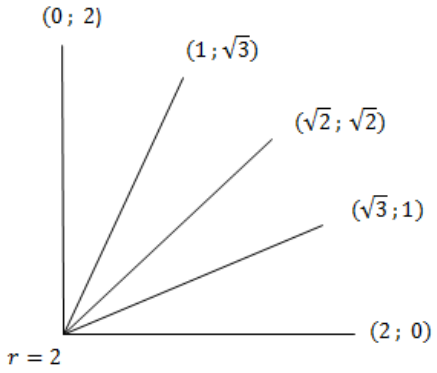
A Guide to Trigonometric Equations

Teaching Approach

There are two basic trig identities that are used at Grade 11 level. These do not appear on the formula sheet and need to be committed to memory. The variations should be able to be recognised so that the learners can 'see' where to use which identities. This skill is not easy and in most cases questions would be classified in the level 3 (complex procedure) range. The context of questions of the 'simplify the following' or 'prove the following identity' is fairly abstract. In order to aid recognition the variations should be tested vigorously before examples are done. Poor algebraic skills will hamper progress in this section.



We also deal with special angles in Grade 11. Deriving the numerical value of the special angles is what the crux of this section. Learners could either use the two triangles as shown in the video, the fan method (shown alongside) or the calculator. The unit circle or calculator could be used to generate the numerical values for the angles found on the axis. Learners need to be cautioned to write decimal as the instruction is usually prefaced with the wording 'without the use of a calculator'. This skill is classified as a level 1 (knowledge) and is seldom examined by its self.

The we move on to reducing ratios and co-functions. This section is split into two parts. The first into reductions of the horizontal axis and the second shows the co-function reductions off the vertical axis. Trig reductions require the knowledge of special angles for numerical examples. The recommended order of reduction is: reduce to a positive angle, reduce to an angle between zero degrees and three hundred and sixty degrees and then lastly reduce, using the ASTC diagram, to an angle between zero degrees and ninety degrees. The learner will have to take care with the positiveness and negativeness of expressions. Accuracy is vital in this section. Learners need to be introduced to the idea of non-special angles being combined with identities in a level 3 type question.

Lastly, we move on to solving equations. General solutions are produced with three simple rules: one for each trig ratio. From these specific solutions within a given range can be provided as answers. This technique can be easily mastered by learners and differs from many textbooks which have not kept up with technological advances and new thinking. The difficulty of question can be increased by making the manipulation before finding the solution more complex.

Video Summaries

Some videos have a 'PAUSE' moment, at which point the teacher or learner can choose to pause the video and try to answer the question posed or calculate the answer to the problem under discussion. Once the video starts again, the answer to the question or the right answer to the calculation is given.

Mindset suggests a number of ways to use the video lessons. These include:

- Watch or show a lesson as an introduction to a lesson
- Watch of show a lesson after a lesson, as a summary or as a way of adding in some interesting real-life applications or practical aspects
- Design a worksheet or set of questions about one video lesson. Then ask learners to watch a video related to the lesson and to complete the worksheet or questions, either in groups or individually
- Worksheets and questions based on video lessons can be used as short assessments or exercises
- Ask learners to watch a particular video lesson for homework (in the school library or on the website, depending on how the material is available) as preparation for the next days lesson; if desired, learners can be given specific questions to answer in preparation for the next day's lesson

1. Introducing Trigonometric Identities

In this video the two basic trig identities are introduced and examples of examination questions are worked through. These are of the form 'simplify the following' and 'prove the following identity'.

2. Working with Special Angles

In this video the idea of special angles is introduced. Examples are worked through. This is as a precursor to the Trig Reduction video which would use this skill in Grade 11 assessment situations.

3. Reducing Trigonometric Ratios

In this video the concept of angles outside the zero degrees to ninety degree range are discussed. The methodology of reducing angles to this range is shown and examples are worked through. The examples are both literal and numeric.

4. Co-function Reductions

This video looks at co-function reductions. This is the last type of reduction and follows from our reductions off the horizontal axis. This reduction is a reduction off the vertical ninety degree axis.

5. Solving Trigonometric Equations

In this video the difference between specific solutions and a general solution is discussed with examples. The importance of generating the general solution first is shown in the context of an exam style question.

Resource Material

1. Introducing Trigonometric Identities	http://www.youtube.com/watch?v=Zktxkfr9zJE	Hints and examples using basic Trigonometric Identities
	http://www.slideshare.net/krillion/proving-trigonometric-identities	A slide show showing worked examples
2. Working with Special Angles	http://sk19math.blogspot.com/2007/05/special-angles-in-trigonometry.html	A simple explanation of what special angles are.
	http://www.youtube.com/watch?v=Ng0NJqQuKC8	An explanation of where the values of special angles come from
	http://www.onlinemathlearning.com/trigonometry-special-angles.html	An explanation of, and some worked examples using special angles.
3. Reducing Trigonometric Ratios	http://www.youtube.com/watch?v=H0QveRy8OgY	A detailed video on sketching graphs using the unit circle and a point-to-point plot
	http://www.purplemath.com/modules/grphtrig.htm	Notes and examples on basic trigonometric graphs
	http://illuminations.nctm.org/ActivityDetail.aspx?ID=174	An interactive sketching facility to see the influence the changing of variables has.
4. Co-function Reductions	http://cnx.org/content/m38870/1.1/	Notes and examples on trigonometric reductions
	http://www.bymath.com/studyguide/tri/sec/tri7.htm	Summary table of the trigonometric reductions
	http://www.bymath.com/studyguide/tri/sec/tri7.htm	Summary table of the trigonometric reductions
5. Solving Trigonometric Equations	http://cnx.org/content/m38870/1.1/	Notes and examples on trigonometric reductions
	http://www.bymath.com/studyguide/tri/sec/tri7.htm	Summary table of the trigonometric reductions
	http://everythingmaths.co.za/grade-11/17-trigonometry/17-trigonometry-04.cnxmlplus	Notes and examples on General solutions
	http://www.youtube.com/watch?v=Ntankql24JQ	Maths 911 live show on Trigonometric Reductions.
	http://www.youtube.com/watch?v=FocBOlJ3PQc	Learn Xtra show on Trigonometric Reductions

Task

Question 1

Without the use of a calculator, determine the value of the following: $\frac{-3\sin 120^\circ}{\cos 180^o(\tan 315^0 - \cos 240^o)}$

Question 2

If $\sin 17^0 = a$, express the following in terms of a:

2.1 $\sin 197^o$

2.2 $\cos 107^o$

2.3 $\tan 343^o$

Question 3

Prove: $\frac{\tan x - \sin x}{\sin^3 x} = \frac{1-\cos x}{\sin^2 x \times \cos x}$

Question 4

4.1 Show that

$$\sin^2\theta + \cos^2\theta + \tan^2\theta = \frac{1}{\cos^2\theta}$$

4.2 Hence, evaluate without using a calculator, and by showing all the relevant steps:

$\sin^2 135^\circ + \cos^2 135^\circ + \tan^2 135^\circ$

Question 5

Given that $\cos\beta = 0{,}123$

5.1 Give the general solution to the equation correct to one decimal place.

5.2 Give all the solutions to the equation for the interval $\beta \in [-300^o; 400^o]$

Question 6

Solve the following equation:

$\sin(2\alpha - 40^\circ) = -\frac{1}{2}$ where $\alpha \in [0^\circ; 360^\circ]$

Question 7

Give the complete general solution, correct to one decimal place, for:

$3\sin A \times \cos A - 2\sin A = 0$

Task Answers

Question 1

$$\frac{3\cos 150^o}{\cos 180^o(\tan 315^0 - \cos 240^o)}$$

$$= \frac{3\cos(180^o - 30^o)}{\cos 180^o(\tan(360^0 - 45^0) - \cos(180^0 + 60^o)}$$

$$= \frac{-3\cos(30^o)}{\cos 180^o(\tan 45^0) + \cos(60^o)}$$

$$= \frac{-3 \cdot \frac{\sqrt{3}}{2}}{-1 \cdot 1 + \frac{1}{2}}$$

$$= 3\sqrt{3}$$

Question 2

2.1 $\sin 197^o$

$= \sin(180^o + 17^o)$

$= -\sin(17^o)$

$= -a$

2.2 $\cos 107^o$

$= \cos(90^o + 17^o)$

$= -\sin 17^o$

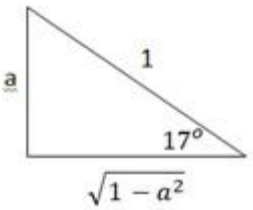
$= -a$

2.3 $\tan 343^o$

$= \tan(360^o - 17^o)$

$= -\tan(17^o)$

$= \frac{a}{\sqrt{1-a^2}}$



$\sin 17^0 = \frac{a}{1}$

Question 3

RTP: $\frac{\tan x - \sin x}{\sin^3 x} = \frac{1-\cos x}{\sin^2 x.\cos x}$

LHS $\frac{\tan x - \sin x}{\sin^3 x}$

$= \frac{\frac{\sin x}{\cos x} - \sin x}{\sin^3 x}$

$= \frac{\sin x(\frac{1}{\cos x} - 1)}{\sin^3 x}$

$= \frac{(\frac{1}{\cos x} - 1)}{\sin^2 x}$

$= \frac{\frac{1-\cos x}{\cos x}}{\sin^2 x}$

$= \frac{1-\cos x}{\sin^2 x.\cos x} = RHS$

Question 4

4.1. RTP $\sin^2\theta + \cos^2\theta + \tan^2\theta = \frac{1}{\cos^2\theta}$

LHS $\sin^2\theta + \cos^2\theta + \tan^2\theta$

$= 1 + \tan^2\theta$

$= 1 + \frac{\sin^2\theta}{\cos^2\theta}$

$= \frac{cos^2\theta + \sin^2\theta}{\cos^2\theta}$

$= \frac{1}{\cos^2\theta}$ = RHS

4.2. $\sin^2 135^o + \cos^2 135^o + \tan^2 135^o$

$= 1 + \tan^2 135^o$

$= 1 + \tan^2(180^o - 45^o)$

$= 1 + \tan^2 45^o$

$= 2$

Question 5

5.1. $\cos\beta = 0{,}123$

5.2. $\therefore \beta = \pm 82{,}9^o + k \cdot 360^o \;;\quad k \in \mathbb{Z}$

For the interval $\beta \in [-300^o; 400^o]$

$\therefore \beta = -227{,}1^o;\; 82{,}9^o;\; -82{,}9°;\; 227{,}1^o$

Question 6

$\sin(2\alpha - 40°) = -\frac{1}{2}$

$\therefore 2\alpha - 40° = -30^o + k \cdot 360^o \quad or \quad 2\alpha - 40° = 180^o - (-30^o) + k \cdot 360^o \;;\quad k \in \mathbb{Z}$

$\therefore 2\alpha = 40° - 30^o + k \cdot 360^o \quad or \quad 2\alpha = 40°+180^o + 30^o + k \cdot 360^o \;;\quad k \in \mathbb{Z}$

$\therefore 2\alpha = 10^o + k \cdot 360^o \quad or \quad 2\alpha = 250° + k \cdot 360^o \;;\quad k \in \mathbb{Z}$

$\therefore \alpha = 5^o + k \cdot 180^o \quad or \quad \alpha = 125° + k \cdot 180^o \;;\quad k \in \mathbb{Z}$

For the interval $\alpha \in [0°; 360°]$

$\therefore \alpha = 5^o;\; 185^o;\; 125°;\; 305^o$

Question 7

$3\sin A \cdot \cos A - 2\sin A = 0$

$\therefore \sin A\,(3\cos A - 2) = 0$

$\therefore \sin A = 0 \quad or \quad \cos A = \frac{2}{3}$

$\therefore A = k \cdot 180^o \quad or \quad A = \pm 48{,}2^o + k \cdot 360^o; \quad k \in \mathbb{Z}$

Acknowledgements

Mindset Learn Executive Head	Dylan Busa
Content Manager Classroom Resources	Jenny Lamont
Content Coordinator Classroom Resources	Helen Robertson
Content Administrator	Agness Munthali
Content Developer	Ian L Atteridge
Content Reviewers	Malindri Eastes
	Helen Robertson

Produced for Mindset Learn by Traffic

Facilities Manager	Belinda Renney
Facilities Coordinator	Cezanne Scheepers
Director	Alriette Gibbs
Editor	Nonhlanhla Nxumalo
	Sipho Mdluli
Presenter	Katleho Serobe
Studio Crew	Abram Tjale
	James Tselapedi
Graphics	Wayne Sanderson
	Jacolene Venter