A Level
Maths

Bronze Set A, Paper 1 (Edexcel version)

Question	Solution	Partial Marks	Guidance
1 (a)	$f(-2)=(-2)^3-3(-2)+2=-8+6+2=0$, therefore $(x+2)$ is a factor	M1 A1 **[2]**	Substitutes ±2 into f **with substitution explicitly shown** Shows f(–2) = 0 and gives a conclusion, i.e. 'therefore, (x + 2) is a factor', 'qed', 'as required', '□', etc.
1 (a) ALT	$$\begin{array}{r} x^2-2x+1 \\ x+2\overline{)x^3+0x^2-3x+2} \\ \underline{x^3+2x^2} \quad \downarrow \\ -2x^2-3x \\ \underline{-2x^2-4x} \quad \downarrow \\ x+2 \\ \underline{x+2} \\ 0 \end{array}$$ therefore (x + 2) is a factor	M1 A1 **[2]**	Attempts long division. Must obtain x^2 term in quotient and $\pm 2x^2$ as first remainder Completely correct long division, obtaining 0 as the remainder, and gives a conclusion, i.e. 'therefore, (x + 2) is a factor', 'qed', 'as required', '□', etc.
1 (b)	Other factor is $x^2 - 2x + 1$ $x^2-2x+1=(x-1)^2$ So $f(x)=(x+2)(x-1)^2$ or $f(x) = (x + 2)(x - 1)(x - 1)$	B1 M1 A1 **[3]**	Correct other factor. **<u>This can be scored in 1(a)</u>**. Attempts to factorise their quadratic factor. M0 if their factor cannot be factorised Express f(x) correctly as a product of linear factors. Must see f(x) = … . Condone f = … . Answer only is 3/3
1 (c)	$3x^2+2x-8=(3x-4)(x+2)$ $\dfrac{3x^2+2x-8}{x^3-3x+2}=\dfrac{(x+2)(3x-4)}{(x+2)(x-1)^2}=\dfrac{3x-4}{(x-1)^2}$	B1 M1 A1 oe **[3]**	Correct factorisation of numerator. Seen or implied Expresses numerator and denominator in terms of linear factors. Allow ft of their 1(b). Correct simplification oe

2 (a)	$\lvert 2x-1\rvert<1 \Leftrightarrow -1<2x-1<1$ So $0<x<1$	M1 A1 **[2]**	States inequality is equivalent to $-1 < 2x - 1 < 1$. Give the M1 for considering $2x - 1$ (=, >, <) ±1 and 'gluing' solutions together Obtains correct solution set. Final answer Answer only is 2/2
2 (b)	$9\left(2^x-1\right)^2=2^{2x}$ OR $3\left(2^x-1\right)=\pm\, 2^x$ $8(2^{2x})-18(2^x)+9=0$ $\Rightarrow 2^x=\frac{3}{2},\ 2^x=\frac{3}{4}$ $\Rightarrow x\log(2)=\log\left(\frac{3}{2}\right),\quad x\log(2)=\log\left(\frac{3}{4}\right)$ $\Rightarrow x=0.585,\quad -0.415$	M1 A1 M1 A1 **[4]**	Attempts to square both sides OR states or implies a set of non-modular equations Obtains both correct values of 2^x Complete method to solve an equation of the form $2^x = k$, $k > 0$. No marks for solving an oversimplified equation, e.g. $2^x = 1, 4, ½$ No marks for solving an equation that reduces to another base, e.g. $2^{2x} = k$, $2^{3x} = k$, etc., but allow the M1 for an equation of the form e.g. $2^{x+1} = k$ Correct values of x. Cao Do not ISW if candidate goes on to reject one of the values **SC1:** allow B1 B1 for each value of 2^x if this leads to a better mark e.g. only considers $3(2^x - 1) = 2^x$, leading to $2^x = 3/2$ can score B1 B0 M1 A0
3 (a)	$4x^2+8x+4y^2-12y=k$ $\Rightarrow x^2+2x+y^2-3y=\frac{k}{4}$ $\Rightarrow (x+1)^2-1+\left(y-\frac{3}{2}\right)^2-\frac{9}{4}=\frac{k}{4}$ $\left(\Rightarrow (x+1)^2+\left(y-\frac{3}{2}\right)^2=\frac{k+13}{4}\right)$ So centre of C is $(-1, 1.5)$	M1 A1 A1ft **[3]**	Obtains coefficients of x^2 and y^2 to be the same and attempts to complete the square on either the x or y terms Obtains terms $(x + 1)^2$ and $(y - 1.5)^2$ in their expression. Ignore other terms Correct centre ft their $(x + 1)^2$ and $(y - 1.5)^2$. Accept $x = -1$, $y = 1.5$ instead of coordinate form.

3 (b)	$\dfrac{k+13}{4}>0$ $\Rightarrow k+13>0$ $\Rightarrow k>-13$	M1 A1 **[2]**	Sets **their** 'radius term' > 0 Correct range of values of k. Final answer
3 (c)	Coefficients of x^2 and y^2 not the same / it is the equation of an ellipse	B1 **[1]**	An explanation for why the equation does not define a circle
4	$\dfrac{1}{2}(4+2\sqrt{3})(BC)=8+5\sqrt{3}$ $BC=\dfrac{16+10\sqrt{3}}{4+2\sqrt{3}}$ $=\dfrac{(16+10\sqrt{3})(4-2\sqrt{3})}{(4+2\sqrt{3})(4-2\sqrt{3})}$ $=\dfrac{64-32\sqrt{3}+40\sqrt{3}-20(3)}{16-4(3)}$ $=\dfrac{4+8\sqrt{3}}{4}$ $=1+2\sqrt{3}$ Hence, $AC^2=(4+2\sqrt{3})^2+(1+2\sqrt{3})^2$ $\Rightarrow AC^2=16+16\sqrt{3}+12+1+4\sqrt{3}+12$ $\Rightarrow AC^2=41+20\sqrt{3}$ $\Rightarrow AC=\left(41+20\sqrt{3}\right)^{\frac{1}{2}}$	M1(*) M1(dep*) A1 A1 M1(dep*) A1 **[6]**	Sets up an equation involving BC using the area of a triangle. Allow omission of the ½ here Complete method to find exact simplified value of BC by rationalising denominator Obtains numerator of 4 + 8√3 or denominator of 4 Correct simplified exact value of BC Uses Pythagoras with their BC to find AC or AC^2. Their BC must be greater than 0 for this mark Correct exact length of the hypotenuse in the required form obtained convincingly. Allow values of a and b stated instead of final answer. Also condone √ instead of power of ½. Final answer

5 (a)	$\frac{d}{dx}(x^3-2x)=\lim_{h\to 0}\frac{(x+h)^3-2(x+h)-x^3+2x}{h}$ $=\lim_{h\to 0}\frac{x^3+3x^2h+3xh^2+h^3-2x-2h-x^3+2x}{h}$ $=\lim_{h\to 0}\frac{3x^2h+3xh^2+h^3-2h}{h}$ $=\lim_{h\to 0}\left(3x^2+3xh+h^2-2\right)$ $=3x^2-2$	M1* M1(dep*) A1 **[3]**	Considers $\frac{(x+h)^3-2(x+h)-x^3+2x}{h}$. No need to consider limit here. Allow use of other symbols for h Expands numerator correctly and attempts to collect like terms. Allow one slip. Complete and convincing proof with correct limiting process seen. No errors allowed.
5 (a) ALT	$\frac{d}{dx}(x^3-2x)=\lim_{x'\to x}\frac{x'^3-2x'-x^3+2x}{x'-x}$ $=\lim_{x'\to x}\frac{x'^3-x^3-2(x'-x)}{x'-x}$ $=\lim_{x'\to x}\frac{(x'-x)(x'^2+x'x+x^2-2)}{x'-x}$ $=\lim_{x'\to x}\left(x'^2+x'x+x^2-2\right)$ $=x^2+x^2+x^2-2$ $=3x^2-2$	M1* M1(dep*) A1 **[3]**	Considers $\frac{x'^3-2x'-x^3+2x}{x'-x}$. No need to consider limit here. Allow use of other symbols (use of x and c is also common) Extracts a factor of $x'-x$ Complete and convincing proof with correct limiting process seen. No errors allowed.
5 (b)	Stationary points are where $3x^2-2=0$ $\Rightarrow x=\pm\sqrt{\frac{2}{3}}$ $\Rightarrow$ stationary points at $\left(\sqrt{\frac{2}{3}},-\frac{4\sqrt{6}}{9}\right)$, $\left(-\sqrt{\frac{2}{3}},\frac{4\sqrt{6}}{9}\right)$	M1 A1ft A1 **[3]**	Sets their derivative or the correct derivative = 0. Allow correct derivative = 0 even if their answer to (a) is different Correct x coordinates of stationary points ft their derivative. Allow decimals Correct coordinates of stationary points. Allow decimals. Allow if answers not given in coordinate form, i.e. in the form $x = \ldots, y = \ldots$

5 (c)	$a=\sqrt{2}$	B1	Correct value of *a* seen or implied
	$\int_0^{\sqrt{2}}(x^3-2x)dx=\frac{1}{4}x^4-x^2\Big\vert_0^{\sqrt{2}}$	M1* A1	Attempts indefinite integration of the equation of the curve Correct indefinite integration of the equation of the curve
	$=\frac{1}{4}(\sqrt{2})^4-(\sqrt{2})^2-0$ $=-1$	M1(dep*)	Substitutes limits into their integral in the correct order (ft their *a*)
	So the area of the shaded region is 1	A1 **[5]**	Correct area of the shaded region. Final answer
6 (a)	$4\times\frac{1}{2}\left(\frac{a}{4}\right)^2\pi=\frac{1}{2}a^2\theta-\frac{1}{2}a^2\sin\theta$	M1	Forms an equation using the information. Allow the incorrect radius to be substituted on the LHS. Allow other expressions for the area of a triangle
		A1	Correct expression
	$2\times\frac{a^2}{16}\pi=\frac{1}{2}a^2\theta-\frac{1}{2}a^2\sin\theta$ $\frac{1}{2}\theta=\frac{1}{8}\pi+\frac{1}{2}\sin\theta$ $\Rightarrow\theta=\frac{1}{4}\pi+\sin\theta$	A1 **[3]**	Complete and convincing proof with no errors seen **NB: Candidates must work in *a* for the A marks, so working in *r* is not a misread and gets M1 A0 A0 unless recovered.**
6 (b)	$\mathrm{f}(\theta)=\theta-\frac{1}{4}\pi-\sin\theta$		OR $\mathrm{f}(\theta)=\frac{1}{4}\pi+\sin\theta-\theta$
	$\mathrm{f}(1)=1-\frac{1}{4}\pi-\sin(1)=-0.626...<0$ $\mathrm{f}(2)=2-\frac{1}{4}\pi-\sin(2)=0.305...>0$	M1	Substitutes 1 and 2 into the function Use of degrees is M0
	Since there has been a change of sign, θ must lie between 1 and 2	A1	Both values correct with sufficient argument. Sufficient argument includes either a statement that there is a change of sign OR > 0 and < 0

<table>
<tr><td>6 (b) ALT</td><td>$f(\theta)=\frac{1}{4}\pi+\sin\theta$

$f(1)=\frac{1}{4}\pi+\sin(1)=1.626...>1$

$f(2)=\frac{1}{4}\pi+\sin(2)=1.694...<2$</td><td>M1

A1

[2]</td><td>Substitutes 1 and 2 into the function
Use of degrees is M0
Both values correct with sufficient argument. Sufficient argument includes either an explanation OR > 1 and < 2</td></tr>
<tr><td>6 (c)</td><td><table><tr><td>1</td><td>1.5</td><td>2</td></tr><tr><td>1.62686</td><td>1.78289</td><td>1.69469</td></tr><tr><td>1.78382</td><td>1.76298</td><td>1.77773</td></tr><tr><td>1.76279</td><td>1.76698</td><td>1.76406</td></tr><tr><td>1.76702</td><td>1.76621</td><td>1.76678</td></tr><tr><td>1.76620</td><td>1.76636</td><td>1.76625</td></tr><tr><td>1.76636</td><td></td><td>1.76635</td></tr><tr><td>1.76633</td><td></td><td></td></tr></table>
So $\theta=1.77$ to 2 dp</td><td>M1

A1
A1

[3]</td><td>The table shows values outputs of the iterative formula $\theta_{n+1}=\frac{1}{4}\pi+\sin(\theta_n)$ for common starting values. Values are truncated to five decimal places. You should of course not penalise candidates who use other starting values

Uses iteration formula $\theta_{n+1}=\frac{1}{4}\pi+\sin(\theta_n)$ correctly once
No need to specify starting value
Use of degrees is M0

Obtains answer of 1.77
Shows sufficient iterations to 4dp to justify answer of 1.77 (need to see two consecutive iterations which round to 1.77)
Not focused on the accuracy of the stated iterations or rounding/truncation errors when giving iterations to 4dp so do not penalise small slips.</td></tr>
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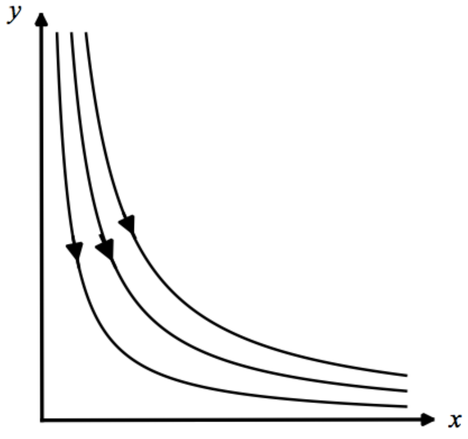
7	Assume that the square root of n is rational, i.e. $\sqrt{n} = \frac{a}{b}$, where $b \neq 0$ and a and b are coprime Then $n^2 = \frac{a^2}{b^2}$. So if p is a prime factor of b, then p must be a prime factor of a^2 and therefore of a. But a and b cannot share any common prime factors, so b cannot have any prime factors, so $b = 1$ Then $\sqrt{n} = a$, which is a contradiction, since n is not a square number, (so n must be irrational)	B1 M1 A1 A1 **[4]**	Makes the assumption with all conditions specified. Allow other descriptions for 'coprime' Considers prime factors (can be implied) Deduces that b must be 1 Obtains a contradiction to provide a complete and convincing proof with no errors seen and complete explanation
8 (i)	$\frac{dy}{dx} = 3x^2 e^x + x^3 e^x$ $\left.\frac{dy}{dx}\right\|_{x=1} = 3(1)^2 e^1 + (1)^3 e^1 = 4e$, so gradient of normal at $x = 1$ is $-\frac{1}{4e}$ At $x = 1$, $y = e$, so equation of normal is $y - e = -\frac{1}{4e}(x-1)$	M1 M1(dep*) A1 B1 A1 **[5]**	Attempts to use the product rule Substitutes $x = 1$ into their gradient function **and** attempts to uses their answer to find the gradient of the normal Obtains correct gradient of the normal States or implies $y = e$ at $x = 1$ Correct equation of the normal in terms of e. Any form is allowed. ISW once a correct answer is seen
8 (ii)	$\frac{dy}{dx} = 2\left(-\frac{1}{2}\right)\left(1+(1-x)^2\right)^{-\frac{3}{2}} \times \frac{d}{dx}\left(1+(1-x)^2\right)$ $= -\frac{1}{\left(1+(1-x)^2\right)^{\frac{3}{2}}} \times (-2)(1-x)$ $= \frac{2(1-x)}{\left(1+(1-x)^2\right)^{\frac{3}{2}}}$	M1* A1 M1(dep*) A1 **[4]**	Attempts to use chain rule Correct workings so far Method to differentiate $1 + (1-x)^2$ using the chain rule Correct differentiation with answer in any form. Accept e.g. $\frac{2(1-x)}{\left(\sqrt{1+(1-x)^2}\right)^3}$ or $\frac{2(1-x)}{\sqrt{\left(1+(1-x)^2\right)^3}}$

9 (a)	If $m = ab^t$, then $\ln m = \ln(ab^t)$ $\Rightarrow \ln(m) = \ln(a) + \ln(b^t)$ $\Rightarrow \ln(m) = t\ln(b) + \ln(a)$ and so the graph of $\ln(m)$ against t is a straight line	M1 A1 **[2]**	Takes logs to both sides and uses product rule Complete and convincing proof
9 (b)	($\ln(a)$ is the y-intercept, so) $\ln(a) = 3.91$ $\Rightarrow a = e^{3.91} = 49.89... \approx 50$	B1 **[1]**	Complete and convincing proof. No need to see the statement $\ln(a) = 3.91$, but we must see $a = e^{3.91}$ in any case. $a = e^{3.91} \approx 50$ is B1
9 (c)	$\frac{1}{2} = b^5$ $\Rightarrow b = \sqrt[5]{\frac{1}{2}} = 0.8705...$ so $b = 0.871$ to 3sf	M1 A1 **[2]**	States correct equation or equivalent. Allow $25 = 50b^5$ Correct value of b to 3 sf
9 (d)	Initial mass of modified substance = mass of old substance after 8 years + 20 kg $= (49.898...)(0.8705...)^8 + 20 = 36.460...$ So $36.460... = P + 40e^{-0.32(0)}$ $\Rightarrow P = 36.460... - 40 = -3.539...$	B1ft M1 A1 **[3]**	Finds initial mass of modified substance using their b. Allow use of $a = 50$ here. Accept awrt 36 Sets up correct equation using their initial mass. Allow 40 instead of $40e^{-0.32(0)}$ Correct value of P . Awrt –3.5

10 (a)	$12\cos x - 4\sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$ So $R\cos\alpha = 12,\ R\sin\alpha = 4$ $\Rightarrow R = \sqrt{12^2+4^2} = 4\sqrt{10}$ $\tan\alpha = \frac{4}{12} \Rightarrow \alpha = 0.3217...$, so $\alpha = 0.322$ to 3sf So $\text{f}(x) = 4\sqrt{10}\cos(x+0.322)$ (not needed)	B1 M1 A1 **[3]**	Correct exact value of R oe Method to find α Correct value of α to 3sf
10 (b)	$4 = 4\sqrt{10}\cos(2x+0.322)$, $\cos(2x+0.322) = \frac{1}{\sqrt{10}}$ $\Rightarrow 2x+0.322 = \cos^{-1}\left(\frac{1}{\sqrt{10}}\right) = 1.2490...$ Other values are 5.0341…, 7.5321… and 11.3172… So $2x = 0.927,\ 4.7121,\ 7.2101,\ 10.9952$ $\Rightarrow x = 0.46,\ 2.36,\ 3.61,\ 5.50$	M1* A1 M1(dep*) A1 **[4]**	Writes down/implies correct equation ft their (a) Correct principal value of $2x + 0.322$ Method to find **all** the other values in range using their principal value Correct values of x. Awrt 0.5, 2.4, 3.6, 5.5
10 (c)	Maximum value is $4\sqrt{10}$ Occurs when $x+0.322 = 0,\ 2\pi$ $\Rightarrow x = -0.322,\ 5.96$	B1ft M1 A1 **[3]**	Correct maximum value ft their R Correct method to find **one** value of x where the maximum occurs, i.e. writes down or implies x + their $\alpha = 0$ or x + their $\alpha = 2\pi$ Correct values of x at which maximum occurs

11 (a)	$\mathbf{v} = \mathbf{i} - 2\mathbf{j}$ So magnitude of $\mathbf{v}$ is $\sqrt{1^2 + (-2)^2} = \sqrt{5}$	B1 B1 **[2]**	States or implies $\mathbf{v} = \mathbf{i} - 2\mathbf{j}$ at the point Correct magnitude Answer only is 2/2
11 (b)	$\frac{1}{y}\frac{dy}{dx} = -\frac{1}{x}$ $\int \frac{1}{y}\frac{dy}{dx}dx = -\int \frac{1}{x}dx$ $\int \frac{1}{y}dy = -\int \frac{1}{x}dx$ $\Rightarrow \ln y = -\ln x + c$ $\Rightarrow y = e^{-\ln x + c} = \frac{A}{x}$, (where A is a constant = e^c)	B1* B1(dep*) M1 A1 **[4]**	Separates variables correctly Attempts to integrate one side Takes exponentials to obtain y in terms of x Simplifies the expression. Accept equivalent forms and any letters/expressions for the constants, i.e. $y = Ax^{-1}$, $y = \frac{e^c}{x}$ etc.
11 (c)		B1 B1 **[2]**	One streamline correctly drawn (no arrows needed) At least three non-overlapping streamlines drawn (no arrows needed) [NB: candidates do not need to specify the equation of each curve they draw, so ignore any labels (and even the wrong labels).] Ignore any streamlines drawn in other quadrants.

12 (a)	$-x = A(2x-3)+B(x-1)$ When $x = 1$, $-1 = A(-1)$, so $A = 1$ When $x = 0$, $0 = -3(1) - B$, so $B = -3$	M1 A1 A1 **[3]**	Method to find one of the coefficients Correct value of A or B Correct value of A and B
12 (b)	$(x-1)^{-1} = -(1-x)^{-1}$ $= -\left(1+(-1)(-x)+\frac{(-1)(-2)}{2!}(-x)^2+\frac{(-1)(-2)(-3)}{3!}(-x)^3+...\right)$ $= -1-x-x^2-x^3+...$ $-3(2x-3)^{-1} = \left(1-\frac{2}{3}x\right)^{-1}$ $= 1+(-1)\left(-\frac{2}{3}x\right)+\frac{(-1)(-2)}{2!}\left(-\frac{2}{3}x\right)^2+\frac{(-1)(-2)(-3)}{3!}\left(-\frac{2}{3}x\right)^3+...$ $= 1+\frac{2}{3}x+\frac{4}{9}x^2+\frac{8}{27}x^3+...$ So $f(x) = -\frac{1}{3}x-\frac{5}{9}x^2-\frac{19}{27}x^3+...$	M1 A1ft + A1ft A1 **[4]**	Finds unsimplfiied expressions for first two terms in expansion of $(1 - x)^{-1}$ **or** $(1 - 2/3x)^{-1}$ Correct unsimplfiied expansion of each partial fraction up to x^3. This is ft their A and B Need to see A or B **multiplied** by the expansion of the denominator to get the A1, so e.g. $1+x+x^2+x^3+...$ for expansion of $(1-x)^{-1}$ is A0 until multiplied by –1. Coefficients must be dealt with correctly, so e.g. expanding $-3(2x-3)^{-1} = \pm 9\left(1-\frac{2}{3}x\right)^{-1}$ is A0 One mark for the expansion of each partial fraction Correct expansion. Ignore any addition terms after x^3. Final answer. If fractions expressed as decimals, these must be given to at least 3 sf
13 (i) (a)	$S_n = a+[a+(2a+1)]+...+[a+(n-1)(2a+1)]$ $S_n = [a+(n-1)(2a+1)]+...+[a+(2a+1)]+a$ $\left.\begin{array}{l}2S_n = [2a+(n-1)(2a+1)]+...+[2a+(n-1)(2a+1)] \\ \Rightarrow 2S_n = n[2a+(n-1)(2a+1)]\end{array}\right\}$ either $\Rightarrow S_n = \frac{n}{2}[2a+(n-1)(2a+1)]$ **AG**	B1 M1 M1(dep*) A1 **[4]**	Writes the series down. Requires at least 3 terms, including the first and last terms, an adjacent term, the dots and + signs Reverses their series. Must be arithmetic in terms of a and n Adds the two series together – one of the two lines is sufficient for the mark Complete and convincing proof with no errors seen Allow the M1 if proof given in terms of d and not $(2a + 1)$. For the B1 and A1, must define/substitute $d = 2a + 1$ explicitly

13 (i) (b)	$11225=\frac{n}{2}\left(8+9(n-1)\right)$ $\Rightarrow 22450=8n+9n^2-9n$ $9n^2-n-22450=0$ $(9n+449)(n-50)=0$ Since $n>0$, $n=50$	M1* M1(dep*) A1 **[3]**	Substitutes a = 4 and S_n = 11225 to form correct equation Forms correct 3TQ and uses a complete method to solve it Chooses n = 50. No need to justify excluding the other solution. If both/any additional solutions given, then A0.
13 (ii)	$\sum_{n=1}^{\infty}\left(\frac{1}{2}\sin x\right)^n=\frac{\frac{1}{2}\sin x}{1-\frac{1}{2}\sin x}=\frac{\sin x}{2-\sin x}$, since $\left\lvert\frac{1}{2}\sin x\right\rvert<1$,	B1 B1 **[2]**	Evaluates series using sum of infinite geometric series formula. Accept answer in any form Justifies validity of answer by stating that $\left\lvert\frac{1}{2}\sin x\right\rvert<1$. Accept stronger inequalities, i.e. $\left\lvert\frac{1}{2}\sin x\right\rvert\le\frac{1}{2}$