A Level Further Mathematics
Curriculum Overview 2020-2021

Core aims of the subject

“Mathematics expresses values that reflect the cosmos, including orderliness, balance, harmony, logic, and abstract beauty.” **Deepak Chopra**

Mathematics is a creative and highly inter-connected discipline that has been developed over centuries, providing the solution to some of history’s most intriguing problems. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject.

Our curriculum in mathematics aims to develop fluency in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that students develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately. We are also striving to allow students to reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, develop mathematical arguments and proofs and make conclusions based on logical inferences. Our intention is also for students to solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions; as such resilience is a crucial skill that we will be cultivating in students. Students also need to be able to utilise technology effectively, such as scientific calculators, to perform increasingly complex problems (as well having strong written and mental mathematical skills, not instead of). As the repertoire of mathematical skills that a student possesses grows increasingly more complex, so should the ability of students to use their mathematics to model real life situations.

At key stage 3, we promote equality by working through the breadth of the curriculum at the same pace for all students so that all students can achieve regardless of their starting point. All students at key stage 3 have the option of attending a number of lunchtime clubs in mathematics (Countdown club, 24 club, TT Rockstars club). We extend our highest attaining students through depth and more challenging problem solving, rather than an acceleration of content. At key stage 4, all students continue their mathematical studies on an appropriate GCSE pathway determined by their prior attainment and performance across key stages 2 and 3 to give them the best chance of achieving well in the subject. The GCSE curriculum is also supplemented for the most able students by offering Additional Maths to inspire, motivate and support students in their transition to key stage 5; in all key stages we also enter our most able students into the annual UKMT maths challenge competition. We intend for a high proportion of our students to go on and study or use mathematics in some form post-16; this means that our key stage 4 curriculum needs to be broad enough to cater for students who will go on to study maths at the highest level in our Further Mathematics and STEP preparation classes to those who will study Core Mathematics to complement their A Level choices.

In addition to the obvious intention of preparing students for more advanced mathematical studies, mathematics also prepares students for future learning in other disciplines and improves the cognitive ability in general of students and, as such, is essential for their personal development. The study of mathematics conditions the brain to see connections and builds neural pathways that make the brain stronger for all other things. Mathematical study enhances students' general intelligence and supports the life-long learning of students by: creating a framework in the brain for systematic thinking, developing the ability to solve and analyse problems, stretching the mind to work on unfamiliar tasks with confidence, developing the sequencing skills critical to arriving at accurate results or logical conclusions, promoting caution and care in thinking and deciphering complex mathematical problems to arrive at an accurate answer and learning through trial and error to integrate different principles to arrive at a logical conclusion.

In addition to these disciplinary aspects of the mathematics curriculum, the actual mathematical knowledge and skills that students learn are also vitally important in allowing students to achieve elsewhere in school. Topics studied in mathematics are prerequisite for several disciplines across key stages 3, 4 and 5 such as geography, psychology and economics (to name only a few). Mathematics also provides a theoretical springboard for the ever-evolving STEM sector. Nationally, there are huge shortfalls in job applicants with strong STEM skills and reports estimate that the cost of this shortfall is £1.5 billion each year. Furthermore, occupations in the STEM sector are growing at a rate that nearly is nearly double other sectors which could see this shortfall exacerbated. Our intention is to develop students' abilities sufficiently so that they are able to rise to the challenging opportunities this sector has to offer. In the 21st century science, technology and engineering are constantly changing and have become increasingly important for society. The mathematical principles that govern these areas, however, have not changed and consequently the breadth and depth of our curriculum aims to future-proof our students in this field. This is all in addition to the inspirational and motivating research that claims that candidates with strong maths skills earn on average 11% more in their lifetime.

Mathematics is a discipline which is universal; transcending language and cultural differences. Throughout its rich history, mathematics has adopted elements from around the world and gives students the opportunity to appreciate fundamental truths and create water-tight arguments based on logic and reasoning; as such it helps contribute to the student's spiritual, moral, spiritual and cultural development.

Ultimately, the intention of the maths curriculum is to provide students with the necessary thinking skills and content to be successful in their next stage of life or education.

Assessment

Each chapter will be assessed by an end of unit test. More formal assessment happens at the end of year 12 and the end of the Autumn term in year 13 where students will sit a full mock exam. Informal assessment happens in lesson through questioning and circulating the classroom. A lot of work at Key Stage 5 is self-assessed as students have access to answers and solutions to the vast majority of questions that they answer. Homework is also used to assess students throughout each unit (see below).

Homework

Students receive a weekly DrFrostMaths homework using the platform listed below. The website automatically marks student work and staff and students can leave each other comments to ask questions/ provide feedback as needed.
Each homework focuses on a specific topic with questions increasing in difficulty through the homework.
Students are also expected to finish exercises started in lessons and complete mixed exercises at the end of each unit. These tasks may not be assessed by the teacher, but the expectation is that these will be completed in students' study periods.

Clubs and/or intervention

Drop-in sessions run each week
UKMT Mentoring runs each week

Parental/Carer support

Parents can check homework online to keep up to date with student work.
Encourage students to attend drop-in sessions

Connections to Learning

Mathematics is a highly inter-connected discipline. A Level Further Mathematics has pure mathematics content which can be divided into the following strands: algebra, linear algebra, coordinate geometry and calculus. A Level Further Mathematics also mechanics content which is an application of the pure mathematics content. These strands can be thought of as symbiotic; advances in one strand allow for further development of the others. Consequently, the connections between the various strands of the mathematics curriculum are myriad and one of the most enjoyable aspects of mathematics is studying seemingly separate areas of study and then making links between them and seeing how these areas interact. In mathematics lessons, depth of understanding is prioritised. This involves taking the time to see how a particular topic links to the other topics that have already been studied. As a consequence of the need to understand all of these various connections within mathematics, students should expect to be given regular opportunity to review prior learning as students will struggle with new concepts if they have not developed fluency with previous concepts.

Below shows the progression of the different strands of A Level mathematics. While these topic areas have been presented as separate strands they should definitely not be viewed as mutually exclusive. As mentioned above each topic area in a given strand links to topics in the other strands. In key stage five, there is an increased emphasis on problem solving and mathematical modelling.

Algebra (The following chapters from A Level Further Mathematics build on the ideas and concepts met in the algebra strand of GCSE mathematics and of A Level Mathematics):
Chapter 1 Complex Numbers
Chapter 3 Series
Chapter 4 Roots of Polynomials
Chapter 8 Proof by Induction
Chapter 1 Complex Numbers
Chapter 2 Series
Chapter 6 Hyperbolic Functions
Chapter 4 Inequalities
Chapter 5 T-formulae

Linear Algebra(The following chapters from A Level mathematics build on the ideas and concepts met in the algebra and geometry strands of GCSE mathematics and of the coordinate geometry strand of A Level Mathematics):
Chapter 6 Matrices
Chapter 7 Linear Transformations
Chapter 9 Vectors
Chapter 1 Vectors

Coordinate Geometry (The following chapters from A Level Further Mathematics build on the ideas and concepts met in the algebra strand of GCSE mathematics and the coordinate geometry strand of A Level Mathematics):
Chapter 2 Argand Diagrams
Chapter 5 Polar Coordinates
Chapter 1 Conic Sections 1
Chapter 2 Conic Sections 2

Calculus (The following chapters from A Level Further Mathematics build on the ideas and concepts met in the algebra strand of GCSE mathematics as well as the coordinate geometry and calculus strands of A Level mathematics):
Chapter 5 Volumes of Revolution
Chapter 3 Methods in Calculus
Chapter 4 Volumes of Revolution
Chapter 7 Methods in Differential Equations
Chapter 8 Modelling with Differential Equations
Chapter 6 Taylor Series
Chapter 7 Methods in Calculus
Chapter 8 Numerical Methods
Chapter 9 Reducible Differential Equations

Mechanics (The following chapters from A Level Further Mathematics build on the ideas and concepts met in the mechanics strand of A Level mathematics):
Chapter 1 Momentum and Impulse
Chapter 2 Work, Energy and Power
Chapter 3 Elastic Springs and Strings
Chapter 4 Elastic Collisions in One Dimension
Chapter 5 Elastic Collisions in Two Dimension

Connections to Future Pathways

Studying maths helps to develop skills in logical thinking, analysis, problem-solving, decision-making and communication, which are valued by employers across many job sectors. Furthermore, mathematical careers are in every business and industry throughout every sector of the economy. Mathematics may not be the central focus of all professions, but it can serve as critical building blocks of a larger and more meaningful whole. Mechanical engineers, for example, work with numbers for the design and production of all types of simple and complex machines. Actuaries use numbers to calculate and assess the consequences of financial risk. And economists analyse and interpret quantitative data to discern macro- and micro-economic patterns. Banking is a world of numbers and mathematics is used in the way accounts are handled, for calculating interest rates and for determining credit scores.

Data - Again, big data plays a major role in the increased demand for skilled data scientists. It is the job of data scientists to immerse themselves in the ocean of big data, bringing structure to it that, in turn, allows for effective analysis of that data. Many employers rate the ability to handle data very highly.

Number – Banking, Accountancy and Finance. For example, Accountants examine financial records and prepare financial documents for businesses, nonprofits, firms and individuals. They are responsible for the accuracy of the documents they create and for making sure that taxes are paid on time.

Geometry – Architecture, Civil Engineering and Astronomers. Geometry is used in astronomy in many, many ways. One of the most common uses, however, is the use of geometry to find the distance between celestial objects, such as stars and planets. ... But other uses of geometry include measuring the speed and velocity of planets orbiting other stars.

Algebra – Air Traffic Controllers, Video Game Designers and Economists. Air traffic controller uses math in order to be able to understand distances and measurements at a moment's notice. They also must be able to do mental math quickly and accurately. Part of their job is directing aircraft at what altitude and speed to fly. For example, air traffic controllers frequently need to calculate the minimum safe level for planes to fly at. To do this they use the equation:

Minimum safe level (measured in feet) = 30 × (1013 - pa)

(*pa* is the atmospheric pressure. This value can change daily, depending on weather systems.)

Other Careers:

• Logistics specialist • Control statistician • Systems operation analyst • Robotics analyst • Actuary • Insurance underwriter • Operations research analyst • Technical mathematical modeller • Financial analyst • Business metrics analyst • Big data analyst • Marketing consultant • Claims adjuster • Database administrator • Cryptographer

The most common destinations for our further mathematicians are: maths degree, physics degree, engineering degree or computer science degree.

Helpful sources of information

https://www.drfrostmaths.com/ provides a lot of the Powerpoint slides and resources that are used in lessons, this is also the platform where students complete their homework.
https://www.examsolutions.net/ and https://www.physicsandmathstutor.com/ are very helpful websites to help students prepare for exams
https://www.pearsonactivelearn.com/ has electronic access to the course textbooks (students will need a login from their class teacher to access these)
https://integralmaths.org/ has a lot of support for students and contains assessments (as well as revision notes) for each unit across years 12 and 13
https://hegartymaths.com/ has video tutorials and quizzes on all of the prerequisite skills from key stages 3 and 4
https://www.desmos.com/ is a free graphing tool which students will find helpful to aid coordinate geometry problems; there is also an app for this that they can install on their phone
https://maths.org/step/ provides resources to help students to prepare for STEP

Year 12 Overview

Term	Knowledge	Assessment
Autumn 1	Chapter 1 Complex Numbers This chapter introduces students to the concept of imaginary and complex numbers which allows their algebraic skills to handle more difficult equations which they previously thought had no solutions. Chapter 2 Argand Diagrams Building on chapter 1, this chapter allows students to see the graphical representation of complex numbers. The topic of loci from GCSE is revisited here in a much more formal and challenging setting. Chapter 6 Matrices The matrix is a new mathematical object for students and has its own rules which challenges some of the assumptions that students have developed so far (such as multiplication being commutative). Once familiarity has been established with matrices they are used to solve systems of simultaneous equations.	

<table>
<tr><td></td><td colspan="2">Chapter 7 Linear Transformations
This chapter builds on chapter 6 and uses matrices to perform transformations such as reflections, rotations and enlargements. These are ideas met at GCSE but they are made much more formal by using matrices. Transformations knowledge is also extended into 3-dimensions.</td></tr>
<tr><td></td><td>➢ Add, subtract and multiply conformable matrices.
➢ Multiply a matrix by a scalar.
➢ Understand and use zero and identity matrices.
➢ Use matrices to represent linear transformations in 2-D.
➢ Successive transformations.
➢ Single transformations in 3-D.
➢ Find invariant points and lines for a linear transformation.
➢ Calculate determinants of 2 x 2 and 3 x 3 matrices and interpret as scale factors, including the effect on orientation.
➢ Understand and use singular and non-singular matrices.
➢ Properties of inverse matrices.
➢ Calculate and use the inverse of non-singular 2 x 2 matrices and 3 x 3 matrices.
➢ Solve three linear simultaneous equations in three variables by use of the inverse matrix.
➢ Interpret geometrically the solution and failure of solution of three simultaneous linear equations
➢ Solve any quadratic equation with real coefficients.
➢ Solve cubic or quartic equations with real coefficients.
➢ Add, subtract, multiply and divide complex numbers in the form x +iy with x and y real.
➢ Understand and use the terms ‘real part’ and ‘imaginary part’.
➢ Understand and use the complex conjugate.
➢ Know that non-real roots of polynomial equations with real coefficients occur in conjugate pairs.
➢ Use and interpret Argand diagrams.</td><td>➢ A baseline assessment will be carried out in September to highlight any areas from GCSE that need to be addressed (this is a different assessment from the A Level maths one).
➢ Each chapter will be assessed by an end of unit test. Informal assessment happens in lesson through questioning and circulating the classroom
➢ A weekly Dr Frost task is also set as homework to assess students.</td></tr>
</table>

	➢ Convert between the Cartesian form and the modulus-argument form of a complex number. ➢ Multiply and divide complex numbers in modulus argument form ➢ Construct and interpret simple loci in the argand diagram such as $\|z – a\| > r$ and $\arg (z – a) = \theta$	
Autumn 2	Chapter 3 Series This chapter introduces students to sigma notation for summations and the standard results for the sum of r, r^2 and r^3. This chapter develops students' algebraic manipulation skills in general. Chapter 4 Roots of Polynomials In this chapter, students study the link between the coefficients in quadratic, cubic, quartic equations and their solutions. This opens the possibility of greater problem solving regarding polynomials and a deeper understanding of the equations covered so far. Chapter 8 Proof by Induction Proof by induction is a difficult and important concept for further mathematicians. Here they will prove results involving divisibility, matrices and series by induction which also gives them chance to recap these topics in the process. Chapter 9 Vectors This chapter utilises vectors to give students a rigorous framework for thinking about coordinate geometry in 3-dimensions. The vector equation of a line is established, and this is extended to planes. Problem solving involving points of intersections between lines and planes is also covered and students should expect this to be a demanding part of the course.	
	➢ Understand and use the vector and Cartesian forms of an equation of a straight line in 3-D. ➢ Understand and use the vector and Cartesian forms of the equation of a plane. ➢ Calculate the scalar product and use it to express the equation of a plane, and to calculate the angle between two lines, the angle between two planes and the angle between a line and a plane. ➢ Check whether vectors are perpendicular by using the scalar product.	➢ Each chapter will be assessed by an end of unit test. Informal assessment happens in lesson through questioning and circulating the classroom ➢ A weekly Dr Frost task is also set as homework to assess students.

	➢ Find the intersection of a line and a plane. Calculate the perpendicular distance between two lines, from a point to a line and from a point to a plane. ➢ Understand and use formulae for the sums of integers, squares and cubes and use these to sum other series. ➢ Understand and use the relationship between roots and coefficients of polynomial equations up to quartic equations ➢ Form a polynomial equation whose roots are a linear transformation of the roots of a given polynomial equation (of at least cubic degree). ➢ Construct proofs using mathematical induction. Contexts include sums of series, divisibility and powers of matrices. ➢ Cartesian and parametric equations for the parabola and rectangular hyperbola ➢ The focus-directrix properties of the parabola ➢ Tangents and normals to these curves. ➢ Loci problems	
Spring 1	Chapter 1 Vectors (Further Pure 1) Extending the work covered on vectors in Core Pure 1, this chapter introduces students to the cross product and its geometric interpretation. The cross product is utilised to find areas and volumes in 3D space, giving students a more efficient manner of tackling these problems. Chapter 2 Conic Sections 1 (Further Pure 1) Prior to this chapter, students will have made familiar with the parabola and hyperbola. In this chapter, these two graphs (and their respective equations) are looked at in much more depth including their parametric equations, foci and directrices. Chapter 5 Volumes of Revolution This chapter extends the calculus work covered in A Level Mathematics which uses integration to calculate an area under a curve. Here, students will learn how to calculate the volume generated when a solid is rotated 360^{o} around either axis. Chapter 5 T-formulae (Further Pure 1)	

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<tr><td></td><td colspan="2">Complementing the work covered in A Level Mathematics, this chapter introduces students to the substitution $t = \tan\frac{\theta}{2}$ and learn how this can be used to rewrite the six trigonometric ratios. Students then explore strategies for solving equations and proving trigonometric identities using this substitution.

Chapter 1 Momentum and Impulse (Further Mechanics 1)
This chapter looks at mathematical formulae for momentum and impulse which requires skills from the kinematics chapter in A Level Mathematics; conservation of momentum is also covered which is an idea students may have met in GCSE Physics. These two ideas are crucial to the subsequent work in Further Mechanics 1, especially the two chapters on collisions.</td></tr>
<tr><td></td><td>➢ Loci problems with conic sections
➢ The vector product a × b of two vectors.
➢ The scalar triple product a.(b × c)
➢ Volumes using the scalar triple product
➢ Momentum and impulse. The impulse momentum principle.
➢ The principle of conservation of momentum applied to two spheres colliding directly.
➢ Derive formulae for and calculate volumes of revolution
➢ The trigonometric ratios sec, cosec and cot in terms of sin, cos and tan
➢ The t-formulae
➢ Applications of t-formulae to trigonometric identities
➢ Applications of t-formulae to solve trigonometric equations
➢</td><td>➢ Each chapter will be assessed by an end of unit test. Informal assessment happens in lesson through questioning and circulating the classroom
➢ A weekly Dr Frost task is also set as homework to assess students.</td></tr>
<tr><td>Spring 2</td><td colspan="2">Chapter 2 Work, Energy and Power (Further Mechanics 1)
A challenging chapter where problems are solved using ideas such as conservation of energy and the work-energy principle. The problems in this chapter draw on several different areas of the Mathematics and Further Mathematics curricula. Students studying Physics should find this complements their work very well.

Chapter 4 Elastic Collisions in One Dimension (Further Mechanics 1)
Building on the work covered on momentum and impulse, this chapter introduces students to Newton’s Law of Restitution. This is a rule that allows for the bounciness of two particles to be taken into consideration when two particles collide. Again, the problem solving demands are high here and students will need skills from elsewhere in the curriculum to tackle them.

Chapter 4 Inequalities (Further Pure 1)</td></tr>
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	This chapter extends the inequalities work in A Level Mathematics to include rational functions. The process of *'cross multiplication'* is challenged here as students learn that this method does not work for inequalities and alternative methods are developed. Chapter 8 Numerical Methods (Further Pure 1) Numerical methods are largely procedural concepts where students need to understand and apply algorithms to solve equations. Here the focus is on finding the solution to differential equations without using the formal calculus techniques which will be covered in year 13.	
	➢ The manipulation and solution of algebraic inequalities and inequations ➢ Numerical solution of first order and second order differential equations ➢ Euler's method ➢ Mid-point method ➢ Direct impact of elastic spheres. ➢ Newton's law of restitution. ➢ Loss of kinetic energy due to impact. ➢ Successive direct impacts of spheres and/or a sphere with a smooth plane surface ➢ Inclined planes and friction ➢ Kinetic and potential energy, work and power. ➢ The work energy principle. ➢ The principle of conservation of mechanical energy.	➢ Each chapter will be assessed by an end of unit test. Informal assessment happens in lesson through questioning and circulating the classroom ➢ A weekly Dr Frost task is also set as homework to assess students.
Summer 1	A Level Mathematics In this half term, students cover differentiation and integration from the year 13 A Level Mathematics Curriculum which are prerequisite for the year 13 Further Mathematics course.	
	The content covered in this half term is from A Level maths but is prerequisite for the A Level Further Maths content for year 13 ➢ Differentiation using the chain, quotient and product rule ➢ Differentiating trigonometric, exponential and logarithmic functions ➢ Integrating functions of linear functions f(ax+b) ➢ Integrating trigonometric functions	➢ Each chapter will be assessed by an end of unit test. Informal assessment happens in lesson through questioning and circulating the classroom ➢ A weekly Dr Frost task is also set as homework to assess students.

		➢ And end of year 12 will be sat which will comprise two papers: one pure paper and one mechanics and statistics paper. These papers will be based on past A Level Mathematics papers.
Summer 2	Chapter 6 Hyperbolic Functions A brief introduction to the hyperbolic functions before the summer holidays. Here students see the parallels between the hyperbolic and trigonometric functions and begin to see their similarities and differences. Chapter 3 Conic Sections 2 (Further Pure 1) This chapter introduces students to the concept of eccentricity of a graph and this is used to formalise the work covered in Conic Sections 1 as well as extend this to the ellipse.	
	➢ Parametric differentiation ➢ Implicit differentiation ➢ Integration using reverse chain rule ➢ Integration by substitution ➢ Integration by parts ➢ Integration using partial fractions ➢ Understand the definitions of hyperbolic functions sinh x, cosh x and tanh x, including their domains and ranges, and be able to sketch their graphs. ➢ Cartesian and parametric equations for the parabola and rectangular hyperbola, ellipse and hyperbola.	➢ Each chapter will be assessed by an end of unit test. Informal assessment happens in lesson through questioning and circulating the classroom ➢ A weekly Dr Frost task is also set as homework to assess students.

Year 13 Overview

Term	Knowledge	Assessment
Autumn 1	Chapter 6 Hyperbolic Functions Continuing the work covered before summer, here students will learn to solve equations using hyperbolics as well as differentiate and integrate them.	

	Chapter 3 Conic Sections 2 (Further Pure 1) Continuing the work covered before summer, here students will progress with their coordinate geometry skills solving increasingly challenging problems involving the hyperbola and ellipse. Chapter 5 Elastic Collisions in Two Dimensions (Further Mechanics 1) This chapter builds on the collisions work in year 12 but extends to oblique collisions (collisions which happen at an angle). This is a challenging part of the course and students should expect to utilise skills from across the curriculum to solve the problems they are given. Strong trigonometry and strong algebra skills are definitely required here in addition to understanding the new content. Chapter 1 Complex Numbers In this chapter, students are introduced to the exponential form of a complex number and see various applications of this such as nth roots and deriving trigonometric identities.	
	➢ Applications of vectors to three-dimensional geometry involving points, lines and planes. ➢ The equation of a line in the form (r – b) × b = 0 ➢ Direction ratios and direction cosines of a line ➢ Oblique impact of smooth elastic spheres and a smooth sphere with a fixed surface. ➢ Loss of kinetic energy due to impact. ➢ Successive oblique impacts of a sphere with smooth plane surfaces. ➢ The definitions of hyperbolic functions sinh x, cosh x and tanh ➢ The domains and ranges of hyperbolic functions and their graphs. ➢ The inverse hyperbolic functions and their domains and ranges. ➢ Cartesian and parametric equations for the ellipse and hyperbola. ➢ The focus-directrix properties of the ellipse and hyperbola, including the eccentricity. ➢ Applications of t-formulae to solve trigonometric equations ➢ Modelling with t-formulae ➢ Representing complex numbers in exponential form	➢ Each chapter will be assessed by an end of unit test. Informal assessment happens in lesson through questioning and circulating the classroom ➢ A weekly Dr Frost task is also set as homework to assess students.

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- DeMoivre's Theorem and its proof
- Deriving trigonometric identities using DeMoivre's Theorem

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<tr><td>**Autumn 2**</td><td colspan="2">Chapter 3 Elastic Springs and Strings (Further Mechanics 1)
In this chapter, students look at Hooke's Law which describes how the tension in an extensible string is linked to the extension of the string. This is then used to tackle more challenging work, energy and power questions by considering the elastic potential energy as well as the gravitational potential and kinetic energy.

Chapter 7 Methods in Differential Equations
Students learn the method of integrating factor for first order differential equations and the complementary function, particular integral approach to a second order differential equation. This is an extremely important chapter for further mathematicians who are going on to study maths or physics at university.

Chapter 9 Reducible Differential Equations (Further Pure 1)
Extending the work covered in the previous chapter, here students develop their calculus skills to allow them to cope with non-linear differential equations by introducing a substitution.</td></tr>
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- Differentiate and integrate hyperbolic functions.
- Integrate functions of the form $\frac{1}{\sqrt{x^2+a^2}}$ and $\frac{1}{\sqrt{x^2-a^2}}$ and be able to choose substitutions to integrate associated functions.
- Using DeMoivre's Theorem to find sums of series
- Calculating nth roots of complex numbers
- Applying complex numbers to geometric problems.
- Elastic strings and springs.
- Hooke's law.
- Energy stored in an elastic string or spring.
- Problem solving with elastic strings and springs
- Find and use an integrating factor to solve differential equations of form dy/dx + P(x)y = Q(x) and recognise when it is appropriate to do so.
- Find both general and particular solutions to differential equations.
- Use differential equations in modelling in kinematics and in other contexts.
- Solve differential equations of form y″ + ay′ + by = 0

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- Each chapter will be assessed by an end of unit test. Informal assessment happens in lesson through questioning and circulating the classroom
- A weekly Dr Frost task is also set as homework to assess students.

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	➢ Solve differential equations of the form y'' + ay' + by=f(x) and recognise the form of the particular integral needed ➢ Differential equations reducible by means of a given substitution. ➢	
Spring 1	Chapter 5 Polar Coordinates This chapter introduces an alternative to Cartesian geometry by describing points as (r, θ) rather than (x, y). All previous Cartesian skills (such as coordinate geometry, differentiation, integration) are revisited using polar coordinates instead. Chapter 3 Methods in Calculus An opportunity to look at some nice calculus results which do not fit into A Level mathematics. Here students look at improper integrals, the mean value theorem and more sophisticated use of partial fractions. Chapter 8 Modelling with Differential Equations Revisiting differential equations from before Christmas, here the skills learnt are applied to specific examples such as simple harmonic motion and damped oscillations. Chapter 5 Volumes of Revolution This is a repeat of the chapter on volumes in year 12. Here the integrals required are more challenging and functions defined parametrically are also covered.	
	➢ Understand and use polar coordinates and be able to convert between polar and Cartesian coordinates. ➢ Sketch curves with r given as a function of θ, including use of trigonometric functions. ➢ Find tangents to polar curves ➢ Find the area enclosed by a polar curve. ➢ Evaluate improper integrals where either the integrand is undefined at a value in the range of integration or the range of integration extends to infinity. ➢ Understand and evaluate the mean value of a function. ➢ Integrate using partial fractions ➢ Differentiate inverse trigonometric functions.	➢ Each chapter will be assessed by an end of unit test. Informal assessment happens in lesson through questioning and circulating the classroom ➢ A weekly Dr Frost task is also set as homework to assess students.

	➢ Integrate functions of the form $\frac{1}{\sqrt{a^2-x^2}}$ and $\frac{1}{x^2+a^2}$ and be able to choose trigonometric substitutions to integrate associated functions ➢ Solve the equation for simple harmonic motion $\ddot{x} = -\omega^2 x$ and relate the solution to the motion. ➢ Model damped oscillations using second order differential equations and interpret their solutions. ➢ Analyse and interpret models of situations with one independent variable and two dependent variables as a pair of coupled first order simultaneous equations and be able to solve them, for example predator-prey models. ➢ Volumes of revolution in parametric form	
Spring 2	Chapter 7 Methods in Calculus (Further Pure 1) A chapter of three separate calculus techniques which can be used to simplify the process of repeatedly applying the product rule, finding limits of functions and performing integrals using the t-formula. Chapter 2 Series This chapter introduces students to the method of differences for finding a summation and also introduces students to Maclaurin Series which is a more general series expansion than the binomial series which students met in A Level Mathematics. Chapter 6 Taylor Series This chapter extends understanding from the previous chapter on Maclaurin series to a Taylor series. This allows students to choose a different point to centre the expansion around besides the origin (the drawback of using Maclaurin series expansion). Chapter 4 Inequalities (Further Pure 1) Revisiting inequalities, here students conclude their inequalities work by solving inequalities graphically which involve the modulus function. A good understanding of curve sketching is required to be successful in this chapter. Chapter 8 Numerical Methods (Further Pure 1) Concluding the work started previously on numerical methods, here students look at Simpson's Rule as a method for finding the area under a curve as an alternative to the trapezium rule.	

	➢ Leibniz's Theorem ➢ L'hopital's Rule ➢ The Weierstrauss substitution for integration ➢ Find the Maclaurin series of a function including the general term. ➢ Recognise and use the Maclaurin series for ex , ln(1+ x) , sin x , cos x and (1+ x) ^{n} , and be aware of the range of values of x for which they are valid ➢ Derivation and use of Taylor series. ➢ Use of series expansions to find limits ➢ Use of Taylor series method for series solution of differential equations ➢ Inequalities involving moduli ➢ Simpson's Rule ➢ Understand and use the method of differences for summation of series including use of partial fractions	➢ Each chapter will be assessed by an end of unit test. Informal assessment happens in lesson through questioning and circulating the classroom ➢ A weekly Dr Frost task is also set as homework to assess students.