

A Guide to Finance, Growth and Decay

Teaching Approach

Finance forms an integral part of the Mathematics syllabus. Financial mathematics has one of the widest applications in everyday life and is important in every aspect, from budgets to home/car loans to investments. It is a dream of most people to own a house, car, retire with enough money and other essential commodities that we do not always have the cash to pay for. Hence most of us will go and make a loan from a bank or from any other financial institutions and invest money so that we can retire in some comfort. Grade 12 finance, growth and decay gives the learner an in depth understanding of the formulae that financial institutions use to calculate interest, loan amounts and investments.

This chapter highlights the importance of saving, investing and loan repayment. If the skills outlined in this guide are mastered by the learner, it will put them in good stead to make sound financial decisions. Therefore it is imperative to ensure that basic principles are well understood. In Grade 12, all financial mathematics concepts are tested, from the mundane simple interest calculations, to timelines to present value and future value annuities or investments.

Teachers must please note that not all the formulae relating to financial mathematics are given on the formula page/s. The following formulae will appear on the formula page:

$$A = P(1+in) \quad A = P(1+i)^n \quad A = P(1-in) \quad A = P(1-i)^n \quad F = \frac{x[(1+i)^n - 1]}{i} \quad P = \frac{x[1 - (1+i)^{-n}]}{i}$$

Hints on solving financial mathematics questions

- If different amounts are invested at irregular intervals draw a timeline.
- Fill in as much detail as possible on the timeline i.e. the amounts invested and when they have been invested (e.g. $T_0 = R1500$), when amounts have been withdrawn, what the interest rate is for a certain time period and how it is compounded.
- Make sure that you know which formulae to use. Incorrect formulae will always result in zero marks.
- Always write down the formula, followed by the substitution of the values.
- Do not round off until you have the final answer of a question.
- Always round off to two decimal places, unless the instructions read otherwise.
- Check that your answer is reasonable. It is impossible to end up with a negative value for n , the investment period or the number of equal payments.
- Care must be taken when punching values into the calculator, especially when it comes to the brackets. If brackets are left out it might lead to a “maths error”.
- Attempts must be made to use real life examples as to ensure that learners will be able to identify with the examples.
- Students need to be reminded about the different compounding of interest and how to change the value of i depending on the compounding.

Video Summaries

Some videos have a 'PAUSE' moment, at which point the teacher or learner can choose to pause the video and try to answer the question posed or calculate the answer to the problem under discussion. Once the video starts again, the answer to the question or the right answer to the calculation is given.

Mindset suggests a number of ways to use the video lessons. These include:

- Watch or show a lesson as an introduction to a lesson
- Watch or show a lesson after a lesson, as a summary or as a way of adding in some interesting real-life applications or practical aspects
- Design a worksheet or set of questions about one video lesson. Then ask learners to watch a video related to the lesson and to complete the worksheet or questions, either in groups or individually
- Worksheets and questions based on video lessons can be used as short assessments or exercises
- Ask learners to watch a particular video lesson for homework (in the school library or on the website, depending on how the material is available) as preparation for the next days lesson; if desired, learners can be given specific questions to answer in preparation for the next day's lesson

1. Introducing Future Value Annuities

This video gives brief description of what future value investment or annuities are and the derivation of the future value formula from the sum of the geometric formula.

2. Working with Future Value Annuities

This video determines the equal regular investment (x). It highlights the fact that if you know the amount that you need (the future value) and the time and interest rate, you will be able to determine the equal regular payments to obtain the future value.

3. Introducing Present Value Annuities

This video starts by covering the loan options available. It explains the pitfalls of taking out loans with organizations that are not legal or trustworthy. This is followed by the derivation of the present value formula from the sum of the geometric series.

4. Working with Present Value Annuities

This video emphasises determining the equal regular payments of the loan. It highlights the fact that if you know the amount that you need and the time and interest rate, you will be able to determine the equal regular payments.

5. Determining the Investment Period

In this video, we discuss how the logarithmic function relates to the exponential function. We extensively use the reflection in the line $y=x$. We explore the asymptote and the x intercept.

6. Balance Outstanding on a Loan

In this lesson students are shown how to determine the Balance Outstanding on a loan after making a certain number of payments.

7. Calculating Sinking Funds

In this lesson we look at what a Sinking Fund is and the calculations involved in Sinking Funds.

Resource Material

Resource materials are a list of links available to teachers and learners to enhance their experience of the subject matter. They are not necessarily CAPS aligned and need to be used with discretion.

1. Introducing Future Value Annuities	http://extralessons.com/Tablet/content_html/gr12L3_1.html	The future value formula is derived and examples of future value are included.
	http://mathsgenius.co.za/qa/?qa=blog&qa_blobid=1466423583358546109	This exercise page gives questions over a variety of topics.
	http://mathsgenius.co.za/qa/?qa=blog&qa_blobid=14367544318654906095	This is the solutions to the questions in the previous link.
	http://www.mindset.co.za/resources/0000068141/0000160821/0000164474/Maths%20Gr%2012%20Session%203%20Topic%201%20LN%20(Fin%20Maths%20A)2011.pdf	Examples of future value and present value annuities.
2. Working with Future Value Annuities	http://www.cqu.edu.au/_data/assets/pdf_file/0008/15947/ANNT.pdf	Gives explanations and examples of future value annuities.
	http://myweb.usf.edu/~grenke/alggebra/interest.pdf	Exercises on annuities
	http://www.freemathhelp.com	Free math Help lessons
3. Introducing Present Value Annuities	http://extralessons.com/Tablet/content_html/gr12L3_3.html	The present value formula is derived and examples of present value are included.
	http://www.mindset.co.za/resources/0000068141/0000160821/0000164474/Maths%20Gr%2012%20Session%203%20Topic%201%20LN%20(Fin%20Maths%20A)2011.pdf	Examples of future value and present value annuities
	https://www.google.co.za/url?sa=t&rct=j&q=&esrc=s&source=web&cd=6&ved=0CEkQFjAF&url=http%3A%2F%2Fwww.thutong.doe.gov.za%2FResourceDownload.aspx%3Fid%3D44279&ei=WwtqUoi6PM-g7Aa4YA&usq=AFQjCNEZMBsMSitS0VDbY_XuN8gLEle_6A	Examples on future value and present value are being covered.
4. Working with Present Value Annuities	https://www.google.co.za/url?sa=t&rct=j&q=&esrc=s&source=web&cd=95&cad=rja&ved=0CD8QFjAE OFo&url=http%3A%2F%2Fmrkha dem.com%2FSanJac%2FFinite%2F	Power point presentation on Present Value annuities and Finance

Task

Question 1

Jason invests R900 each month at 12% p.a. compounded monthly starting on the 1st January 2013, ending on 1st January 2020. How much will he receive immediately after his final investment?

Question 2

Peter needs R190 000 in 10 years' time to study for 4 years at a university. What quarterly amount must his parents invest to pay for his university fees if they are offered 11% p.a. compounded quarterly for 10 years?

Question 3

Lumka secures a bond for a house. Interest is 11,5% p.a. compounded monthly over 20 years. She pays 18% of her monthly salary of R25 000 each month on her bond. Determine her bond amount.

Question 4

I need R75 000 to buy the car of my dreams. I have a choice of two finance agreements, both to be repaid in 48 equal monthly instalments: 1) 14% p.a. simple interest or 2) 21% p.a. compounded monthly. Determine the equal monthly instalments of the two finance agreements and decide which option is the better one to take.

Question 5

Jack won R500 000, paid off his bond and invested R250 000 at 8,5% p.a. compounded monthly. One month after the investment he made equal monthly withdrawals of R8 000 to cover his expenses. How long (to the nearest year) will his money last?

Question 6

Esihle takes out a loan of R120 000 at 17% p.a. compounded quarterly to start a business. He will repay the loan with equal quarterly payments over 5 years, starting 9 months after the loan was granted.

6.1 Determine the equal quarterly payments.

6.2 Determine the balance outstanding on the loan at the end of 3 years

Question 7

AA Photocopiers bought a Photostat machine for R150 000. It depreciates at 12% p.a. compounded half-yearly. The cost of a brand new machine appreciates at 4% p.a. compounded quarterly.

7.1 Determine the scrap value of the machine after 5 years.

7.2 Determine the cost a new machine in 5 years' time.

7.3 A sinking fund at 9% p.a. compounded monthly is opened. The scrap value of the machine will be used as a deposit. What equal monthly amounts must be invested to be able to buy the new machine in 5 years?

Task Answers
Question 1

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$F = ?$$

$$x = 900$$

$$i = \frac{0,12}{12} = \frac{1}{100}$$

$$n = 7 \times 12 + 1 = 85$$

$$\begin{aligned} \therefore F &= \frac{900 \left[\left(1 + \frac{1}{100} \right)^{85} - 1 \right]}{\frac{1}{100}} \\ &= R119\,681,10 \end{aligned}$$

Question 2

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$x = ?$$

$$F = 190\,000$$

$$i = \frac{0,11}{4} = \frac{11}{400}$$

$$n = 10 \times 4 = 40$$

$$\therefore 190\,000 = \frac{x \left[\left(1 + \frac{11}{400} \right)^{40} - 1 \right]}{\frac{11}{400}}$$

$$\therefore 190\,000 = x \times 71,26814499$$

$$\therefore x = \frac{190\,000}{71,26814499} = R2\,665,99$$

Question 3

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$P = ?$$

$$x = 18\% \text{ of } R25\,000 = R4\,500$$

$$i = \frac{0,115}{12} = \frac{23}{2400}$$

$$n = 20 \times 12 = 240$$

$$\begin{aligned} \therefore P &= \frac{4\,500 \left[1 - \left(1 + \frac{23}{2400} \right)^{-240} \right]}{\frac{23}{2400}} \\ &= R421\,968,77 \end{aligned}$$

Question 4

Option 1:

$$A = P(1 + in)$$

$$A = ?$$

$$P = 75\,000$$

$$i = \frac{14}{100} = \frac{7}{50}$$

$$n = 4$$

$$\therefore A = 75\,000 \left(1 + \frac{7}{50} \times 4 \right) \\ = R117\,000$$

$$\therefore \text{Monthly instalments} = \frac{117\,000}{48} = R2\,437,50$$

Option 2:

$$P = \frac{x \left[1 - (1+i)^{-n} \right]}{i}$$

$$x = ?$$

$$P = 75\,000$$

$$i = \frac{0,21}{12} = \frac{7}{400}$$

$$n = 48$$

$$\therefore 75\,000 = \frac{x \left[1 - \left(1 + \frac{7}{400} \right)^{-48} \right]}{\frac{7}{400}}$$

$$\therefore 75\,000 = x \times 32,29380129$$

$$\therefore x = \frac{75\,000}{32,29380129} = R2\,322,43$$

Therefore, option 2 is the better finance deal. It is R115,07 per month cheaper than option 1.

Question 5

$$P = \frac{x \left[1 - (1+i)^{-n} \right]}{i}$$

$$n = ?$$

$$P = R250\,000$$

$$x = R8\,000$$

$$i = \frac{0,085}{12} = \frac{17}{2400}$$

$$\therefore 250\,000 = \frac{8\,000 \left[1 - \left(1 + \frac{17}{2400} \right)^{-n} \right]}{\frac{17}{2400}}$$

$$\therefore \frac{250\,000 \times \frac{17}{2400}}{8\,000} = 1 - \left(1 + \frac{17}{2400} \right)^{-n}$$

$$\therefore 1 - \frac{250\,000 \times \frac{17}{2400}}{8\,000} = \left(1 + \frac{17}{2400} \right)^{-n}$$

$$\therefore \frac{299}{384} = \left(\frac{2417}{2400} \right)^{-n}$$

$$\therefore -n = \log_{\left(\frac{2417}{2400} \right)} \left(\frac{299}{384} \right)$$

$$\therefore -n = -35,44716\dots$$

$$\therefore n = 35,44716\dots \text{ months}$$

$$\therefore n = \frac{35,44716\dots}{12} = 2,95\dots \text{ years} = 3 \text{ years}$$

Question 6

6.1

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$\Rightarrow P(1+i)^2 = \frac{x[1 - (1+i)^{-n}]}{i}$$

payment started after 9 months = 3 quarters \Rightarrow skipped 2 quarters $\therefore P \Rightarrow P(1+i)^2$

Remember, n also changes $\Rightarrow n = 5 \times 4 - 2 = 18$ for the 2 missed payments

$$\therefore x = ?, \quad P = 120\,000, \quad i = \frac{0,17}{4} = \frac{17}{400}, \quad n = 5 \times 4 - 2 = 18$$

$$\therefore 120\,000 \left(1 + \frac{17}{400}\right)^2 = \frac{x \left[1 - \left(1 + \frac{17}{400}\right)^{-18}\right]}{\frac{17}{400}}$$

$$\therefore 130\,416,75 = x \times 12,40589985$$

$$\therefore x = \frac{130\,416,75}{12,40589985} = R10\,512,48 = \text{quarterly payments}$$

6.2

$$B/O = \frac{x[1 - (1+i)^{-n}]}{i} \quad B/O = ?$$

$$x = R10\,512,48$$

$$i = \frac{0,17}{12} = \frac{17}{400}$$

$$n = 2 \times 4 = 8$$

$$\begin{aligned} \therefore B/O &= \frac{10\,512,48 \left[1 - \left(1 + \frac{17}{400}\right)^{-8}\right]}{\frac{17}{400}} \\ &= R70\,052,88 \end{aligned}$$

OR

$$B/O = P(1+i)^n - \frac{x[(1+i)^n - 1]}{i} \dots\dots\dots P \Rightarrow P(1+i)^2 \dots\dots\dots P = R120\,000$$

$$x = R10\,512,48$$

$$i = \frac{0,17}{12} = \frac{17}{400}$$

$$n = 3 \times 4 - 2 = 10 \dots (\text{number of payments made, skipped 2 payments})$$

$$\therefore B/O = P(1+i)^2 (1+i)^n - \frac{x[(1+i)^n - 1]}{i}$$

$$\therefore B/O = 120\,000 \left(1 + \frac{17}{400}\right)^2 \left(1 + \frac{17}{400}\right)^{10} - \frac{10\,512,48 \left[\left(1 + \frac{17}{400}\right)^{10} - 1\right]}{\frac{17}{400}}$$

$$= R70\,052,84$$

Question 7

7.1

$$A = P(1-i)^n \quad A = ?$$

$$P = R150\,000$$

$$i = \frac{0,12}{2} = \frac{3}{50}$$

$$n = 5 \times 2 = 10$$

$$\therefore A = 150\,000 \left(1 - \frac{3}{50}\right)^{10}$$

$$= R80\,792,27$$

7.2

$$A = P(1+i)^n \quad A = ?$$

$$P = R150\,000$$

$$i = \frac{0,04}{4} = \frac{1}{100}$$

$$n = 5 \times 4 = 20$$

$$\therefore A = 150\,000 \left(1 + \frac{1}{100}\right)^{20}$$

$$= R183\,028,51$$

7.3

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$x = ?$

$$F = 183\,028,51 - 80\,792,27 = R102\,236,24$$

$$i = \frac{0,09}{12} = \frac{3}{400}$$

$$n = 5 \times 12 = 60$$

$$\therefore 102\,236,24 = \frac{x \left[\left(1 + \frac{3}{400} \right)^{60} - 1 \right]}{\frac{3}{400}}$$

$$\therefore 102\,236,24 = x \times 75,42413693$$

$$\therefore x = \frac{102\,236,24}{75,42413693} = R1\,355,48$$

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