A Guide to Introducing Analytical Geometry

Teaching Approach

Analytical geometry is really an easy chapter to teach. The learners also find it rather easy. Before starting, it is suggested to start by revising some concepts on the Cartesian Plane, making sure that the learners remember how to plot coordinates on the Cartesian Plane and also that a set of coordinates consists of a x and y value.

The video lessons have a natural progression from the easy to more complicated concepts. They can be watched in the intended order, but will also work if they are watched out of order. The task video contains 12 questions based on a combination of the skills found in the video lessons.

The learners should be encouraged to always write down the formulas before starting a question as this helps them to remember the formulas and to use them correctly.
It is very important that the learners are able to use and apply the formulas to calculate the distance, gradient and midpoint of geometric figures on the Cartesian plane. It is not important to know how the formulas are derived. However, the learners must know these formulas off by heart.

The importance of acquiring skills in order to apply the concepts and formula in analytical geometry with confidence, while also developing a sound understanding of distance, gradient and midpoint should be emphasized.

Video Summaries

Some videos have a 'PAUSE' moment, at which point the teacher or learner can choose to pause the video and try to answer the question posed or calculate the answer to the problem under discussion. Once the video starts again, the answer to the question or the right answer to the calculation is given.

Mindset suggests a number of ways to use the video lessons. These include:

- Watch or show a lesson as an introduction to a lesson
- Watch of show a lesson after a lesson, as a summary or as a way of adding in some interesting real-life applications or practical aspects
- Design a worksheet or set of questions about one video lesson. Then ask learners to watch a video related to the lesson and to complete the worksheet or questions, either in groups or individually
- Worksheets and questions based on video lessons can be used as short assessments or exercises
- Ask learners to watch a particular video lesson for homework (in the school library or on the website, depending on how the material is available) as preparation for the next days lesson; if desired, learners can be given specific questions to answer in preparation for the next day's lesson

1. The Cartesian Plane

This starts with the revision of some Grade 9 concepts on the Cartesian plane. These include plotting points on this Cartesian plane and how to represent geometric figures.

2. The Distance between Two Points

This video teaches learners how to calculate the distance - or length of a line segment - between two points using the distance formula.

3. The Gradient of Lines

This video shows how to calculate the gradient or slope of a line segment on the Cartesian plane.

4. Gradients of Parallel and Perpendicular Lines

This video investigates the gradients of parallel and perpendicular lines. It also compares the gradients of parallel lines with those of perpendicular lines.

5. The Midpoint Formula

This video shows us how to calculate the midpoint of a line segment using the midpoint formula.

6. Using Analytical Geometry

This video shows the usage of the distance, midpoint and gradient formulae in integrated problems.

7. Properties of Quadrilaterals

This video looks at the properties of some quadrilaterals and uses these properties to in problems involving the distance, midpoint and gradient formulae.

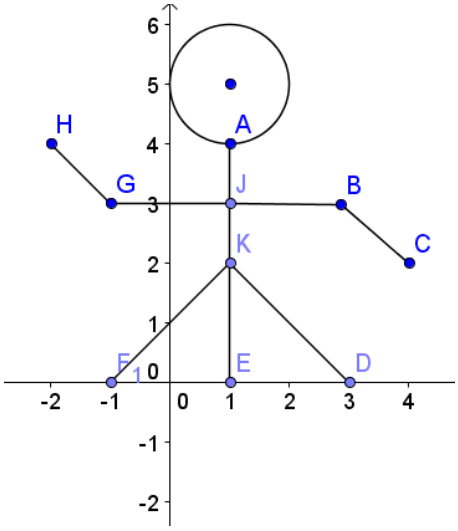
Resource Material

1. The Cartesian Plane	http://www.mathsteacher.com.au/year8/ch15_graphs/01_cartesian/plane.htm	Explaining the Cartesian plane and plotting of coordinates on the Cartesian plane
	http://armorgames.com/play/13405/snakes-on-a-cartesian-plane	Playing a game on the Cartesian Plane using "snakes".
2. The Distance between Two Points	http://www.purplemath.com/modules/distform.htm	Revision and explanation of the distance formula
	http://www.freemathhelp.com/distance-formula.html	Revision and explanation of the distance formula
3. The Gradient of Lines	http://www.videojug.com/film/how-to-be-the-change	How to find the equation and gradient of a line
	http://m.everythingmaths.co.za/grade-10/08-analytical-geometry	Summary of the gradient formula and examples of calculating the gradient
	http://www.youtube.com/watch?v=MGidkGDFsVA	Gradients and straight lines
4. Gradients of Parallel and Perpendicular Lines	http://www.mathcentre.ac.uk/resources/uploaded/mc-ty-gradstlnseg-2009-1.pdf	A summary on the gradients of parallel and perpendicular lines. Including some examples on how to calculate the gradient using the gradient formula.
5. The Midpoint Formula	http://www.purplemath.com/modules/midpoint.htm	Revision and explanation of the midpoint formula
6. Using Analytical Geometry	http://www.youtube.com/watch?v=wnWoc8gUXvE	A video on Analytical Geometry
7. Properties of Quadrilaterals	http://www.mathsteacher.com.au/year7/ch09_polygons/04_quad/quad.htm	A summary of the properties of quadrilaterals
	https://www.teachingchannel.org/videos/geometry-lesson-quadrilaterals	The Teaching Channel. Discovering the properties of quadrilaterals
	http://www.youtube.com/watch?v=CBAbQDX7K5Y&noredirect=1	A You Tube video. Verifying the properties of quadrilaterals by using Analytical geometry.

Task

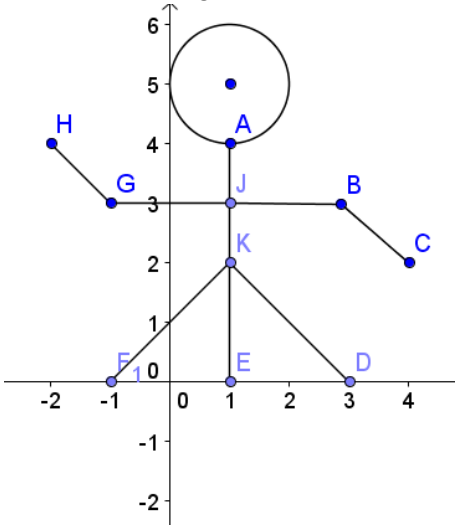
Question 1

Below is a stick figure, Fred. Determine the lengths of Fred's legs KF and KD.



Question 2

Determine the gradient of Fred's arm BC.

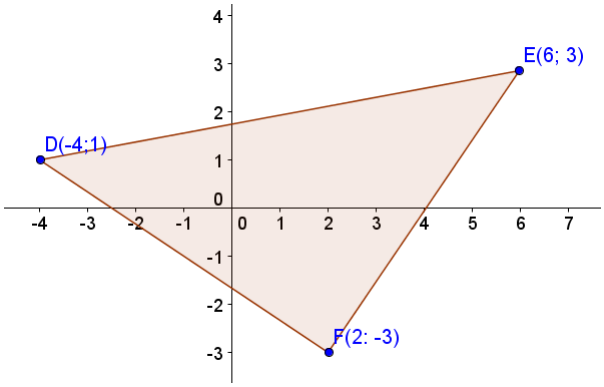


Question 3

Given are the points D(1; m) and E(n; 1). If F(3; 2) is the midpoint of line DE, determine the values of m and n.

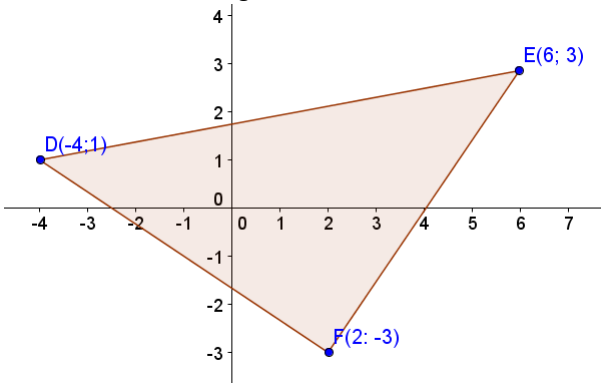
Question 4

Calculate the coordinates of G the midpoint of DF.



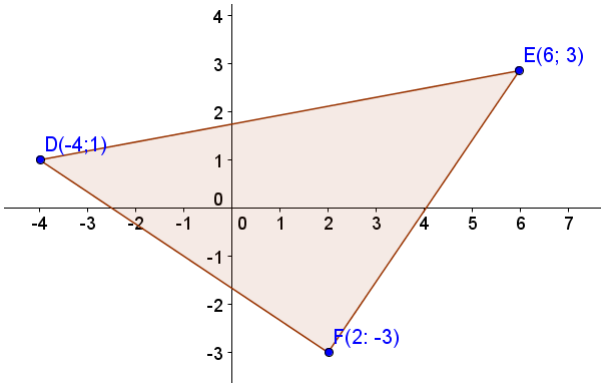
Question 5

Determine the length of DE.



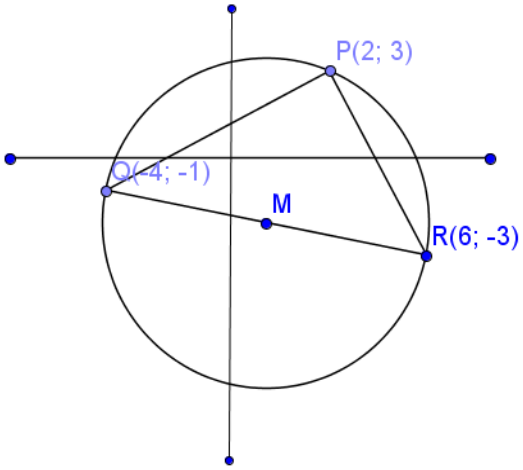
Question 6

Show that triangle DEF is a right-angled triangle. Show all working.



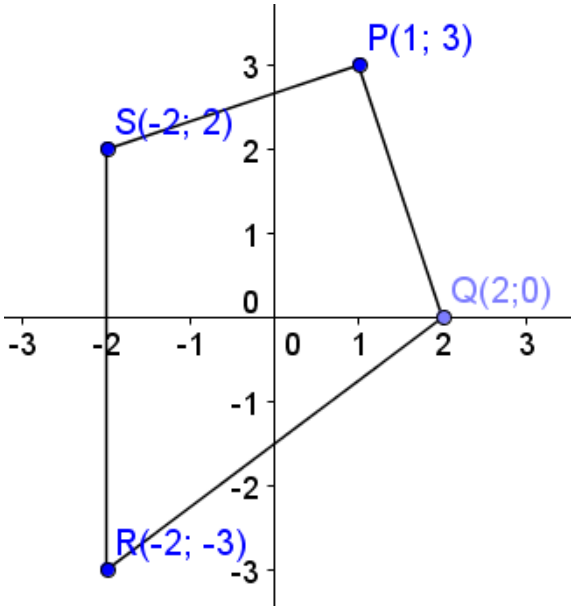
Question 7

ΔPQR has vertices on the circumference of a circle with centre M. The coordinates of the vertices are:
P(2;3)
Q(-4;-1)
R(6;-3)
Determine the length of the radius of the circle.



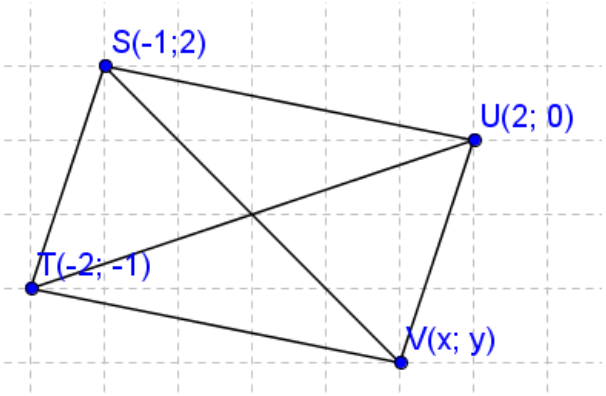
Question 8

Use the diagram of kite PQRS to show that the adjacent sides PS and PQ are equal and that the diagonals are perpendicular.



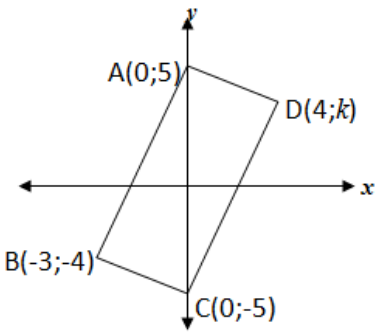
Question 9

SUVT is a parallelogram with vertices S(-1;2), T(-2;-1), U(2;0) and V(x;y).
Find the coordinates of V by using the fact that the diagonals of a parallelogram bisect each other.



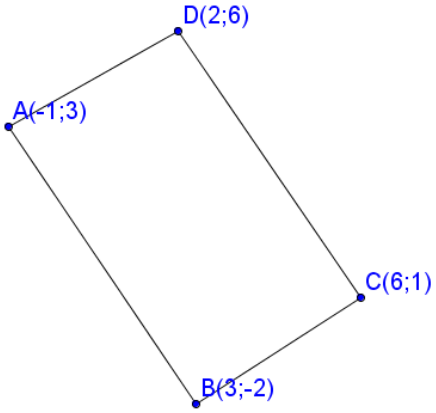
Question 10

This sketch shows a quadrilateral ABCD with vertices A(0;5), B(-3;-4), C(0;-5) and D(4;k), where $k \geq 0$. What should k be so that AD $\perp$ DC ?



Question 11

Prove that ABCD is a parallelogram.



Question 12

Determine the midpoint of line ST with coordinates S$\left(3p^2; 2n\right)$ and T$\left(-7p^2; -4\right)$.

Task Answers

Question 1

$$KF = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-1-1)^2 + (0-2)^2}$$
$$= \sqrt{(-2)^2 + (-2)^2}$$
$$= \sqrt{8}$$

$$KD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(3-1)^2 + (0-2)^2}$$
$$= \sqrt{(-2)^2 + (0)^2}$$
$$= \sqrt{4}$$
$$= 2$$

Question 2

$$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{2-3}{4-3}$$
$$= -1$$

Question 3

Midpoint: $\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$\left(\frac{x_1 + x_2}{2}\right)$

$\frac{1+n}{2} = 3$

$1 + n = 6$

$n = 5$

and

$\left(\frac{y_1 + y_2}{2}\right)$

$\frac{m+1}{2} = 2$

$m + 1 = 4$

$m = 3$

Question 4

D (- 4; 1) F (2; -3)

Midpoint = $\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$\left(\frac{-4+2}{2}; \frac{1-3}{2}\right)$

$\therefore$ Midpoint = (-1; -1)

Question 5

$$DE = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(6+4)^2 + (3-1)^2}$$
$$= \sqrt{(10)^2 + (2)^2}$$
$$= \sqrt{104}$$

Question 6

$$DE_m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{6+4} = \frac{2}{10}$$

$$EF_m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3+3}{6-2} = \frac{3}{2}$$

$$DF_m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-1}{2+4} = \frac{-2}{3}$$

$\therefore \quad EF_m \cdot DF_m = -1$

$\therefore \quad EF \perp DF \quad \therefore \Delta DEF$ is a right angled triangle.

OR

$$DE = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6+4)^2 + (3-1)^2} = \sqrt{(10)^2 + (2)^2} = \sqrt{104}$$

$$EF = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(2-6)^2 + (-3-3)^2} = \sqrt{(-4)^2 + (-6)^2} = \sqrt{52}$$

$$DF = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(2+4)^2 + (-3-1)^2} = \sqrt{(6)^2 + (-4)^2} = \sqrt{52}$$

$\therefore \left(\sqrt{104}\right)^2 = \left(\sqrt{52}\right)^2 + \left(\sqrt{52}\right)^2$ Theorem of Pythagoras

$\therefore \Delta DEF$ is a right angled triangle

Question 7

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right) = \left(\frac{-4+6}{2}; \frac{-1-3}{2}\right) = \left(\frac{2}{2}; \frac{-4}{2}\right) = (1; -2)$$

M (1; -2) and R (6; -3)

$$MR \text{ (radius)} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6-1)^2 + (-3+2)^2} = \sqrt{(5)^2 + (-1)^2} = \sqrt{26}$$

Question 8

$$PS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-2-1)^2 + (2-3)^2}$$
$$= \sqrt{(-3)^2 + (-1)^2}$$
$$= \sqrt{10}$$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(2-1)^2 + (0-3)^2}$$
$$= \sqrt{(1)^2 + (-3)^2}$$
$$= \sqrt{10}$$

$\therefore PS = PQ$

$$SQ_m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{0-2}{2+2}$$
$$= \frac{-1}{2}$$

$$PR_m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-3-3}{-2-1}$$
$$= 2$$

$\therefore \quad SQ_m \cdot PR_m = -1$

$\therefore \quad SQ \perp PR$

Question 9

Midpoint VS $\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$\left(\frac{x-1}{2}; \frac{y-2}{2}\right)$

Midpoint UT $\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$\left(\frac{2-2}{2}; \frac{0-1}{2}\right)$

$\left(0; \frac{-1}{2}\right)$

Midpoint VS = Midpoint UT, because diagonals of a parallelogram bisect each other

$\therefore \frac{x-1}{2} = 0$

$x = 1$

$\therefore \frac{y+2}{2} = \frac{-1}{2}$

$y = -3$

Question 10

$$AD_m = \frac{y_2 - y_1}{x_2 - x_1} \qquad DC_m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{k-5}{4-0} \qquad = \frac{k+5}{4-0}$$

$$= \frac{k-5}{4} \qquad = \frac{k+5}{4}$$

$$\therefore \left(\frac{k-5}{4}\right)\left(\frac{k+5}{4}\right) = -1$$

$$\left(\frac{k^2-25}{16}\right) = -1$$

$$k^2 - 25 = -16$$

$$k^2 - 9 = 0$$

$$(k-3)(k+3) = 0$$

$k = 3$ or $k \neq -3$

$\therefore k = 3$

Question 11

$$DC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(6-2)^2 + (1-6)^2}$$

$$= \sqrt{(4)^2 + (-5)^2}$$

$$= \sqrt{41}$$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(3+1)^2 + (-2-3)^2}$$

$$= \sqrt{(4)^2 + (-5)^2}$$

$$= \sqrt{41}$$

$\therefore$ DC = AB

$$m_{DC} = \frac{y_2 - y_1}{x_2 - x_1} \qquad m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1-6}{6-2} \qquad = \frac{-2-3}{3+1}$$

$$= \frac{-5}{4} \qquad = \frac{-5}{4}$$

$\therefore m_{DC} = m_{AB}$

Therefore ABCD is a parallelogram, one pair of opposite sides equal and parallel.

Question 12

$$\begin{aligned} Midpoint &= \left(\frac{3p^2+(-7p^2)}{2}; \frac{2n+(-4)}{2}\right) \\ &= \left(\frac{-4p^2}{2}; \frac{2n-4}{2}\right) \\ &= (-2p^2; n-2) \end{aligned}$$

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