

# A Guide to Trigonometry for Beginners

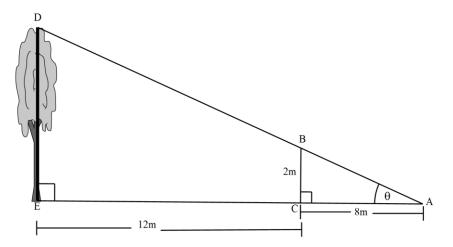
#### **Teaching Approach**

When teaching trigonometry, start with a recap the theorem of Pythagoras followed by defining the trigonometric ratios in a right angles triangle. A lot of examples are recommended to ensure proper understanding in recognizing the opposite, adjacent and hypotenuse sides. Triangles with the numerical length of sides and algebraic lengths of sides are recommended.

The proper use of the calculator must be stressed. Learners must not be restricted to just remembering the important keys, but a proper explanation of the reasoning behind pressing the sin, cos and tan keys, compared to Shift sin, Shift cos and Shift tan. The responsibility lies with the learners to ensure that their calculator is set in degrees.

Once learners are able to solve angles and sides in right angled triangles, the link to similar triangles can be bridged. The important concept of sides in the same ratio does not affect the angle. This concept can be further developed by using similar triangles to determine angles and sides of triangles which otherwise would be difficult to determine. The following example will illustrate this point:

We can use similar triangles to determine the height of the tree in the example



The shadow that a tree casts is used in the following way: we put a pole of a certain height (example 2m) in the line of the shadow, we measure the distance from the tree to where the shadow hits the horizontal (example 20m) and measuring the distance from the pole to where the shadow hits the horizontal (8m), we can use similar triangles and trigonometric ratios in the following way to determine the height of the tree:





 $In \ \Delta ABC : \ \tan \theta = \frac{2}{8}$  $In \ \Delta ADE : \ \tan \theta = \frac{x}{20}$  $\therefore \ \frac{x}{20} = \frac{2}{8} \ \therefore \ x = \frac{20 \times 2}{8}$  $\therefore \ x = 5m$ 

Other uses of trigonometry and similar triangles must be highlighted to ensure learners see the relevance of trigonometric definitions. Trigonometry and Similar triangles are used in engineering, architecture, construction etc.

#### Hints on solving trigonometry problems:

- If no diagram is given, draw one yourself.
- Mark the right angles in the diagram.
- Show the sizes of the other angles and the lengths of any lines that are known
- Mark the angles or sides you have to calculate.
- Consider whether you need to create right triangles by drawing extra lines. For example, divide an isosceles triangle into two congruent right triangles.
- Decide whether you will need Pythagoras theorem, sine, cosine or tangent.
- Check that your answer is reasonable.
- The hypotenuse is the longest side in a right triangle





#### Video Summaries

Some videos have a 'PAUSE' moment, at which point the teacher or learner can choose to pause the video and try to answer the question posed or calculate the answer to the problem under discussion. Once the video starts again, the answer to the question or the right answer to the calculation is given.

Mindset suggests a number of ways to use the video lessons. These include:

- Watch or show a lesson as an introduction to a lesson
- Watch of show a lesson after a lesson, as a summary or as a way of adding in some interesting real-life applications or practical aspects
- Design a worksheet or set of questions about one video lesson. Then ask learners to watch a video related to the lesson and to complete the worksheet or questions, either in groups or individually
- Worksheets and questions based on video lessons can be used as short assessments or exercises
- Ask learners to watch a particular video lesson for homework (in the school library or on the website, depending on how the material is available) as preparation for the next days lesson; if desired, learners can be given specific questions to answer in preparation for the next day's lesson

#### 1. Introduction to Trigonometry

This video gives brief description of how trigonometry was first discovered and used. It also describes the practical application of trigonometry through the theodolite, as used by land surveyors.

#### 2. Introduction to Sin, Cos and Tan

This video covers the fundamental definitions of the trigonometry. It explains that trigonometry is ultimately the relationship between ratios of sides of triangles with respect to an angle in that triangle.

#### 3. Basic Use of Sin, Cos and Tan

In this lesson we will use sin, cos and tan ratios in right angled triangles. We start by revising the definitions.

#### 4. Using to Calculating a Side

This video covers the first of the application videos in which we use the trigonometric ratios to determine the length of a side in a right angled triangle. The emphasis is on choosing the correct trigonometric ratio.

## 5. Using Trigonometric to Calculate an Angle

This video covers the second of the application videos in which we use the trigonometric ratios to determine the size of an angle, given at least two sides in the right angled triangle.

## 6. Introducing Trigonometry on the Cartesian Plane

In this lesson we look at we will be looking at the trigonometric ratios on the Cartesian Plane.

3

mindset





#### 7. Application of Trigonometry on the Cartesian Plane

In this video we apply what we know about trigonometric ratios on the Cartesian Plane. We determine lengths of sides by sketching a right angle triangle in the correct quadrants according the given information.

#### 8. Introducing Cosec, Sec and Cot

This lesson introduces learners to the three reciprocals of sign, cos and tan, namely cosec, sec and cot

#### 9. Introducing Special Angles

In this video we introduce special angles of which the trigonometric ratios can be derived without the use of a calculator. We derive these special triangles by using the Cartesian Plan with a circle with centre at the origin and with radius 2.

#### 10. Solving Basic Trigonometric Equations

In this video we solve trigonometric equations. Learners are exposed to solving equations using a calculator and solving equations without the use of a calculator. The important calculator steps are outlined.

#### 11. Angles of Elevation and Depression

In this video we use trigonometric ratios to find the length of sides and the size of angles in two dimensional sketches. An angle of elevation and an angle of depression are also discussed in this lesson.





## **Resource Material**

Resource materials are a list of links available to teachers and learners to enhance their experience of the subject matter. They are not necessarily CAPS aligned and need to be used with discretion.

	Introduction to	http://www.mathopenref.com/trigin	This page defines trigonometry.
1	Introduction to Trigonometry	tro.html http://www.ehow.com/m/list_7769 833_real-life-applications- trigonometry.html http://www.learner.org/courses/lea rningmath/geometry/session8/part c/	This script explains to us the uses of trigonometry in the industry and in everyday life. A resource on trigonometry.
2	Introduction to Sin, Cos and Tan	http://en.wikipedia.org/wiki/Trigon ometric_Ratios http://www.purplemath.com/modul es/basirati.htm http://teachertube.com/viewVideo. php?video_id=13980	Summary of the definitions of trigonometric ratios The definition of the sin, cos and tan ratios Video on the definition of the sin, cos and tan ratios.
3	Basic Use of Sin, Cos and Tan	http://www.mathwarehouse.com/tr igonometry/sine-cosine-tangent- practice3.php http://cshsyear10maths.edublogs. org/trigonometry/i-application-of- trigonometric-ratios/ http://www.mathsisfun.com/algebr a/trig-finding-side-right-	This video gives examples of finding the lengths of sides using the trigonometric ratios. This page covers the calculations of sides of a right angled triangle using trig ratios Gives us examples of how to find the size of a side in a right angled
4	Using Trigonometry to Calculate a Side	aring informs side right         triangle.html         http://www.mathsangel.co.uk/free/         rp-ss2.pdf         http://www.youtube.com/watch?v         =b1PBnTjWSiA&feature=related	triangle. This page gives useful hints on how to calculate the lengths and angles in right angled triangles. This video gives examples of how to calculate the angle in a right angled triangle.
5	Using Trigonometric to Calculate an Angle	http://www.mathsisfun.com/algebr a/trig-four-quadrants.html http://www.intmath.com/trigonome tric-functions/5-signs-of- trigonometric-functions.php http://www.onlinemathlearning.co	This page gives a summary of the trig ratios in all 4 quadrants. This page gives us a summary of the trig ratios in all 4 quadrants. This page gives us a summary of the trig ratios in all 4 quadrants.
6	Introducing Trigonometry on the Cartesian Plane	m/trigonometric-ratios.html http://www.doctortang.com/PureM ath10/Trigonometry%20Notes%2 0%28answers%29.pdf http://www.intmath.com/trigonom etric-functions/5-signs-of-	A summary of the trig ratios in all 4 quadrants as well as examples
7.	Application of Trigonometry on the Cartesian Plane	trigonometric-functions.php http://www.regentsprep.org/Rege nts/math/algtrig/ATT1/trigsix.htm http://www.intmath.com/trigonom etric-functions/2-sin-cos-tan-csc- sec-cot.php	of trig ratios in all 4 quadrants. This page defines the reciprocal ratios of sin, cos and tan. This page defines the reciprocal ratios of sin, cos and tan.
8.	Introducing Cosec, Sec and Cot	http://www.algebralab.org/lessons /lesson.aspx?file=Trigonometry_T rigSecCscCot.xml http://m.everythingmaths.co.za/gr ade-10/07-trigonometry/07- trigonometry-05.cnxmlplus	This page defines the reciprocal ratios of sin, cos and tan. It also includes some useful examples. This page gives a summary of all the special angles and the value of sin, cos and tan ratios of the special angles. An exercise with



mindset



	http://www.youtube.com/watch?v	This video explains the special
	=nVTtSE5nv7c	angles in a right angled triangle as
		well as the values of the trig ratios
		of the special angles.
	http://m.everythingmaths.co.za/gr	This page explains the solution to
0 Introducing Cooco Coo and		trigonometric equations with some
9. Introducing Cosec, Sec and	ade-10/07-trigonometry/07-	<b>U</b>
Cot	trigonometry-04.cnxmlplus	useful examples
	http://everythingmaths.co.za/grad	This page includes a video and an
	e-10/07-trigonometry/07-	exercise page on solving
	trigonometry-06.cnxmlplus	trigonometric equations
	http://m.everythingmaths.co.za/gr	This page gives some good
10.Solving Basic Trigonometric	ade-10/07-trigonometry/07-	examples of solving two
Equations	trigonometry-08.cnxmlplus	dimensional problems
	http://www.mathwarehouse.com/t	This page gives some good
	rigonometry/sohcahtoa-real-	examples of solving two
	world-applications.php	dimensional problems and step by
		step explanations.
	http://www.analyzemath.com/high	This page gives some good
	_school_math/grade_10/trigonom	examples of solving two
	etry.html	dimensional problems. It also
		includes exercise on two
		dimensional problems
	http://www.doctortang.com/PureM	Page 60 – 62 : this page gives
	ath10/Trigonometry%20Notes%2	some good examples of solving
	0%28answers%29.pdf	two dimensional problems
	http://www.doctortang.com/PureM	Gives us a summary plus
11.Angles of Elevation and	ath10/Trigonometry%20Notes%2	examples of trigonometric
Depression	0%28answers%29.pdf	functions in the Cartesian Plane -
Depression	07020413We137029.pdf	Page 63
	http://www.onlinemathlearning.co	This page includes a video plus a
	m/trigonometric-ratios.html	detailed explanation of
		trigonometric functions in the Cartesian Plane.
		Cartesian Plane.

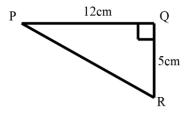


# Task

# **Question 1**

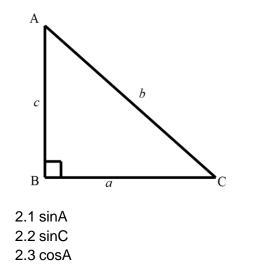
In  $\Delta PQR$ ,  $\hat{Q} = 90^{\circ}$ , PQ = 12cm, QR = 5cm. Write down the value of each of the following:

- 1.1 sin P
- 1.2 cos P
- 1.3 tan P



## **Question 2**

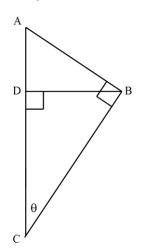
In  $\triangle ABC$ ,  $\hat{B} = 90^{\circ}$ , side AB = *c*, BC= *a* and AC = *b*. Write in each of the following in terms of *a*, *b* and *c*.



$2.4 \cos C$
2.5 tan A
2.6 tan C

# **Question 3**

Refer to the given sketch.

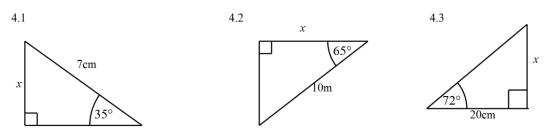


Write down two ratios for: 3.1 sin  $\theta$ 3.2 cos  $\theta$ 3.3 tan  $\theta$ 



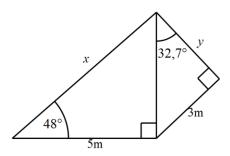


All answers must be given correct to two decimal places, where applicable. Determine the length of x in the given sketches:



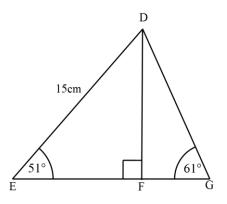
## **Question 5**

Determine the value of *x* and *y* in the given diagram.



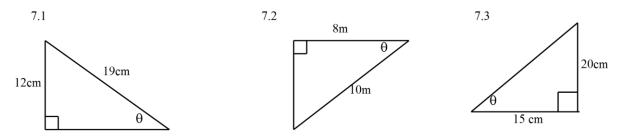
## **Question 6**

In  $\Delta DEG$ , DE=15cm. Determine the length of DG



# **Question 7**

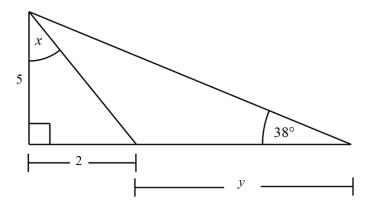
Determine the size of  $\theta$  (correct to one decimal place) in each of the following sketches.







Determine the value of x and y in the diagram below (correct to one decimal value)



# **Question 9**

If  $\cos P = -\frac{3}{5}$  and  $0^0 \le P \le 180^0$ , determine with the aid of a sketch

and without the use of a calculator:

- 9.1 sin *P*
- 9.2 tan *P*

9.3  $\sin^2 P + \cos^2 P$ 

# **Question 10**

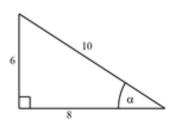
Given:  $\tan \theta = \frac{6}{\sqrt{13}}$ , with  $90^\circ \le \theta \le 360^\circ$ .

Determine with the aid of a sketch and without the use of a calculator:

10.1  $\cos\theta$ 10.2  $7\sin\theta - 14\cos^2\theta$ 10.3  $\cos\theta \times \tan\theta$ 

# **Question 11**

Use the given triangle to determine the value of:



11.1 cosec α 11.2 sec α 11.3 cot α



a



Use the given triangle to answer the following questions. Determine the value of:

- 12.1 Cosec 75°
- **12.2** Sec 149°
- 12.3  $\cot 256^{\circ}$

# **Question 13**

Determine the value of each of the following without the use of a calculator:

- 13.1 sin 30°+cos 60°
- 13.2 cos 30° × sec 30°
- 13.3 cosec 30°×cos60°×cot30°×tan45°

# **Question 14**

Determine the value of  $\theta$  for  $0^{\circ} \le \theta \le 90^{\circ}$  if: 14.1 3sin  $\theta - 2 = 0,3$ 14.2 tan  $2\theta = 9$ 14.3 cosec  $\theta = 2,5$ 

# **Question 15**

If 
$$\cos \alpha = \frac{-\sqrt{3}}{2}$$
 and  $\sin \alpha > 0$ , find  $\tan \alpha$  without using a calculator

# **Question 16**

If  $\sin \theta = -\frac{2}{3}$  and  $\cos \theta > 0$ , determine without finding the value of  $\theta$  and with the aid of a electric the value of

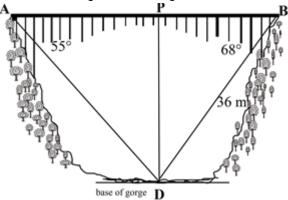
sketch, the value of

16.1  $\cos\theta$ 

16.2  $\tan^2 \theta \times \sin \theta$ 

# **Question 17**

Marilyn and Vijay are standing on opposite sides of a bridge. They want to calculate the length of the bridge. Marilyn is at point A and Vijay at point B. The angle of depression from A to the bottom of the gorge, D, is 55° and the angle of depression from B to D is 68°. If the length of BD 36m, help them to calculate the length of the bridge.







# **Task Answers**

#### **Question 1**

1.1 PR<sup>2</sup> = 12<sup>2</sup> + 5<sup>2</sup>  
= 144 + 25 = 169  
∴ PR = 
$$\sqrt{169}$$
 = 13  
∴ sin P =  $\frac{O}{H} = \frac{5}{13}$   
1.2 cos P =  $\frac{A}{H} = \frac{12}{13}$   
1.3 tan P =  $\frac{O}{A} = \frac{5}{12}$ 

## **Question 2**

2.1 sin $A = \frac{a}{b}$	2.2	$\sin C = \frac{c}{b}$
2.3 $\cos A = \frac{c}{b}$	2.4	$\cos C = \frac{a}{b}$
2.5 $\tan A = \frac{a}{c}$	2.6	$\tan C = \frac{c}{a}$

# **Question 3**

3.1	$\sin\theta = \frac{BD}{BC}$	$\sin\theta = \frac{AB}{AC}$
3.2	$\cos\theta = \frac{CD}{BC}$	$\cos\theta = \frac{BC}{AC}$
3.3	$\tan\theta = \frac{BD}{CD}$	$\tan\theta = \frac{AB}{BC}$

# **Question 4**

$$4.1 \sin 35^{\circ} = \frac{x}{7}$$
  

$$\therefore 7 \sin 35^{\circ} = x$$
  

$$\therefore x = 4,02$$
  

$$4.2 \cos 65^{\circ} = \frac{x}{10}$$
  

$$\therefore 10 \cos 65^{\circ} = x$$
  

$$\therefore x = 4,23$$
  

$$4.3 \tan 72^{\circ} = \frac{x}{20}$$
  

$$\therefore 20 \tan 72^{\circ} = x$$
  

$$\therefore x = 61,55$$





 $\cos 48^{0} = \frac{5}{x}$   $\times x \qquad \therefore x \cos 48^{0} = 5$   $\div \cos 48^{0} \therefore \qquad x = \frac{5}{\cos 48^{0}}$   $\therefore \qquad x = 7,47$   $\tan 32,7^{0} = \frac{3}{y}$   $\times y \qquad \therefore y \tan 32,7^{0} = 3$   $\div \tan 32,7^{0} \therefore \qquad y = \frac{3}{\tan 32,7^{0}}$   $\therefore \qquad y = 4,67$ 

#### **Question 6**

$$\sin 51^{0} = \frac{DF}{15}$$

$$\times 15 \quad \therefore 15 \sin 51^{0} = DF$$

$$\therefore DF = 11,66cm$$

$$\cos 20^{0} = \frac{DF}{DG} = \frac{11,66}{DG}$$

$$\times DG \quad \therefore DG \cos 20^{0} = 11,66$$

$$\div \cos 20^{0} \quad \therefore DG = \frac{11.66}{\cos 20^{0}} = 12,41cm$$

## **Question 7**

7.1 
$$\sin \theta = \frac{12}{19}$$
  
 $\therefore \theta = 39, 2^{0}$   
7.2  $\cos \theta = \frac{8}{10}$   
 $\therefore \theta = 36, 9^{0}$   
7.3  $\tan \theta = \frac{20}{15}$   
 $\therefore \theta = 53, 1^{0}$ 





$$\tan x = \frac{2}{5}$$
  

$$\therefore x = 21, 8^{0}$$
  

$$\tan 38^{0} = \frac{5}{2 + y}$$
  

$$\times (2 + y) \therefore (2 + y) \tan 38^{0} = 5$$
  

$$\div \tan 38^{0} \therefore (2 + y) = \frac{5}{\tan 38^{0}}$$
  

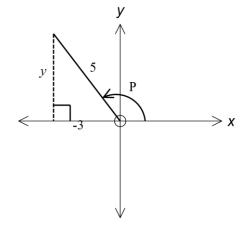
$$\therefore 2 + y = 6,3997...$$
  

$$\therefore y = 6,3997... - 2$$
  

$$\therefore y = 4, 4$$

# **Question 9**

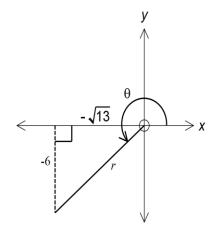
9.1 
$$5^2 = y^2 + (-3)^2$$
  
 $\therefore 25 = y^2 + 9$   
 $\therefore 25 - 9 = y^2$   
 $\therefore 16 = y^2$   
 $\therefore y = \pm \sqrt{16} = \pm 4$   
*but*  $y = 4$ .....Quad 2  
 $\therefore \sin P = \frac{4}{5}$   
9.2  $\tan P = \frac{4}{-3}$   
9.3  $\sin^2 P + \cos^2 P$   
 $= \left(\frac{4}{5}\right)^2 + \left(\frac{-3}{5}\right)^2$   
 $= \frac{16}{25} + \frac{9}{25} = 1$ 



# **Question 10**

10.1 
$$\tan \theta = \frac{6}{\sqrt{13}} = \frac{0}{A}$$
, but  $\theta$  in Quad 3  
∴ opposite side = -6  
and adjacent side =  $-\sqrt{13}$ 

$$r^{2} = (-6)^{2} + (-\sqrt{13})^{2}$$
  
$$\therefore r^{2} = 36 + 13 = 49$$
  
$$\therefore r = 7$$
  
$$\cos \theta = \frac{-\sqrt{13}}{7}$$







10.2 
$$7\sin\theta - 14\cos^2\theta$$

$$=7\left(\frac{-6}{7}\right) - 14\left(\frac{-\sqrt{13}}{7}\right)$$
$$=\frac{-68}{7}$$
$$\cos\theta \times \tan\theta$$

$$= \left(\frac{-\sqrt{13}}{7}\right) \times \left(\frac{-6}{-\sqrt{13}}\right)$$
$$= \frac{-6}{7}$$

10.3

11.1	$\cos ec\alpha = \frac{10}{6} = \frac{5}{3}$
11.2	$\sec \alpha = \frac{10}{8} = \frac{5}{4}$
11.3	$\cot \alpha = \frac{8}{6} = \frac{4}{3}$

# **Question 12**

12.1 
$$\cos ec75^{\circ} = \frac{1}{\sin 75^{\circ}} = 1,04$$
  
12.2  $\sec 149^{\circ} = \frac{1}{\cos 149^{\circ}} = -1,17$   
12.3  $\cot 256^{\circ} = \frac{1}{\tan 256^{\circ}} = 0,25$ 

# **Question 13**

13.1  $\sin 30^{\circ} + \cos 60^{\circ}$ 

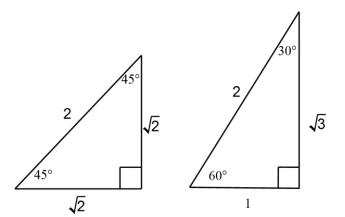
$$=\frac{1}{2}+\frac{1}{2}=1$$

13.2  $\cos 30^\circ \times \sec 30^\circ$ 

$$=\frac{\sqrt{3}}{2}\times\frac{2}{\sqrt{3}}=1$$

13.3  $\cos ec30^{\circ} \times \cos 60^{\circ} \times \cot 30^{\circ} \times \tan 45^{\circ}$ 

$$= \frac{2}{1} \times \frac{1}{2} \times \frac{\sqrt{3}}{1} \times \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{3}$$







14.1 
$$3\sin \theta - 2 = 0,3$$
  
 $3\sin \theta = 2,3$   
 $\div 3 \therefore \sin \theta = \frac{2,3}{3}$   
 $\therefore \theta = 50,1^{0}$   
14.2  $\tan 2\theta = 9$   
 $\therefore 2\theta = 83,66^{0}$   
 $\div 2 \therefore \theta = 41,83^{0}$   
14.3  $\cos ec\theta = 2,5$   
 $\cos ec\theta = \frac{1}{\sin \theta}$   
 $\therefore \frac{1}{\sin \theta} = 2,5$   
 $\therefore \sin \theta = \frac{1}{2,5}$   
 $\therefore \theta = 23,58^{0}$ 

# **Question 15**

$$y^{2} = 2^{2} - (-\sqrt{3})^{2}$$

$$y^{2} = 1$$

$$\therefore y = \pm 1$$

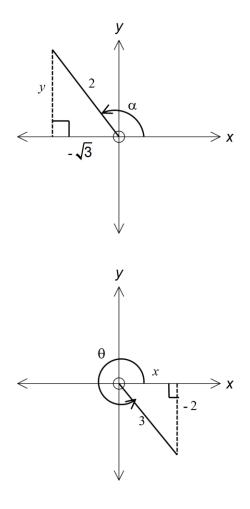
$$\therefore y = 1.....quad 2$$

$$\therefore \tan \alpha = \frac{1}{-\sqrt{3}}$$

# **Question 16**

16.1 
$$x^{2} = 3^{2} - (-2)^{2}$$
$$x^{2} = 9 - 4 = 5$$
$$\therefore x = \pm \sqrt{5}$$
$$\therefore x = \sqrt{5} \dots Quad 4$$
$$\cos \theta = \frac{\sqrt{5}}{3}$$
16.2 
$$\tan^{2} \theta \times \sin \theta$$

.2 
$$\tan^2 \theta \times \sin \theta$$
  
=  $\left(\frac{-2}{\sqrt{5}}\right)^2 \times \frac{-2}{3} = \frac{-8}{15}$ 







$$\cos 68^{\circ} = \frac{BP}{36} \therefore BP = 36 \cos 68^{\circ}$$
$$\therefore BP = 13, 49m$$
$$\sin 68^{\circ} = \frac{PD}{36} \therefore PD = 36 \sin 68^{\circ}$$
$$\therefore PD = 33, 38m$$
$$\tan 55^{\circ} = \frac{33, 38}{AP}$$
$$\times AP \qquad \therefore AP \tan 55^{\circ} = 33, 38$$
$$\div \tan 55^{\circ} \qquad \therefore AP = \frac{33, 38}{\tan 55^{\circ}} = 23, 37m$$
$$\therefore AB = AP + BP$$
$$= 23, 37 + 13, 49 = 36, 86m$$





## Acknowledgements

Mindset Learn Executive Head Content Manager Classroom Resources Content Coordinator Classroom Resources Content Administrator Content Developer Content Reviewer Dylan Busa Jenny Lamont Helen Robertson Agness Munthali Ronald Jacobs Malindri Eastes Helen Robertson

#### **Produced for Mindset Learn by Traffic**

Facilities Coordinator Production Manager Director

Editor Presenter Studio Crew

Graphics



Cezanne Scheepers Belinda Renney Alriette Gibbs Belinda Renney Nonhlanhla Nxumalo Lebo Mazibuko Abram Tjale James Tselapedi Wayne Sanderson

This resource is licensed under a <u>Attribution-Share Alike 2.5 South Africa</u> licence. When using this resource please attribute Mindset as indicated at<u>http://www.mindset.co.za/creativecommons</u>

