A Framework for Incorporating Model-Based Inquiry into Physics Laboratory Courses

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Abstract

In an effort to align lab curricula with recent calls for increasing students' engagement with scientific practices important for research, we present a pedagogical approach for developing or redesigning labs using existing equipment in order to accomplish this goal. In the context of a polarization of light lab, we give examples of how these principles were used to transform a classic optics experiment with familiar apparatus. We also provide specific examples of lab guide questions and student work. Based on a review of the literature, examples from professional research, and interviews with leading experimentalists, we introduce a framework for including scientific modeling practices (including quantitative reasoning) into undergraduate laboratories. This framework was used to redesign a polarization of light lab to include modeling using the Jones matrix formalism and standard optics equipment such as lasers, polarization optics, and photodetectors; however, the framework and pedagogical approach is designed to enable instructors to transform their own labs at their own institution.

I. INTRODUCTION: THE PURPOSE OF LAB COURSES

The laboratory classroom is an educational environment with unique opportunities. Typically, the lab classroom is a dedicated space with sophisticated equipment where classes of small numbers of students work on extended activities under the guidance of experimental physicists. Those same opportunities make the lab class a complicated environment for instructors. The instructor needs to manage and assist in using sophisticated equipment, while at the same time giving attention to pedagogy, such as identifying learning goals, developing activities that align with goals, and providing evidence of student competency. The instructional lab community has often addressed questions such as: *What experiment involves physics topic X?* or *How do I build apparatus Y?*, but equally important and often not explicitly discussed is: *How do we construct a meaningful learning experience with this equipment?*

Throughout our work we believe the key role of lab courses in the undergraduate physics curriculum is to prepare students for participation in undergraduate research and beyond. There is widespread acceptance of the unique value of undergraduate research. Many faculty believe a good undergraduate research experience (URE) is essential preparation for graduate school. Also, outcomes of UREs can be a good indicators of future success.[1] The National Science Foundation provides significant funding for these experiences through the Research Experience for Undergraduates program. Although such experiences are valuable, they are also complex and resource intensive. A URE often starts with the identification of an interesting original question within a wider field (itself a substantial endeavor) and extends to the planning, execution, and eventual communication of the project as a scientific argument. A URE is not a strictly linear process, and they are complicated to teach or mentor. Even many years in graduate school may not prepare students for developing, managing, executing, and communicating their own research projects. The authors agree that UREs offer some unique and essential preparation for professional research, but believe that lab courses play an essential role in preparing students in a way that is scalable and efficient and serves as a precursor to UREs. Table I clarifies some of the differences between a good undergraduate research experience and a good lab course.

As Table I shows, the lab course should not attempt to emulate every aspect of a URE, but rather should focus on the elements that are easily scalable and most widely applicable for research. Even the most well-funded laboratory courses cannot include the diversity of physics content and apparatus that are used in professional physics. The priority then shifts from equipment and content to scientific practices, such as modeling, communication, and design that are used in every context. These particular scientific practices emerged through discussions with faculty, a review of STEM education literature, input from the physics lab community, and reflection on professional practice. Key learning goals developed from these sources are summarized in Fig. 1.[2,3]

The role of the lab is to engage students in a number of scientific practices aligned with the learning goals in Fig. 1 using widely-used equipment to explore interesting and important physics phenomena. All aspects of the experience (practices, apparatus, physics phenomenon) are chosen to be of high value; however, the set of core scientific practices are the priority, while the choice of apparatus and physics phenomena is based upon the local context, including available resources or interests of the students and instructor.

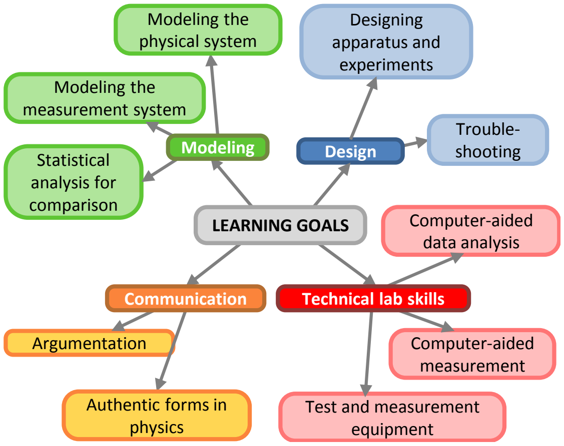


FIG. 1. Learning goals for the advanced physics lab course. The full detailed learning goals document can be accessed at http://tinyurl.com/Advanced-Lab-LGs.

II. MANY TYPES OF INQUIRY

One long-standing option for lab pedagogy is the “cookbook” verification lab which directs students through a series of steps, often aimed at demonstrating or verifying a well-known result.[6] Cookbook verification labs have been maligned for over a century[7] because students are able to complete the exercises without deeply engaging with the physics content or prac-

TABLE I. A comparison of attributes of good undergraduate research experiences and lab courses. Attributes of good research experiences are based on the literature on UREs.[1] Attributes of a research-preparatory lab course are adapted from URE guidelines in a way that is suitable for a lab classroom environment.[4,5]

A good research experience	**A good lab course**
Authentic problems of interest to the broader community. New results anticipated.	Results will not be new to the broader community, but may still be important to know as a member of that community.
Situated in the workplace.	Situated in the lab classroom.
Apparatus and content sophisticated enough to do original research.	Apparatus and content generally useful in the field. Sophistication may be good enough for original research, but often more limited.
Open-ended problems, multiple solutions.	More limited open-ended problem solving. Guidance from a lab guide document.
1-1 mentoring relationships. A true apprenticeship with a master scientist.	Many-to-one teaching relationship with an expert experimentalist.
Part of community of professional physicists: graduate students, post-docs, faculty.	Community is still relevant, but primarily other undergraduate students.
The most important goal is answering the research question.	The most important goal is engaging in good scientific practices which can be applied later in research.

tices. In recent decades, significant attention has been given to the idea of *inquiry*, which, as defined by the National Science Education Standards refers "to the diverse ways in which scientists study the natural world and propose explanations based on the evidence derived from their work. Inquiry also refers to the activities of students in which they develop knowledge and understanding of scientific ideas, as well as an understanding of how scientists study the natural world."[8] In particular, inquiry has a dual meaning of being student-directed and also referring to a set of "fundamental abilities necessary to do scientific inquiry."[9] Within K-12 and undergraduate science education, inquiry can take on a range of meanings.[10] Very frequently inquiry is viewed as an alternative to "cookbook" emphasizing open-ended lab activities with unknown or unexpected final results. The widely varying definitions of inquiry cause it to be understood and implemented by instructors in varying ways,[11] and the variation has limited the adoption of inquiry at the undergraduate level.[12] Various schemes for characterizing the level of inquiry at the college level have been produced by various authors.[5,6,10,12] The recently released K-12 science frameworks,[13] which are an input to the Next Generation Science Standards, have replaced the language of "fundamental abilities necessary to do scientific inquiry" with "Scientific and Engineering Practices." These scientific practices now form one of the key areas of the science education standards along with "core disciplinary ideas" and "crosscutting concepts."

Although many ideas of inquiry have been profitably applied at the introductory level,[14–17] there are many apparent obstacles to a wholesale adoption of inquiry in the upper-division. An example of an inquiry lab appropriate for the introductory level is a pendulum lab where students *discover* the mathematical relationships between the oscillation period and the string length, mass, and amplitude.[8] Another introductory physics curriculum has students discovering models of motion and force though a series of guided inquiry laboratory activities.[15] However, the mathematical models frequently used in the upper-division labs are sufficiently sophisticated (e.g., polarized light described by the Jones formalism) that it is not clearly possible or desirable to have students *discover* general unknown relationships between particular quantities. Further, if we view labs as preparation for research, the most important criteria is that the activity accurately characterizes authentic research practices and students build on their prior knowledge—just as scientists do. We argue below that inquiry should begin in the context of an existing theoretical model, a form of inquiry known as *model-based inquiry*.[18]

III. MODEL-BASED GUIDED INQUIRY

A. Guided Inquiry

In our advanced lab course at the University of Colorado Boulder (CU), a conscious decision was made to provide a guided lab experience that engages students in a number of best practices while conducting experimental investigations. The proposed lab experience should focus on: (1) particular big-picture experimental goals (e.g., optimize the production of circularly polarized light and model your non-ideal components to quantitatively describe any remaining non-circularity in the polarization), (2) meaningful physics ideas (e.g., polarized light and its interaction with materials), (3) and common laboratory apparatus (e.g., lasers, optomechanics, polarization optics, photodetectors, data-acquisition). The lab class is then viewed as a learning environment where many of the hidden aspects of scientific expertise are revealed, modeled, and practiced, as students work towards greater levels of proficiency and independence. This is the basic idea of cognitive apprenticeship,[19] which has been employed in other transformed curricula such as the Investigative Science Learning Environment (ISLE).[20] Research from cognitive science[21,22] and attempts to scaffold the learning of scientific practices[23,24] also advocate for such guidance in the teaching of science content and practices.

The resulting guided inquiry lab activities are "procedural" in the sense that the questions appear in a particular order and sometimes include significant detail. However, since the primary goal is engaging students in best experimental practices, the questions are all designed to prompt students to engage in important scientific practices. If students are routinely practicing these approaches in a number of experimental contexts (using different concepts and equipment), the expectation is that students will continue to engage in these practices later on in a more independent setting. The expectation of transfer of scientific abilities fits well with how we think students learn and agrees with existing research.[4] The questions in the lab guide also act as a form of assessment that provides feedback to students and instructors about students' level of proficiency in the practices. Fig. 2 situates the guided inquiry activities described here in terms of two dimensions of inquiry: the *level of guidance* and the *engagement in scientific practices*.

The most significant practice that is not directly prompted by the guided lab, as presented here, is the planning of steps needed to reach the goal. This may include designing the overall experiment, breaking a large project into manageable sub-components, and setting short-range and long-range goals. That said, we do incorporate this organization and planning aspect of doing science into the course in two ways: (1) Prioritizing lab notebooks as a means to have students tell the story of their experiment—a narrative of the students' thoughts, results, interpretation, difficulties, conclusions, future directions. (2) Creating structure to transition students to more open-ended projects later in the semester. This second strategy

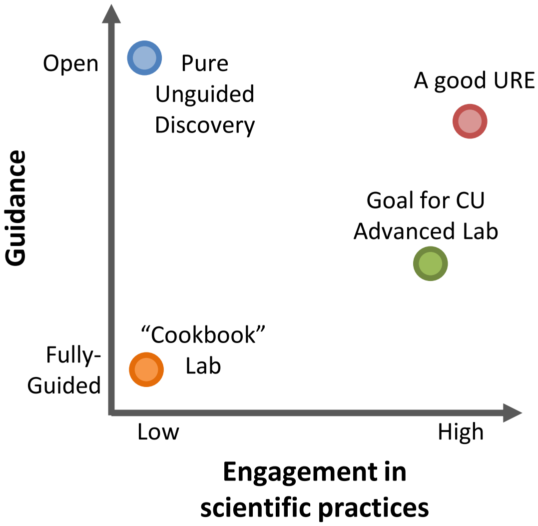


FIG. 2. The two dimensions of "inquiry."

is accomplished during the final five weeks of the CU advanced lab course, which involves a final project that requires significant planning in addition to the scientific practices used throughout the semester. Students begin their projects by identifying their own interests and then continue on to forming testable research questions, designing the apparatus and experiment, making predictions, analyzing results, and communicating results and possible future directions to the class.

IV. MOTIVATION FOR MODEL-BASED GUIDED INQUIRY

A. Modeling as a consensus learning goal

Learning goals for the advanced lab class at CU were developed using input from: (1) faculty, (2) the literature on science education, and (3) an analysis of authentic practice of physicists.[3] These learning goals represents a target outcome for our physics majors. What is interesting is that *scientific practices* emerged as primary learning goals for the course.[25] Physics content and particular apparatus were viewed by faculty as important, but the options are simply too broad to include more than a small sampling of different sub-domains of physics and common techniques. From discussion with faculty, one common theme that emerged was that good labs, and student research projects, should be quantitative in both measurement and theory and involve interesting physics content. In particular, we found that the practice of modeling—the act of developing, testing, and refining models (see Fig. 3)—provides a good descriptive framework for the use of quantitative reasoning in the laboratory.

Model - An abstract representation of a real physical system that: (1) is simplified, (2) is predictive, and (3) has specified limits to its applicability.

Modeling - The process of developing, testing, and refining models.

FIG. 3. Definitions of *model* and *modeling* drawn from literature.[26,27]

B. Modeling as an authentic practice

Another means for assessing the relevance of particular scientific practices is to find examples of these activities in professional practice. For example, we asked the question, "What are the marks of a convincing experiment in a professional physics setting?" Regardless of whether the content is cutting edge quantum computing or the simple pendulum, we wanted to identify key aspects about what distinguishes a skillful treatment of the experiment from a novice treatment. Drawing from our experience as researchers and interactions with faculty (in-class and during interviews), one key aspect of convincing physics experiments is the reliance on quantitative reasoning. In the laboratory this manifests itself in some particular ways:

- Many of the best experiments combine quantitative predictive models with quantitative measurement. The models are as important as the data.
- Experimental physicists do night lose sight of the basic physics ideas and principles that govern the system. It is never as simple as matching results to an equation.
- Experimental physicists recognize the idealizations and simplifications that limit the accuracy of the model, and can speak to the validity of applying the model to the experimental apparatus.
- Experimental physicists do not treat measurement devices as "black boxes," but understands how the measurement tools work and their limitations.

- Experimental physicists consider both random and systematic sources of uncertainty.
- Experiments are typically iterative, systematic investigations where models and apparatus are refined in many stages.

Similarly, the relevance of modeling in professional practice can be gleaned from perusing the pages of any journal, such as *Physical Review Letters.* A typical paper answers a number of questions about the research: *What are the interesting phenomena under investigation? What physical principles describe it? What predictions can be made? How do the apparatus and measurement tools work? How does the physical phenomena create the measured signal and how does the raw data become processed into a value comparable with the predictions? What assumptions are made and how could this account for systematic deviation in the data? What future steps are needed to improve the agreement?*

Particularly for quantitative sciences, like physics, using predictive models based on widely applicable theories is an essential component of scientific inquiry. Willis Lamb, winner of the 1955 Nobel Prize in Physics for measuring the energy difference between the $2S_{1/2}$ and $2P_{1/2}$ levels of hydrogen, had this to say when reflecting on his introductory physics course at University of California, Berkeley:

> “In 1930, I wondered how Newton’s laws of motion could give such a good description of phenomena studied in the undergraduate laboratory which was an integral part of Physics 1A. After some fruitless speculations, I decided that the most important object of physics was to study interesting laboratory phenomena, and to try to make a mathematical model in which the mathematical symbols imitated, in a way to be determined, the motions of the physical system. I regarded this as a game, to be taken seriously only if it worked well.”[28]

To Willis Lamb, an experimental physicist, modeling was both a key part of doing physics, and one of the most important parts of his laboratory experience.

C. Modeling in the physics education literature.

Science education literature has a strong emphasis in modeling, including ideas of model-based inquiry in K-12,[18] and the “Modeling Instruction” curriculum in high school[14] and college physics.[27,29] David Hestenes, an early proponent and developer of Modeling Instruction, firmly believed that modeling theory was part of authentic research and went so far as to claim that "Much of the knowledge [modeling theory] explicates is so basic and well known to physicists that they take it for granted and fail to realize that it should be taught to students."[27]

The close link between theoretical models and measurements is nothing new in physics or in the undergraduate laboratory, but this paper provides a way to redesign labs to increasingly engage students in the same scientific practices that have been used in research in our discipline.

V. MODELING FRAMEWORK

As part of revising the CU advanced lab course to incorporate modeling, we developed a framework to describe how models are used in the process of experimental physics, which is shown in Fig. 4. The framework was inspired by the use of computation modeling in introductory physics,[30] but was expanded for use in the upper-division labs especially by reflecting on our own research experiences.

The top of the modeling framework shown in Fig. 4 starts with the real equipment that makes up the experiments. This is typically divided into two parts: (1) the *physical system* being studied (right side of Fig. 4), and (2) the *measurement probes* and tools that gather and record information about the physical system (left side of Fig. 4). Both the physical system and the measurement probes are modeled. The physical system is often modeled from a set of important physical *principles* (e.g., Maxwell's equations) applied to a *specific situation* (e.g., monochromatic, linearly polarized plane waves reflecting from an semi-infinite dielectric surface). The measurement system also obeys the laws of physics, and modeling helps us understand this part of the experiment as well. Often the measurement system is not the object of study in lab courses; however understanding the measurement tools is essential for many practicing researchers. Modeling the measurement system is especially important in physics, where research often proceeds in directions that demand newly constructed apparatus and measurement tools. However, when manufactured apparatus or measurement tools are available, it is common for this model to be derived from manufacturer's data sheets, and the model is often refined through calibration. Regardless of whether a model comes from first principles or a data sheet, a physicist understand the principles of operation for their apparatus and can quantitatively explain performance specifications and key design parameters. An experimenter who cannot do these things is unlikely to have designed a good experiment and may have difficulty convincing other researchers of the validity of their results and conclusions.

In the end, the model of the physical system must make *predictions* that can be compared with analyzed data (the *comparison* phase in Fig. 4). The modeling approach does not end with the comparison, however. Nearly all complicated measurements require an iterative series of tests of both the measurement apparatus and physical system. The framework offers numerous suggestions for an iterative approach, which could occur by refining models or the actual apparatus (dashed arrows in Fig. 4). The modeling framework offers a way to envision the process of model development and refinement in the context of an experiment. It is designed to emphasize quantitative and physical reasoning in the context of experimentation, to give significant attention to often-overlooked issues of systematic error sources, and to allow for open-ended outcomes through iterative refinement of models.

VI. INTEGRATING SYSTEMATIC ERROR

One of the most elegant aspects of this model-based perspective is how tightly it integrates an analysis of systematic error into the experiment. Statistical uncertainty and error analysis have long been a favored part of physics lab classes because the analysis is quantitative and, to a large extent, can be conducted without a detailed knowledge of the experiment. On the other hand, systematic error depends on all the specific details of the experiment, and without explicit reference to the models of physical and measurement systems, can seem extremely daunting. Taylor, in *An Introduction to Error Analysis* states, “Obviously, tracking down the source of systematic errors is difficult and has defied the best efforts of many great scientists. In all probability, your instructors are not going to penalize you too severely if you fail to do so. Nevertheless, they will expect an intelligent discussion of the problem and at least an honest admission that there appear to have been systematic errors that you were unable to identify.”[31] This common opinion about systematic error used to be the perspective of the authors as well, and only changed when we began incorporating modeling into the lab.

A sophisticated discussion of systematic error can be accomplished in multiple ways within this new framework. First, one of the core aspects of modeling is articulating the idealizations, simplifications, and assumptions in a model. Each of these assumptions can be analyzed for its appropriateness, and also suggests possible sources of systematic error. One key attribute of the modeling framework in Fig. 4 is that models of both the physical system and the measurement tools are developed, tested, and refined. For instance, a model of a photodetector may assume a linear response over a particular range of input signals. When that assumption of linearity is explicitly identified it raises a number of questions: *To what degree is the response linear over this range? Is the desired measured signal within the specified range of linear response? Is there a simple experiment (calibration) that can be done to test the measurement device?* Using this modeling point-of-view, systematic error becomes a limitation in the validity of a model itself. To minimize discrepancies due to systematic error, the error sources can be modeled or made negligible by redesigning the apparatus (dashed arrows in Fig. 4).

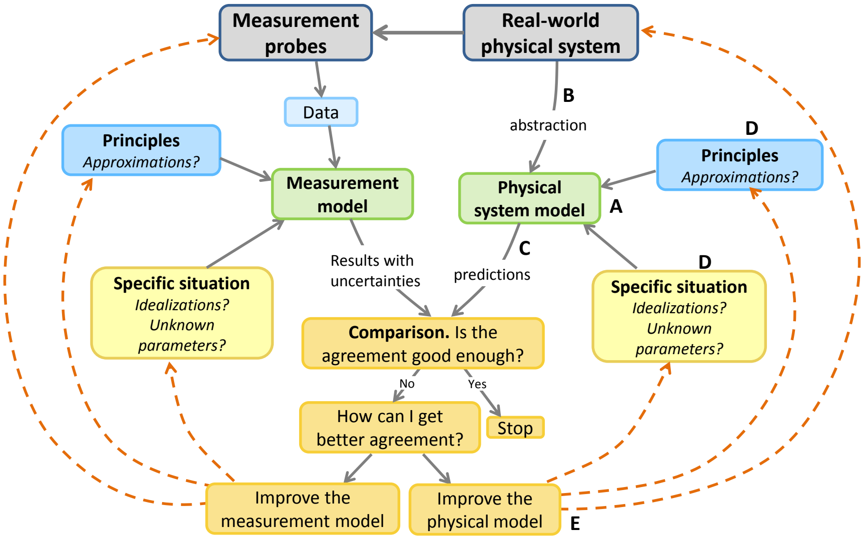


FIG. 4. A framework for the use of models in a physics experiment connecting real world apparatus, theoretical principles, and measurements. Labels A-E refer to five steps in this modeling process: A) Identifying the physical principles, B) Mapping the model onto the physical system, C) Making predictions with the model, D) Identifying limitations of the model, and E) Refining the model.

VII. APPLYING MODELING TO A POLARIZATION OF LIGHT LAB

Although the process described in this paper can be applied to nearly any experiment, we chose a polarization of light lab for a number of reasons. Conceptually, the lab builds on students' models of electromagnetic waves and their interactions with dielectric media, which is covered in standard electricity and magnetism lecture courses. Also, polarization optics are part of the standard optical tool set, and are essential for our students working in many CU research labs. Finally, there are many everyday examples involving polarization optics, including technologies such as liquid crystal displays and some 3D theater projectors.

A. Old Lab

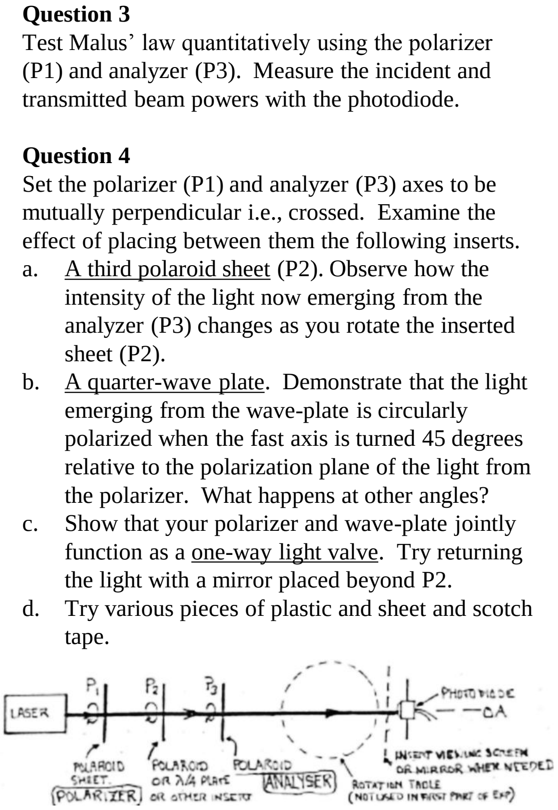
In order to motivate some of the changes to the polarization of light lab, we briefly discuss the prior version. The apparatus for the previous lab is identical to the lab presented here: HeNe lasers, photodetectors, linear polarizers, quarter-wave plates, and rotatable dielectric slabs. The physics concepts are similar too: Malus' Law, circularly polarized light, and the Fresnel equations. The major limitations of the previous lab are two-fold. Some questions, like Question 4 shown in Fig. 5, are qualitative and omit quantitative reasoning. On the other hand, other questions, like Question 3, are explicitly quantitative, but the kind of quantitative reasoning or modeling that is invoked depends on student interpretation and may be quite limited.[32]

Question 4 prompts students to observe some of the most sophisticated polarization phenomena in the experiment. However, question 4 does not meet many of the learning goals we established for the course, particularly involving modeling. The main limitation is that Question 4, as it is written, asks for a primarily qualitative response. A reasonable student response focuses on observing, and does not require any prediction or reflection on the results. In addition, although the phenomena are easy to observe in the lab, students do not have the theoretical tools to actually predict what should happen (especially in 4.b, 4.c, and 4.d). Also absent is a discussion of the limitations of the measurement of circularly polarized light (Question 4.b), or of the one-way light valve (Question 4.c). Students completing the lab likely demonstrate only a qualitative, partially predictive model of the quarter-wave plate and elliptically polarized light.

Question 3, on the other hand, is explicitly quantitative ("Test Malus' Law quantitatively..."). Also, the question is very open-ended—it does not specify as to what constitutes a "test." The independent and dependent variables are not specified, the desired data and presentation format are not specified, the appropriate functional form is not specified, and the level of agreement desired is not specified. For the student with little expectation of what constitutes a high quality experimental investigation in physics, it may be interpreted to produce a graph of data that goes approximately like $\cos^2\theta$. For a professional physicist, or a mature student who has an idea about the nature of a good physics experiment, s/he may take it to mean much more. It could mean: converting the photodetector measurements into powers and plot the transmitted power vs. polarizer angle; or fitting the data to a functional form predicted by a theoretical model of the light and polarizing filter; or explaining the physical meaning of the different fit parameters and why they cannot be determined except by fitting; or refining the model to include absorption in the polarizer or background levels of light in the room. The precise list of expert practices will vary from physicist to physicist, but a well-executed physics experiment should be much more than a graphical or numerical comparison between an evaluated equation and data. The goal of the guidance in model-based guided inquiry is to clarifying the meaning of "test" and to lead all the students through a series of valuable scientific practices that ultimately become standard practices they can use in research.

When the lab guide lacks guidance, instructors, through their interactions with students, may redirect students to focus on practices that are absent from the lab guide. Unfortunately, dealing productively with such open-ended prompts can be difficult for instructors. At CU and at other institutions, multiple experiments are conducted simultaneously by different groups during the lab class. The instructor needs to be aware of particular challenges for a wide range of experiments simultaneously, which is especially difficult for someone teaching the course for the first time. The guided prompts used in the new lab guides at CU are not designed to offload responsibility from instructors, but rather to foster good conversations between the instructor and students around complex ideas in experimental physics. For example, a prompt in the model-based lab guide such as, "use your predictive model to make a fit function for your data and explain the physical meaning of each of your fit parameters," encourages students to think about fitting data in an expert-like way and naturally guides students and instructors toward having a discussion about their their data, the apparatus, and the experiment.

The goal of many of the transformations made to our optics labs was to emphasize the modeling learning goals by transitioning a number of qualitative questions into quantitative ones, and to prompt more expert-like quantitative thinking, like making predictions, understanding limitations in the models, and reflecting on results.



Question 3
Test Malus' law quantitatively using the polarizer (P1) and analyzer (P3). Measure the incident and transmitted beam powers with the photodiode.

Question 4
Set the polarizer (P1) and analyzer (P3) axes to be mutually perpendicular i.e., crossed. Examine the effect of placing between them the following inserts.

a. <u>A third polaroid sheet</u> (P2). Observe how the intensity of the light now emerging from the analyzer (P3) changes as you rotate the inserted sheet (P2).
b. <u>A quarter-wave plate</u>. Demonstrate that the light emerging from the wave-plate is circularly polarized when the fast axis is turned 45 degrees relative to the polarization plane of the light from the polarizer. What happens at other angles?
c. Show that your polarizer and wave-plate jointly function as a <u>one-way light valve</u>. Try returning the light with a mirror placed beyond P2.
d. Try various pieces of plastic and sheet and scotch tape.

FIG. 5. Two of five questions from the polarization of light lab before it included modeling.

VIII. NEW MODEL-BASED INQUIRY LAB

By redesigning the polarization of light lab to focus on our process-oriented learning goals, especially modeling, we can change the educational lab experience with almost no additional equipment. The experiment investigates a number of phenomena (e.g. polarization change after passing through a quarter-wave plate or reflection from a dielectric surface) that can be quantitatively measured and modeled using undergraduate-level physics and mathematics.

The transformed polarization of light lab has students investigate two primary research questions that are addressed during the two-week period: (1) Create circularly polarized light, measure the elliptical polarization parameters, determine how deviations from the simplified model might quantitatively account for deviations between the model predictions and data, and identify which error sources best account for the deviation, and (2) Measure the standard ellipsometry parameters Ψ and Δ for a Lucite slab as a function of the angle of incidence and compare with predictions. Typically, ellipsometry would be used for measuring the thickness and index of refraction of a thin film, but in this case we use a slab of dielectric with unknown index of refraction as a simple test of the method.

Each of these primary research questions involves modeling and measurements, and they become the storyline upon which a number of smaller exercises, which focus on scientific practices, are introduced during the two-week period.

The modeling framework shown in Fig. 4 provides a guide on how to stepwise modify a laboratory to include a scientific modeling approach. The five steps are A) Identifying the physical principles, B) Mapping the model onto the physical system, C) Making predictions with the model, D) Identifying limitations of the model, and E) Refining the model.

A. Identifying the physical principles

The modeling framework in Fig. 4 begins by creating an abstract model of the *real-world physical system.* The model takes two inputs: A set of *physical principles* (e.g., Maxwell's vector wave equation in an anisotropic linear medium) and a *specific situation* where the principles are applied (propagation of monochromatic plane waves of arbitrary polarization through crystalline quartz). Many advanced lab experiments involve phenomena that cannot directly be seen, so identifying the important aspects of the system that need to be modeled and describing the physical principles that govern their behavior is an essential aspect of understanding the experiment. This focus on the big ideas and principles prioritizes a qualitative understanding of the model before it is potentially obscured by lots of equations.

Example: *What basic physics ideas explain why the polarizing filter only absorbs one polarization? What makes a quarter-wave plate different than glass?* It would be totally possible to complete the lab without understanding how the polarizing filter works, but an experimentalist loses credibility when s/he cannot explain how key elements of the apparatus function. A brief response from a student is shown in Fig. 6A.

B. Mapping the model onto the physical system

A key part of the abstraction from the *real-world physical system* to the model (see Fig. 4) is connecting all the parameters and quantities in the model to aspects of the real system. This step is especially important when the model is provided by the instructor because the model still needs to be interpreted by the student. This strategy of mapping between the real world and abstract model is commonly employed in problem solving in introductory physics.[14] This mapping is often done with diagrams that identify all the important information in a problem as an early step in setting up the problem, similar to the use of force diagrams when solving mechanics problems.

Example: *Draw a diagram which explains the following quantities for reflection from dielectric slab: (a) The plane of incidence (b) electric field polarization normal to the plane of incidence (c) electric field polarization parallel to the plane of incidence (d) θ_t (e) θ_i.* The diagram is essential for sense-making of any of the equations, and for taking measurements. A student's response is shown in Fig. 6B.

C. Making predictions with the model

Another essential aspect of understanding a model is having the ability to use it to make accurate quantitative predictions. Foremost, an experiment is a test of the principles and assumptions underlying a physical model, but the test is accomplished through comparing measurements with *specific quantitative predictions* about the behavior of the system. This aspect of modeling moves beyond the basic identification and qualitative description, and requires students to make specific predictions that can be compared with measurements in the lab. In the modeling framework shown in Fig. 4, the arrow labeled *predictions* is the output of the *physical system model*. The predictions and measurements meet in the *comparison*. Setting up this comparison is not a trivial task because there are usually multiple ways to represent data and theory, and the student must choose the representations based on what is most convenient or will make the most sense when trying to communicate the results to colleagues.

Example: *Using the Jones formalism, predict Malus' law (the transmitted power through two crossed polarizers). This exercise will give you confidence in applying the Jones formal-*

ism to more complicated models, like those using the quarter-wave plate. Fig. 6C shows example student work of Jones matrices coded in Mathematica and used to generate plots.

D. Identifying limitations of the model

All models in physics are simplified. Because the model intentionally does not capture every detail, it is important to provide justification for why a model is appropriate to use to describe a measurement of a particular apparatus in the lab. The Modeling framework in Fig. 4 shows that assumptions may arise in the *principles*, which may be approximate, or in the *specific situation*, which may contain idealizations and unknown parameters. An expert uses these simplifications as staring points for identifying possible sources of systematic error in the measurements or predictions. In addition, in its most useful form, a discussion of simplifications takes on a quantitative character. For example, when a simplification assumes a particular quantity is "small" it is important to identify what scale counts as "small enough." For instance, one assumption for the simple pendulum is that the string is "massless," which means, to an experimenter, that the mass of the string m_{string} should small compared to the mass suspended at its end M ($m_{\text{string}}/M << 1$).

Example: *Suppose two beams of light of different polarization* $\begin{pmatrix} E_x \\ E_y \end{pmatrix}$ *and* $\begin{pmatrix} E'_x \\ E'_y \end{pmatrix}$ *are combined using a beam splitter. The Jones matrix formalism suggests that the final polarization state after a 50/50 beamsplitter would be proportional to* $\begin{pmatrix} E_x + E'_x \\ E_y + E'_y \end{pmatrix}$. *Under what experimental conditions would this use of the Jones formalism be valid, meaning it would accurately describe the final polarization state of the combined beams of light after the beamsplitter?* Another example of identifying and testing the limitations of a model are shown in student work shown in Fig. 6D. The student is commenting on the idealizations that might explain why they have difficulty producing circularly polarized light.

E. Refining the model

The final phase of the Modeling framework in Fig. 4 is the the refinement of the model or apparatus in order to reconcile any differences between the predictions and measurements. This step is shown by the dashed arrows that suggest a refinement of the model or the apparatus, for either the measurement or physical systems. Refining the models includes activities like calibrating the measurement system, making the model more sophisticated to include a particular systematic error source, or modifying the apparatus to make a simpler model more valid. Through this modeling emphasis, a deep quantitative analysis of the experiment, including systematic error, is naturally accomplished.

In addition to including systematic error effects in the lab, an additional benefit is that the experiment can take on a more open-ended character. Inquiry is feasible in an introductory mechanics courses, for example, where students develop a model of motion under constant velocity or acceleration from observations.[15] However, the upper-division labs start with systems that involve complicated theory and apparatus. The refinement of models gives one way to incorporate inquiry into the upper-division labs, for example, by starting with a basic model for the system (e.g, a plane wave is incident upon an ideal quarter-wave plate), and then refining the model through experimentation and analysis.

Example: *This question explores the systematic error effects that could limit your ability to produce circularly polarized light. (a) Predict how a small violation of the idealization would change the result. (b) Can you distinguish between the systematic error sources? (c) Could this systematic error account for your non-ideal result? (d) Is the violation of the idealization within tolerances on our ability to measure angles or the specifications on the quarter-wave plate? (e) Which error source, if any, is most likely?* A partial student lab notebook response is given in Fig. 6, which shows the Jones formalism extended to make predictions about the effect of two different systematic error sources.

IX. MODELING THE MEASUREMENT SYSTEM

Modeling plays an important role in understanding both the phenomena of interest and the measurement tools used to measure its behavior. The left side of Fig. 4, which describes the measurement system, is intentionally drawn to be symmetric with the physical system. Physicists tend to build much of the apparatus they use and experiments studying similar phenomena can differ in both in construction and how measurements are taken. As such, it is a necessary part of any scientific communication to explain the principles of operation and calibration of the measurement system just as much as it is to understand the physical system. A key part of this is quantitatively explaining the chain that extends from the phenomena under study, to a measurable signal that is recorded, to the results of further analysis, and finally to the comparison with predictions. Similarly, knowing the limits of the measurement tools and refining (calibrating) models of the measurement tools are essential aspects of experimentation. While not discussed here, all of these aspects of modeling the measurement system are also incorporated into lab guides, and the prompts follow a similar form to those for modeling the physical system that are described above. Other labs go into detail about particular aspects of the measurement system, such as the operation and model of the photodetector.

X. CONCLUSIONS AND FUTURE WORK

We describe and justify a pedagogical approach for designing labs that engage students in scientific practices like modeling, while using widely-used apparatus to investigate interesting physical systems. The scientific practices, the physics concepts, and the apparatus should all offer valuable preparation for future professional work in physics, but it is the scientific practices that will have the most wide reaching impact independent of the particular research domain. The proposed approach advocates for guided inquiry labs that engage students in a sequence of scientific practices that parallel those of an expert researcher. Examples in the context of a polarization of light lab demonstrate feasibility, and how how the ideas can be adapted into any physics experiment. As students gain expertise in these practices, the level of guidance can be reduced to allow students greater independence.

A large emphasis is placed on the scientific practice of quantitative reasoning in the context of the experiment. *Model-based inquiry*, which was first described for K-12 science education,[18] is proposed as an authentic way to engage in inquiry and quantitative reasoning in the context of experimental physics. We present a new framework for modeling in the context of experimentation. The framework has two intersecting components that emphasize both the model of the physical phenomenon and the measurement apparatus. Both models are systematically investigated by identifying the principles, connecting with the physical apparatus, making predictions of behavior, investigating limitations, and refining models.

This approach raises natural research questions about pedagogy that should be addressed, in particular regarding the amount of guidance that is optimal for the development of scientific practices (not just conceptual learning), and how to most effectively reduce the level of guidance throughout the course. Now that the educational environment and curriculum have been developed, we can compare how students engage in scientific practices like modeling when given different sets of curricula: those designed on the principles described here, and those with more either more structure or more open-ended prompts. Further, we could explore how students, after engagement in the different curricula, are able to transfer those skills to other experimental research problems.

XI. ACKNOWLEDGMENTS

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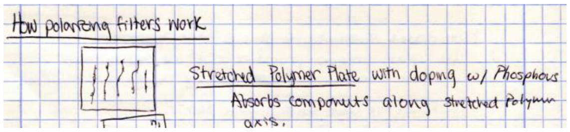
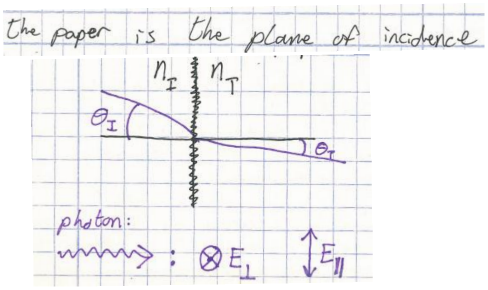
A. IDENTIFYING PHYSICAL PRINCIPLES



How polarizing filters work

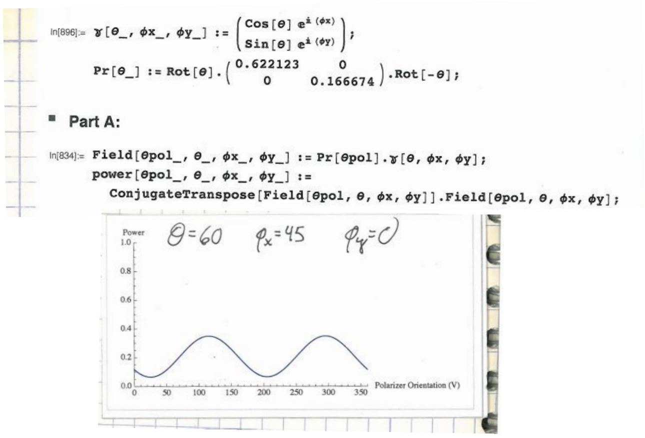
Stretched Polymer Plate with doping w/ Phosphous
Absorbs components along stretched Polymer axis.

B. MAPPING THE MODEL ONTO THE PHYSICAL SYSTEM



the paper is the plane of incidence

C. MAKING PREDICTIONS



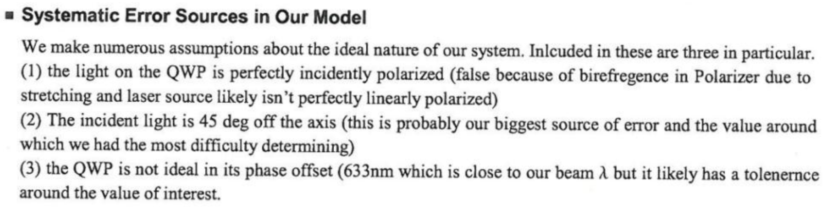
```
In[896]:= ɣ[θ_, ϕx_, ϕy_] := ( Cos[θ] ⅇ^ⅈ (ϕx) ) ;
                             ( Sin[θ] ⅇ^ⅈ (ϕy) )
          Pr[θ_] := Rot[θ].( 0.622123     0      ).Rot[-θ];
                           (    0      0.166674 )
```

- **Part A:**

```
In[834]:= Field[θpol_, θ_, ϕx_, ϕy_] := Pr[θpol].ɣ[θ, ϕx, ϕy];
          power[θpol_, θ_, ϕx_, ϕy_] :=
            ConjugateTranspose[Field[θpol, θ, ϕx, ϕy]].Field[θpol, θ, ϕx, ϕy];
```

$\theta=60$ $\phi_x=45$ $\phi_y=0$

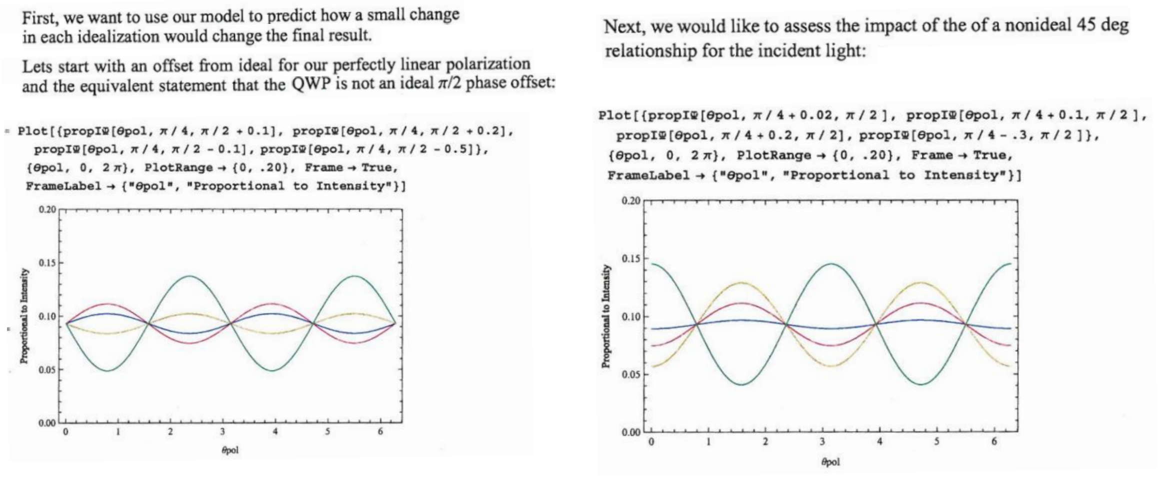
D. LIMITATIONS OF THE MODEL



- **Systematic Error Sources in Our Model**

We make numerous assumptions about the ideal nature of our system. Inlcuded in these are three in particular.
(1) the light on the QWP is perfectly incidently polarized (false because of birefregence in Polarizer due to stretching and laser source likely isn't perfectly linearly polarized)
(2) The incident light is 45 deg off the axis (this is probably our biggest source of error and the value around which we had the most difficulty determining)
(3) the QWP is not ideal in its phase offset (633nm which is close to our beam λ but it likely has a tolenernce around the value of interest.

E. REFINING THE MODEL



First, we want to use our model to predict how a small change in each idealization would change the final result.

Lets start with an offset from ideal for our perfectly linear polarization and the equivalent statement that the QWP is not an ideal π/2 phase offset:

```
= Plot[{propIΨ[θpol, π / 4, π / 2 + 0.1], propIΨ[θpol, π / 4, π / 2 + 0.2],
    propIΨ[θpol, π / 4, π / 2 - 0.1], propIΨ[θpol, π / 4, π / 2 - 0.5]},
   {θpol, 0, 2 π}, PlotRange → {0, .20}, Frame → True,
   FrameLabel → {"θpol", "Proportional to Intensity"}]
```

Next, we would like to assess the impact of the of a nonideal 45 deg relationship for the incident light:

```
Plot[{propIΨ[θpol, π / 4 + 0.02, π / 2 ], propIΨ[θpol, π / 4 + 0.1, π / 2 ],
   propIΨ[θpol, π / 4 + 0.2, π / 2], propIΨ[θpol, π / 4 - .3, π / 2 ]},
  {θpol, 0, 2 π}, PlotRange → {0, .20}, Frame → True,
  FrameLabel → {"θpol", "Proportional to Intensity"}]
```

FIG. 6. Examples of student work in lab notebooks. Work is either handwritten, or recorded in a digital lab notebook and printed and taped into the lab notebook. The labels A-E correspond to the five aspects of modeling described in Secs. VIII A–E

.