A Guide to Advanced Number Patterns

Teaching Approach

The section number patterns is an incredibly important part of the current CAPS curriculum. Number patterns should not be taught in isolation but rather linked to the algebraic concepts already taught. Learners should be taught how quadratic equations, factorizing and simultaneous equations form part of this section. During teaching it is a good idea to emphasize these links.

The series begins by reviewing the basic knowledge that learners should know from previous grades. There are 6 lessons in this series and one task video at the end. The first lesson covers in depth all the knowledge that learners should have learnt in grade eleven regarding linear or arithmetic number patterns as well as quadratic number patterns.

The series then continues to cover the grade 12 content. We begin with lesson two which introduces learners to sigma notation and teaches learners how to use and work with this very important notation. The series then continues by investigating the geometric sequence, also known as the exponential sequence. Lessons 4, 5 and 6 focus on series where sigma notation is covered in depth and the concepts of infinite series is introduced.

Many of the new resources that are CAPS compliant have still included geometric sequences as part of the grade eleven work. It is important to show learners the differences so that they are not confused when using older resources such as study guides or past papers.

To get the full benefit of the lessons, your learners need to engage actively with the concepts presented. So, when you preview the videos, think about how to introduce each lesson and what follow up activities will be useful. We have used this pause icon to suggest some of these places to you.

Video Summaries

Some videos have a 'PAUSE' moment, at which point the teacher or learner can choose to pause the video and try to answer the question posed or calculate the answer to the problem under discussion. Once the video starts again, the answer to the question or the right answer to the calculation is given.

Mindset suggests a number of ways to use the video lessons. These include:

- Watch or show a lesson as an introduction to a lesson
- Watch of show a lesson after a lesson, as a summary or as a way of adding in some interesting real-life applications or practical aspects
- Design a worksheet or set of questions about one video lesson. Then ask learners to watch a video related to the lesson and to complete the worksheet or questions, either in groups or individually
- Worksheets and questions based on video lessons can be used as short assessments or exercises
- Ask learners to watch a particular video lesson for homework (in the school library or on the website, depending on how the material is available) as preparation for the next days lesson; if desired, learners can be given specific questions to answer in preparation for the next day's lesson

1. Revising Number Patterns

In this lesson we revise number patterns which display a constant difference and a constant second difference between consecutive terms. Learners are taught how to identify arithmetic or linear sequences as well as quadratic sequences.

2. Using Sigma Notation

In this lesson we introduce the notation that denotes summation. Learners are taught how to use sigma notation and how to develop sigma notation.

3. Arithmetic Series

In this lesson learners are taught how to identify arithmetic series and how to work with questions involving arithmetic series. Sigma notation is introduced and learners are shown how to use this notation and more importantly how to interpret it.

4. Investigating Geometric Number Patterns

In this lesson the focus is on teaching learners what a geometric sequence is and how to identify geometric or exponential number patterns. Learners are shown how to solve problems involving number patterns that lead to geometric sequences.

5. Geometric Series

In this lesson learners are taught how to identify geometric series and how to work with questions involving this series. Sigma notation is once again used and learners are shown how to use this notation and more importantly how to interpret it.

6. Infinite Geometric Series

In this lesson learners are guided through an investigation to help them understand the concept of diverging and converging series. In addition the lesson works questions with infinite series that converge and diverge.

Resource Material

Resource materials are a list of links available to teachers and learners to enhance their experience of the subject matter. They are not necessarily CAPS aligned and need to be used with discretion.

1. Revising Number Patterns	http://www.mathsisfun.com/numberpatterns.html	The web reference below is good resources to use with weaker learners who have not yet understood the concepts of number patterns in earlier grades.
	http://www.education.gov.za/LinkClick.aspx?fileticket=wZRXkSStB0A%3D&tabid=621&mid=1735	This resource contains examples and detailed solutions of linear patterns and could be used by learners who want more practice or used by educators to develop quick spot tests.
	http://m.everythingmaths.co.za/grade-11/03-number-patterns/03-number-patterns-02.cnxmlplus	The resource below contains good theory notes on quadratic number patterns that could be used both by educators and learners.
	http://www.teachingideas.co.uk/maths/contents_nopatterns.htm	This website below could expose learners to other patterns that exist in the real world but are not part of the curriculum. One could possibly develop a project where learners use this resource to present to their peers the different patterns that occur around us every day.
2. Using Sigma Notation	http://www.youtube.com/watch?v=meMJBqamL1E	This is a video clip note on how to use sigma notation and how it links to series.
3. Arithmetic Series	http://mathworld.wolfram.com/ArithmeticSeries.html	These web references are good resources to use as presentations using a data projector with weaker learners who have not yet understood the concepts of number patterns in earlier grades.
	http://www.purplemath.com/modules/series4.htm	
	http://www.mathcentre.ac.uk/resources/uploaded/mc-ty-sigma-2009-1.pdf	
	http://www.mathcentre.ac.uk/resources/uploaded/mc-ty-apgp-2009-1.pdf	This resource is best for educators, it can be used to assist educators when making notes for their learners of for the educator who wants to improve their knowledge and understanding of this exciting topic.
4. Investigating Geometric Number Patterns	http://www.mathsisfun.com/numberpatterns.html	These are notes which you can work through with weaker learners who have not yet understood the concepts of number patterns in earlier grades.
	http://www.purplemath.com/modules/series3.htm	
5. Geometric Series	https://people.richland.edu/james/lecture/m116/sequences/geometric.html	Theory notes that cover content and practice examples that can be used as a recap lesson.
	http://www.math.ufl.edu/~vatter/teaching/m8w10/m8l05.pdf	This resource is best for educators, it can be used to assist

6. Infinite Geometric Series	http://formulas.tutorvista.com/math/infinite-geometric-series-formula.html http://www.ltcconline.net/greenl/courses/103b/seqSeries/INFGEO.HTM http://hotmath.com/hotmath_help/topics/infinite-geometric-series.html	educators when making notes for their learners of for the educator who wants to improve their knowledge and understanding of this exciting topic. A revised lesson containing theory and examples on infinite series. Best used as a consolidation lesson.

Task

Question 1

1.1 Determine the 15^{th} term of the sequence: 5; 8; 11; …

1.2 The sequence $1; \frac{3}{2}; \frac{9}{4}; ...$ is given. Calculate which term in the sequence is equal to $\frac{243}{32}$

Question 2

2.1 Determine which term of the sequence
25; 14; 3; … is equal to -52

2.2 Calculate the value of $T_5 + T_8$ in the sequence
9; 3; 1; …

Question 3

The 1^{st} term of an arithmetic sequence is 10 and the 6^{th} term is 85. Calculate the first three terms of the sequence.

Question 4

Evaluate

$$\sum_{k=1}^{13} 3 + \frac{3}{2}(k-1)$$

Question 5

Consider the pattern: 5; -2; -7;-10; …

5.1 Write down the next two terms

5.2 Determine an expression for the n^{th} terms.

5.3 Show that the sequence will never have a term with a value less than -11

Question 6

A tunnel is built by digging 1km each day. Each evening the workers return to their base camp which was built at the starting point of the tunnel. After how many days will the workers have travelled a total of 1260km in the tunnel?

Question 7

The first term of an arithmetic sequence is 5 and the 6^{th} term is ten times the 3^{rd} term.

7.1 Calculate the common difference

7.2 Calculate the sum of the first 20 terms in this sequence.

Question 8

The first three terms of a convergent geometric series are: $\mathrm{k}+1;\ \mathrm{k}-1;\ 2\mathrm{k}-5$

8.1 Calculate the value of k

8.2 Calculate the sum to infinity of the series

Task Answers

Question 1

1.1 $T_{15} = 5 + (15 - 1)(3)$

$T_{15} = 47$

1.2 $T_n = 1\left(\frac{3}{2}\right)^{n-1}$

$\frac{243}{32} = \left(\frac{3}{2}\right)^{n-1}$

$\left(\frac{3}{2}\right)^5 = \left(\frac{3}{2}\right)^{n-1}$

$\therefore n - 1 = 5$

$n = 6$

Question 2

2.1 $T_n = 25 + (n - 1)(-11)$

$-52 = 25 - 11n + 11$

$-52 = 36 - 11n$

$-88 = -11n$

$n = 8$

Term 8 is equal to – 52

2.2 $T_5 = 9\left(\frac{1}{3}\right)^{5-1}$

$T_5 = \frac{1}{9}$

$T_8 = 9\left(\frac{1}{3}\right)^{8-1}$

$T_8 = \frac{1}{243}$

$\therefore T_5 + T_8 = \frac{28}{243}$

Question 3

$T_1 = 10$

$T_6 = 85$

$\therefore T_6 = 10 + (6 - 1)(d)$

$85 = 10 + 5d$

$d = 15$

$\therefore 10\,; 25; 40; \ldots$

Question 4

$3 + \frac{9}{2} + 6 + \cdots + 21$

$d = \frac{3}{2} \quad \therefore arithmetic$

$S_{13} = \frac{13(3 + 21)}{2}$

$S_{13} = 156$

Question 5

5.1 -11; -10

5.2 $a = 1$

$3(1) + b = -7$

$b = -10$

$1 + (-10) + c = 5$

$c = 14$

$\therefore T_n = n^2 - 10n + 14$

5.3 $n^2 - 10n + 14 < -11$

$n^2 - 10n + 25 < 0$

$(n-5)^2 < 0$

This is not true for any values of n thus the sequence will not have a term less than -11

Question 6

$2 + 4 + 6 + \cdots = 1260$

$$S_n = \frac{n(2a + (n-1)d)}{2}$$

$$1260 = \frac{n(2(2) + (n-1)2)}{2}$$

$2520 = n(4 + 2n - 2)$

$2520 = 2n + 2n^2$

$n^2 + n - 1260 = 0$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1260)}}{2}$$

$x = 35 \text{ or } x \neq -36$

$\therefore \textit{It takes the workers } 35 \textit{ days to travel a total of } 1260km$

Question 7

7.1 $T_1 = a = 5$

$T_3 = 5 + 2d$

$T_6 = 5 + 5d$

$5 + 5d = 10(5 + 2d)$

$\therefore d = -3$

7.2 $S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{20} = \frac{20}{2}(10 + (20-1)(-3))$

$S_{20} = -470$

Question 8

8.1 $\frac{k-1}{k+1} = \frac{2k-5}{k-1}$r

$(k-1)^2 = (2k-5)(k+1)$

$k^2 - k - 6 = 0$

$(k-3)(k+2) = 0$

$\therefore k = 3 \quad or \quad k = -2$

$\frac{3-1}{3+1} = \frac{1}{2}$

Therefore K = 3 as the ratio is half and will then converge

8.2 $S_\infty = \frac{a}{1-r}$

$S_\infty = \frac{4}{1-\frac{1}{2}}$

$S_\infty = 8$

Acknowledgements

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