## A Guide to Trigonometry for Beginners

## Teaching Approach

When teaching trigonometry, start with a recap the theorem of Pythagoras followed by defining the trigonometric ratios in a right angles triangle. A lot of examples are recommended to ensure proper understanding in recognizing the opposite, adjacent and hypotenuse sides. Triangles with the numerical length of sides and algebraic lengths of sides are recommended.

The proper use of the calculator must be stressed. Learners must not be restricted to just remembering the important keys, but a proper explanation of the reasoning behind pressing the sin, cos and tan keys, compared to Shift sin, Shift cos and Shift tan. The responsibility lies with the learners to ensure that their calculator is set in degrees.

Once learners are able to solve angles and sides in right angled triangles, the link to similar triangles can be bridged. The important concept of sides in the same ratio does not affect the angle. This concept can be further developed by using similar triangles to determine angles and sides of triangles which otherwise would be difficult to determine. The following example will illustrate this point:

We can use similar triangles to determine the height of the tree in the example


The shadow that a tree casts is used in the following way: we put a pole of a certain height (example 2 m ) in the line of the shadow, we measure the distance from the tree to where the shadow hits the horizontal (example 20m) and measuring the distance from the pole to where the shadow hits the horizontal (8m), we can use similar triangles and trigonometric ratios in the following way to determine the height of the tree:

> In $\triangle A B C: \tan \theta=\frac{2}{8}$
> In $\triangle A D E: \tan \theta=\frac{x}{20}$
> $\therefore \frac{x}{20}=\frac{2}{8} \therefore x=\frac{20 \times 2}{8}$
> $\therefore x=5 m$

Other uses of trigonometry and similar triangles must be highlighted to ensure learners see the relevance of trigonometric definitions. Trigonometry and Similar triangles are used in engineering, architecture, construction etc.

## Hints on solving trigonometry problems:

- If no diagram is given, draw one yourself.
- Mark the right angles in the diagram.
- Show the sizes of the other angles and the lengths of any lines that are known
- Mark the angles or sides you have to calculate.
- Consider whether you need to create right triangles by drawing extra lines. For example, divide an isosceles triangle into two congruent right triangles.
- Decide whether you will need Pythagoras theorem, sine, cosine or tangent.
- Check that your answer is reasonable.
- The hypotenuse is the longest side in a right triangle


## Video Summaries

Some videos have a 'PAUSE' moment, at which point the teacher or learner can choose to pause the video and try to answer the question posed or calculate the answer to the problem under discussion. Once the video starts again, the answer to the question or the right answer to the calculation is given.

Mindset suggests a number of ways to use the video lessons. These include:

- Watch or show a lesson as an introduction to a lesson
- Watch of show a lesson after a lesson, as a summary or as a way of adding in some interesting real-life applications or practical aspects
- Design a worksheet or set of questions about one video lesson. Then ask learners to watch a video related to the lesson and to complete the worksheet or questions, either in groups or individually
- Worksheets and questions based on video lessons can be used as short assessments or exercises
- Ask learners to watch a particular video lesson for homework (in the school library or on the website, depending on how the material is available) as preparation for the next days lesson; if desired, learners can be given specific questions to answer in preparation for the next day's lesson

1. Introduction to Trigonometry

This video gives brief description of how trigonometry was first discovered and used. It also describes the practical application of trigonometry through the theodolite, as used by land surveyors.

## 2. Introduction to Sin, Cos and Tan

This video covers the fundamental definitions of the trigonometry. It explains that trigonometry is ultimately the relationship between ratios of sides of triangles with respect to an angle in that triangle.
3. Basic Use of Sin, Cos and Tan

In this lesson we will use sin, cos and tan ratios in right angled triangles. We start by revising the definitions.

## 4. Using to Calculating a Side

This video covers the first of the application videos in which we use the trigonometric ratios to determine the length of a side in a right angled triangle. The emphasis is on choosing the correct trigonometric ratio.

## 5. Using Trigonometric to Calculate an Angle

This video covers the second of the application videos in which we use the trigonometric ratios to determine the size of an angle, given at least two sides in the right angled triangle.

## 6. Introducing Trigonometry on the Cartesian Plane

In this lesson we look at we will be looking at the trigonometric ratios on the Cartesian Plane.

## 7. Application of Trigonometry on the Cartesian Plane

In this video we apply what we know about trigonometric ratios on the Cartesian Plane. We determine lengths of sides by sketching a right angle triangle in the correct quadrants according the given information.

## 8. Introducing Cosec, Sec and Cot

This lesson introduces learners to the three reciprocals of sign, cos and tan, namely cosec, sec and cot
9. Introducing Special Angles

In this video we introduce special angles of which the trigonometric ratios can be derived without the use of a calculator. We derive these special triangles by using the Cartesian Plan with a circle with centre at the origin and with radius 2.
10. Solving Basic Trigonometric Equations

In this video we solve trigonometric equations. Learners are exposed to solving equations using a calculator and solving equations without the use of a calculator. The important calculator steps are outlined.

## 11. Angles of Elevation and Depression

In this video we use trigonometric ratios to find the length of sides and the size of angles in two dimensional sketches. An angle of elevation and an angle of depression are also discussed in this lesson.

## Resource Material

Resource materials are a list of links available to teachers and learners to enhance their experience of the subject matter. They are not necessarily CAPS aligned and need to be used with discretion.

| 1 | Introduction to Trigonometry | http://www.mathopenref.com/trigin tro.html | This page defines trigonometry. |
| :---: | :---: | :---: | :---: |
|  |  | http://www.ehow.com/m/list 7769 <br> 833 real-life-applications- <br> trigonometry.html | This script explains to us the uses of trigonometry in the industry and in everyday life. |
|  |  | http://www.learner.org/courses/lea rningmath/geometry/session8/part c/ | A resource on trigonometry. |
| 2 | Introduction to Sin, Cos and Tan | http://en.wikipedia.org/wiki/Trigon ometric Ratios | Summary of the definitions of trigonometric ratios |
|  |  | http://www.purplemath.com/modul es/basirati.htm | The definition of the sin, cos and tan ratios |
|  |  | http://teachertube.com/viewVideo. php?video id=13980 | Video on the definition of the sin, cos and tan ratios. |
| 3 | Basic Use of Sin, Cos and Tan | http://www.mathwarehouse.com/tr igonometry/sine-cosine-tangentpractice3.php | This video gives examples of finding the lengths of sides using the trigonometric ratios. |
|  |  | http://cshsyear10maths.edublogs. org/trigonometry/i-application-of-trigonometric-ratios/ | This page covers the calculations of sides of a right angled triangle using trig ratios |
|  |  | http://www.mathsisfun.com/algebr a/trig-finding-side-righttriangle.html | Gives us examples of how to find the size of a side in a right angled triangle. |
| 4 | Using Trigonometry to Calculate a Side | http://www.mathsangel.co.uk/free/ rp-ss2.pdf | This page gives useful hints on how to calculate the lengths and angles in right angled triangles. |
|  |  | http://www.youtube.com/watch?v =b1PBnTiWSiA\&feature=related | This video gives examples of how to calculate the angle in a right angled triangle. |
| 5 | Using Trigonometric to Calculate an Angle | http://www.mathsisfun.com/algebr a/trig-four-quadrants.html | This page gives a summary of the trig ratios in all 4 quadrants. |
|  |  | http://www.intmath.com/trigonome tric-functions/5-signs-of-trigonometric-functions.php | This page gives us a summary of the trig ratios in all 4 quadrants. |
|  |  | http://www.onlinemathlearning.co $\mathrm{m} /$ trigonometric-ratios.html | This page gives us a summary of the trig ratios in all 4 quadrants. |
| 6 | Introducing Trigonometry on the Cartesian Plane | http://www.doctortang.com/PureM ath10/Trigonometry\%20Notes\%2 0\%28answers\%29.pdf | A text book section |
|  |  | http://www.intmath.com/trigonom etric-functions/5-signs-of-trigonometric-functions.php | A summary of the trig ratios in all 4 quadrants as well as examples of trig ratios in all 4 quadrants. |
| 7. Application of Trigonometry on the Cartesian Plane |  | http://www.regentsprep.org/Rege nts/math/algtrig/ATT1/trigsix.htm | This page defines the reciprocal ratios of sin, cos and tan. |
|  |  | http://www.intmath.com/trigonom etric-functions/2-sin-cos-tan-csc-sec-cot.php | This page defines the reciprocal ratios of sin, cos and tan. |
|  |  | http://www.algebralab.org/lessons /lesson.aspx?file=Trigonometry T rigSecCscCot.xml | This page defines the reciprocal ratios of sin, cos and tan. It also includes some useful examples. |
|  | Introducing Cosec, Sec and Cot | http://m.everythingmaths.co.za/gr ade-10/07-trigonometry/07-trigonometry-05.cnxmlplus | This page gives a summary of all the special angles and the value of $\sin , \cos$ and tan ratios of the special angles. An exercise with full solutions is included. |


|  | http://www.youtube.com/watch?v =nVTtSE5nv7c | This video explains the special angles in a right angled triangle as well as the values of the trig ratios of the special angles. |
| :---: | :---: | :---: |
| 9. Introducing Cosec, Sec and Cot | http://m.everythingmaths.co.za/gr ade-10/07-trigonometry/07-trigonometry-04.cnxmlplus | This page explains the solution to trigonometric equations with some useful examples |
|  | http://everythingmaths.co.za/grad <br> e-10/07-trigonometry/07- <br> trigonometry-06.cnxmlplus | This page includes a video and an exercise page on solving trigonometric equations |
| 10.Solving Basic Trigonometric Equations | http://m.everythingmaths.co.za/gr ade-10/07-trigonometry/07-trigonometry-08.cnxmlplus | This page gives some good examples of solving two dimensional problems |
|  | http://www.mathwarehouse.com/t rigonometry/sohcahtoa-real-world-applications.php | This page gives some good examples of solving two dimensional problems and step by step explanations. |
|  | http://www.analyzemath.com/high school math/grade 10/trigonom etry.htm | This page gives some good examples of solving two dimensional problems. It also includes exercise on two dimensional problems |
|  | http://www.doctortang.com/PureM ath10/Trigonometry\%20Notes\%2 0\%28answers\%29.pdf | Page 60-62: this page gives some good examples of solving two dimensional problems |
| 11.Angles of Elevation and Depression | http://www.doctortang.com/PureM ath10/Trigonometry\%20Notes\%2 0\%28answers\%29.pdf | Gives us a summary plus examples of trigonometric functions in the Cartesian Plane Page 63 |
|  | http://www.onlinemathlearning.co $\mathrm{m} /$ trigonometric-ratios.htm | This page includes a video plus a detailed explanation of trigonometric functions in the Cartesian Plane. |

## Task

## Question 1

In $\triangle P Q R, \hat{Q}=90^{\circ}, P Q=12 \mathrm{~cm}, Q R=5 \mathrm{~cm}$. Write down the value of each of the following:
$1.1 \sin P$
$1.2 \cos P$
$1.3 \tan P$


## Question 2

In $\triangle A B C, \hat{B}=90^{\circ}$, side $\mathrm{AB}=c, \mathrm{BC}=a$ and $\mathrm{AC}=b$. Write in each of the following in terms of $a, b$ and $c$.

$2.1 \sin A$
$2.4 \cos C$
$2.2 \sin C$
$2.5 \tan \mathrm{~A}$
$2.3 \cos A$
$2.6 \tan \mathrm{C}$

## Question 3

Refer to the given sketch.


Write down two ratios for:
$3.1 \sin \theta$
$3.2 \cos \theta$
$3.3 \tan \theta$

## Questions 4

All answers must be given correct to two decimal places, where applicable.
Determine the length of $x$ in the given sketches:
4.1

4.2



## Question 5

Determine the value of $x$ and $y$ in the given diagram.


## Question 6

In $\triangle \mathrm{DEG}, \mathrm{DE}=15 \mathrm{~cm}$. Determine the length of DG


## Question 7

Determine the size of $\theta$ (correct to one decimal place) in each of the following sketches.
7.1

7.2

7.3


## Question 8

Determine the value of $x$ and $y$ in the diagram below (correct to one decimal value)


## Question 9

If $\cos \mathrm{P}=-\frac{3}{5}$ and $0^{\circ} \leq P \leq 180^{\circ}$, determine with the aid of a sketch and without the use of a calculator:
$9.1 \quad \sin P$
$9.2 \tan P$
$9.3 \quad \sin ^{2} P+\cos ^{2} P$

## Question 10

Given $: \tan \theta=\frac{6}{\sqrt{13}}$, with $90^{\circ} \leq \theta \leq 360^{\circ}$.
Determine with the aid of a sketch and without the use of a calculator:
$10.1 \cos \theta$
$10.2 \quad 7 \sin \theta-14 \cos ^{2} \theta$
$10.3 \cos \theta \times \tan \theta$

## Question 11

Use the given triangle to determine the value of:


## Question 12

Use the given triangle to answer the following questions.
Determine the value of:
12.1 Cosec $75^{\circ}$
$12.2 \mathrm{Sec} 149^{\circ}$
$12.3 \cot 256^{\circ}$

## Question 13

Determine the value of each of the following without the use of a calculator:
$13.1 \sin 30^{\circ}+\cos 60^{\circ}$
$13.2 \cos 30^{\circ} \times \sec 30^{\circ}$
$13.3 \operatorname{cosec} 30^{\circ} \times \cos 60^{\circ} \times \cot 30^{\circ} \times \tan 45^{\circ}$

## Question 14

Determine the value of $\theta$ for $0^{\circ} \leq \theta \leq 90^{\circ}$ if:
$14.13 \sin \theta-2=0,3$
$14.2 \tan 2 \theta=9$
$14.3 \operatorname{cosec} \theta=2,5$

## Question 15

If $\cos \alpha=\frac{-\sqrt{3}}{2}$ and $\sin \alpha>0$, find $\tan \alpha$ without using a calculator.

## Question 16

If $\sin \theta=-\frac{2}{3}$ and $\cos \theta>0$, determine without finding the value of $\theta$ and with the aid of a sketch, the value of
$16.1 \cos \theta$
$16.2 \tan ^{2} \theta \times \sin \theta$

## Question 17

Marilyn and Vijay are standing on opposite sides of a bridge. They want to calculate the length of the bridge. Marilyn is at point A and Vijay at point B . The angle of depression from A to the bottom of the gorge, $D$, is $55^{\circ}$ and the angle of depression from $B$ to $D$ is $68^{\circ}$. If the length of $B D 36 \mathrm{~m}$, help them to calculate the length of the bridge.


## Task Answers

## Question 1

$1.1 \quad \mathrm{PR}^{2}=12^{2}+5^{2}$

$$
=144+25=169
$$

$$
\therefore P R=\sqrt{169}=13
$$

$$
\therefore \sin P=\frac{O}{H}=\frac{5}{13}
$$

$1.2 \cos P=\frac{A}{H}=\frac{12}{13}$
$1.3 \tan P=\frac{O}{A}=\frac{5}{12}$

## Question 2

$2.1 \sin A=\frac{a}{b}$
$2.2 \quad \sin C=\frac{c}{b}$
$2.3 \cos A=\frac{c}{b}$
$2.4 \quad \cos C=\frac{a}{b}$
$2.5 \tan A=\frac{a}{c}$
$2.6 \tan C=\frac{c}{a}$

## Question 3

$3.1 \sin \theta=\frac{B D}{B C} \quad \sin \theta=\frac{A B}{A C}$
$3.2 \cos \theta=\frac{C D}{B C} \quad \cos \theta=\frac{B C}{A C}$
$3.3 \tan \theta=\frac{B D}{C D} \quad \tan \theta=\frac{A B}{B C}$

## Question 4

$4.1 \sin 35^{\circ}=\frac{x}{7}$
$\therefore 7 \sin 35^{\circ}=x$
$\therefore x=4,02$
$4.2 \cos 65^{\circ}=\frac{x}{10}$
$\therefore 10 \cos 65^{\circ}=x$
$\therefore x=4,23$
$4.3 \tan 72^{\circ}=\frac{x}{20}$
$\therefore 20 \tan 72^{\circ}=x$
$\therefore x=61,55$

## Question 5

$$
\begin{gathered}
\cos 48^{0}=\frac{5}{x} \\
\times x \quad \therefore x \cos 48^{0}=5 \\
\div \cos 48^{0} \therefore \quad x=\frac{5}{\cos 48^{0}} \\
\therefore \quad x=7,47 \\
\tan 32,7^{0}=\frac{3}{y} \\
\times y \quad \therefore y \tan 32,7^{0}=3 \\
\therefore \tan 32,7^{0} \therefore y=\frac{3}{\tan 32,7^{0}} \\
\therefore y=4,67
\end{gathered}
$$

## Question 6

$$
\begin{gathered}
\sin 51^{0}=\frac{D F}{15} \\
\times 15 \quad \therefore 15 \sin 51^{0}=D F \\
\therefore D F=11,66 \mathrm{~cm} \\
\cos 20^{\circ}=\frac{D F}{D G}=\frac{11,66}{D G} \\
\times D G \quad \therefore D G \cos 20^{\circ}=11,66 \\
\div \cos 20^{\circ} \quad \therefore D G=\frac{11.66}{\cos 20^{\circ}}=12,41 \mathrm{~cm}
\end{gathered}
$$

## Question 7

$7.1 \sin \theta=\frac{12}{19}$
$\therefore \theta=39,2^{0}$
$7.2 \cos \theta=\frac{8}{10}$
$\therefore \theta=36,9^{0}$
$7.3 \tan \theta=\frac{20}{15}$
$\therefore \theta=53,1^{0}$

## Question 8

$$
\begin{gathered}
\tan x=\frac{2}{5} \\
\therefore x=21,8^{0} \\
\tan 38^{0}=\frac{5}{2+y} \\
\times(2+y) \therefore(2+y) \tan 38^{0}=5 \\
\div \tan 38^{0} \quad \therefore(2+y)=\frac{5}{\tan 38^{0}} \\
\therefore 2+y=6,3997 \ldots \\
\therefore y=6,3997 \ldots-2 \\
\therefore y=4,4
\end{gathered}
$$

## Question 9

$9.1 \quad 5^{2}=y^{2}+(-3)^{2}$
$\therefore 25=y^{2}+9$
$\therefore 25-9=y^{2}$
$\therefore 16=y^{2}$
$\therefore y= \pm \sqrt{16}= \pm 4$
but $y=4 . \ldots .$. Quad 2
$\therefore \sin P=\frac{4}{5}$
$9.2 \tan P=\frac{4}{-3}$

$9.3 \sin ^{2} P+\cos ^{2} P$

$$
\begin{aligned}
& =\left(\frac{4}{5}\right)^{2}+\left(\frac{-3}{5}\right)^{2} \\
& =\frac{16}{25}+\frac{9}{25}=1
\end{aligned}
$$

## Question 10

$10.1 \tan \theta=\frac{6}{\sqrt{13}}=\frac{O}{A}$, but $\theta$ in Quad 3
$\therefore$ opposite side $=-6$
and adjacent side $=-\sqrt{13}$

$$
\begin{aligned}
& r^{2}=(-6)^{2}+(-\sqrt{13})^{2} \\
& \therefore r^{2}=36+13=49 \\
& \therefore r=7 \\
& \cos \theta=\frac{-\sqrt{13}}{7}
\end{aligned}
$$

$10.27 \sin \theta-14 \cos ^{2} \theta$

$$
\begin{aligned}
& =7\left(\frac{-6}{7}\right)-14\left(\frac{-\sqrt{13}}{7}\right)^{2} \\
& =\frac{-68}{7}
\end{aligned}
$$

$10.3 \cos \theta \times \tan \theta$

$$
\begin{aligned}
& =\left(\frac{-\sqrt{13}}{7}\right) \times\left(\frac{-6}{-\sqrt{13}}\right) \\
& =\frac{-6}{7}
\end{aligned}
$$

## Question 11

$11.1 \cos e c \alpha=\frac{10}{6}=\frac{5}{3}$
$11.2 \sec \alpha=\frac{10}{8}=\frac{5}{4}$
$11.3 \cot \alpha=\frac{8}{6}=\frac{4}{3}$

## Question 12

$12.1 \operatorname{cosec} 75^{\circ}=\frac{1}{\sin 75^{0}}=1,04$
$12.2 \sec 149^{0}=\frac{1}{\cos 149^{0}}=-1,17$
$12.3 \cot 256^{\circ}=\frac{1}{\tan 256^{\circ}}=0,25$

## Question 13

$13.1 \sin 30^{\circ}+\cos 60^{\circ}$

$$
=\frac{1}{2}+\frac{1}{2}=1
$$

$13.2 \cos 30^{\circ} \times \sec 30^{\circ}$

$$
=\frac{\sqrt{3}}{2} \times \frac{2}{\sqrt{3}}=1
$$

$13.3 \operatorname{cosec} 30^{\circ} \times \cos 60^{\circ} \times \cot 30^{\circ} \times \tan 45^{\circ}$

$$
=\frac{2}{1} \times \frac{1}{2} \times \frac{\sqrt{3}}{1} \times \frac{\sqrt{2}}{\sqrt{2}}=\sqrt{3}
$$



## Question14

$14.13 \sin \theta-2=0,3$
$3 \sin \theta=2,3$
$\div 3 \therefore \sin \theta=\frac{2,3}{3}$
$\therefore \theta=50,1^{0}$
$14.2 \tan 2 \theta=9$
$\therefore 2 \theta=83,66^{\circ}$
$\div 2 \therefore \theta=41,83^{\circ}$
$14.3 \operatorname{cosec} \theta=2,5$
$\operatorname{cosec} \theta=\frac{1}{\sin \theta}$
$\therefore \frac{1}{\sin \theta}=2,5$
$\therefore \sin \theta=\frac{1}{2,5}$
$\therefore \theta=23,58^{0}$

## Question 15

$y^{2}=2^{2}-(-\sqrt{3})^{2}$
$y^{2}=1$
$\therefore y= \pm 1$
$\therefore y=1 \ldots . .$. quad 2
$\therefore \tan \alpha=\frac{1}{-\sqrt{3}}$

## Question 16

$16.1 \quad x^{2}=3^{2}-(-2)^{2}$
$x^{2}=9-4=5$
$\therefore x= \pm \sqrt{5}$
$\therefore x=\sqrt{5}$ .Quad 4
$\cos \theta=\frac{\sqrt{5}}{3}$
$16.2 \tan ^{2} \theta \times \sin \theta$

$$
=\left(\frac{-2}{\sqrt{5}}\right)^{2} \times \frac{-2}{3}=\frac{-8}{15}
$$



## Question 17

$$
\begin{gathered}
\cos 68^{0}=\frac{B P}{36} \therefore B P=36 \cos 68^{0} \\
\therefore B P=13,49 m \\
\sin 68^{\circ}=\frac{P D}{36} \therefore P D=36 \sin 68^{0} \\
\therefore P D=33,38 m \\
\tan 55^{\circ}=\frac{33,38}{A P} \\
\therefore A P \tan 55^{0}=33,38 \\
\times A P \\
\therefore \tan 55^{0} \quad \therefore A P=\frac{33,38}{\tan 55^{0}}=23,37 m \\
\therefore A B=
\end{gathered} \begin{gathered}
\therefore P+B P \\
=
\end{gathered}
$$

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