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A Methodology for Microchannel Heat Sink Design Based on Topology Optimization

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Abstract
Microchannel heat sink design consists in an innovative technology which has been studied as alternative to increase cooling efficiency of small electronic devices, such as high-power LEDs (Light Emitting Diodes), and high-end microprocessors of CPUs. As time passes, these electronic devices become smaller and more powerful, and as consequently, they dissipate a large amount of heat which requires very efficient cooling systems. In order to achieve an optimized heat sink design, its cooling systems must have high efficient fluid flow channels, by minimizing pressure drops along its extension, and providing the largest amount of heat dissipation possible. Microchannels constructed on a conductive body allow us to obtain an efficient heat sink design having better thermal dissipation with small mass and volume, and large convective heat transfer coefficient, and, thus, suitable for cooling compact areas of small electronic devices. Thus, the main objective of this work is the study of a methodology for microchannel heat sink design through the application of the Topology Optimization Method, which allows the distribution of a limited amount of material, inside a given design domain, in order to obtain an optimized system design. This method combines the Finite Element Method (FEM) and Sequential Linear Programming (SLP) to find, systematically, an optimized layout design for microchannels in heat sinks. Essentially, the topology optimization problem applied to channel fluid flow consists of determining which points of a given design domain (small heat sink) should be fluid, and which points should be solid to satisfy a multi-objective function that maximizes the heat dissipation, with minimum pressure drop. In this multi-physics analysis, channel fluid flow operates at low Reynolds number, thus, the Stokes flow equations are considered. Some results are shown to illustrate the methodology, and computational simulations of some optimized channel layouts are employed to validate the implemented topology optimization algorithm.
Keywords: microchannels, topology optimization, finite element, fluid flow, heat transfer.

1. Introduction
Microchannel heat sink is an innovative technology which has been long studied as alternative to increase cooling efficiency of small electronic devices, which do not have large areas for heat exchanging. This technology was introduced by Tuckerman and Pease [1] who performed experiments on silicon based microchannel heat sink for electronic cooling. Microchannels constructed on a conductivity body can provide large convective heat transfer coefficient, and small mass and volume for heat sink designs, which makes it very suitable for cooling compact electronics devices such as high-end microprocessors applied to general computation. The microprocessors dissipate a large amount of heat, and need very efficient dissipation system to avoid malfunction or even product damage. As long as these microprocessors become more powerful and smaller, the need of efficiency in heat dissipation is even more highlighted. Heat sink design for high-power LED's (Light Emitting Diodes) should also be other great potential application of this technology. This kind of LED is characterized by a high luminosity that yields a high heat generation, which it is only removed by an efficient heat sink [2].
In fluid flow systems, one important matter is power dissipation along channels which leads to a pressure drop, compromising their correct operation and their efficiency. Especially in small scale applications, such as micro-channel devices, which operate under low pressure conditions, the fluid flow can be greatly influenced and compromised by any pressure drop.
In order to achieve a better heat sink design, it is crucial that these cooling systems have a very efficient fluid flow channel, minimizing pressure drops along its extension, and consequently allowing an efficient full capacity operation. This kind of application is directly affected by the channel performance.
In the past decades, many studies have been conducted, in order to achieve fluid flow channels with better configuration for minimizing power dissipation. These studies take advantage of numerical methods application to analyze the fluid behavior, allowing the study of more complex cases. The basis of numerical studies on channel flow optimization was given by Pironneau [3], who conducted a shape optimization analysis in airfoils and other devices, such as diffusers. In its studies he applied the shape optimization process to obtain minimum drag profiles and minimum pressure drop diffusers. Other studies in this field were conducted later by Mohammadi and Pironneau [4].

The main objective of this work is to present a methodology for obtaining optimal microchannel layouts applied to a heat sink design using Topology Optimization Method. Later in the 90's, there has been a great development of the Topology Optimization Method, which essentially distributes limited amount of material inside a design domain to optimize a cost function requirement, satisfying some specified constraints [5]. First, Topology Optimization Method was developed for structural analysis field [6]. Nowadays, it has been applied by several researches in other field applications, such as Borrvall and Petersson [7] who use this method for fluid flow problems field. In this case, instead of controlling only solid material and void regions, as often performed for structural analysis field, the interest is focused on distributing liquid and solid materials, and then, creating optimized fluid flow systems.
One of the great advantages of the Topology Optimization Method is the possibility for analyzing a much wider range of solutions, due to the "free" material distribution method. By applying this method, ones can achieve an optimal solution with no need of proposing a "pre-structured" initial guess, which tends to limit the final solution, by directing the optimization process. This issue may be a problem in parametric and shape optimization, where a preliminary solution model must be stated. Thus, "not-so-intuitive" solution can be recovered, as can be seen in reference [7], where it is concluded that for longer domains, the pressure drop on a "single-merged" channel is lower than a "double-way" separated channel, which is not an intuitive solution. Lately, other studies in this field have been conducted, such as [8, 9, 10, 11].
The methodology proposed in this work is presented in the next sections. Section 2 describes the fundamental theory. Section 3 presents the Finite Element (FE) model. Section 4 shows an overview of the optimization procedures. Section 5 details the results obtained at this moment. Finally, in Section 6 some discussion about obtained results and conclusion are given.

2. Fundamental Theory

The fundamental theory is given by the constitutive equations for Newtonian fluid flow, based on the well-known Navier-Stokes equations, given by:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t}+\mathbf{u}\cdot\nabla\mathbf{u}\right)=-\nabla p+\mu\Delta\mathbf{u}+\mathbf{f} \tag{1}$$

$$\frac{\partial \rho}{\partial t}+\nabla\cdot\left(\rho\mathbf{u}\right)=0 \tag{2}$$

where ρ is the fluid specific mass, μ is the fluid viscosity, $\mathbf{u}$ is the velocity vector, p is the pressure and $\mathbf{f}$ is the body load. Equation (1) refers to the conservation of momentum, and equation (2) refers to conservation of mass, or continuity equation.
In this work, the Navier-Stokes equations are simplified to a linear form [7], considering a steady-state, incompressible fluid flow (Newtonian fluid) at low Reynolds, where the viscous effects overlap the inertia effects, obtaining the following Stokes flow equations:

$$-\mu\Delta\mathbf{u}+\nabla p=\mathbf{f} \tag{3}$$

$$\nabla\cdot\mathbf{u}=0 \tag{4}$$

Equation (3) dictates the fluid flow, coupled with equation (4) that acts similar to a constraint in the velocity field to ensure the incompressibility condition.
Besides the fluid flow in free regions of the domain, it is necessary to model the solid region behavior. This problem is solved by combining the standard Stokes flow equation with a contribution from a porous medium flow, known as Darcy flow, given by [7, 8, 10]:

$$\begin{gathered}\mathbf{u}=-\frac{\kappa}{\mu}\left(\nabla p-\rho_f\mathbf{b}\right)\\ \nabla\cdot\mathbf{u}=0\end{gathered} \tag{5}$$

where κ is the porous media permeability.
The main idea is to apply the Stokes equations to model the fluid flow behavior, and to control the velocity field in solid regions through the Darcy equation, by assuming it to be a porous medium with nearly-zero flow permeability.
The combination of equations (3) and (5) results in the Brinkman's equation form, given by:

$$\begin{gathered}\mu\Delta\mathbf{u}+\alpha\mathbf{u}=\nabla p-\mathbf{f}\\ \nabla\cdot\mathbf{u}=0\end{gathered} \tag{6}$$

where α is the fluid inverse permeability.
In equation (6), it is included a penalization term controlled by α, which is the inverse permeability of the porous medium region, and penalizes the velocity, enforcing a very small flow in solid regions. This approach allows the optimization to work with a continuum variation on the liquid-solid model, which is relevant during the optimization phase, discussed ahead in this manuscript.

Additionally to the fluid movement equations, the convection-diffusion heat transfer governing equation is:

$$\rho_f c_p \left(\mathbf{u} \cdot \nabla T\right) = k_f \nabla^2 T + Q \tag{7}$$

where T is the temperature, Q is the heat generation. ρ_f, c_p, and k_f represent the fluid density, the specific heat, and the thermal conductivity, respectively. In the energy equation (7) steady state flow is considered.

3. Finite Element Modeling

The finite element method (FEM) is applied to solve the equations presented in the previous section. The design domain is divided by using rectangular bilinear elements, which have four nodes for velocity field and one node for pressure field. Although it is recognized as a not full-stable element according to the LBB or div-stability-condition [12], the adopted element has shown a good accuracy for velocity field calculation, with expected spurious oscillation in the pressure field caused by the velocity and pressure fields coupling, which in this particular application do not affect the results decisively [7].

By applying the FEM to Brinkman's equation, and writing it to the discrete matrix form, the following equation system is obtained [13]:

$$\begin{bmatrix} \mathbf{K} & -\mathbf{G}^T \\ \mathbf{-G} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \mathbf{p} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f} \\ \mathbf{0} \end{Bmatrix} \tag{8}$$

where $\mathbf{K}$ is the velocity stiffness matrix and $\mathbf{G}$ represents the divergent operator, $\mathbf{u}$ and $\mathbf{p}$ are the nodal velocity and nodal pressure distribution respectively, and $\mathbf{f}$ is the nodal body load component. By solving the system presented in equation (8), considering a correct set of boundary conditions, the velocity and pressure fields can be determined.

4. Topology Optimization Problem

The topology optimization method [5] is applied to make each of finite elements of the discretized domain to assume either fluid or solid material, according to the material model. In this work, topology optimization problem is solved by using the sequential linear programming [14].

The material model combines the characteristics of both materials (solid and fluid), and allows defining which kind of material is placed on each element of the discretized domain. This is controlled by the design variable ρ, in such way that for $\rho=0$ one retains a solid material, and for $\rho=1$ one retains a fluid material, characterizing a discrete 0-1 problem. Although there is not physical application for intermediate values of ρ and as it is not desirable to have them at the final design, it is very common to work with a continuous problem, allowing ρ to assume these intermediate values, preventing well known solution problems in the discrete model [5].

The Brinkman's equation (6), describes the material model, where the inverse permeability (α) is a continuous function of the design variable ρ. The material model is the same applied by Borrvall and Petersson [7] and later by Gersborg-Hansen [8], in which for solid elements the combined porous medium model predominates, with permeability controlled by the design variable ρ, such as for a full solid element ($\rho \rightarrow 1$), the velocity is nearly zero ($u \rightarrow 0$).

The total potential power evaluated at the solution obtained by the FEM analysis is adopted as objective function for the topology optimization problem. Considering a common case, where there are no body forces over the fluid domain, the total potential power represents the power dissipation on the fluid in the design domain, given by:

$$\Phi = \mathbf{U}^T \mathbf{K} \, \mathbf{U} \tag{9}$$

where $\mathbf{U}$ represents the nodal velocity field vector and $\mathbf{K}$ is the velocity stiffness matrix. Equation (9) has the same form of the well-known mean-compliance used very often in structural optimization [5], and may represent the mean pressure drop over the channel. The goal is to minimize power dissipation, and consequently to minimize the pressure drop.

Thus, the topology optimization problem is stated for the fluid flow channel optimization, in a discrete form, as:

$$\begin{aligned} &\underset{\boldsymbol{\rho}}{\text{minimize:}} \quad \Phi = \mathbf{U}^T \mathbf{K} \, \mathbf{U} \\ &\text{such as:} \quad \begin{bmatrix} \mathbf{K} & -\mathbf{G}^T \\ \mathbf{-G} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \mathbf{p} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f} \\ \mathbf{0} \end{Bmatrix} \\ &\qquad \sum_{i=1}^{N} \rho_i \leq V \, ; \quad 0 \leq \rho_i \leq 1 \end{aligned} \tag{10}$$

A volume fraction constraint (V) is adopted in the optimization problem. The volume fraction is the ratio of fluid material volume over the whole domain volume.

For controlling the design heat transfer, another objective function must be stated. The chosen cost function utilizes the temperature distribution, obtained from the energy equation (7), to evaluate the system heat transfer performance. It has also the same discrete form as the mean-compliance problem, as follows:

$$\Gamma = \mathbf{T}^T \mathbf{K}_t \mathbf{T} \tag{11}$$

where **T** is the thermal field distribution and $\mathbf{K}_t$ is the thermal stiffness matrix.

These two optimization problems (to minimize the pressure drop and to maximize heat transfer) are evaluated through a multi-objective function which allows the design process to give priority to one of them, or treat both equally, as follows:

$$\Psi = u \log(\Phi) + w \log(\Gamma) \tag{12}$$

where u and w are weighting factors.

The sensitivity analysis is performed by calculating the gradients of mean-compliances Φ and Γ in the objective function of equation (12) in relation to the design variables [5].

5. Heat sink design

Here, topology optimization problem shown before is applied to a heat sink design. Essentially, optimization process is carried out to achieve a channel with both low pressure drop and high heat transfer attributes, increasing a heat sink device.

The design domain of the heat sink is shown in figure 1. This domain has one inlet and two outlet regions, with direction defined as follows in figure 1. At the inlet region, fluid velocity is set to a constant value equal to 0.01m/s and fixed temperature of 20 ^{o}C. The pressure at outlet regions is set to zero, acting similar to a sink. A uniform heat source (1 W/m^2) is distributed along the whole domain. All external walls of the domain are prescribed as non-slip adiabatic boundaries. For this example, thermodynamic constants of water (fluid material) and Aluminum (solid material) are considered.

The results presented in figure 2 are obtained for a domain discretized by 40x40 elements. White regions represent fluid material and black regions represent solid regions. This test shows how the optimization problem is stated and illustrates also the weighting factors (u and w) influence over the solution. Figure 3 shows the temperature distribution and the heat flux for the obtained channel configuration shown in figure 2b.

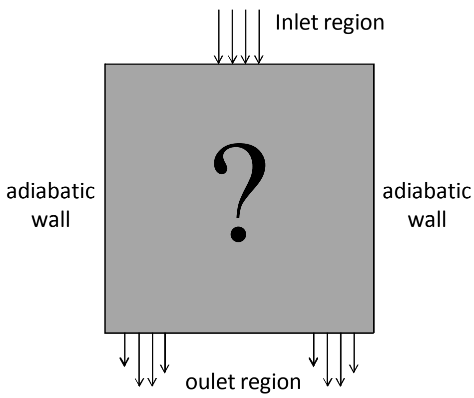


Figure 1 – Heat sink design domain

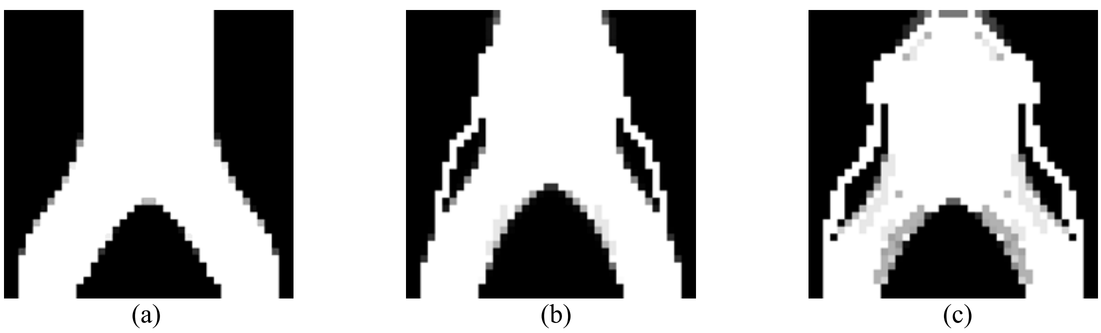
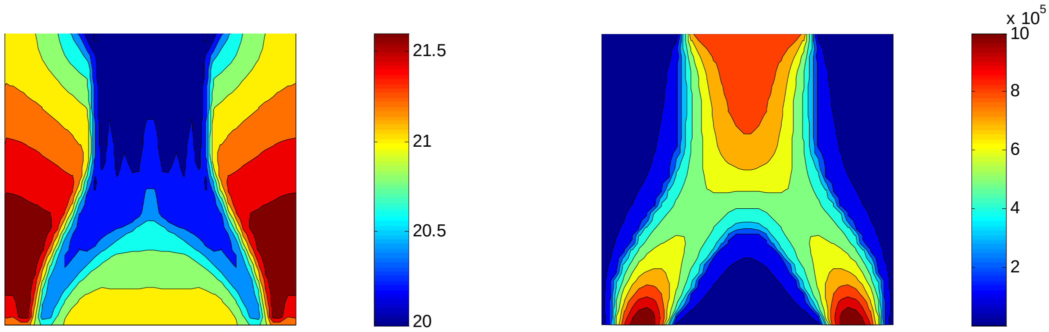


Figure 2 – Heat sink optimization results: (a) u larger than w (fluid flow priority); (b) equal priority for u and w; (c) w larger than u (heat transfer priority).



(a) (b)

Figure 3 – (a) Temperature distribution; and (b) heat flux for configuration shown in the figure 2b.

6. Discussion and conclusion

The application of the topology optimization in fluid mechanics and heat transfer is practical and it allows the systematic design of fluid flow channels. This approach could be used for more efficient microchannel heat sink design, which has a lot of possible applications. An example of application of the combined fluid flow and heat transfer characteristics for channel design is shown. This example illustrates the viability of applying the topology optimization process to achieve a channel design combining these two distinct characteristics (fluid flow and heat transfer) at the same time.

Weighting factors (*u* and *w*) which allows to control tuning for fluid flow or heat transfer behavior has been performed and it is noticed its influence over the results. As there is a tradeoff between fluid flow layout that maximize the heat transfer, the algorithm tries to increasing the heat exchanging area by introducing some small channels around the principal (major) channel, as can seen in figure 2. According some tests, it is also verified that the implemented topology optimization algorithm is mesh-independent [7], even with no applied filtering technique.

The application of combined Stokes-Darcy flow equations has been shown very efficient within the topology optimization algorithm. Although it has certain limitations, as it is a linear approach of the full Navier-Stokes equations, and suitable only for low Reynolds fluid flow, this model is applicable for many different problems, from bend-pipes and diffusers to more complex cases.

As conclusion, authors consider the application of the topology optimization in fluid flow channel design a very promising field. Thus, experimental characterization and manufacturing prototypes of some microchannel heat sink configurations will be executed as next steps of this work, in order to validate the results obtained through implemented optimization algorithm.

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