goal of this NCES project was to allow for useful interpretation and comparison of the results from each assessment. By engaging a panel of experts, NCES approached this project by cross-classifying items and framework dimensions across the three assessments. Considering the structure and content of the frameworks at hand, our current study built on NCES's strategy and borrowed methodology modeled after the alignment work developed by Norman Webb, senior research scientist at the Wisconsin Center for Education Research, University of Wisconsin-Madison.

Population

The target population for the TIMSS Advanced 2015 assessment is defined as advanced students in their final year of secondary schooling who have taken or are taking courses in advanced math and physics. For a school to be considered eligible, they must offer at least one advanced math and physics course. From a high-level perspective, the unit of focus on TIMSS assessments is the participating country. Student incentives are not provided for taking part in TIMSS. By comparison, the unit of focus for AP Exams is the individual student. Typically, AP students who are motivated to receive college credit or stand out in a competitive college admission process self-select into AP's rigorous course work. Accordingly, a sample of AP students is not representative of the U.S. student population.

Purpose

The purpose of the current study is to provide an alignment report, which will show to what degree the TIMSS Advanced and AP frameworks are aligned. This report will also lay the foundation for a future 2016 performance report, which will compare performance of students who took AP Physics 1 and 2 and AP Calculus to students who did not take those courses as well as to students who participated in the international sample on TIMSS Advanced 2015. By conducting this alignment study first, we will be able to explore how AP students could be meaningfully compared internationally and to ensure U.S. overall performance on TIMSS Advanced is not conflated with the performance of AP students, and AP Physics and AP Calculus students, in particular. Moreover, this research will explore how appropriate, from a content and skill alignment perspective, this comparison of AP student performance to international student performance is as well as will determine the claims that can be made regarding AP student performance on TIMSS Advanced.

Country vs. AP Sampling

By conducting this study first, we will be able to explore how AP students could be meaningfully compared internationally and to ensure U.S. country overall performance on TIMSS Advanced is not conflated with the performance of AP students, and AP Physics and AP Calculus, in particular.

Overview of AP and TIMSS Advanced Frameworks

The AP Physics and AP Calculus frameworks comprise different domains and are at different stages of being rewritten at the time of our access to TIMSS Advanced 2015 data. As a result, the frameworks do differ, as does our approach to aligning. To the extent to which it is possible, we will bring together important concepts and findings, but without conflating the alignment by domain. In the following section, we will outline Physics, followed by Calculus.

AP Physics 1 and 2 Framework

Purpose

In 2015, the College Board will offer two algebra-based AP Physics courses: AP Physics 1 and AP Physics 2. These two courses comprise a two-year sequence equivalent to the first and second semesters of a typical introductory, algebra-based, college physics course. The 2014-2015 school year was the first year that the redesigned AP Physics 1 and Physics 2 courses and exams were offered by the College Board.

The purpose of redesigning the AP Physics 1 and AP Physics 2 science courses was to:

1. Emphasize a deep understanding rather than comprehensive content coverage (addressing the "mile-wide and inch-deep" approach).
2. Reflect current understanding of how students learn in a discipline.
3. Reflect current research directions within the disciplines.
4. Emphasize the development of inquiry and reasoning skills.

The redesigned courses differ from the traditional content coverage model of instruction in that they focus on the "big ideas" of an introductory college-level physics sequence. These "big ideas" outline the core disciplinary knowledge and conceptual understandings of foundational physics principles. The principal difference in this model of instruction is that it requires students to spend less time on mathematical routines and more time engaged in inquiry-based learning of essential concepts. These differences focus students on the critical thinking and reasoning skills necessary to engage in an AP Physics course or subsequent course work in science disciplines.

Structure and Organization

Big Ideas. The redesigned AP Physics 1 and AP Physics 2 concepts are articulated together in the curriculum framework. This framework provides the full scope of conceptual understandings a student should acquire by the end of an introductory sequence in college-level algebra-based physics course. The key concepts and related content that define the AP Physics 1 and AP Physics 2 courses and exams are organized around seven "big ideas," which encompass the core scientific principles, theories, and processes of physics that cut across traditional content boundaries and provide students with a broad way of thinking about the physical world.

Table 1.
Big Ideas for AP Physics 1 and AP Physics 2

Big Idea 1:	Objects and systems have properties such as mass and charge. Systems may have internal structure.
Big Idea 2:	Fields existing in space can be used to explain interactions.
Big Idea 3:	The interactions of an object with other objects can be described by forces.
Big Idea 4:	Interactions between systems can result in changes in those systems.
Big Idea 5:	Changes that occur as a result of interactions are constrained by conservation laws.
Big Idea 6:	Waves can transfer energy and momentum from one location to another without the permanent transfer of mass and serve as a mathematical model for the description of other phenomena.
Big Idea 7:	The mathematics of probability can be used to describe the behavior of complex systems and to interpret the behavior of quantum mechanical systems.[1]

Enduring Understandings and Essential Knowledge. For each of the seven big ideas, enduring understandings, which incorporate the core concepts that students should retain from the learning experience, are also identified. Statements of the essential knowledge follow each enduring understanding. The essential knowledge statements support the corresponding enduring understanding and delineate the conceptual targets for student learning. These statements provide a more detailed description of the broader knowledge outlined in the enduring understanding. An example of two enduring understandings and several associated essential knowledge statements for Big Idea 1 is provided below in Table 2.

Table 2.
Big Idea 1: Objects and systems have properties such as mass and charge. Systems may have internal structure

Enduring Understanding 1.A: The internal structure of a system determines many properties of the system.	**Essential Knowledge 1.A.1:** A system is an object or a collection of objects. Objects are treated as having no internal structure.
	Essential Knowledge 1.A.2: Systems have properties determined by the properties and interactions of their constituent atomic and molecular substructures. In AP Physics, when the properties of the constituent parts are not important in modeling the behavior of the macroscopic system, the system itself may be referred to as an *object*.
Enduring Understanding 1.B: Electric charge is a property of an object or system that affects its interactions with other objects or systems containing charge.	**Essential Knowledge 1.B.1:** Electric charge is conserved. The net charge of a system is equal to the sum of the charges of all the objects in the system.
	Essential Knowledge 1.B.2: There are only two kinds of electric charge. Neutral objects or systems contain equal quantities of positive and negative charge, with the exception of some fundamental particles that have no electric charge.
	Essential Knowledge 1.B.3: The smallest observed unit of charge that can be isolated is the electron charge, also known as the elementary charge.

1. Big Idea 7 is only covered in Physics 2.

The Emphasis on Science Practices. Having a deep understanding of physics principles implies the ability to reason about physical phenomena using science process skills such as explaining causal relationships, applying and justifying the use of mathematical routines, designing experiments, analyzing data, and making connections across multiple topics within the course. Therefore, in addition to the content components that are outlined in the big ideas, enduring understandings, and essential knowledge statements, the AP framework also outlines a set of science practices. These science practices articulate a way to coordinate knowledge and skills to accomplish a goal or task.

Science Practices

The science practices enable students to establish lines of evidence and use them to develop and refine testable explanations and predictions of natural phenomena.

The science practices enable students to establish lines of evidence and use them to develop and refine testable explanations and predictions of natural phenomena. Because content, inquiry, and reasoning are equally important in AP Physics 1 and 2, each learning objective described in the concept outline combines content with inquiry and reasoning skills described in the science practices. The science practices outlined in Table 3 capture important aspects of the work in which scientists engage, at the level of competence expected of AP Physics 1 and 2 students.

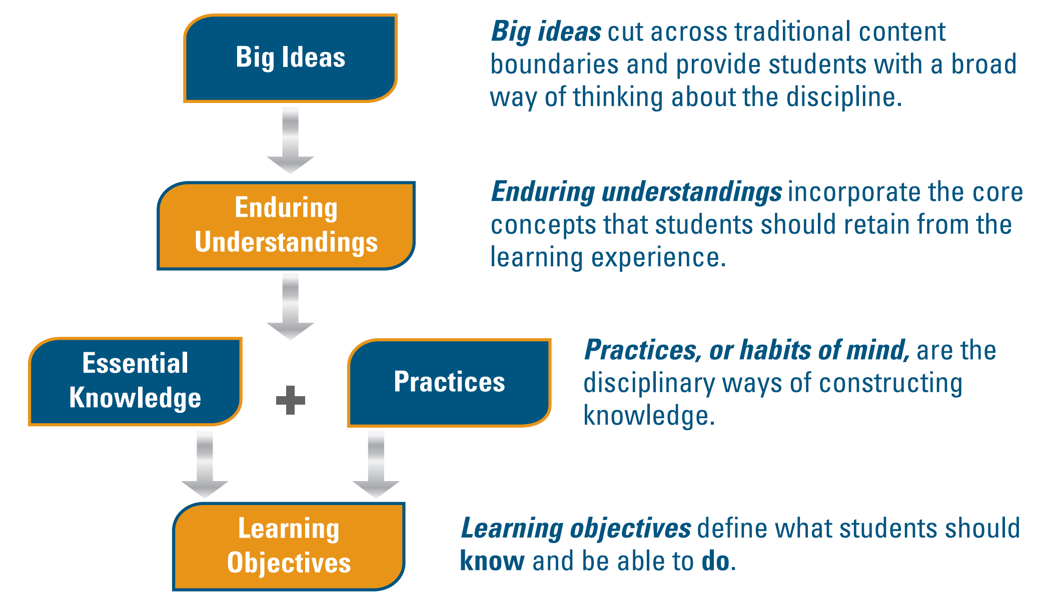
Table 3.
AP Science Practices

Science Practice 1:	The student can use representations and models to communicate scientific phenomena and solve scientific problems.
Science Practice 2:	The student can use mathematics appropriately.
Science Practice 3:	The student can engage in scientific questioning to extend thinking or to guide investigations within the context of the AP course.
Science Practice 4:	The student can plan and implement data collection strategies in relation to a particular scientific question.
Science Practice 5:	The student can perform data analysis and evaluation of evidence.
Science Practice 6:	The student can work with scientific explanations and theories.
Science Practice 7:	The student is able to connect and relate knowledge across various scales, concepts, and representations in and across domains.

Learning Objectives. The learning objectives provide detailed articulation of what students should know and be able to do. Each learning objective is designed to integrate science practices with a specific essential knowledge statement to provide information about how students will be expected to demonstrate their knowledge and abilities. These learning objectives in conjunction with the science practices define what will be assessed on the AP Physics 1 and AP Physics 2 Exams; questions that do not correspond to one or more learning objectives will not appear on the exam.

Figure 1.

Hierarchy of the AP Physics Framework.



AP Calculus Framework

Purpose

AP Calculus includes two courses, AP Calculus AB and AP Calculus BC. The curriculum for AP Calculus AB is equivalent to that of a first-semester college calculus course; AP Calculus BC extends the work from AP Calculus AB to include subsequent work in single-variable calculus. The AP Calculus courses are designed to develop students' understanding of calculus concepts and provide experience with its methods and applications. The courses represent a multidimensional approach to calculus, with concepts, results, and problems expressed graphically, numerically, analytically, and verbally. The connections between the representations are also important. Even though facility with manipulation and computational competence are important outcomes, they are not the core of these courses. It is expected that teachers and students will use technology regularly and purposefully to investigate relationships among the multiple representations of functions, to confirm written work, and to assist with the interpretation of results.

Structure and Organization

Using the unifying themes of derivatives, integrals, limits, approximation, and applications and modeling, the AP Calculus courses are intended to be a cohesive whole rather than a collection of unrelated topics. These themes are developed using a variety of functions from previous mathematics courses including linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric, and piecewise-defined functions. Each theme is addressed using a set of related topics that highlight the content topics used to guide planning, instruction, and assessment.

The topics that address the unifying themes are organized into content categories and sub-categories. However, the sequence of topics presented in the AP Calculus framework is not necessarily the order in which the content needs to be taught; the sequence of instruction is at the discretion of the teacher. Each unifying theme includes topics that can be assessed using knowledge, application, and reasoning. As an example, the unifying theme of derivatives, which is addressed in both the AP Calculus AB and AP Calculus BC courses, includes the following content categories and subcategories (College Board, 2012a, pp. 7–8):

Concept of the derivative

- Derivative presented graphically, numerically, and analytically.
- Derivative interpreted as an instantaneous rate of change.
- Derivative defined as the limit of the difference quotient.
- Relationship between differentiability and continuity.

Derivative at a point

- Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
- Tangent line to a curve at a point and local linear approximation.
- Instantaneous rate of change as the limit of average rate of change.
- Approximate rate of change from graphs and tables of values.

Derivative as a function

- Corresponding characteristics of graphs of f' and f''.
- Relationship between the increasing and decreasing behavior of f and the sign of f'.
- The Mean Value Theorem and its geometric interpretation.
- Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Second derivatives

- Corresponding characteristics of the graphs of f, f', and f''.
- Relationship between the concavity of f and the sign of f''.
- Points of inflection as places where concavity changes.

Applications of derivatives

- Analysis of curves, including the notions of monotonicity and concavity.
- Optimization, both absolute (global) and relative (local) extrema.
- Modeling rates of change, including related rates problems.
- Use of implicit differentiation to find the derivative of an inverse function.
- Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.

- Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations.

Computation of derivatives

- Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.
- Derivative rules for sums, products, and quotients of functions.
- Chain rule and implicit differentiation.

The theme of derivatives, therefore, includes various ways to compute, represent, and apply the concept of the derivative. Functions are analyzed using derivatives as an interpretive tool for rates of change, which leads to further investigation of the relationship between position, velocity, and acceleration, for example. The unifying themes of limits and integrals, while related to derivative concepts, also include specific topics that are defining characteristics and applications of those themes.

The AP Calculus BC course includes all AP Calculus AB topics, and the topic of polynomial approximations and series. This includes the use of the Taylor and Maclaurin series to approximate other functions, and the use of techniques to determine convergence or divergence. The AP Calculus BC course also includes the analysis of parametric, polar, and vector functions and further evaluation and application of integrals.

New Framework for 2016

Several changes have been made to the AP Calculus Framework for 2016-17 academic year. These changes include using an *Understanding by Design* (Wiggins & McTighe, 2005) structure to replace the topic list, aligning course content with demonstrable content topics and making essential mathematical practices explicit. The purpose of these changes to the framework are, in part, to promote greater conceptual understanding and broaden the focus on a range of mathematical practices as part of the course experience. A few additional topics have been added to each course: L'Hospital's rule has been moved from Calculus BC to Calculus AB; and the limit comparison test, absolute and conditional convergence, and the alternating series error bound have been added to Calculus BC. No topics have been deleted from the existing AP Calculus program.

The AP Calculus AB and BC Exams continue to share the same format, which consists of a multiple-choice section and a free-response section. The structure of the free-response section has not changed. Multiple-choice questions now have four answer choices instead of five and the distribution of questions and relative timing for each part of the multiple-choice section have been adjusted based on feedback from teachers and administrators.

New Calculus Framework for 2016

The purpose of these changes to the framework is, in part, to promote greater conceptual understanding and broaden the focus on a range of mathematical practices as part of the course experience.

Prerequisite Knowledge

Before studying calculus, all students should complete four years of secondary mathematics designed for college-bound students: courses in which they study algebra, geometry, trigonometry, analytic geometry, and elementary functions. These functions include linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric, and piecewise-defined functions. In particular, before studying calculus, students must be familiar with the properties of functions, the algebra of functions, and the graphs of functions. Students must also understand the language of functions (e.g., domain and range, odd and even, periodic, symmetry, zeros, intercepts, etc.) and key values of trigonometric functions.

AP Calculus Exams

The Calculus AB and BC Exams assess how well a student has mastered the concepts and techniques of the respective courses. Each exam consists of two sections:

> Section I: a multiple-choice section testing proficiency in a wide variety of topics.
> Section II: a free-response section requiring the student to demonstrate the ability to solve problems involving a more extended chain of reasoning.

Section I is organized into two parts. Part A does not allow the use of a calculator and Part B requires the use of a calculator. Similarly, one part of Section II is calculator-free and the other part requires the use of a graphing calculator. When analyzing the AP Calculus Exams, the categorization of a section as calculator-free or calculator-required was taken into account to determine categorical congruence and cognitive demand.

TIMSS Advanced 2015 Frameworks

Purpose

TIMSS Advanced is an international assessment of the physics and mathematics achievement of students who are enrolled in pre-university physics or advanced mathematics courses in their final year of high school (Mullis and Martin, 2014). In most countries that participate in TIMSS Advanced, these students are typically those who are planning further study in mathematics or physics at a university or other institute of higher education (Mullis, Martin, & Foy, 2014). TIMSS Advanced is a companion assessment to TIMSS, the Trends in Mathematics and Science Study, which assesses the mathematics and science achievement of fourth- and eighth-grade students in participating countries across the world. Both TIMSS and TIMSS Advanced are projects of IEA (the International Association for the Evaluation of Educational Achievement), which is headquartered in Amsterdam. IEA has conducted an array of international comparative studies of student educational achievement since 1959.

TIMSS Advanced was first administered in 1995, and then again in 2008. Twenty-one countries, including the United States, participated in TIMSS Advanced in 1995. Ten countries participated in TIMSS Advanced in 2008 and nine participated in the 2015 administration. Those countries participating in the 2015 administration include France, Italy, Lebanon, Norway, Portugal, Russian Federation, Slovenia, Sweden, and the United States. The United States did not participate in 2008 but did participate in 2015. The results of TIMSS Advanced provide information about the comparative achievement of students from the various participating countries for each of the three administrations of TIMSS Advanced, and trend information about the achievement of students from countries that participated in two or more administrations over the 20 years from 1995 to 2015.

In each administration, TIMSS Advanced also collects contextual data about curriculum, instruction, school resources, and student characteristics from curriculum specialists, school principals, science teachers, and the students themselves in each participating country (Hooper, Mullis, & Martin, 2014). These data provide important contexts for interpreting the science achievement data and for improving teaching and learning in mathematics and physics in the participating countries.

Structure and Organization — TIMSS Advanced 2015 Mathematics Framework

The TIMSS Advanced 2015 Mathematics assessment was based on an assessment framework that was developed collaboratively with the countries participating in TIMSS Advanced 2015 (Gronmo, Lindquist, & Arora, 2014). This 2015 assessment framework was similar to the framework for the 2008 assessment, with minor updates to reflect changes in the pre-university mathematics curricula, standards, and frameworks of the participating countries. Updates to the TIMSS Advanced 2015 Mathematics Framework also took into account current initiatives in mathematics education, including the *Common Core Standards for Mathematics* (National Governors Association Center for Best Practices, 2010) and the *AP Calculus Course Description* (College Board, 2012a).

The TIMSS Advanced 2015 Mathematics Framework consists of a content dimension that specifies the mathematics content domains to be assessed and a cognitive dimension specifying the cognitive domains to be assessed. As described in the TIMSS Advanced 2015 Mathematics Framework, the cognitive domains define "the thinking processes students are expected to use when confronting the mathematics items developed for the TIMSS Advanced 2015 assessment" (Gronmo, Lindquist, & Arora, 2014).

The TIMSS Advanced 2015 Mathematics Framework specifies three content domains:

1. Algebra
2. Calculus
3. Geometry

Each of these three content domains is organized into topic areas, and each topic area is further divided into subtopics. The framework specifies that each topic should receive approximately equal weight in the 2015 Mathematics assessment, based on time devoted to testing each topic.

The TIMSS Advanced 2015 Mathematics Framework also specifies three cognitive domains:

1. Knowing
2. Applying
3. Reasoning

Each of these three domains is in turn described by a set of specific thinking skills and behaviors. The framework states that while there is some hierarchy across these three cognitive domains, from knowing to applying to reasoning, each domain is assessed with items representing a full range of difficulty.

The TIMSS Advanced 2015 Mathematics Framework also shows target percentages of total testing time to be allocated to the three mathematics content domains and the three cognitive domains. These target percentages are shown in Table 4. Each mathematics item in the TIMSS Advanced 2015 Assessment assesses both a content domain and a cognitive domain, and the framework explains that each content domain includes items that assess each of the three cognitive domains, i.e., some items that assess geometry require students to use knowing, some require students to use applying, and some require students to use reasoning.

Table 4.

Target Percentages of the TIMSS Advanced 2015 Mathematics Assessment

Content Domains	Percentages
Algebra	35%
Calculus	35%
Geometry	30%
Cognitive Domains	**Percentages**
Knowing	35%
Applying	35%
Reasoning	30%

Structure and Organization — TIMSS Advanced 2015 Physics Framework

As was the case for the TIMSS Advanced 2015 Mathematics Framework, the TIMSS Advanced 2015 Physics Framework was based on an assessment framework that was developed collaboratively with the countries participating in TIMSS Advanced 2015 (Jones, Wheeler, & Centurino, 2014). This 2015 Physics Assessment Framework was updated from the framework for the 2008 assessment, using recommendations from participating countries that were based on changes in the content and instructional emphases of pre-university physics curricula, standards, and frameworks in their countries since 2008. Updates to the TIMSS Advanced 2015 Physics Framework also took into consideration recent initiatives in science education, including the *Framework for K–12 Science Education* (National Research Council, 2012) and the *AP Physics 1 and 2 Course Description* (College Board, 2012b). Thus, the TIMSS Advanced 2015 Physics Framework assessment evolved somewhat since 2008, while still maintaining the continuity from assessment to assessment required for reporting trend data.

Similar to the TIMSS Advanced 2015 Mathematics Framework, the TIMSS Advanced 2015 Physics Framework consists of a content dimension that specifies the physics content domains to be assessed and a cognitive dimension specifying the cognitive domains to be assessed. As described in the TIMSS Advanced 2015 Physics Framework, the cognitive domains define "the thinking processes students are expected to use when encountering the physics items developed for the TIMSS Advanced 2015 Assessment" (Jones, Wheeler, & Centurino, 2014).

The TIMSS Advanced 2015 Physics Framework specifies three content domains:

1. Mechanics and Thermodynamics
2. Electricity and Magnetism
3. Wave Phenomena and Atomic/Nuclear Physics

Each of these three content domains is organized into topic areas, and each topic area is further divided into topics, as shown in figures 2, 3, and 4. The framework specifies that each topic should receive equal weight in the 2015 Physics assessment, based on time devoted to testing each topic.

The TIMSS Advanced 2015 Physics Framework also specifies three cognitive domains:

1. Knowing
2. Applying
3. Reasoning

Each of these three domains is in turn described by a set of specific types of thinking processes, as shown in Table 6. The framework states that while there is some hierarchy across these three cognitive domains, from knowing to applying to reasoning, each domain is assessed with items representing a full range of difficulty.

The TIMSS Advanced Physics Framework also shows target percentages of total testing time to be allocated to the three content domains and the three cognitive domains. These target percentages are shown in Table 5. Each physics item in the TIMSS Advanced 2015 Assessment assesses both a content domain and a cognitive domain, and the framework explains that each content domain includes items that assess each of the three cognitive domains, i.e., some items that assess mechanics and thermodynamics require students to use knowing, some require students to use applying, and some require students to use reasoning.

Table 5.

Target Percentages of the TIMSS Advanced 2015 Physics Assessment

Content Domains	Percentages
Mechanics and Thermodynamics	40%
Electricity and Magnetism	25%
Wave Phenomena/Atomic and Nuclear Physics	35%
Cognitive Domains	**Percentages**
Knowing	30%
Applying	40%
Reasoning	30%

In addition to describing a content dimension and a cognitive dimension for the 2015 physics assessment, the TIMSS Advanced 2015 Physics Framework also describes a set of five science practices to be assessed. These science practices are described in the framework as "skills from across mathematics and science course work that students use in a systematic way to conduct scientific inquiry." The five practices are:

1. Asking questions based on observations.
2. Generating evidence.
3. Working with data.
4. Answering the research question.
5. Making an argument from evidence.

The framework does not specify a target percentage for items assessing these practices, but states that "some items in TIMSS Advanced Physics will assess one or more of these science practices as well as content specified in the content domains and thinking processes specified in the cognitive domains" (Jones, Wheeler, & Centurino, 2014).

Alignment Methodology

Introduction

The methodology used to produce this alignment study is modeled after the methodology developed by Norman Webb (Webb, 1997), senior research scientist at the Wisconsin Center for Education Research, University of Wisconsin-Madison. The alignment study was conducted by content specialists who rated the alignment of the AP Calculus AB and BC and AP Physics 1 and 2 frameworks (College Board, 2012a and 2012b, respectively) to the TIMSS Advanced Assessment Framework (Mullis & Martin, 2014). The alignment rating process includes three rating criteria: (1) Categorical Concurrence, (2) Depth of knowledge, and (3) Balance of representation. While Norman Webb's methodology includes further alignment criteria, the decision to include only the mentioned criteria in the current report was driven by the fit and appropriateness of such analyses afforded by the framework documents.

To conduct framework comparisons, a College Board team specializing in international assessments, curriculum, and educational policy met with a group of external specialists over a two-day period to review the frameworks and organize methodology. The specialists were divided into two groups, mathematics and science. The math and science groups approached the alignment criteria commensurate with their respective frameworks. The process undertaken by each group is detailed in the following sections.

Categorical Concurrence — Content Areas

Categorical concurrence considers whether both alignment frameworks address the same content categories. The criterion of categorical concurrence between frameworks is met if the same or consistent categories of content appear in both documents.

Physics

As described in previous sections of this report, the TIMSS Advanced Physics Framework and the AP Physics 1 and 2 frameworks have different high-level structures. The content of the TIMSS Advanced Physics Framework is organized around three primary physics content domains. These content domains represent a traditional organization of physics content at the high school and introductory college level of instruction. The AP Physics 1 and 2 frameworks are organized around seven big ideas, each of which includes content that cuts across the traditional physics content domains. Despite this fundamental distinction in the high-level organization of physics content between the TIMSS and AP frameworks, at the most granular level of content the frameworks are organized in a much more similar manner. Both the learning objectives specified in the AP Physics 1 and 2 curriculum frameworks and the topics specified in the TIMSS Advanced Physics Framework provide descriptions of the physics

Alignment Methodology

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knowledge expected of students and how students are expected to demonstrate their ability to use that knowledge. Thus, the AP learning objectives and the TIMSS topic provided logical units of analyses for undertaking the categorical concurrence analysis of the frameworks.

The categorical concurrence analysis of the TIMSS Advanced Physics Framework and the AP Physics 1 and 2 Curriculum Framework occurred in two steps:

Step 1) Each of the 250 AP Physics learning objectives was evaluated by the group of science specialists to determine if the content of the learning objective is covered within one or more of the 23 TIMSS Advanced Physics topics. (It is important to note here that the TIMSS topics typically, but not always, tend to be more general and encompass a broader range of content than does each individual AP learning objective). Each AP learning objective was assigned one of four ratings:

- Complete alignment — The learning objective aligns with the full description of the TIMSS topic.
- Partial alignment — The learning objective aligns to a part of the TIMSS topic description.
- Extended alignment — The learning objective aligns to all or part of the TIMSS topic description but also includes knowledge and/or uses of that knowledge that extend beyond what is described in the TIMSS topic.
- Not covered — The learning objective does not align with any of the 23 TIMSS topics.

Step 2) The alignment ratings of each of the AP learning objectives to the TIMSS topics were identified. As a next step, all of the AP learning objectives that aligned to a given TIMSS topic (either partial, complete, or extended alignment) were reviewed again by the group of science specialists and a holistic judgment was made as to whether this collection of AP learning objectives represented partial, complete, or extended alignment with the TIMSS topic as a whole. This review and holistic judgment was completed for each of the 23 TIMSS topics. If the overall rating for a TIMSS topic is partial alignment, then only part of the TIMSS topic is covered in the AP Physics 1 and 2 Framework. If the overall rating for a TIMSS topic is complete alignment, then all of the TIMSS topic is covered in the AP Physics 1 and 2 curriculum framework. If the overall rating for a TIMSS topic is extended alignment, then all of the TIMSS topic is covered in the AP Physics 1 and 2 curriculum framework, and the AP Physics 1 and 2 curriculum framework also requires students to demonstrate additional knowledge that is closely connected to the TIMSS topic but requires additional conceptual knowledge and/or additional applications of that knowledge.

Calculus

The TIMSS Advanced Mathematics Framework, by design, addresses a broader range of mathematical content than the AP Calculus AB and BC frameworks. The TIMSS Advanced Mathematics Framework includes calculus as just one of three primary content domains. The TIMSS Advanced Mathematics Framework includes algebra and geometry as the other two domains. Due to this difference in content domains in these two frameworks, we should expect to observe categorical concurrence between only the calculus portion of the TIMSS Advanced Mathematics Framework and the AP Calculus framework. However, to identify any partial alignment of the AP Calculus framework to the TIMSS Advanced Mathematics algebra and geometry domains, we completed an analysis of all content domains in both frameworks. Both the content topics specified in the AP Calculus framework and the topics specified in the TIMSS Advanced Mathematics Framework provide descriptions of the mathematics knowledge expected of students. Thus, the AP content topics and the TIMSS topics provided logical units of analyses for undertaking the categorical concurrence analysis of the frameworks. The categorical concurrence analysis of the TIMSS Advanced Mathematics Framework and the AP Calculus AB and BC Framework occurred in two steps:

Step 1) Each of the 67 AP Calculus content topics was evaluated by a group of mathematics specialists to determine if the content of the content topic was covered within one or more of the 22 TIMSS Advanced Mathematics topics. (As with the TIMSS Physics Framework, the TIMSS Advanced Mathematics topics tend to be more general and encompass a broader range of content than does each individual AP content topic.) To interpret statements in each of the frameworks the specialists drew from experience with instructional materials and/or instruction commonly used to address the topics described. Any differences in the initial evaluation of the alignment of AP Calculus content topics and TIMSS Advanced Mathematics topics were discussed to determine a final alignment rating. Each AP content topic was assigned one of four ratings:

Complete alignment — The content topic aligns with the full description of the TIMSS topic.

Partial alignment — The content topic aligns to a part of the TIMSS topic description.

Extended alignment — The content topic aligns to all or part of the TIMSS topic description but also includes knowledge and/or uses of that knowledge that extends beyond what is described in the TIMSS topic.

Not covered — The content topic does not align with any of the 22 TIMSS topics.

Step 2) The alignment ratings of each of the AP Calculus content topics to the TIMSS Advanced Mathematics topics were identified. As a next step, all of the AP content topics that aligned to a given TIMSS topic (either partial, complete, or extended alignment) were reviewed again by the group of mathematics specialists and a decision was made as to whether this collection of AP content topics represented partial, complete, or extended alignment with the TIMSS topics as a whole. This review and decision was completed for each of the 22 TIMSS topics. If the overall rating for a TIMSS topic is Partial Alignment, then only part of the TIMSS topic is covered in the AP Calculus framework. If the overall rating for a TIMSS topic is Complete Alignment, then all of the TIMSS topic is covered in the AP Calculus Framework. If the overall rating for a TIMSS topic is Extended Alignment, then all of the TIMSS topic is covered in the AP frameworks, and the AP frameworks also requires students to demonstrate additional knowledge that is closely connected to the TIMSS topic but requires additional conceptual knowledge and/or additional applications of that knowledge.

Depth of Knowledge — Cognitive Domains

Physics

In physics, the depth of knowledge alignment was measured by considering the cognitive domains outlined in the TIMSS Advanced Physics Framework and the science practices outlined in the AP Physics 1 and 2 learning objectives. As previously noted, each AP Physics 1 and 2 learning objective is a cross between the content outlined in the enduring understandings and one of the science practices. These science practices allowed the science specialists to determine the appropriate alignment between the TIMSS cognitive domain and the AP Physics 1 and 2 learning objectives. While some of the AP Physics 1 and 2 learning objectives did not align in content to the TIMSS framework, all of the AP Physics 1 and 2 learning objectives were aligned to at least one of the TIMSS cognitive domains. The TIMSS cognitive domains in physics can be found in Table 6.

Table 6.
TIMSS Advanced Physics Cognitive Domains

TIMSS Cognitive Domain	Subdomain	Description
Knowing		
	Recall/Recognize	Identify or state facts, relationships, processes, phenomena, and concepts; identify the appropriate uses for scientific equipment and procedures; and recognize and use scientific vocabulary, symbols, abbreviations, units, and scales.
	Describe	Describe or identify descriptions of materials, structures, phenomena, processes, properties, interactions, and relationships.
	Provide Examples	Provide or identify examples of processes and phenomena that possess certain specified characteristics; and clarify statements of facts or concepts with appropriate examples.
Applying		
	Use Models	Use a diagram or other model to demonstrate knowledge of physics concepts and principles or to illustrate a structure, process, relationship, or system (e.g., electrical circuit, or atomic structure).
	Interpret Information	Use knowledge of physics concepts and principles to interpret relevant textual, tabular, pictorial, or graphical information.
	Find Solutions	Apply a physical relationship, equation, or formula to find a qualitative or quantitative solution.
	Explain	Provide or identify an explanation for an observation or a natural phenomenon using a physics concept, principle, law, or theory.
Reasoning		
	Analyze	Identify the elements of a scientific problem and use relevant information, concepts, relationships, and data patterns to answer questions to solve the problem.
	Synthesize	Solve problems that require consideration of a number of different factors or related concepts; and integrate mathematical concepts in the solutions to physics problems.
	Design Investigations	Plan investigations or procedures appropriate for answering scientific questions or testing hypotheses; and describing or recognize the characteristics of well-designed investigations in terms of variables to be measured and controlled as well as cause-and-effect relationships.
	Formulate Questions/ Hypothesize/Predict	Formulate questions that can be answered by investigation and formulate testable assumptions based on theory, analysis of scientific information, and/or knowledge from observations; and use evidence and conceptual understanding to make predictions about the effects of changes in physical conditions.
	Evaluate	Evaluate alternative explanations; and evaluate results of investigations with respect to sufficiency of data to support conclusions.
	Draw Conclusions	Make valid inferences on the basis of observations, evidence, and/or understanding of physics concepts; and draw appropriate conclusions that address questions or hypotheses.
	Generalize	Make general conclusions that go beyond the experimental or given conditions; and apply conclusions to new physics contexts.
	Justify	Use evidence and physics understanding to support the reasonableness of explanations, solutions to problems, and conclusions from investigations.

The TIMSS assessment contains items specifically developed to address all three of the cognitive in each of the content domains. Also, as the TIMSS framework notes on page 22, "while there is some hierarchy across the three domains (from knowing to applying to reasoning), each domain contains items representing a full range of difficulty." Given this element, the science specialists determined that the appropriate level of analysis should focus on the three TIMSS cognitive domains, not the subdomain level.

To conduct the depth of knowledge alignment, each of the 250 AP Physics 1 and 2 learning objectives was evaluated by the science specialists to determine if the science practice of the learning objective was covered within one or more of the 23 TIMSS Advanced Physics cognitive domain subtopics. Each AP learning objective was either considered "aligned" or "aligned to multiple subtopics." If the AP learning objective was considered "aligned," it was fully aligned to only one of the TIMSS Advanced Physics cognitive domain subtopics. If it was considered "aligned to multiple subtopics," then the AP Physics 1 and 2 learning objective was aligned to more than one TIMSS Advanced Physics cognitive domain subtopics. While there were several instances of AP learning objectives aligned to multiple subtopics, when these data were analyzed at the TIMSS cognitive domain topic level, each AP Physics 1 and 2 learning objective only aligns once to a TIMSS cognitive domain topic level.

The analysis of the depth of knowledge is reported as the overall percentages of AP learning objectives that are aligned to each TIMSS cognitive domain topic level. The reported percentage is not weighted even though an uneven number of subtopics for each TIMSS cognitive domain exist. This unevenness would be problematic if there were equal chances that the data could align across any of the TIMSS subtopic areas, but this is not the case. The alignment of the AP Physics 1 and 2 learning objectives illustrates where AP places emphasis within the TIMSS cognitive domain.

Calculus

To determine the distribution of levels of cognitive demand as described by the TIMSS Advanced Mathematics Framework, each of the 80 unique tasks from the released 2008 AP Calculus AB and BC Exams (College Board, 2012a) was examined by a group of mathematics experts according to the TIMSS descriptions of thinking skills and behaviors for knowing, applying, and reasoning. The thinking processes for these three cognitive domains are identified in the TIMSS Advanced Mathematics Framework according to the definitions in Table 7.

Table 7.

TIMSS Advanced Mathematics Cognitive Domain and Thinking Processes

	Knowing	Applying	Reasoning
Descriptions	Refers to students' knowledge of mathematical facts, concepts, and procedures	Involves the application of mathematics in a range of contexts, including real-life and purely mathematical situations	Involves logical, systematic thinking, including formulating conjectures, making logical deductions, and justifying results
Thinking Processes	Recall Recognize Compute Retrieve	Determine Represent/Model Implement	Analyze Integrate/Synthesize Evaluate Draw Conclusions Generalize Justify

Citation: (International Association for the Evaluation of Educational Achievement [IEA], 2014, p. 14–15)

According to the TIMSS Advanced Mathematics Framework, tasks are categorized into three cognitive domains: knowing, applying, and reasoning, based on the thinking processes involved. A task classified as knowing relies on student knowledge of mathematical facts, concepts, and procedures. It often involves student recall or computation as its primary goal. A task classified as applying involves students incorporating mathematical knowledge of facts, skills, and procedures or understanding to mathematical concepts to create representations and solve problems. While this is certainly relevant to problems set in real-life situations, it does also include problems purely mathematical in nature. Finally, a task classified as reasoning involves formulating conjectures, making logical deductions, and justifying results. The primary purpose of these problems is to have students analyze, evaluate, or justify their solutions. Table 8 illustrates a task from AB Calculus for each domain.

Table 8.
Examples of AP Calculus Tasks Within TIMSS Advanced Mathematics Framework

Cognitive Domain	AP Calculus Task Example	Thinking Processes
Knowing	AB, Section I, Part A, #6	Recall (S)
	6. If $f(x) = \sin^2(3-x)$, then $f'(0) =$ (A) $-2\cos 3$ (B) $-2\sin 3\cos 3$ (C) $6\cos 3$ (D) $2\sin 3\cos 3$ (E) $6\sin 3\cos 3$	Compute (P)
Applying	AB, Section I, Part B, #15 15. A particle travels along a single straight line with a velocity of $v(t) = 3e^{(-t/2)}\sin(2t)$ meters per second. What is the total distance, in meters, travelled by the particle during the time interval $0 \le t \le 2$ seconds? (A) 0.835 (B) 1.850 (C) 2.055 (D) 2.261 (E) 7.025	Recall (S) Determine (P)
Reasoning	AB, Section I, Part B, #24 24. If $f'(x) = \sin\left(\frac{\pi e^x}{2}\right)$ and $f(0) = 1$, then $f(2) =$ (A) -1.819 (B) -0.843 (C) -0.819 (D) 0.157 (E) 1.157	Determine (S) Analyze (P)

Note. P = Primary; S = Secondary

Experts reviewed the tasks at the thinking process level and identified the primary or secondary reasoning goals for each task. To be identified by as a primary reasoning goal, the main purpose of the task had to align with the descriptors for this thinking process. To be identified as a secondary reasoning goal, the range of possible responses could include features of the described thinking process, but it was not the main purpose of the task. A task could have multiple primary and secondary goals. For example, on the 2008 AP Calculus AB Exam, Problem 6 from Section I states, "if $f(x) = \sin^2(3-x)$, then $f'(x) =$" (College Board, 2012a, page 19). Even though students are expected to recall the derivative of $\sin(x)$ and the chain rule, the primary goals for this task are to: (a) determine the appropriate method needed to solve this problem (in this case, use of the chain rule), and (b) to compute the derivative.

Recall of related mathematical facts and procedures were determined to be secondary goals. In contrast, Problem 24 from Section I requires students to analyze the given trigonometric function and determine an alternative method to find its integral.

Table 9.

Number and Description of Tasks in AP Calculus AB and AP Calculus BC Exams

Task Section	Description of Section	Number of Tasks in Section
AB Calculus, Section I, Part A	Multiple Choice, No Calculators	14
AB Calculus, Section I, Part B	Multiple Choice, Calculators Allowed	10
AB Calculus, Section II, Part A	Free Response, Calculators Allowed	11
AB Calculus, Section II, Part B	Free Response, No Calculators	11
BC Calculus, Section I, Part A	Multiple Choice, No Calculators	14
BC Calculus, Section I, Part B	Multiple Choice, Calculators Allowed	10
BC Calculus, Section II, Part A	Free Response, Calculators Allowed	11
BC Calculus, Section II, Part B	Free Response, No Calculators	11

It is worth noting that even though the framework of AP Calculus BC consists of all topics covered in AP Calculus AB, plus some topics not covered in AP Calculus AB (e.g., different coordinate systems and series), Section I of the AP Calculus AB and AP Calculus BC Exams share no common tasks. However, in Section II, there are three shared questions (12 tasks), two of which are in the calculator section (8 tasks). The three questions (10 tasks) that are different for AP Calculus BC are topics that are unique to the course. Both exams consist of the same number of tasks in each section.

While the two examples provided for Applying and Reasoning appear to be straightforward computational problems, note that both tasks are in the calculator active portion of Section I of the AP Calculus AB Exam. Therefore, it is expected that students perform a higher level of thinking to determine the appropriate methods to solve the tasks and to use their calculators to complete any necessary computations.

Negotiation Between Experts

Mathematics education experts evaluated each task from the released 2008 AP Calculus AB and AP Calculus BC Exams independently. After the evaluations, many of the primary and secondary goals assigned to each task were in agreement, and the experts reconciled any differences in categorization of tasks. These results were also reviewed by another group of content experts. The most common reasons for these differences are listed below:

- Differences in the experts' classroom experience teaching calculus, resulting in different interpretations of what would be expected of students, given a task.
- Differences in the categorization of tasks in calculator-active sections.
- Overemphasizing the importance of Recall. It was understood that every item requires some measure of recall, but the experts frequently negotiated how much a given task relied on Recall as a thinking process.

Deliberations over the primary goals were resolved based on inferences regarding instructional goals and commonly expected responses for tasks. There were also discrepancies when assigning secondary goals. Even though the secondary goals were not used in assigning the overall distribution of cognitive domains in the depth of knowledge analysis, the identification of secondary goals helped to distinguish the difference between primary and secondary goals.

Balance of Representation — Item Distribution

In addition to having comparable depth of knowledge and content coverage, frameworks that are properly aligned will require that knowledge be distributed equally in both frameworks. The balance of representation criterion is used to indicate the degree to which one performance indicator is given more emphasis on one assessment than the other. The distribution of items across content categories stated in the frameworks should be proportional across assessments to conclude proper alignment.

Physics

The analysis of the balance of representation across content categories for TIMSS Advanced 2015 Physics and AP Physics 1 and AP Physics 2 was organized around the seven topic areas within the three physics content domains from the TIMSS Advanced 2015 Physics Framework:

1. Forces and Motion
2. Laws of Conservation
3. Heat and Temperature
4. Electricity and Electrical Circuits
5. Magnetism and Electromagnetic Induction
6. Wave Phenomena
7. Atomic and Nuclear Physics

These seven topic areas represent traditional areas of physics study in high school and introductory college-level physics, and thus are familiar to physics educators and policymakers.

Balance of representation data were obtained from two sources. Data regarding the numbers of items from the TIMSS Advanced 2015 Physics assessment that are categorized in each of the seven TIMSS Advanced Physics topic areas were obtained from the TIMSS International Study Center at Boston College, which develops, administers, and directs TIMSS for the IEA. Data regarding the distribution of items in the AP Physics 1 and 2 exams across these same seven topic areas were determined using the exam specifications provided in *AP Physics 1 and AP Physics 2: Partial Form Assessment Specification*s (College Board, 2015). This document provides detailed information about the numbers of items that the multiple-choice sections of the AP Physics 1 and AP Physics 2 Exams are to include from each of the seven big ideas and from each of the seven science practices described in the AP Physics 1 and 2 frameworks along with additional detail prescribing the learning objectives that can be used to satisfy the requirements for covering big ideas and science practices.

Using the data generated in the physics categorical concurrence analysis, the AP Physics 1 and 2 Exam specifications for the multiple-choice sections were converted to specifications organized by the seven TIMSS Advanced Physics topic areas, and then compared directly with balance of representation data for the TIMSS Advanced 2015 Physics assessment. (The physics content covered by the free-response section of the AP Physics Exams can vary from exam to exam, thus this balance of representation analysis is based only on the specifications for the multiple-choice sections of the AP Physics 1 and 2 exams).

This approach to the balance of representation analysis compares the actual percentages of items from the TIMSS Advanced 2015 Physics Assessment categorized in each of the seven topic areas with percentages of items in the AP Physics 1 and 2 exams that would be categorized in the seven topic areas if the AP Physics 1 and 2 exams reflect the precise, proportional representation of learning objectives and science practices specified in the Partial Form Assembly Specifications. It is not possible to use a categorization of the actual multiple-choice items used on the 2015 AP Physics 1 and 2 exams since these are still secure.

Calculus

The balance of representation of content for the TIMSS Advanced Mathematics and AP Calculus frameworks were organized according to the eight TIMSS Advanced Mathematics topics:

1. Expressions and Operations
2. Equations and Inequalities
3. Functions
4. Limits
5. Derivatives
6. Integrals
7. Non-Coordinate and Coordinate Geometry
8. Trigonometry

These eight topics are often identified in courses organized as advanced algebra[2], pre-calculus and calculus courses.

Similar to the approach used with the physics analysis of balance of representation, data were obtained from two sources. Data regarding the number of items from the TIMSS Advanced Mathematics assessment that were categorized in each of the eight TIMSS Advanced Mathematics topic areas were obtained from the TIMSS International Study Center at Boston College, which develops, administers, and directs TIMSS for the IEA. Data regarding the distribution of items in the AP Calculus AB and BC Exams across these same eight topic areas were determined using the content topics described in *Calculus AB and BC Course Description* (College Board, 2012a).

In contrast to the physics analysis, the AP Calculus content topics were compared to the TIMSS Advanced Mathematics Framework to determine the best match of the AP Calculus content topic to one of eight TIMSS topics. Each content topic was, therefore, matched to only one TIMSS topic. Some AP Calculus content topics were found to have no match to any of the TIMSS topics.

The percentage of items from the TIMSS Advanced Mathematics Framework categorized in each of the eight topic areas was compared to the percentage of content topics in the AP Calculus AB and BC framework aligned to the same topic areas.

2. This is also identified as Algebra 2 in many school districts in the United States.

Alignment Analysis

Physics

Categorical Concurrence

The categorical concurrence analysis comparing the AP Physics 1 and 2 Curriculum Framework with the TIMSS Advanced Physics Framework showed that the large majority of physics content described in the TIMSS Advanced Physics Framework is also described in the AP Physics 1 and 2 Curriculum Framework. However, in some portions of each of the three TIMSS Advanced Physics content domains, the content coverage of the AP Physics 1 and 2 Curriculum Framework extended beyond the content coverage of the TIMSS Advanced Physics Framework to include additional conceptual knowledge and applications of that knowledge. This section of the report presents a detailed description of the results of the categorical concurrence analysis, and delineates the similarities and differences in physics content coverage between the TIMSS Advanced Physics Framework and the AP Physics 1 and 2 Curriculum Framework.

The results of the categorical concurrence analysis of the AP Physics 1 and 2 Curriculum Framework and TIMSS Advanced 2015 Physics Framework show the alignment of the AP Physics 1 and 2 learning objectives with the TIMSS Advanced Physics topic. Of the 250 AP Physics learning objectives, 180 (72%) are covered within one or more of the 23 TIMSS Advanced Physics topics, while 70 (28%) of the learning objectives are not covered in the TIMSS Advanced Physics Framework.

The results of the evaluation of the degree of alignment between the AP Physics 1 and 2 learning objectives and science practices and each of the TIMSS Advanced Physics topics is presented in Figures 2, 3, and 4. For each TIMSS Advanced Physics topic, this figure shows whether there is partial alignment, complete alignment, extended alignment, or not covered between the TIMSS Advanced Physics topic and the AP Physics 1 and 2 Curriculum Framework (as described in the Methodology section of this report, extended alignment indicates that the TIMSS Advanced Physics topic is completely covered within the AP Physics 1 and 2 Curriculum Framework, but also that some AP learning objectives aligned to the topic require additional conceptual knowledge and/or additional applications of that knowledge than are included in the topic description.) None of the 23 TIMSS Advanced Physics topics had no alignment with the AP Physics 1 and 2 Curriculum Framework, two (9%) had partial alignment, 14 (61%) had complete alignment, and seven (30%) had extended alignment. The following section now examines the degree of alignment between the TIMSS Advanced Physics Framework and the AP Physics 1 and 2 Curriculum Framework for each of the content domains and topic areas in the TIMSS Advanced Physics Framework.

Physics Alignment Results

None of the 23 TIMSS Advanced Physics topics had no alignment with the AP Physics 1 and 2 Curriculum Framework, two (9%) had partial alignment, 14 (61%) had complete alignment, and seven (30%) had extended alignment.

Figure 2.

Evaluation of degree of alignment: mechanics and thermodynamics.

Mechanics and Thermodynamics	Partial Alignment	Complete Alignment	Extended Alignment	Key Differences
Forces and Motion				
1. Predict and determine the position, displacement, and velocity of bodies given initial conditions; and use Newton's laws of motion to explain the dynamics of different types of motion and to calculate displacement, velocity, acceleration, distance traveled, or time elapsed.		●		---
2. Identify forces, including frictional force, acting on a body at rest, moving with constant velocity, or moving with constant acceleration and explain how their combined action influences the body's motion; and find solutions to problems involving forces.		●		---
3. Determine the forces acting on a body moving in a circular path at constant velocity, the body's centripetal acceleration, its velocity, and the time for it to complete a full revolution.			►	AP also includes: *Rotational motion *Angular momentum
4. Use the law of gravitation to determine the motion of celestial objects and the forces acting on them.		●		TIMSS focuses on the motion of celestial objects; AP focuses on the motion of objects generally.
Laws of Conservation				
1. Apply the law of conservation of mechanical energy in practical contexts, including finding solutions to problems involving the transformation of potential to kinetic energy and vice versa.			►	AP also includes: *Work in the conservation of mechanical energy *Open and closed systems and defining boundaries of systems
2. Apply the law of conservation of linear momentum in elastic and inelastic collisions.			►	AP also includes: *Open and closed systems *Conservation of momentum in two dimensions
3. Solve problems using the first law of thermodynamics.		●		---
Heat and Temperature				
1. Demonstrate understanding of mechanisms of heat transfer and the mechanical equivalent of heat (work); and use specific heats or heat capacities to predict equilibrium temperature when bodies of different temperature are brought together.			►	AP also includes: *Calculation of energy transfer and work done
2. Determine the expansion of solids in relation to temperature change; and use the ideal gas law (in the form pV/T = constant) to solve problems and demonstrate an understanding of the limitations of this law.		●		---

Figure 3.

Evaluation of degree of alignment: electricity and magnetism.

Electricity and Magnetism	Partial Alignment	Complete Alignment	Extended Alignment	Key Differences
Electricity and Electrical Circuits				
1. Calculate the magnitude and direction of the electrostatic attraction or repulsion between isolated charged particles by application of Coulomb's law.				---
2. Predict the force on and the path of a charged particle moving in a homogeneous electric field.				---
3. Solve problems relating current in electrical circuits (and components of circuits) to voltage, resistance, and energy transformation, including using Ohm's law and Joule's law.				AP also includes: *Properties of capacitors and problem solving involving capacitors *Kirchoff's loop rule and Kirchhoff's junction rule
Magnetism and Electromagnetic Induction				
1. Predict the force on and the path of a charged particle moving in a homogeneous magnetic field.				AP includes only "express the force exerted on a moving charged object by a magnetic field."
2. Demonstrate understanding of the relationship between magnetism and electricity in phenomena such as magnetic fields around electric conductors (Ampere's law), electromagnets, and electromagnetic induction.				---
3. Solve problems using Faraday's and Lenz's laws of induction.				---

Figure 4.

Evaluation of degree of alignment: wave phenomena and atomic and nuclear physics.

Wave Phenomena and Atomic and Nuclear Physics	Partial Alignment	Complete Alignment	Extended Alignment	Key Differences
Wave Phenomena				
1. Apply knowledge of mechanical wave phenomena and the relationship between speed, frequency, and wavelength to solve problems.	●	●	→	AP also includes: *Calculating the period and amplitude of a wave *Using wave front diagrams
2. Demonstrate understanding of electromagnetic radiation in terms of waves caused by the interplay between variations in electric and magnetic fields; and identify various types of waves (radio, infrared, visible light, x-rays, gamma rays) by wavelength and frequency.	●	●		---
3. Demonstrate an understanding of thermal radiation in terms of temperature and wavelength of emitted electromagnetic radiation.	●			AP does not explicitly include this topic regarding thermal radiation.
4. Demonstrate understanding of reflection, refraction, interference, and diffraction.	●	●	→	AP also includes: *Wave pulses *Superposition of standing waves *Circular wave fronts *Model of specular reflection
Atomic and Nuclear Physics				
1. Apply knowledge of the structure of atoms and isotopes, atomic number and atomic mass to solve problems; and relate light emission and absorption in the spectrum to the behavior of electrons.	●	●		---
2. Demonstrate understanding of wave-particle duality, including applying knowledge of the photoelectric effect to predict the consequence of changing the incoming intensity or wavelength of light and solving problems involving the wave nature of matter.	●	●		---
3. Demonstrate understanding of nuclear reactions and solve problems involving radioactive decay, such as finding the half-life of a radioactive isotope; and describe the role of nuclear reactions in nature (such as in stars), and explain their practical applications, such as in nuclear reactors.	●	●		---
4. Demonstrate understanding of mass-energy equivalence in nuclear reactions and particle transformations.	●	●		---

Mechanics and Thermodynamics

Forces and Motion (4 Topics)

This TIMSS topic area covers much of the content generally included in classical mechanics. There is complete alignment of the AP Physics 1 and 2 Curriculum framework with three of the four topics in forces and motion, and extended alignment with the fourth topic. Key points of differentiation between the TIMSS and AP frameworks include:

- The AP Physics 1 and 2 Curriculum Framework extends beyond problem solving involving objects moving in a circular path to include rotational motion, the conservation of angular momentum, and the calculation of angular momentum; this content is not covered in the TIMSS Advanced Physics Framework.
- The TIMSS framework focuses on the specific use of the law of gravitation to determine the motion of celestial objects; the AP framework focuses on the general application of this law to determine the motion of objects, including those close to the Earth's surface.

Laws of Conservation (3 Topics)

There is a high degree of categorical concurrence between the laws of conservation topic area in the TIMSS Advanced Physics Framework and the AP Physics 1 and 2 Curriculum Framework. In the topic addressing the first law of thermodynamics, there is complete alignment between the TIMSS and AP Physics 1 and 2 Curriculum frameworks, and in the topics addressing the law of conservation of mechanical energy and the law of conservation of linear momentum there is extended alignment. With respect to these latter two topics, the content of the AP Physics 1 and 2 Curriculum Framework extends beyond that of the TIMSS Advanced Physics Framework in the following ways:

- The AP Physics 1 and 2 Curriculum Framework explicitly emphasizes (1) the concept of work and how work is accounted for in the conservation of mechanical energy, (2) the application of the conservation of mechanical energy in both open and closed systems, and (3) defining boundaries of systems in which the conservation of mechanical energy occurs, and (4) recognizing that changes in the internal structure of the system result in internal changes in the energy of the system.
- The AP Physics 1 and 2 Curriculum Framework also includes an emphasis on the application of the conservation of linear momentum in both open and closed systems, and a semiquantitative treatment of the conservation of momentum in two dimensions.

Heat and Temperature (2 Topics)

The TIMSS Advanced Physics Framework and the AP Physics 1 and 2 Curriculum Framework are completely aligned in much of the coverage of the heat and temperature topic area, including understanding mechanisms of heat transfer, problem solving using specific heats and heat capacities, predicting equilibrium temperatures, determining the expansion of solids due to temperature change, and the understanding and use of the ideal gas law. In addition, the AP framework includes learning objectives that extend beyond the content included in this TIMSS topic area. These extensions include the following:

- The TIMSS Advanced Physics Framework requires students to demonstrate understanding of the mechanisms of heat transfer and the mechanical equivalent of heat (work). The AP Physics 1 and 2 Curriculum Framework also includes these understandings but extends to include several learning objectives involving the calculation of energy transfer to or work done on an object or system.

- The AP framework also requires students to analyze graphical data showing the area under a pressure-volume curve to determine work done on or by an object or system; this requirement is not explicit in the TIMSS framework.

Electricity and Magnetism

Electricity and Electrical Circuits (3 Topics)

In this topic area, two of the three topics in the TIMSS Advanced Physics Framework were determined to be completely aligned with the AP Physics 1 and 2 Curriculum Framework, and for the third topic, the content of the AP framework was determined to extend beyond that of the TIMSS framework in the following ways:

- The AP Physics 1 and 2 Curriculum Framework includes an understanding of the properties of capacitors and the effects of capacitors in electrical circuits, and problem solving relating to electrical circuits that include capacitors; this content is not included in the TIMSS Advanced Physics Framework.

- The AP framework includes the understanding and various applications of Kirchhoff's loop rule and Kirchhoff's junction rule; this content is not included in the TIMSS framework.

Magnetism and Electromagnetic Induction (3 Topics)

Upon evaluation, the TIMSS Advanced Physics Framework and the AP Physics 1 and 2 Curriculum Framework were completely aligned across the content that traditionally forms the core of instruction in magnetism (Ampere's law, electromagnets and electromagnetic induction, Faraday's and Lenz's laws of induction). Two exceptions to the complete alignment between the frameworks are described below:

- The TIMSS framework includes "predict the force on and the path of a particle moving in a homogeneous magnetic field" as a full topic. In the AP Physics 1 and 2 Curriculum Framework, only Learning Objective 2.D.1.1 and Science Practice 2.2 address part of this highly specific topic ("express the force exerted on a moving charged object by a magnetic field").

- The AP Physics 1 and 2 Curriculum Framework does not explicitly include Faraday's law of induction but do include Learning Objective 4.E.2.1 and Science Practice 6.4, strongly related to the conceptual understanding of Faraday's law.

Magnetism and Electromagnetic Induction

Upon evaluation, the TIMSS Advanced Physics Framework and the AP Physics 1 and 2 Curriculum Framework were completely aligned across the content that traditionally forms the core of instruction in magnetism.

Wave Phenomena and Atomic and Nuclear Physics

Wave Phenomena (4 Topics)

There is significance congruence between this topic area in the TIMSS Advanced Physics Framework and Big Idea 6 in the AP Physics 1 and Physics 2 Curriculum Framework. Also, the TIMSS framework and the AP Physics 1 and Physics 2 Curriculum Framework were evaluated to be completely aligned in content covering the causes, types, and characteristics of electromagnetic radiation. However, the AP Physics 1 and 2 Curriculum Framework extended beyond the TIMSS framework in content related to mechanical wave phenomena and the processes of reflection, refraction, interference, and diffraction, and there was only partial alignment between the TIMSS and AP frameworks in a specific TIMSS topic covering thermal radiation. These distinctions between the TIMSS Advanced Physics Framework and AP Physics 1 and 2 Curriculum Framework are elucidated further below:

- The AP Physics 1 and 2 Curriculum Framework explicitly requires students to calculate the period and amplitude of waves and to use wave front diagrams to demonstrate or interpret the observed frequency of a wave; these are not explicitly included in the TIMSS Advanced Physics Framework.
- The AP Physics 1 and 2 Curriculum Framework includes several conceptual areas that extend beyond what typically would be encapsulated in the "demonstration of understanding of reflection, refraction, interference, and diffraction" described in the TIMSS framework, including wave pulses, the superposition of standing waves, circular wave fronts, and the model of specular reflection.
- The TIMSS Advanced Physics Framework includes a topic requiring students to understand thermal radiation in terms of temperature and wavelength of emitted radiation. This requirement is not explicitly stated in the AP Physics 1 and 2 Curriculum Framework, although the concepts described in AP Learning Objective 5.B.8.1[3] and Science Practices 1.2 and 7.2 build on much of the same knowledge that is required for the understanding of this TIMSS topic.

Atomic and Nuclear Physics (4 Topics)

The TIMSS Advanced Physics Framework and the AP Physics 1 and 2 Curriculum Framework delineate coverage of similar core content from this TIMSS Advanced Physics topic area, including, for example: light emission and absorption by electrons, wave-particle duality, the photoelectric effect, nuclear reactions, radioactive decay and solving half-life problems, and mass-energy equivalence. All four TIMSS Advanced Physics topics in this topic area were evaluated to be completely aligned with the AP Physics 1 and 2 Curriculum Framework, but there are modest differences between the frameworks that are worth noting:

- The TIMSS Advanced Physics Framework requires students to "apply knowledge of the structure of atoms and isotopes, atomic number and atomic mass to solve problems." This requirement is not stated in the AP Physics 1 and 2 Curriculum Framework although it logically can be implied to be prerequisite for other knowledge and skills that are included in the AP Physics 1 and 2 Curriculum Framework.

3. Learning Objective 5.B.8.1: The student is able to describe emission or absorption spectra associated with electronic or nuclear transitions as transitions between allowed energy states of the atom in terms of the principle of energy conservation, including the characterization of the frequency of radiation emitted or absorbed.

- The AP Physics 1 and 2 Curriculum Framework explicitly requires students to "construct or interpret representations of transitions between atomic energy states involving the emission and absorption of photons," which likely extends beyond the intention of the TIMSS framework's requirement that students "relate light emission and absorption in the spectrum to the behavior of electrons."

AP Physics 1 and 2 Content Not Covered in the TIMSS Advanced Physics Framework

The categorical concurrence analysis presented in this section supports the conclusion that nearly all of the physics content described in the TIMSS Advanced Physics Framework is covered in the AP Physics 1 and 2 Curriculum Framework, but that for some TIMSS Advanced Physics topics, the AP Physics 1 and 2 Curriculum Framework extends beyond the TIMSS topic descriptions to include additional conceptual knowledge and additional applications of that understanding.

However, there were also 70 learning objectives in the AP Physics 1 and 2 Curriculum Framework that were not covered in the TIMSS Advanced Physics Framework. Each of the seven big ideas in the AP Physics 1 and 2 Curriculum Framework included some of these 70 learning objectives. For each big idea, the physics content that is not included in the TIMSS Advanced Physics Framework is summarized below:

Big Idea 1

AP Physics 1 and 2, Big Idea 1 includes 19 learning objectives, 12 (63%) of which are covered in the TIMSS Advanced Physics Framework, and seven (37%) of which are not. The Big Idea 1 content that is not included in the TIMSS framework includes:

- The understanding of the concept of fundamental particles and some of the specific types of fundamental particles (neutrinos and quarks).
- The understanding of relativistic mass-energy equivalence and special relativity.
- Predicting densities of objects under various physical conditions and using data to determine densities and resistivities of various objects.

Physics Categorical Concurrence Analysis

The categorical concurrence analysis presented in this section supports the conclusion that nearly all of the physics content described in the TIMSS Advanced Physics Framework is covered in the AP Physics 1 and 2 Framework, but that for some TIMSS Advanced Physics topics, the AP Physics 1 and 2 Curriculum Framework extends beyond the TIMSS topic descriptions.

Big Idea 2

Of the 22 learning objectives included in AP Physics 1 and 2, Big idea 2, only seven (32%) are aligned with the TIMSS Advanced Physics Framework, and 14 (68%) are not. The content that is not aligned with the TIMSS framework includes:

- Distinguishing monopole fields and dipole fields and determining the electric field around a distribution of point charges.
- Calculating and determining the direction of electric fields between two charged perpendicular plates.
- Understanding and applying the concepts of magnetic dipoles and magnetic domains.
- Applying the concept of isolines to describe and determine scalar fields (e.g., gravitational equipotential, electric potential).
- Calculating the magnitude of an electric field from the change in electric potential and the displacement.

Big Idea 3

AP Physics 1 and 2, Big Idea 5 includes 52 learning objectives, 38 (73%) of which are covered in the TIMSS Advanced Physics Framework. The 14 Big Idea 5 learning objectives not covered by the TIMSS framework include the following content:

- Understanding oscillatory motion and solving problems involving simple harmonic oscillation.
- Comparing similarities and differences between gravitational force and electric force.
- Explaining the microscopic cause of contact forces.
- Understanding how torque changes the angular momentum of an object and solving problems involving angular momentum.
- Understanding the concept of fundamental forces and the scales at which these forces are exerted.

Big Idea 4

AP Physics 1 and 2, Big Idea 4 includes 35 learning objectives. Twenty-four (69%) are aligned with the TIMSS Advanced Physics Framework, and 11 (31%) are not. The Big Idea 4 learning objectives not aligned with the TIMSS framework include the following content:

- Understanding torque, angular velocity, angular acceleration, and angular momentum and the relationships between them.
- Determining the change in angular momentum of a system due to interactions with other objects or systems, and solving problems involving angular momentum.
- Explaining the redistribution of electric charges that occurs during conduction and induction.

Big Idea 5

The large majority of physics content included in AP Physics 1 and 2, Big Idea 5 is aligned with the TIMSS Advanced Physics Framework. Fifty-seven (88%) of the Big Idea 5 learning objectives are aligned with the TIMSS Advanced Physics Framework, and eight (12%) are not. The content not covered in the TIMSS Advanced Physics Framework includes:

- Understanding and applying the principle of conservation of electric charge.
- Understanding Bernoulli's equation and using it to solve problems involving moving fluids.
- Calculating quantities related to fluid flow using the continuity equation.

Big Idea 6

Of the 44 learning objectives included in AP Physics 1 and 2 Big Idea 6, 32 (73%) are covered in the TIMSS Advanced Physics Framework. The 12 (27%) AP Physics 1 and 2 learning objectives not covered in the TIMSS Advanced Physics Framework focused on the following content:

- Determining the amplitude of a wave and understanding the relationship between amplitude and energy.
- Understanding the concept of standing waves, explaining how standing waves are formed, and solving problems involving standing waves.
- Explaining how beats are formed.
- Understanding the core concepts of quantum mechanics.

Big Idea 7

AP Physics 2, Big Idea 7 includes 13 learning objectives, which only are included in the AP Physics 2 Framework. Nine (69%) of these learning objectives are aligned with the TIMSS Advanced Physics Framework and four (31%) are not. These specific learning objectives include the following:

- Understanding the second law of thermodynamics and the concept of entropy.
- Understanding how matter can be described as a wave function at the quantum scale, and specifically how the allowed states for an electron can be calculated from the wave model of an electron.

Depth of Knowledge

The depth of knowledge alignment was measured by considering the cognitive domains outlined in the TIMSS Advanced Physics Framework and the science practices outlined in the AP Physics 1 and 2 learning objectives. Overall, this section highlights how the AP Physics 1 and 2 Curriculum Framework places a significant amount of emphasis on the applying and reasoning categories of the TIMSS cognitive domains, but less of an emphasis on the knowing cognitive domain. Table 10 highlights the differences between the cognitive emphasis between the TIMSS Advanced Physics Framework and AP Physics 1 and 2 Curriculum Framework using the construct that has been defined by TIMSS Advanced, which is the outline of the cognitive domains. This research aligns the AP Physics 1 and 2 learning objectives to those appropriate areas. Because this is the same construct, it is possible to determine the emphasis that the AP Physics learning objectives placed on the TIMSS cognitive domains.

Table 10.

Emphasis on Cognitive Domains Across Frameworks

Cognitive Domains	AP Learning Objectives Percentages	TIMSS Percentages
Knowing	11.6%	30%
Applying	40.8%	40%
Reasoning	47.6%	30%

The results of the depth of knowledge analysis of the AP Physics 1 and 2 Framework and TIMSS Advanced 2015 Physics Framework show the alignment of the AP Physics 1 and 2 learning objectives with the TIMSS Advanced Physics cognitive domains. The following sections outline each of the three TIMSS cognitive domain topic areas and describe where the AP Physics 1 and 2 learning objectives place emphasis. Included in this analysis are tables depicting the overall percentage of the AP Physics 1 and 2 learning objectives that fall within each cognitive domain. While this analysis is focused on the topic level of the TIMSS Advanced Physics cognitive domain, each topic level is broken down further to highlight the subtopics that are of notable interest to this alignment.

Knowing

TIMSS Cognitive Domain	Percentage of AP Learning Objectives
Knowing	11.6%

Overall, the domain of knowing only contains 11.6% of the total cognitive alignments. This is by far the least aligned domain among the three TIMSS cognitive domains. One possible explanation for this difference in focus might be that the AP Physics 1 and 2 Curriculum Framework, and more specifically the AP Physics 1 and 2 redesign, heavily focuses on engaging students in the practices of science. Also, while students in AP Physics 1 and 2 are not expected to have specific physics content knowledge prior to entering the course, the AP Physics 1 and 2 courses do expect students to have considerable knowledge within the domain of science. Rather than memorizing a lot of facts and examples, the focus of the AP Physics 1 and 2 courses stress applying and reasoning as students engage with the content of the courses. The observed differences in focus around knowing may be a result of TIMSS Advanced Physics assessing a broader array of knowledge within physics, some of which is procedural and based on core disciplinary facts within the discipline.

Another notable aspect in the domain of knowing is that the alignment is not well distributed across the subtopics. TIMSS outlines three subtopics in the domain of knowing, Recall/Recognize, Describe, and Provide Examples. Of all of the subtopics in the three cognitive domains, Provide Examples is the only subtopic that does not contain any alignment with the AP Physics 1 and 2 learning objectives. Recall/Recognize is only represented by two AP Physics 1 and 2 learning objectives, with the other learning objectives aligned to Describe within the domain of knowing. This is the most uneven distribution among the three cognitive domains.

Applying

TIMSS Cognitive Domain	Percentage of AP Learning Objectives
Applying	40.8%

Applying was the domain with the most evenly distributed alignments. Subtopics such as Find Solutions, Explain, and Use Models fit well with many of the AP Physics 1 and 2 learning objectives, and there was quite a bit of overlap in the language used by TIMSS Advanced Physics and AP Physics 1 and 2 to describe these components of the frameworks. A strong example of this overlap in language between the AP Physics 1 and 2 science practices and the TIMSS Advanced Physics cognitive domains appears in the science practice related to students' use of representations and models. In this practice AP Physics 1 and 2 outlines the following:

Physics Depth of Knowledge Analysis

Applying was the domain with the most evenly distributed alignments. Subtopics such as Find Solutions, Explain, and Use Models fit well with many of the AP Learning Objectives, and there was quite a bit of overlap in the language used by TIMSS and AP to describe these components of the frameworks.

The student can use representations and models to communicate scientific phenomena and solve scientific problems.

The student can *create representations and models of natural or man-made phenomena and systems* in the domain. The student can *describe representations and models* of natural or man-made phenomena and systems in the domain.

The student can *refine representations and models of natural or man-made phenomena and systems* in the domain.

The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.

The student can *re-express key elements of natural phenomena across multiple representations* in the domain.

In addition to Use Models, there is language in several other AP science practices that aligns well with the subtopics of explain and find solutions. This precision in language that AP Physics 1 and 2 allows for 40.8% of the AP Physics 1 and 2 learning objectives to align with the TIMSS Advanced Physics cognitive domain of applying.

Reasoning

TIMSS Cognitive Domain	Percentage of AP Learning Objectives
Reasoning	47.6%

The cognitive domain of reasoning was also strongly aligned with many of the AP Physics 1 and 2 learning objectives. In particular, designing investigations, formulate questions/hypothesize/predict, and analyze were the most aligned of the eight subtopics. Much of the language, especially in these three subtopics, overlaps with parts of the AP science practices. Again, as with the previous cognitive domain, the precision in language of the AP Science Practices illustrates a clear alignment between AP and TIMSS. Most notable was the language in the subtopic formulate questions/hypothesize/predict:

The student can engage in scientific questioning to extend thinking or to guide investigations within the context of the AP course.

The student can pose scientific questions.

The student can refine scientific questions.

The student can evaluate scientific questions.

Balance of Representation

The TIMSS Advanced 2015 Physics assessment includes 110 items, approximately evenly divided between multiple-choice items and short constructed-response items. The multiple-choice sections of the AP Physics 1 and AP Physics 2 Exams each include 40 items. The percentages of these items that are categorized in each of the seven TIMSS Advanced Physics topic areas are shown in Figures 5, 6, and 7.

Figure 5.

Topic area weighting in TIMSS Advanced 2015 Physics and AP Physics 1.

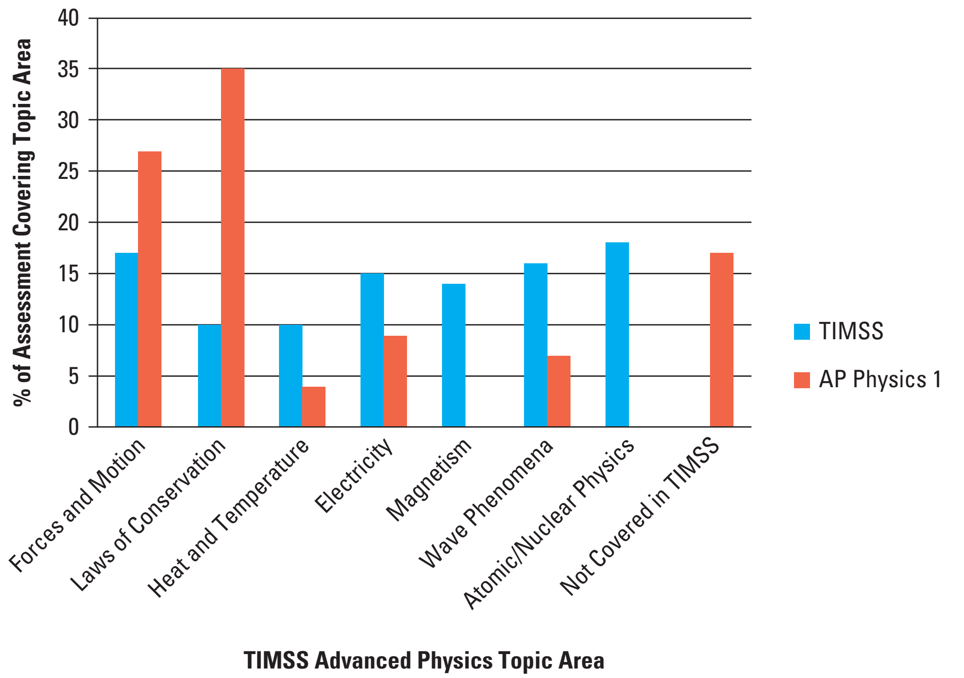


Figure 6.

Topic area weighting in TIMSS Advanced 2015 Physics and AP Physics 2.

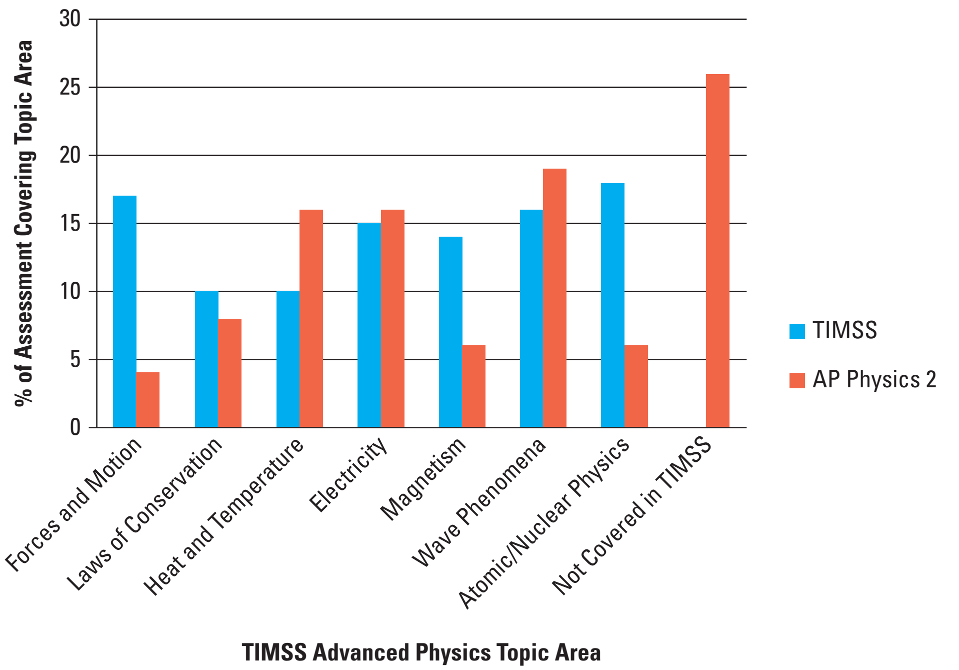


Figure 7.

Topic area weighting in TIMSS Advanced 2015 Physics and AP Physics 1 and 2 combined.

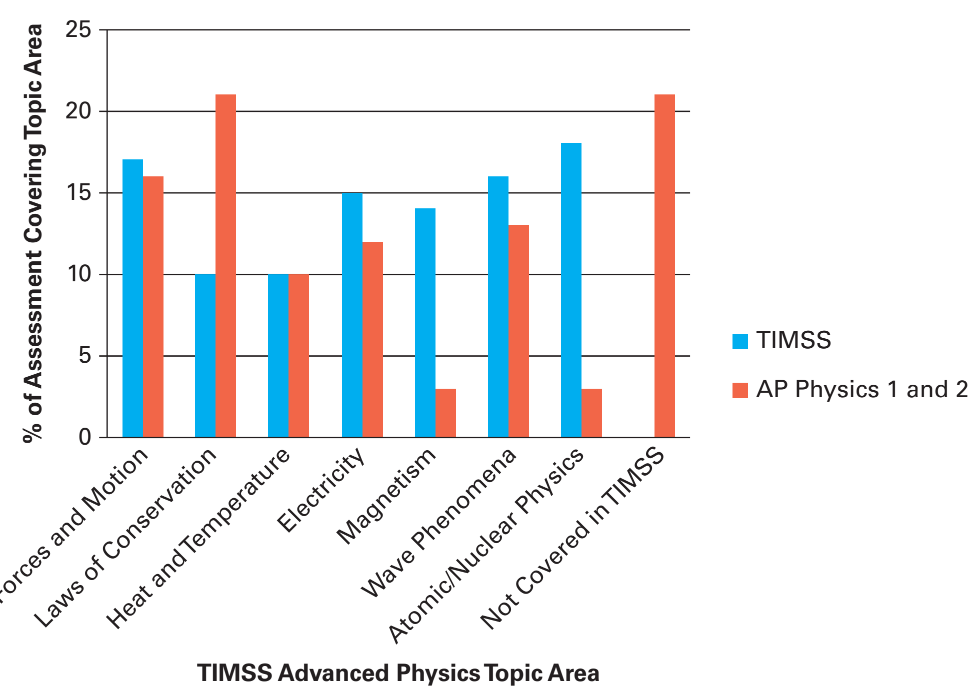


Figure 5 compares the TIMSS Advanced Physics assessment data to the percentages of items from AP Physics 1 Exam that are categorized in the seven topic areas, as well as the percentage of items in AP Physics 1 that are not covered in the TIMSS Advanced Physics assessment. Figure 6 shows a similar comparison between the TIMSS Advanced Physics assessment and the AP Physics 2 Exam, and Figure 7 shows a similar comparison between the TIMSS Advanced Physics assessment and the AP Physics 1 and 2 Exams combined.

There are notable differences between the TIMSS Advanced Physics assessment and AP Physics 1 in the proportions of items in several of the seven topic areas that were the focus of the analysis:

- The proportions of items covering forces and motion and laws of conservation are much higher on the AP Physics 1 Exam than on the TIMSS Advanced Physics assessment.
- The proportions of items covering heat and temperature, electricity and electrical circuits, and wave phenomena are much lower on the AP Physics 1 Exam than on the TIMSS Advanced Physics assessment.
- There is no coverage of magnetism and electromagnetic induction and atomic and nuclear physics in the AP Physics 1 Exam; these topic areas represent 16% and 18% of the TIMSS Advanced Physics assessment, respectively.
- 17% of the AP Physics 1 items are not covered in any of the seven TIMSS Advanced Physics topic areas.

There are also important comparisons in the balance of representation among topic areas between the TIMSS Advanced Physics assessment and the AP Physics 2 Exam:

- The proportion of items covering heat and temperature is much higher on the AP Physics 2 Exam than on the TIMSS Advanced Physics assessment.
- The proportion of items covering forces and motion, magnetism and electromagnetic induction, and atomic and nuclear physics is much lower on the AP Physics 2 Exam than on the TIMSS Advanced Physics assessment.

Physics Balance of Representation Analysis

In conclusion, while the categorical concurrence analysis shows that nearly all of the physics content covered in the TIMSS Advanced Physics assessment will be covered by students who have completed both AP Physics 1 and 2, the balance of representation analysis shows that there are important differences between the TIMSS Advanced Physics assessment and the AP Physics Exams in the proportional emphasis on certain topic areas.

- The proportions of items covering laws of conservation, electricity and electrical circuits, and wave phenomena on the AP Physics 2 Exam and the TIMSS Advanced Physics assessment are similar.
- 26% of the AP Physics 2 items are not covered in any of the seven TIMSS Advanced Physics topic areas.

When data from the AP Physics 1 and AP Physics 2 Exams are combined, there is closer alignment between the AP Physics Exams and the TIMSS Advanced Physics assessment, although some important differences still occur:

- The AP Physics 1 and 2 Exams together have a higher proportion of items covering laws of conservation than does the TIMSS Advanced Physics assessment (21% vs. 10%), and have a lower proportion of items covering magnetism and electromagnetic induction (3%, vs. 14% on TIMSS) and atomic and nuclear physics (3%, vs. 18% on TIMSS).
- 21% of the combined set of items from the AP Physics 1 and 2 Exams are not covered in any of the TIMSS Advanced Physics topic areas.

In conclusion, while the categorical concurrence analysis shows that nearly all of the physics content covered in the TIMSS Advanced Physics assessment will be covered by students who have completed both AP Physics 1 and 2, the balance of representation analysis shows that there are important differences between the TIMSS Advanced Physics assessment and the AP Physics 1 and 2 Exams in the proportional emphasis on certain topic areas. Most notably, AP students who have completed only AP Physics 1 and then take the TIMSS Advanced Physics assessment will encounter numerous items in magnetism and electromagnetic induction and atomic and nuclear physics (comprising 34% of the TIMSS assessment) that assess content not included in AP Physics 1.

Calculus

Categorical Concurrence

In contrast to the categorical concurrence analysis for physics, the comparison of the AP Calculus AB and AP Calculus BC Framework and the TIMSS Advanced Mathematics Framework found significant discrepancies between the mathematics content described in the TIMSS Advanced Mathematics Framework and the AP Calculus Framework. This is due, in large part, to calculus being one of three topic areas in the TIMSS Advanced Mathematics Framework. The other two topics are algebra and geometry, which typically are prerequisites for taking pre-calculus. However, when comparing the calculus topics on the AP Calculus AB and BC Framework to the TIMSS Advanced Mathematics Framework, there is either complete alignment, or the AP Calculus topics extend beyond the content coverage of the TIMSS Advanced Mathematics Framework to include advanced applications of expected procedural and conceptual knowledge. This section includes a detailed description of the results of this comparative analysis of categorical concurrence analysis, and outlines the similarities and differences in content coverage between the TIMSS Advanced Mathematics and AP Calculus AB and BC frameworks.

The results of the categorical concurrence analysis of the AP Calculus AB and BC Framework and TIMSS Advanced Mathematics Framework shows the alignment of the AP Calculus AB and BC content topics with the TIMSS Advanced Mathematics topics. Of the 44 AP Calculus AB content topics, all at least partially cover or extend beyond one or more of the 22 TIMSS Advanced Mathematics topics, and 42 (95%) of the AP Calculus AB content topics completely cover the TIMSS topics. Of the 67 AP Calculus BC content topics, all at least partially cover or extend beyond one or more of the 22 TIMSS Advanced Mathematics topics, and 56 (84%) of the AP Calculus BC content topics completely cover the TIMSS topics.

The degree of alignment between the AP Calculus AB and BC content topics with respect to the algebra, calculus, and geometry topics described in the TIMSS Advanced Mathematics Framework are presented, respectively, in 8, 9, and 10. For each TIMSS Advanced Mathematics topic, these three figures illustrate whether there is no alignment, partial alignment, complete alignment, or extended alignment between each TIMSS Advanced Mathematics topic and the AP Calculus AB and BC content topics. When the AP Calculus AB Framework was compared to the 22 TIMSS Advanced Mathematics topics, three TIMSS topics (14%) had no alignment, seven (32%) had partial alignment, 10 (45%) had complete alignment, and two (9%) had extended alignment. The comparison of the AP Calculus BC Framework to the TIMSS Advanced Mathematics topics resulted in two TIMSS topics (9%) had no alignment, five (23%) had partial alignment, nine (41%) had complete alignment, and six (27%) had extended alignment. The section that follows examines the degree of alignment between the TIMSS Advanced Mathematics Framework and the AP Calculus AB and BC Framework for each of the content domains and topic areas in the TIMSS Advanced Mathematics Framework (IEA, 2014).

Figure 8.

Evaluation of degree of alignment: algebra.

Algebra	Partial Alignment	Complete Alignment	Extended Alignment	Key Differences
Expressions and Operations				
1. Operate with exponential, logarithmic, polynomial, rational, and radical expressions; and perform operations with complex numbers.				AP Calculus AB and BC use operations of many types of expressions as a tool to solve more advanced problems.
2. Evaluate algebraic expressions (e.g., exponential, logarithmic, polynomial rational, and radical).				AP Calculus AB and BC use evaluation of algebraic expressions as a tool to solve more advanced problems, but not as a primary goal for the task.
3. Determine the nth term of arithmetic and geometric series and the sums of finite and infinite series.				AP Calculus BC also includes: * Taylor and Maclaurin series with the formation of new series from known series, and functions defined by power series.
Equations and Inequalities				
1. Solve linear and quadratic equations and inequalities as well as systems of linear equations and inequalities.				AP Calculus AB and BC use solving linear and quadratic equations as a tool to solve more advanced problems. AP Calculus AB and BC also have little alignment with inequalities.
2. Solve exponential, logarithmic, polynomial, rational, and radical equations.				AP Calculus AB and BC use solving equations as a tool to solve more advanced problems, but not as a primary goal for the task.
3. Use equations and inequalities to solve contextual problems.				---
Functions				
1. Interpret, relate, and generate equivalent representations of functions, including composite functions, as ordered pairs, tables, graphs, formulas, or words.				AP Calculus AB also includes: * Verbal descriptions translated into equations involving derivatives and vice versa. * Interpretation of differential equations using slope fields. AP Calculus BC also includes: * Analysis of curves in parametric, polar and vector form.
2. Identify and contrast distinguishing properties of exponential, logarithmic, polynomial, rational, and radical functions.				AP Calculus AB also includes: * Geometric understanding of graphs of continuous functions. * Analysis of curves, including notions of monotonicity. AP Calculus BC also includes: * Analysis of curves in parametric, polar, and vector form.

Note. Orange represents AP Calculus AB and blue represents AP Calculus BC.

Figure 9.

Evaluation of degree of alignment: calculus.

Calculus	Partial Alignment	Complete Alignment	Extended Alignment	Key Differences
Limits				
1. Determine limits of functions, including rational functions.				---
2. Recognize and describe the conditions for continuity and differentiability of functions.				---
Derivatives				
1. Differentiate polynomial, exponential, logarithmic, trigonometric, rational, radical, and composite functions; and differentiate products and quotients of functions.				---
2. Use derivatives to solve problems in optimization and rates of change.				---
3. Use first and second derivatives to determine slope, extrema, and points of inflection of polynomial and rational functions.				---
4. Use first and second derivatives to sketch and interpret graphs of functions.				---
Integrals				
1. Integrate polynomial, exponential, trigonometric, and simple rational functions.				---
2. Evaluate definite integrals, and apply integration to compute areas and volumes.				---

Note. Orange represents AP Calculus AB and blue represents AP Calculus BC.

Figure 10.

Evaluation of degree of alignment: geometry.

Geometry	Partial Alignment	Complete Alignment	Extended Alignment	Key Differences
Non-Coordinate and Coordinate				
1. Use non-coordinate geometry to solve problems in two and three dimensions.				While some topics may be represented through non-coordinate geometry, the predominant method uses the coordinate plane.
2. Use coordinate geometry to solve problems in two dimensions.				AP Calculus AB and BC use coordinate geometry as a tool to solve more advanced problems.
3. Apply the properties of vectors and their sums and differences to solve problems.				AP Calculus BC also includes: * Finding the derivative of vector functions and related analyses. * Analysis of curves in vector form, including the context of velocity and acceleration.
Trigonometry				
1. Use trigonometry to solve problems involving triangles.				Trigonometry is used extensively in AP Calculus, but the TIMSS Advanced framework specifies the use of trigonometry involving triangles. This considerably diminishes the alignment to a select few topics (e.g., finding the derivative of inverse trigonometric functions).
2. Recognize, interpret, and draw graphs of sine, cosine, and tangent functions.				AP Calculus AB does not emphasize graphing of trigonometric functions. AP Calculus BC also includes: * The use of calculus to predict and explain the local and global behavior of a function. * Analysis of curves in parametric form, which may include trigonometric functions.
3. Solve problems involving trigonometric functions.				AP Calculus AB includes the application of trigonometric functions in the context of derivatives and integrals; however, it is rarely used in solving problems in context. AP Calculus BC also includes: * Modeling rates of change and related rates problems, and the use of implicit differentiation. * Use of the Maclaurin series for trigonometric functions.

Note. Orange represents AP Calculus AB and blue represents AP Calculus BC.

Algebra

Expressions and Operations (3 Topics)

The algebra topic areas described in the TIMSS Advanced Mathematics Framework include those typically addressed in an advanced algebra course offered in most high schools in the United States. The evaluation of AP Calculus topics with respect to this TIMSS Advanced Mathematics topic resulted in partial alignment of the AP Calculus AB framework with all three Expressions and Operations topics and extended alignment between the AP Calculus BC and one of the three topics. Key points of differentiation between the TIMSS and AP Calculus AB and BC frameworks include the following:

Even though the AP Calculus AB Framework partially addresses the TIMSS topic determining the *n*th term of arithmetic and geometric series and the sums of finite and infinite series through the knowledge and application of Riemann sums, the AP Calculus BC Framework includes the Taylor and Maclaurin series and analysis of related error bounds, which is an extension beyond this topic.

The TIMSS framework focuses on operating with exponential, logarithmic, polynomial, rational, and radical expressions; and performing operations with complex numbers. However, these topics are only partially addressed within the context of AP Calculus content topics and often involve operations with expressions. Similarly, the TIMSS topic of evaluating algebraic expressions is indirectly addressed in AP Calculus in the context of solving calculus problems. In addition, the TIMSS topic of operations with complex numbers is rarely addressed in AP Calculus AB or BC.

Equations and Inequalities (3 Topics)

There is an uneven degree of categorical concurrence between the equations and inequalities topic area in the TIMSS Advanced Mathematics Framework and the AP Calculus AB and BC Framework. For two of the topics — *solve linear and quadratic equations and inequalities; and solve exponential, logarithmic, polynomial, rational, and radical equations* — there is only partial alignment between the TIMSS and AP Calculus frameworks. The partial alignment for these two topics was determined due to the regular use of equations and finding solutions of equations when solving derivative and integral calculus problems. For the third topic, *use equations and inequalities to solve contextual problems*, there is complete alignment between the TIMSS and AP Calculus frameworks since this is a common feature of many AP Calculus content topics, although inequalities are rarely used in calculus for problem solving.

Functions (2 Topics)

The AP Calculus AB and BC Framework extends beyond the TIMSS Advanced Mathematics functions topics. In fact, 17 of the 44 (39%) AP Calculus AB content topics and 17 of the 67 (25%) AP Calculus BC content topics were determined to extend beyond the two TIMSS functions topics, which include interpreting, relating, and generating equivalent representations of functions; and identifying and contrasting distinguishing properties of functions. These extensions include the following:

The AP framework requires students to interpret, relate, and generate representations of functions using techniques that involve continuity and the relationships between the first and second derivative of a function in ways that are not described in the TIMSS framework. The types of functions used in AP Calculus BC also extend beyond those mentioned in the TIMSS framework (e.g., parametric, polar, etc.).

The TIMSS Advanced Mathematics Framework requires students to interpret, relate, and generate equivalent representations of functions, including composite functions, as ordered pairs, tables, graphs, formulas, or words. The AP Calculus framework uses these various representations for the interpretation and generation of functions but also includes techniques such as finding the slope of a curve at a point, finding the equation of a line tangent to a curve at a given point, and local linear approximation.

Calculus

Limits (2 Topics)

For the calculus topic of limits, both topics in the TIMSS Advanced Mathematics Framework were determined to be completely aligned with the AP Calculus AB framework. TIMSS Calculus topics such as determine limits of functions, and recognize and describe the conditions for continuity and differentiability of functions are addressed, almost verbatim, in the AP Calculus AB framework. Some aspects of AP Calculus, however, go beyond recognizing and describing conditions for continuity and differentiability since students are often expected to use that knowledge to determine, for example, if an integral can be taken or if a theorem can be used.

Limits

For the calculus topic of limits, both topics in the TIMSS Advanced Mathematics Framework were determined to be completely aligned with the AP Calculus AB framework.

Derivatives (4 Topics)

The TIMSS Advanced Mathematics Framework includes four topics for derivatives, and the AP Calculus framework was deemed to be completely aligned across all four topics. In both frameworks students are expected to differentiate polynomial, exponential, logarithmic, trigonometric, rational, radical, and composite functions; and differentiate products and quotients of functions. Both frameworks call for the use of derivatives to solve problems involving optimization and rates of change, and use first and second derivatives to determine slope, extrema, and points of inflection of polynomial and rational functions. While there are some exceptions with respect to the AP Calculus BC framework (which also includes the derivatives of parametric, polar, and vector functions), the differences were minor and not enough to evaluate the AP Calculus BC as extended beyond the TIMSS Advanced Mathematics Framework.

Integrals (2 Topics)

As with the other calculus topic areas, the TIMSS Advanced Mathematics Framework and the AP Calculus framework were evaluated to be completely aligned across the content that traditionally involves the integration of functions and the evaluation of integrals. Several exceptions to the complete alignment between the frameworks are described below:

The AP Calculus framework includes the use of the fundamental theorem of calculus to evaluate definite integrals and to represent a particular antiderivative. However, the TIMSS Advanced Mathematics Framework does not explicitly mention use of the fundamental theorem of calculus. Similarly, AP Calculus AB and BC includes the development of the integral using Riemann sums, but the TIMSS Advanced Mathematics Framework does not explicitly address Riemann sums and their relationship to limits and definite integrals.

The AP Calculus AB and BC Framework includes applications of integrals in a variety of contexts such as modeling physical, biological, or economic situations. The TIMSS framework does not mention explicit applications beyond using integrals to compute areas and volumes.

The AP Calculus BC framework includes solving logistic differential equations and using them in modeling, and finding the solution to improper integrals. The TIMSS framework does not mention these advanced topics. The TIMSS Advanced Mathematics Framework may indirectly address the use of Riemann sums with respect to integral calculus but does not explicitly mention this method.

Geometry

Non-Coordinate and Coordinate (3 Topics)

There is varied concurrence between this topic area in the TIMSS Advanced Mathematics Framework and the AP Calculus framework. Even though calculus tasks often involve the use of geometric principles, there is only partial alignment between the AP Calculus framework and the use of coordinate geometry to solve problems in two dimensions; there was no alignment between the TIMSS topic use of non-coordinate geometry in two and three dimensions and the AP Calculus framework. AP Calculus AB has no alignment to the TIMSS topic applying the properties of vectors and their sums and differences to solve problems; however, AP Calculus BC had extended alignment beyond the same topic. The differences between the TIMSS Advanced Mathematics and AP Calculus frameworks for these topics are described further below:

The AP Calculus framework includes content topics such as analysis of curves, optimization with global and local extrema, and the analysis of characteristics of functions using first and second derivatives. All of these involve some application of coordinate geometry to solve problems, and were evaluated as partially aligned with this topic.

The use of vectors as described in the AP Calculus BC framework includes several concepts that extend beyond the TIMSS topic of applying the properties of vectors and their sums and differences to solve problems. These AP content topics include the use of derivatives and analysis of curves in vector form.

Trigonometry (3 Topics)

The trigonometry topics described in the TIMSS Advanced Mathematics had varied alignment with the AP Calculus framework. The AP Calculus framework does not include the use of trigonometry to solve triangle problems, and was not aligned with this topic. However, the AP Calculus AB framework was partially aligned to the other two TIMSS trigonometry topics involving the interpretation of trigonometric functions and solving problems involving trigonometric functions. The AP Calculus BC framework was found to have extended alignment beyond the same two topics. Even though several AP Calculus content topics may involve interpreting, drawing, and solving problems involving basic trigonometric functions, the use of these functions are not required and their use was viewed as incidental in AP Calculus. Other differences between the AP Calculus and TIMSS frameworks worth noting include:

The TIMSS Advanced Mathematics Framework requires students to "recognize, interpret, and draw graphs of sine, cosine, and tangent functions," yet there is no explicit mention of the same activity in the AP Calculus framework. Students may or may not be asked to draw trigonometric functions when modeling or solving calculus problems, resulting in a determination of partial alignment. The use of the Taylor and Maclaurin series in AP Calculus BC to model and solve problems involving trigonometric functions was determined to have extended alignment beyond two of the three TIMSS Advanced Mathematics topics for trigonometry. In addition, the use of derivatives to analyze relationships among standard trigonometry functions also supported an evaluation of extended alignment.

AP Calculus AB and BC Content Not Covered in the TIMSS Advanced Mathematics Framework

The categorical concurrence analysis presented in this section supports the conclusion that all of the calculus content described in the TIMSS Advanced Mathematics Framework, and some of the algebra and geometry content, is addressed in the AP Calculus AB and BC Framework. In addition, several content topics in the AP Calculus Framework extend beyond the TIMSS topic descriptions in their use of applications and focus on specialized topics.

TIMSS Advanced Mathematics Topics Addressed in the AP Calculus Framework

In this section, we investigate how the TIMSS Advanced Mathematics Framework aligns to the AP Calculus framework. In other words, given a content topic for AP Calculus, which topics in the TIMSS Advanced Mathematics Framework does the content topic fully cover or go beyond?

As one would expect, the content topics in AP Calculus that result in the most topics in TIMSS Advanced Mathematics that are covered or extend beyond are typically general AB Calculus topics, such as analyzing graphs, finding tangent lines, characteristics of the derivative, and modeling rates of change. However, there was one notable content topic from

Calculus Categorical Concurrence Analysis

The categorical concurrence analysis presented in this section supports the conclusion that all of the calculus content described in the TIMSS Advanced Mathematics Framework, and some of the algebra and geometry content is addressed in the AP Calculus AB and BC Framework.

BC Calculus — analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration — that went beyond coverage for many topics in TIMSS Advanced Mathematics due to its breadth, including functions, derivatives, and geometry. The following is a list of content topics in AP Calculus AB and BC that do not have complete coverage in the TIMSS Advanced Mathematics.

AP Calculus AB:

- Basic properties of definite integrals (examples include additivity and linearity).

AP Calculus BC:

- Terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of p-series.
- The ratio test for convergence and divergence.
- Comparing series to test for convergence or divergence.
- Taylor polynomial approximation with graphical demonstration of convergence (for example, viewing graphs of various Taylor polynomials of the sine functions approximating the sine curve).
- Maclaurin series and the general Taylor series centered at $x = a$.
- Functions defined by power series.
- Lagrange error bound for Taylor polynomials.

The large number of topics listed here from AP Calculus BC shows that while some of the added topics are aligned well with TIMSS Advanced Mathematics (e.g., functions in polar form, parametric form, or vector form), there are a number of AP Calculus BC content topics not addressed in TIMSS Advanced Mathematics.

An element of the list is the AP Calculus AB content topic of basic properties of definite integrals. Even though this is a very specific topic that has little to do with most of the TIMSS Advanced Mathematics Framework, especially those not related to calculus directly, the TIMSS framework explicitly addresses integrals. However, upon further evaluation, it was determined that TIMSS Advanced Mathematics expects integration of polynomial, exponential, trigonometric, and simple rational functions, along with evaluating the definite integrals and computing areas and volumes. These two TIMSS topics are only partially aligned with the AP Calculus content topic, *basic properties of definite integrals* — the former is only concerned with integrating specific types of functions and applying them to area and volume, while the latter includes integration of algebraic manipulations of various types of functions.

Depths of Knowledge

As described in the methodology section, the tasks from the released 2008 AP Calculus AB and BC Exams were compared to the TIMSS Advanced Mathematics Framework, particularly focusing on the cognitive domains of knowing, applying, and reasoning. Table 11 shows the number of tasks for each section of each exam:

Table 11.

Total Number of Tasks Corresponding to Each Domain in TIMSS Advanced Mathematics

	Knowing	Applying	Reasoning
AB, Section I	17 (71%)	5 (21%)	2 (8%)
BC, Section I	18 (75%)	2 (8%)	4 (17%)
AB, Section II	9 (41%)	8 (36%)	5 (23%)
BC, Section II	10 (45%)	5 (23%)	7 (32%)
TIMSS Distribution	**35%**	**35%**	**30%**

Note the subtle differences in the categorization of tasks between the TIMSS Advanced cognitive domains and the AP Calculus AB and BC Exams. While the number of tasks is precisely the same, the distribution of the tasks is slightly different. The AP Calculus AB Exam has three more tasks in the applying cognitive domain than AP Calculus BC. Table 12 below illustrates the proportion of knowing, applying, and reasoning tasks for each section:

Table 12.

Total Number of Tasks Corresponding to Each Domain in TIMSS Advanced, Expanded

	Knowing	Applying	Reasoning
AB, Section I, Part A	11 (79%)	2 (14%)	1 (7%)
BC, Section I, Part A	13 (93%)	0 (0%)	1 (7%)
AB, Section I, Part B	6 (60%)	3 (30%)	1 (10%)
BC, Section I, Part B	5 (50%)	2 (20%)	3 (30%)
AB, Section II, Part A	3 (27%)	7 (64%)	1 (9%)
BC, Section II, Part A	5 (45%)	5 (45%)	1 (9%)
AB, Section II, Part B	6 (55%)	1 (9%)	4 (36%)
BC, Section II, Part B	5 (45%)	0 (0%)	6 (55%)
TIMSS Distribution	**35%**	**35%**	**30%**

Recall that in both AP Calculus Exams, Section I Part A is multiple choice, no calculators and Part B is multiple choice, with calculators; Section II Part A is free response, with calculators and Part B is free response, no calculators. Reviewing how the tasks are distributed throughout the entire exams tells a much different story. The small differences within each section are exemplified through differences when looking between calculator active and non-calculator parts. For example, the two tasks related to the applying cognitive domain in Section I of the AP Calculus BC Exam are both found in the calculator-active part. In fact, almost all tasks in the AP Calculus AB and BC Exams that are classified as applying according to the TIMSS Advanced Mathematics Framework are in the calculator-active component of each section (Section I, Part B and Section II, Part A). Similarly, almost all of the tasks classified as reasoning in Section II of the AB and BC Calculus Exams do not allow students to use calculators.

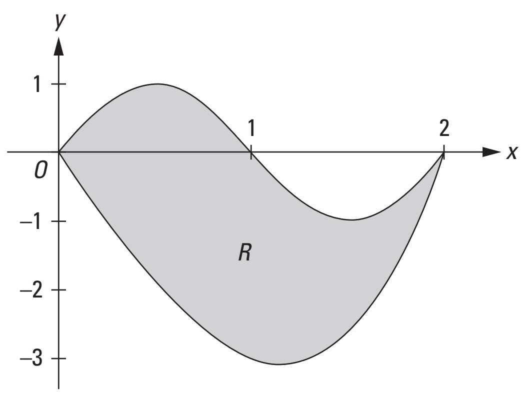
Even though Section I of each exam consists of different tasks, Section II shared 12 tasks out of 22 between the AB and BC Calculus Exams. Of the 10 tasks in Section II that were different between the two exams, the tasks in BC Calculus were unique to the BC Calculus content topics. It was observed that all five tasks aligned with the applying domain in Section II of the AP Calculus BC Exam were among the shared tasks. That is, no free-response questions unique to the AP Calculus BC curriculum were aligned with the applying domain. Moreover, no additional tasks were aligned with the reasoning cognitive domain in Section II of the AP Calculus AB Exam were unique to that exam (i.e., all five reasoning tasks in Section II of the AP Calculus AB Exam were shared with the AP Calculus BC Exam). The AP Calculus BC Exam, however, has two additional reasoning tasks.

Representation of Thinking Processes

The most-used thinking processes identified in the AP Calculus tasks were Recall and Compute, as many of the non-calculator tasks required a combination of the two. Since a majority of the tasks in Section I in each of the AP Calculus Exams involve solving problems, often by finding derivatives, the result is a preponderance of item representation in the knowing domain. Similarly, the sizable number of tasks within the applying domain are attributed to two problems in the free-response section (Section II) that rely heavily on the Represent/Model thinking process (generate an equation or diagram that models problem situations and generate equivalent representations for a given mathematical entity). For example, all four tasks of question one of Section II in the AB and BC Calculus Exams were determined to be aligned with the Application cognitive domain (see Figure 11 below).

Figure 11.

Example of an application task from AP Calculus Exam.



Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.

(a) Find the area of R.

(b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

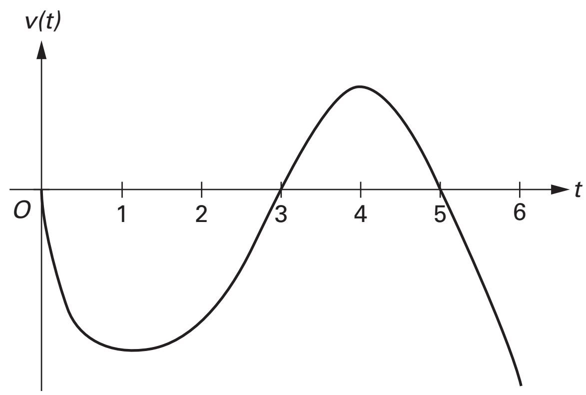
(d) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

Even though it was determined that Analysis (i.e., identify the elements of a problem and determine the information, procedures, and strategies necessary to solve the problem) from the reasoning domain and retrieval (i.e., retrieve information from graphs, tables, texts, or other sources) from the knowledge domain was involved in completing this problem, the primary goal of each task in the question above is to generate equations that model the given information (i.e., Represent/Model from the applying cognitive domain).

Lastly, the tasks attributed to reasoning are due to a relatively equal distribution between Analyze (see description above), Integrate/Synthesize (link different elements of knowledge, related representations, and procedures), Draw Conclusions (make valid inferences on the basis of information and evidence), and Justify (provide mathematical arguments or proofs to support a strategy, solution, or statement). For example, all four tasks in question four of Section II in the AB and BC exams were determined to be aligned with the reasoning cognitive domain, but for different reasons (see Figure 12 below):

Figure 12.

Example of a reasoning task from AP Calculus Exam.



Graph of v

A particle moves along the x-axis so that its velocity at time t, for $0 \le t \le 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t-axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

(a) For $0 \le t \le 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.

(b) For how many values of t, where $0 \le t \le 6$, is the particle at $x = -8$? Explain your reasoning.

(c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

(d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

The primary goal of tasks 4A and 4B is to Analyze; the primary goal in task 4C is to Draw Conclusions; and the primary goal in task 4D is to Integrate/Synthesize. Even though secondary goals were found, they had no impact on determining the cognitive domain for each task.

The following problem from Section II (Part B) of the AP Calculus BC Exam gives an example of Justification as the primary thinking process (5A). Tasks 5A and 5B involve a combination of Justification, Analysis, and Drawing Conclusions. Since 5C involves rather straightforward integration by parts it is a basic Computation problem and was categorized in the Knowledge cognitive domain.

Figure 13.

Example of a justification task from AP Calculus Exam.

Question 5

The derivative of a function f is given by $f'(x) = (x-3)e^x$ for $x > 0$, and $f(1) = 7$.

(a) The function f has a critical point at $x = 3$. At this point, does f have a relative minimum, a relative maximum, or neither? Justify your answer.

(b) On what intervals, if any, is the graph of f both decreasing and concave up? Explain your reasoning.

(c) Find the value of $f(3)$.

Lack of Representation of Thinking Processes

Also worth noting is the lack of representation of some of the thinking processes described in the TIMSS Advanced Mathematics framework within the AP Calculus tasks. The following thinking processes were not identified as primary reasoning goals in all of the tasks reviewed:

- Recognize (recognize entities that are mathematically equivalent, from the knowing cognitive domain).
- Implement (implement strategies to solve problems in familiar concepts and procedures, from the applying cognitive domain).
- Evaluate (determine appropriateness of alternative strategies and solutions, from the reasoning cognitive domain).
- Generalize (make statements that represent relationships in more general terms, from the reasoning cognitive domain).

While the first three thinking processes incorporate many secondary reasoning goals within the set of AP Calculus tasks, there were no AP Calculus tasks that featured Generalize as either a primary or secondary reasoning goal. The most common usage of the thinking process, Recognize, is in the multiple-choice sections of the AB and BC Calculus Exams, where students are relate their solutions to the given choices. Finally, Question 23 from Section I, Part B of the AP Calculus AB Exam is an example of Recognize as a secondary goal (the primary goal is Retrieve):

Calculus Depth of Knowledge Analysis

Also worth noting is the lack of representation of some of the thinking processes described in the TIMSS Advanced Mathematics Framework within the AP Calculus tasks.

Figure 14.

Example of a recognize task from AP Calculus Exam.

23. Oil is leaking from a tanker at the rate of $R(t) = 2{,}000e^{-0.2t}$ gallons per hour, where t is measured in hours. How much oil leaks out of the tanker from time $t = 0$ to $t = 10$?

(A) 54 gallons
(B) 271 gallons
(C) 865 gallons
(D) 8,647 gallons
(E) 14,778 gallons

Balance of Representation

Two separate analyses of AP Calculus AB and the AP Calculus BC were completed to determine the balance of representation of content compared to the TIMSS Advanced Mathematics assessment. As discussed in the methodology section, AP Calculus content topics were categorized according to the following TIMSS topics:

- Expressions and Operations
- Equations and Inequalities
- Functions
- Limits
- Derivatives
- Integrals
- Non-Coordinate and Coordinate Geometry
- Trigonometry

In Figure 15, the distribution of AP Calculus AB content topics is compared to the distribution of TIMSS Advanced Mathematics tasks for these same eight topics.

The balance of representation for each topic in the TIMSS Advanced Mathematics Framework ranges from 8% to 18%. The AP Calculus AB balance of representation ranges from 2.3% to 43.2% across five of the eight topics, with no representation for Expressions and Operations and the two geometry topics. Of the AP Calculus AB content topics, 4.5% did not match any of the TIMSS topics. When focusing on the balance of representation for both TIMSS and AP Calculus AB for the three calculus topics, the derivatives representation (43.2%) is almost double comparing to the representation of limits and integrals (20.5%).

Figure 16 compares the distribution of AP Calculus BC content topics to the distribution of TIMSS Advanced Mathematics tasks. The AP Calculus BC balance of representation ranges from 1.5% to 28.4% across seven of the eight topics, with no representation for trigonometry. Given the greater proportion of advanced topics that extended beyond the TIMSS framework, it is not surprising that a much greater proportion of AP Calculus BC content topics (19.4%) did not match any of the TIMSS topics. Similar to the AP Calculus AB, when focusing on just the three calculus topics the derivatives representation is almost double (28.4%) compared to the representation of limits and integrals (14.9%).

Figure 15.

Comparison of topic area weighting in TIMSS Advanced Mathematics and AP Calculus AB.

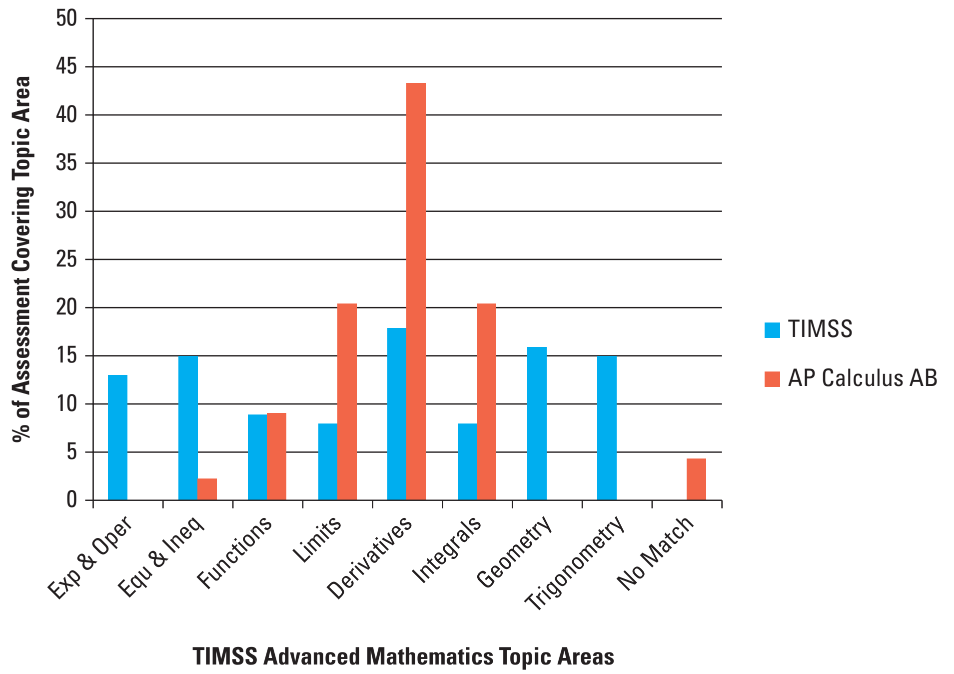
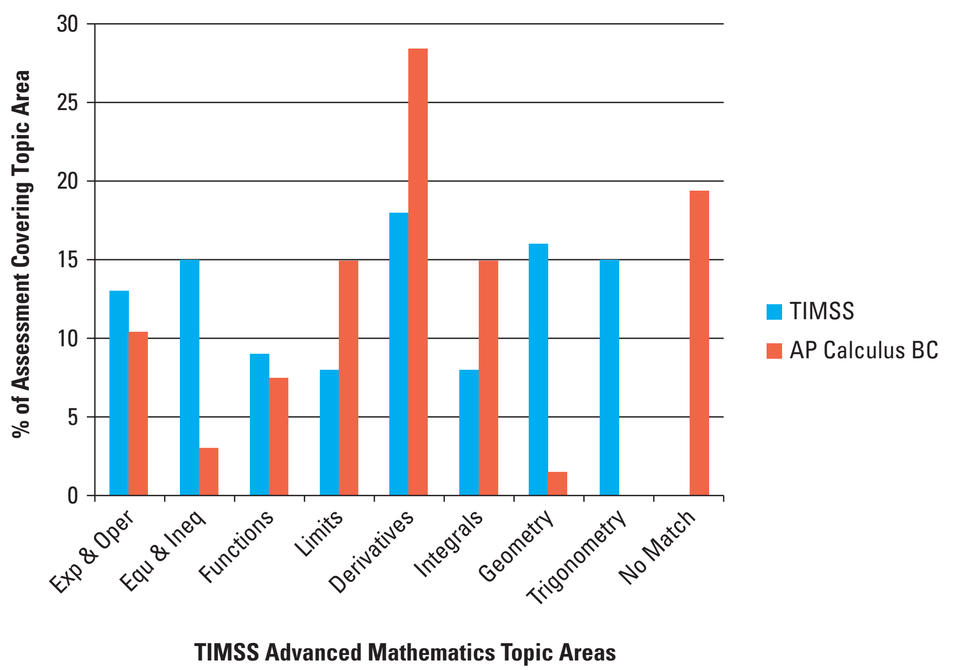


Figure 16.

Comparison of topic area weighting in TIMSS Advanced Mathematics and AP Calculus BC.



Future Studies and Limitations

Performance Report

The purpose of this alignment report is to lay the necessary foundation for a future performance report, which will compare performance of U.S. students who took AP Physics 1 and 2 and AP Calculus to U.S. students who participated in TIMSS Advanced. The purpose of the Advanced Placement to TIMSS Advanced 2015 Performance Report is to explore AP Calculus and AP Physics student performance on TIMSS Advanced 2015. Through this analysis, we will be able to see how the AP Calculus and AP Physics students perform as compared to international peers. Although TIMSS Advanced reports performance at the country level, our analysis will allow for interpreting performance at the student level as well as sample level, whether country or other subgroup, albeit with respective limitations. To date, there has been only a post hoc analysis in 2001 but, otherwise, we do not know the degree to which AP students are competitive when compared to their international peers on TIMSS Advanced. Moreover, although we will have a strong understanding about the framework alignment, we will better understand this alignment when analyzing performance data. Specifically, we will analyze AP Physics 1 and 2 as well as AP Calculus AB and BC student performance on TIMSS Advanced 2015.

Future Performance Report

We do not know the degree to which AP students are competitive when compared to their international peers on TIMSS Advanced.

Assessments Not Taken Concurrently

One issue with the proposed analysis is that the two exams will not be taken concurrently. When exams are not taken concurrently, there are many factors that may come into play that could impact the performance differential of students. For example, what students know at one point in the year will differ at another point in the year. Fortunately, the TIMSS Advanced 2015 Exam will be administered in April and May, giving AP students the benefit of completing nearly their entire AP course prior to taking the exam. While it is not possible to say with certainty that a student will have covered all of the content outlined in the AP curriculum framework prior to taking the TIMSS Advanced 2015 exam, or which content the student might be missing, we are confident that the exam is being administered late enough in the year that this should not significantly impact student performance on the exam. In addition, there are other environmental or motivational factors that also need to be considered when comparing students' performance on two different exams that are not taken concurrently. While we do not believe this fact to impact the way in which the alignment study or subsequent performance report will be interpreted, contextualized, or utilized, we do acknowledge that there will be some uncertainty in the performance report that cannot be accounted for.

Findings and Conclusion

Overall Summary

This alignment report compares the AP Physics 1 and 2 and the AP Calculus curriculum frameworks with the TIMSS Advanced 2015 Frameworks to show alignment in both the content and cognitive domains for each subject area. We believe that this analysis is important in laying the necessary foundation for a future performance report comparing performance of U.S. students who took AP Physics 1 and 2 and AP Calculus to U.S. students who participated in the TIMSS Advanced. The findings of this alignment report reveal considerable alignment between the TIMSS Advanced Framework and the AP Physics and Calculus curriculum frameworks in terms of Categorical Concurrence, Cognitive Demand, and Balance of Representation. The most notable findings are:

Categorical Concurrence

Physics. None of the 23 TIMSS Advanced Physics topics had no alignment with the AP Physics 1 and 2 Curriculum Framework, two (9%) had partial alignment, 14 (61%) had complete alignment, and seven (30%) had extended alignment. The categorical concurrence analysis supports the conclusion that nearly all of the physics content described in the TIMSS Advanced Physics Framework is covered in the AP Physics 1 and 2 Framework, but that for some TIMSS Advanced Physics topics, the AP Physics 1 and 2 Curriculum Framework extends beyond the TIMSS topic descriptions.

Math. The categorical concurrence analysis supports the conclusion that all of the calculus content described in the TIMSS Advanced Mathematics Framework, and some of the algebra and geometry content is addressed in the AP Calculus AB and BC Framework.

Depth of Knowledge

Physics. Applying was the domain with the most evenly distributed alignments. Subtopics such as Find Solutions, Explain, and Use Models fit well with many of the AP learning objectives, and there was quite a bit of overlap in the language used by TIMSS and AP to describe these components of the frameworks.

Math. There is a lack of representation of some of the thinking processes described in the TIMSS Advanced Mathematics Framework within the AP Calculus tasks.

Balance of Representation

Physics. The balance of representation analysis shows that there are important differences between the TIMSS Advanced Physics assessment and the AP Physics Exams in the proportional emphasis on certain topic areas. Most notably, AP students who have completed only AP Physics 1 and then take the TIMSS Advanced Physics assessment will encounter numerous items in magnetism and electromagnetic induction and atomic and nuclear physics (comprising 34% of the TIMSS assessment) that assess content not included in AP Physics 1.

Math. The balance of representation for each topic in the TIMSS Advanced Mathematics Framework ranges from 8% to 18%. The AP Calculus AB balance of representation ranges from 2.3% to 43.2% across five of the eight topics. Of the AP Calculus AB content topics, 4.5% did not match any of the TIMSS topics. When focusing on the balance of representation for both TIMSS and AP Calculus AB for the three calculus topics, the derivatives representation (43.2%) is almost double compared to the representation of limits and integrals (20.5%).

Conclusion

While it is not the aim of an AP Physics or AP Calculus course to cover content outlined in the TIMSS Advanced Exam this overall strong alignment between the TIMSS Advanced framework and AP Physics 1 and 2 and AP Calculus suggests that AP students taking the TIMSS Advanced Exam in the Spring of 2015 should be very familiar with the content being assessed. Further, AP students should also be familiar with the context of the assessment items and the way in which the assessment items are presented to the students on the exam. Knowing that AP students are prepared for the TIMSS 2015 Exam will allow for deeper claims to be made in the subsequent performance report, as it will not be necessary to speculate whether or not the material on the TIMSS Advanced 2015 Exam is relevant to AP students. In addition, the level of detail this alignment report provides will allow researchers writing the performance report to deconstruct different sections of the TIMSS 2015 performance report to identify sections of the exam where we would expect AP students to excel and where AP students are less prepared for the content being assessed.[4]

Appendixes available upon request.

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